

1. Show that for every integer $n \in \mathbb{N}^+$:

$$\sum_{i=1}^n \frac{1}{(4n+1)(4n-3)} = \frac{n}{4n+1}.$$

Solution. Use mathematical induction to establish that for every integer $n \in \mathbb{N}^+$:

$$\sum_{i=1}^n \frac{1}{(4n+1)(4n-3)} = \frac{n}{4n+1}.$$

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Here is a theorem:

Theorem 1. Let $a, b \in \mathbb{R}$. For every integer $n \in \mathbb{N}$:

$$\sum_{m=0}^n (a + mb) = \frac{(n+1)(2a+nb)}{2}.$$

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Proof. Let $a, b \in \mathbb{R}$. Observe that the identity in question readily follows from the well-known identity for evaluating the sum of the first n natural numbers:

$$\sum_{m=0}^n m = \frac{n(n+1)}{2}.$$

Indeed, for every $n \in \mathbb{N}$:

$$\begin{aligned} \sum_{m=0}^n (a + mb) &= a \sum_{m=0}^n 1 + b \sum_{m=0}^n m \\ &= a(n+1) + b \frac{n(n+1)}{2} \\ &= \frac{2a(n+1) + nb(n+1)}{2} \\ &= \frac{(n+1)(2a+nb)}{2}. \end{aligned}$$

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