Supplementary File for "Minimizing the energy consumption of flexible assembly systems with tool change processes using hybrid heuristic search"

Theorem 1: h_1, h_2, h_3, and h_5 are admissible.

Proof: Let β^* denote the actual optimal schedule from (M, α) to a final vertex, i.e., $M[\beta^* > M_f$. Let $h^*(M, \alpha)$ denote the total energy consumption of β^*. Let $EW(\alpha, \beta^*)$, $EO(\alpha, \beta^*)$, $EI(\alpha, \beta^*)$, $EH(\alpha, \beta^*)$ and $EC(\alpha, \beta^*)$ denote the total energy consumption of working, occupied, idle, hold and tool change state from α to $\alpha\beta^*$, respectively. Under E_1, $h^*(M, \alpha) = EH(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*)$, while under E_2, $h^*(M, \alpha) = EW(\alpha, \beta^*) + EO(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*)$.

Under E_2, $h_1(M, \alpha) = \sum_{p\in P_o\cup P_s}\sum_{j\in \boldsymbol{N}_{M(p)}}(RE(\zeta(p, j), M, \alpha) + PE(p))$ is the minimum working energy consumption of all parts from (M, α) to the end, thus $h_1(M, \alpha) \le EW(\alpha, \beta^*)$ under E_2. Since $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha)$ is only part of the occupied energy, $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le EO(\alpha, \beta^*)$. Since $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha)$ is only part of the idle energy, $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) \le EI(\alpha, \beta^*)$. Since $EC(\alpha, \beta^*) \ge 0$, $h_5(M, \alpha) = h_1(M, \alpha) + \sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le EW(\alpha, \beta^*) + EO(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*) = h^*(M, \alpha)$, hence $h_5(M, \alpha)$ is admissible under E_2. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence $h_1(M, \alpha)$, $h_2(M, \alpha)$ and $h_3(M, \alpha)$ are admissible under E_2.

Since $(h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha))$ is only part of the hold energy consumption of all parts from (M, α) to the end, $h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le EH(\alpha, \beta^*)$. $h_5(M, \alpha) \le EH(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*) = h^*(M, \alpha)$, hence $h_5(M, \alpha)$ is admissible under E_1. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence $h_1(M, \alpha)$, $h_2(M, \alpha)$ and $h_3(M, \alpha)$ are admissible under E_1. ■

Theorem 2: D^{2}WS can always output a solution

Proof: All vertexes in D^{2}WS are generated under a DAP, which prohibit transition firings that lead APNS from safe markings to deadlock. Thus all markings of generated vertexes are safe. That is to say, from any markings except for M_f, there is at least one enabled transition that can fire and lead to a safe marking.

There are two termination conditions for D^{2}WS: the final marking is reached or $OPEN = \varnothing$. If D^{2}WS terminates with the final marking reached, then D^{2}WS yields a solution. Assume the final marking has not been reached but D^{2}WS terminates with $OPEN = \varnothing$. Then, all vertexes in the current window (upon termination of the algorithm) are explored. Let (M, α) be an explored vertex with the deepest depth among all the vertexes in the current window. Since $M_f \neq M$, there must be a vertex (M_1, α_1) in the current window generated from (M, α) and $|\alpha_1| = |\alpha| + 1$. (M_1, α_1) will be kept in the current window unless there is a vertex at the depth of $|\alpha_1|$ with marking M_1 or the number of successor vertexes of (M, α) is more than *max_size*. That is a contradiction to the assumption that (M, α) is an explored vertex with the deepest depth among all vertexes in the current window. Thus, D^{2}WS can always end with the final marking. ■

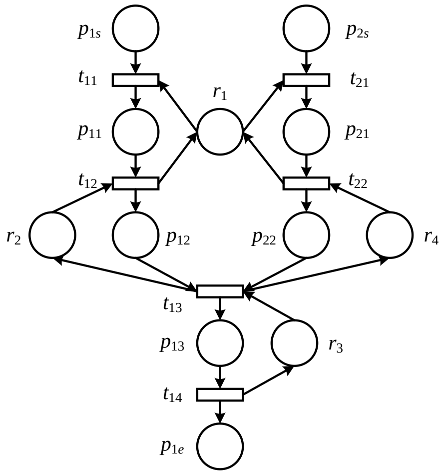

Fig. 1. APNS of FAS in Example 1.

Example 1: For the APNS processing two types of parts in Fig. 1, suppose $M_0 = p_{1s} + p_{2s} + r_1 + r_2 + r_3 + r_4$. The processing time of operations in places p_{11}, p_{12}, p_{13}, p_{21} and p_{22} are 10, 5, 5, 30 and 20, respectively. The energy consumption per unit time in different kinds of states is listed in Table I. For tool change processes, $\mu = 5$ and $e_c = 2$.

TABLE I

ENERGY CONSUMPTION PER UNIT TIME OF RESOURCES IN FIG. 1.

	r_1	r_2	r_3	r_4
hold state	2.00	10.00	2.00	2.00
working state	2.00	10.00	2.00	2.00
occupied state	1.00	5.00	1.00	1.00
idle state	0.20	1.00	0.20	0.20

Let's calculate $h_4(M_0, \alpha_0)$ under E_1 and E_2. The first part of $h_4(M_0, \alpha_0)$ is $h_1(M_0, \alpha_0)$. $RE(\zeta(p_{1s}, 1), M_0, \alpha_0) = RE(\zeta(p_{2s}, 1), M_0, \alpha_0) = 0$, $PE(p_{1s}) = d(p_{11}) \times e_0(r_1) + d(p_{12}) \times e_0(r_2) + d(p_{13}) \times e_0(r_3) = 10 \times 2 + 5 \times 10 + 5 \times 2 = 20 + 50 + 10 = 80$ and $PE(p_{2s}) = d(p_{21}) \times e_0(r_1) + d(p_{22}) \times e_0(r_4) = 30 \times 2 + 20 \times 2 = 100$. Then $h_1(M_0, \alpha_0) = RE(\zeta(p_{1s}, 1), M_0, \alpha_0) + PE(p_{1s}) + RE(\zeta(p_{2s}, 1), M_0, \alpha_0) + PE(p_{2s}) = 180$.

The rest of $h_4(M_0, \alpha_0)$ under E_1 is calculated as follows. $\Pi = \{t_{12}, t_{22}, t_{13}\}$, for t_{12} and t_{22}, since $M(p_{11}) = M(p_{21}) = 0$, $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$. For assembly transition t_{13}, since $p_{12} \in {}^{(o)}t_{13}$, $p_{1s} \in I(p_{12})$ and $M(p_{1s}) = 1 > 0$, $\theta(t_{13}, M_0, \alpha_0) = 1$.

$\Pi = \{t_{12}, t_{13}, t_{22}\}$. Since $|{}^{(o)}t_{12}| = |{}^{(o)}t_{22}| = 1$ and $M_0(p_{11}) = M_0(p_{21}) = 0$, $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$. $EO_1(t_{13}, M_0, \alpha_0)$ and $EO_2(t_{13}, M_0, \alpha_0)$ are calculated as follows. Since $M_0(r_3) > 0$, $RT(r_3, M_0, \alpha_0) = 0$. $OT(t_{13}, M_0, \alpha_0) = \max\{OT(p_{12}, M_0, \alpha_0), OT(p_{22}, M_0, \alpha_0)\} = \max\{d(p_{11}) + d(p_{12}), d(p_{21}) + d(p_{22})\} = \max\{10 + 5, 30 + 20\} = 50$. Then $EO_1(t_{13}, M_0, \alpha_0) = (e_0(r_2) + e_0(r_4)) \times (\max\{OT(t_{13}, M_0, \alpha_0), RT(r_3, M_0, \alpha_0)\} - OT(t_{13}, M_0, \alpha_0)) = (10 + 2) \times (\max\{50, 0\} - 50) = 0$, and $EO_2(t_{13}, M_0, \alpha_0) = (OT(t_{13}, M_0, \alpha_0) - OT(p_{12}, M_0, \alpha_0)) \times e_0(r_2) + (OT(t_{13}, M_0, \alpha_0) - OT(p_{22}, M_0, \alpha_0)) \times e_0(r_4) = (50 - 15) \times 10 + (50 - 50) \times 2 = 350$. Thus $h_4(M_0, \alpha_0) = h_1(M_0, \alpha_0) + \theta(t_{13}, M_0, \alpha_0)(EO_1(t_{13}, M_0, \alpha_0) + EO_2(t_{13}, M_0, \alpha_0)) = 180 + 1 \times (0 + 350) = 530$ under E_1.

Similarly, under E_2, $EO_2(t_{13}, M_0, \alpha_0) = (OT(t_{13}, M_0, \alpha_0) - OT(p_{12}, M_0, \alpha_0)) \times e_1(r_2) + (OT(t_{13}, M_0, \alpha_0) -

$OT(p_{22}, M_0, \alpha_0)) \times e_1(r_4) = (50 – 15) \times 5 + (50 – 50) \times 1 = 175$, hence $h_4(M_0, \alpha_0) = 180 + 1 \times (0 + 175) = 355$.

Let's calculate $h_6(M_0, \alpha_0)$ under E_1 and E_2. Since $^{(o)}(r_1^{\bullet}) = \varnothing$, $\delta(r_1, M_0, \alpha_0) = 0$. Since $^{(o)}(r_2^{\bullet}) = \{p_{11}\}$, $^{(o)}(r_4^{\bullet}) = \{p_{21}\}$, $^{(o)}(r_3^{\bullet}) = \{p_{12}, p_{22}\}$, and $M(p_{11}) = M(p_{12}) = M(p_{21}) = M(p_{22}) = 0$, $\delta(r_2, M_0, \alpha_0) = \delta(r_3, M_0, \alpha_0) = \delta(r_4, M_0, \alpha_0) = 0$. Then $h_2(M_0, \alpha_0) = h_1(M_0, \alpha_0)$ under both E_1 and E_2. $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) = 530$ under E_1 and $h_6(M_0, \alpha_0) = 355$ under E_2.

$$T(\alpha) = \begin{pmatrix} 35 & 50 & 60 & 70 \\ 0 & 35 & 60 & \end{pmatrix}$$

The energy consumption of a feasible schedule $\alpha = t_{21}t_{22}t_{11}t_{12}t_{13}t_{14}$ is calculated as follows. It can be checked that $M_0[\alpha > M_f$. To calculate the energy consumption during α, matrix $T(\alpha)$ is generated. Let q_{ij} denote the j-th operation for q_i, then the Gantt chart of α is shown in Fig. 2.

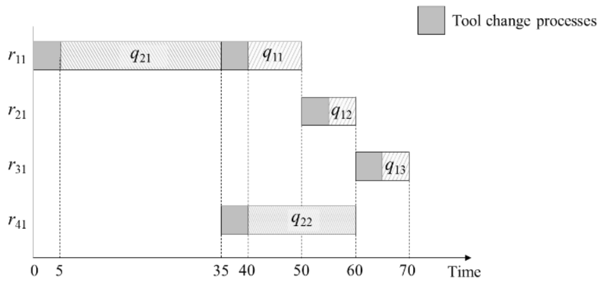

Fig. 2. Gantt chart of schedule α

Under E_1, $EH(\alpha) = e_0(r_1)HT(\alpha, p_{11}) + e_0(r_2)HT(\alpha, p_{12}) + e_0(r_3)HT(\alpha, p_{13}) + e_0(r_1)HT(\alpha, p_{21}) + e_0(r_4)HT(\alpha, p_{22}) = 2 \times 10 + 10 \times 5 + 2 \times 5 + 2 \times 30 + 2 \times 20 = 180$. From Fig. 2, it is easy to find that the idle time of r_1, r_2, r_3 and r_4 are 20, 60, 60 and 45, respectively. Thus, $EI(\alpha) = 20 \times 0.2 + 60 \times 1 + 60 \times 0.2 + 45 \times 0.2 = 85$. Tool change processes occur before q_{11}, q_{12}, q_{13}, q_{21} and q_{22}, thus $EC(\alpha) = 50$. Then $E_1(\alpha) = EH(\alpha) + EI(\alpha) + EC(\alpha) = 180 + 85 + 50 = 315$. Since there is no occupied time during α under E_2, $E_2(\alpha) = E_1(\alpha) = 315$.

Let $h_a^*(M_0, \alpha_0)$ and $h_b^*(M_0, \alpha_0)$ be the actual optimal total energy consumption from (M_0, α_0) to M_f under E_1 and E_2. For a feasible solution α, $E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$ and $E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$. Since $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$ under E_1 and $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$ under E_2, h_4 and h_6 are not admissible under both E_1 and E_2.