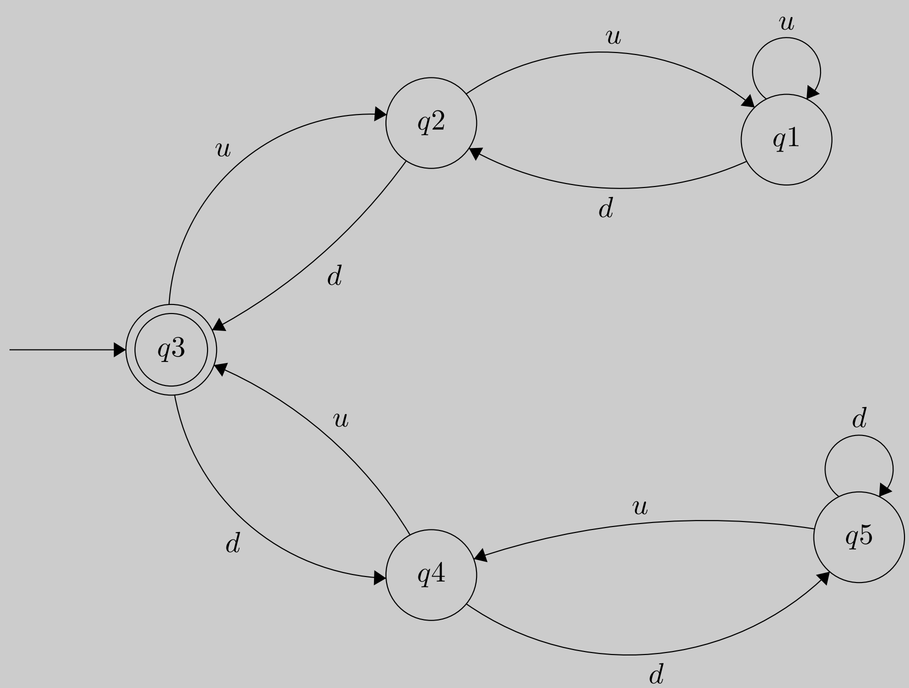
1.3 The formal description of a DFA M is $(\{q1, q2, q3, q4, q5\}, \{u, d\}, \delta, q3.\{q3\})$, where δ is given by the following table. Give the state diagram of this machine.

	u	d
$q1$	$q1$	$q2$
$q2$	$q1$	$q3$
$q3$	$q2$	$q4$
$q4$	$q3$	$q5$
$q5$	$q4$	$q5$

Solution.



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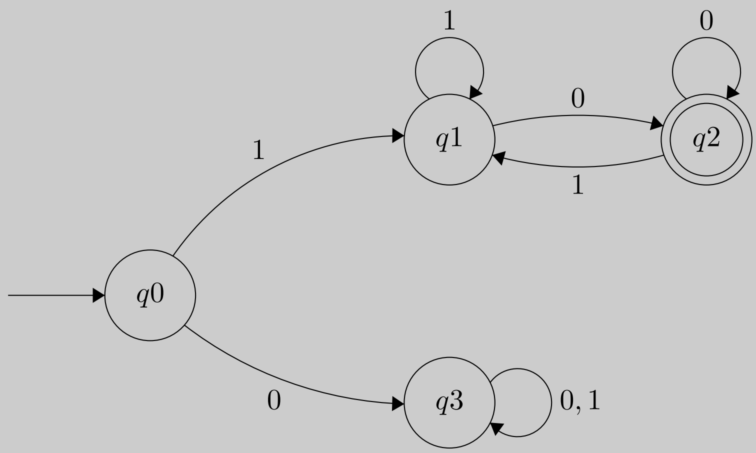
1.6 Given state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is {0,1}.

a. $\{w|w$ begins with a 1 and ends with a 0$\}$

Solution.



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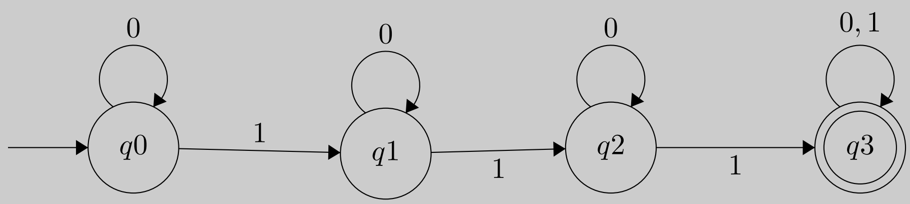
b. $\{w|w \text{ contains at least three 1s}\}$

Solution.



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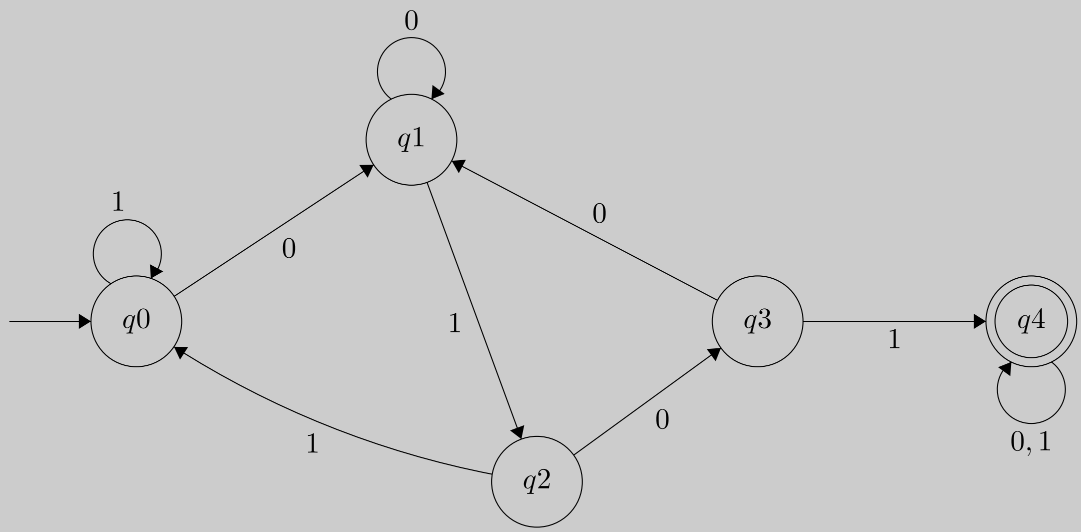
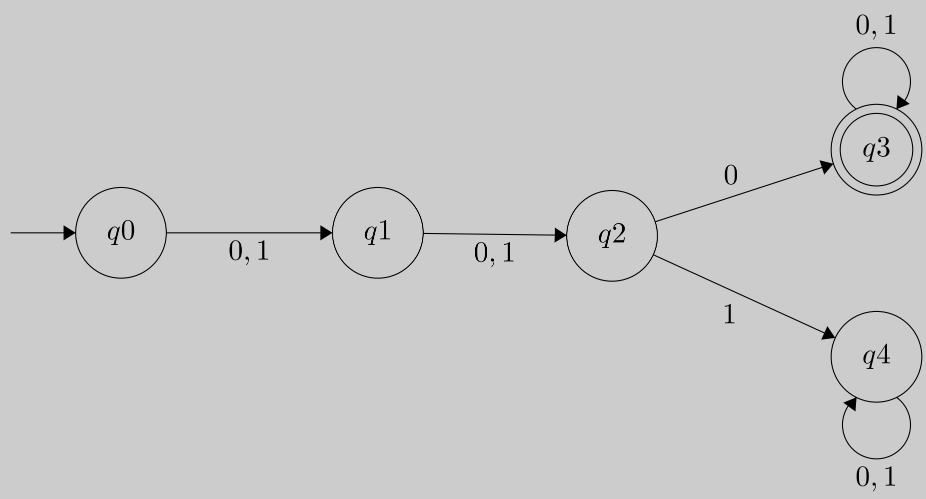
c. $\{w|w \text{ contains the substring 0101 (i.e., } w = x0101y \text{ for some } x \text{ and } y\}$

Solution.



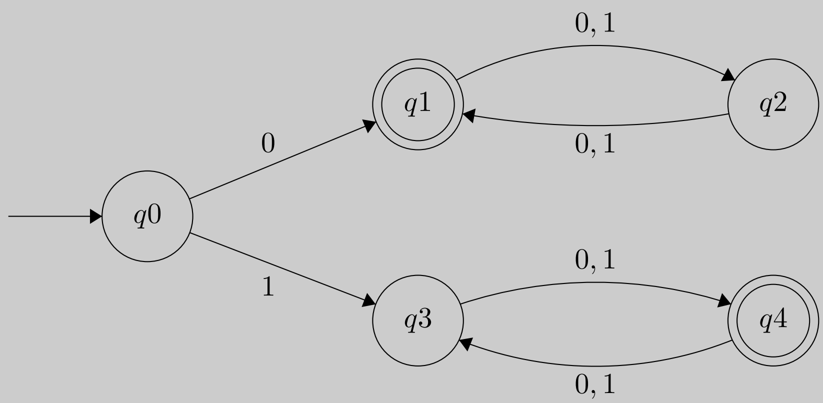
d. $\{w|w$ has length at least 3 and its third symbol is a $0\}$

Solution.



e. $\{w|w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$

Solution.



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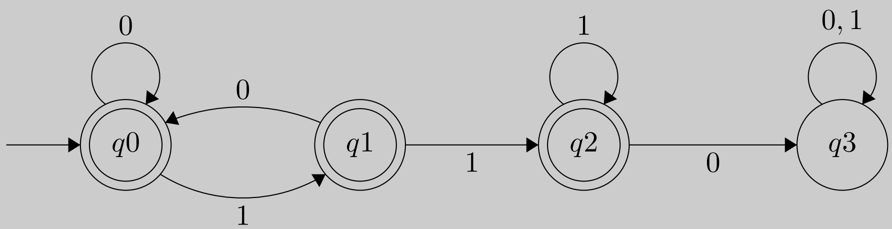
f. $\{w|w$ doesn't contain the substring 110$\}$

Solution.



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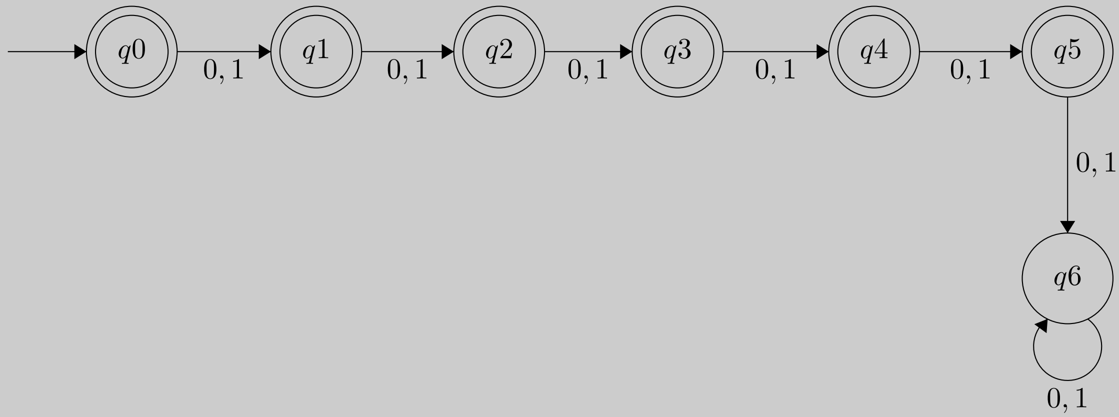
g. $\{w|$ the length of w is at most 5$\}$

Solution.



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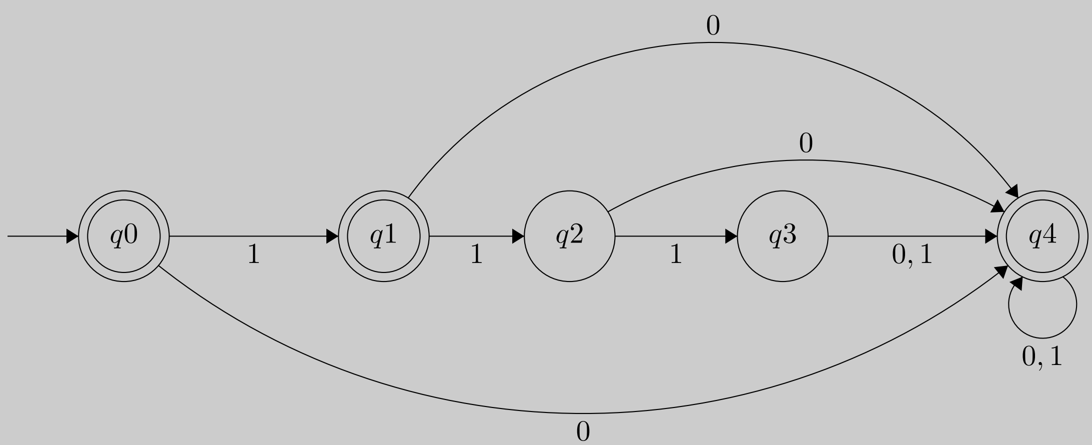
h. $\{w|w$ is any string except 11 and 111$\}$

Solution.



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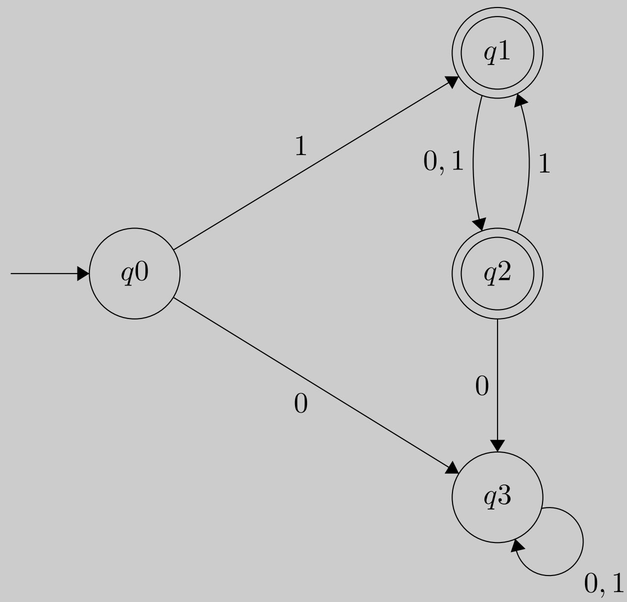
i. $\{w|$ every odd position of w is a 1$\}$

Solution.



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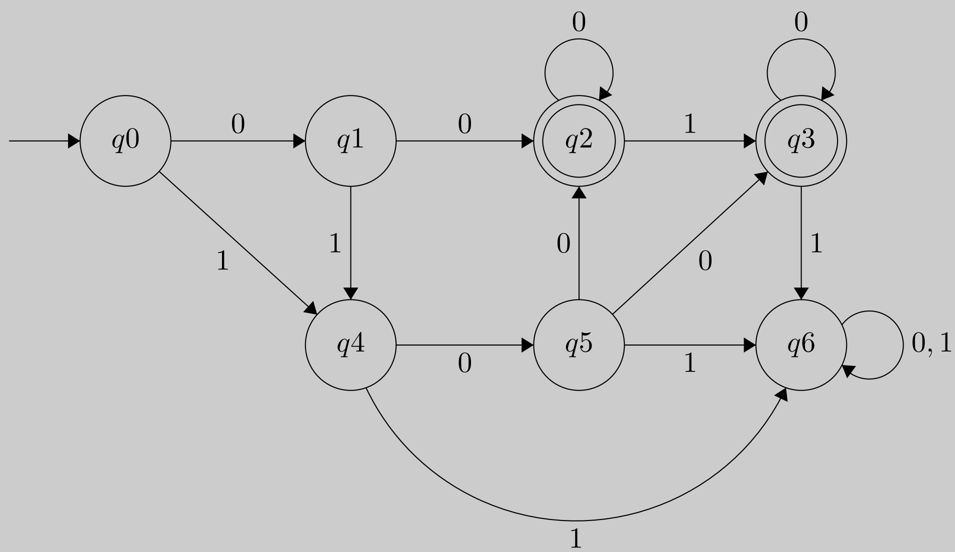
j. $\{w|w$ contains at least two 0s and at most one $1\}$

Solution.



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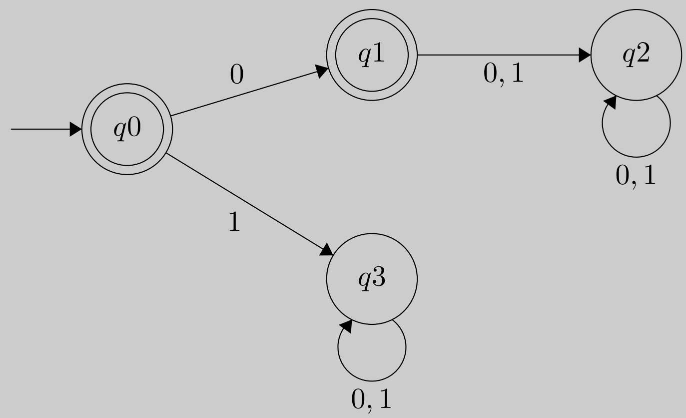
k. $\{\epsilon, 0\}$

Solution.



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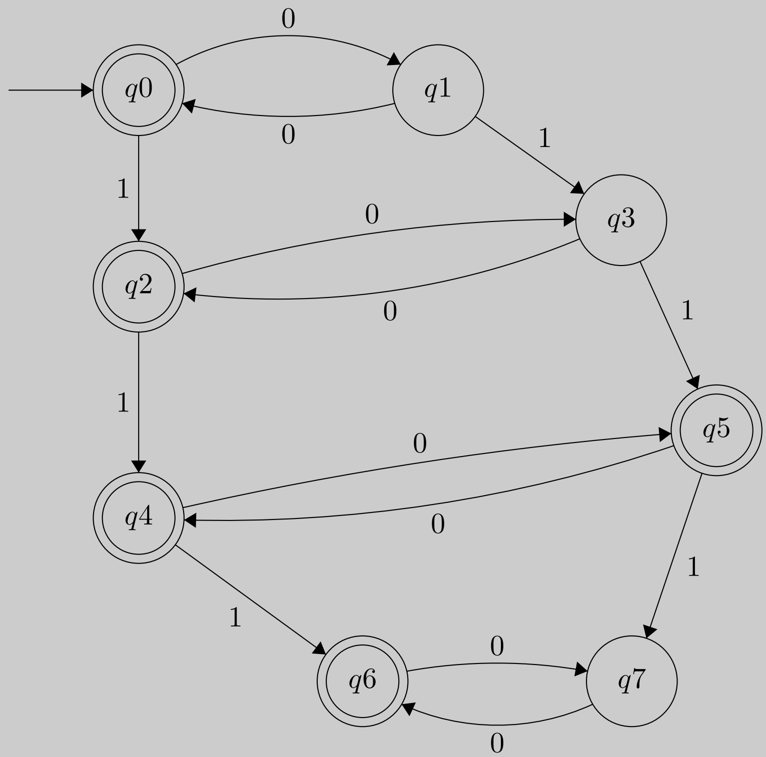
l. $\{w|w$ contains an even number of 0s, or contains exactly two 1s$\}$

Solution.



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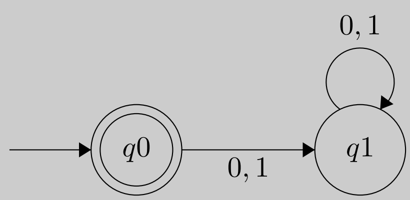
m. The empty set

Solution.



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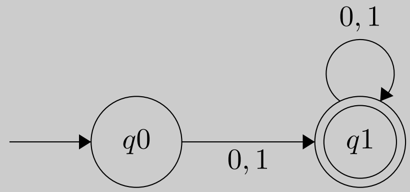
n. All strings except the empty string

Solution.



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1.66 A *homomorphism* is a function $f : \sum \to \Gamma^*$ from one alphabet to strings over another alphabet. We can extend f to operate on strings by defining $f(w) = f(w_1)f(w_2)\dots f(w_n)$, where $w = w_1w_2\dots w_n$ and each $w_i \in \sum$. We further extend f to operate on languages by defining $f(A) = \{f(w)|w \in A\}$, for any language A.

a. Show, by giving a formal construction, that the class of regular languages is closed under homomorphism. In other words, given a DFA M that recognizes B and a homomorphism f, construct a finite automaton M' that recognizes $f(B)$. Consider the machine M' that you constructed. Is it a DFA in every case?

b. Show, by giving an example, that the class of non-regular languages is not closed under homomorphism.

Solution. **a.** Let $M = (Q, \sum, \delta, q_0, F)$ be a DFA that recognizes A and let $f : \sum \to \Gamma^*$ be the homomorphism of strings. Then let $N = (Q, \Gamma, \delta', q_0, F)$ recognize $B = f(A)$, where δ' is a function from $Q \times \Gamma^*$ to $P(Q)$ given with $\delta'(q, f(\alpha)) = \{\delta(q, \alpha)\}$ for every $q \in Q$ and $\alpha \in \sum$. For every $w \in \Gamma^*$ where w is not $f(\beta), \beta \in \sum$, there is $\delta'(q, w) = \emptyset$.

b. We can take a nonregular language A and a non homomorphism $f(\alpha) = \epsilon$, and the result will end up as $B = f(A) = \{\epsilon\}$, which is a regular language.

Source for this problem:
`https://www.cs.colorado.edu/~astr3586/courses/csci3434/lec05.pdf` ■