Supplementary File for "Minimizing energy consumption of flexible assembly systems with tool change processes using hybrid heuristic search"

Theorem 1: h_1, h_2, h_3, and h_5 are admissible.

Proof: Let β^* denote the actual optimal schedule from (M, α) to a final vertex, i.e., $M[\beta^* > M_F$. Let $h^*(M, \alpha)$ denote the total energy consumption of β^*. Let $E_W(\alpha, \beta^*)$, $E_O(\alpha, \beta^*)$, $E_I(\alpha, \beta^*)$, $E_H(\alpha, \beta^*)$ and $E_C(\alpha, \beta^*)$ denote the total energy consumption of working, occupied, idle, hold and tool change state from α to $\alpha\beta^*$, respectively. Under E_1, $h^*(M, \alpha) = E_H(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*)$, while under E_2, $h^*(M, \alpha) = E_W(\alpha, \beta^*) + E_O(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*)$.

Under E_2, $h_1(M, \alpha) = \sum_{p\in P_A\cup P_B}\sum_{j\in \boldsymbol{N}_{M(p)}}(E_R(p, j, M, \alpha) + E_P(p))$ is the minimum working energy consumption of all parts from (M, α) to the end, thus $h_1(M, \alpha) \le E_W(\alpha, \beta^*)$ under E_2. Since $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha)$ is only part of the occupied energy, $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le E_O(\alpha, \beta^*)$. Since $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha)$ is only part of the idle energy, $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) \le E_I(\alpha, \beta^*)$. Since $E_C(\alpha, \beta^*) \ge 0$, $h_5(M, \alpha) = h_1(M, \alpha) + \sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le E_W(\alpha, \beta^*) + E_O(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*) = h^*(M, \alpha)$, hence $h_5(M, \alpha)$ is admissible under E_2. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence $h_1(M, \alpha)$, $h_2(M, \alpha)$ and $h_3(M, \alpha)$ are admissible under E_2.

Since $(h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha))$ is only part of the hold energy consumption of all parts from (M, α) to the end, $h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le E_H(\alpha, \beta^*)$. $h_5(M, \alpha) \le E_H(\alpha, \beta^*) + E_I(\alpha, \beta^*) + E_C(\alpha, \beta^*) = h^*(M, \alpha)$, hence $h_5(M, \alpha)$ is admissible under E_1. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence $h_1(M, \alpha)$, $h_2(M, \alpha)$ and $h_3(M, \alpha)$ are admissible under E_1. ■

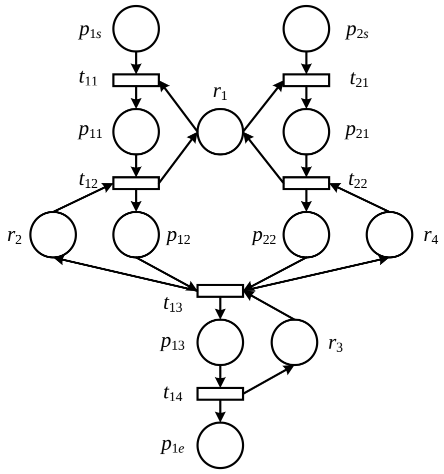

Fig. 1. APNS of FAS in Example 1.

Example 1: For the APNS processing two types of parts in Fig. 1, suppose $M_0 = p_{1s} + p_{2s} + r_1 + r_2 + r_3 + r_4$. The processing time of operations in places p_{11}, p_{12}, p_{13}, p_{21} and p_{22} are 10, 5, 5, 30 and 20, respectively. The energy consumption per unit time in different kinds of states is listed in Table I. For tool change processes, $\mu = 5$ and $e_c = 2$.

TABLE I

ENERGY CONSUMPTION PER UNIT TIME OF RESOURCES IN FIG. 1.

	r_1	r_2	r_3	r_4
Hold state	2.00	10.00	2.00	2.00
Working state	2.00	10.00	2.00	2.00
Occupied state	1.00	5.00	1.00	1.00
Idle state	0.20	1.00	0.20	0.20

Let's calculate $h_4(M_0, \alpha_0)$ under E_1 and E_2. The first part of $h_4(M_0, \alpha_0)$ is $h_1(M_0, \alpha_0)$. $E_R(p_{1s}, 1, M_0, \alpha_0) = E_R(p_{2s}, 1, M_0, \alpha_0) = 0$, $E_P(p_{1s}) = d(p_{11}) \times e_0(r_1) + d(p_{12}) \times e_0(r_2) + d(p_{13}) \times e_0(r_3) = 10 \times 2 + 5 \times 10 + 5 \times 2 = 20 + 50 + 10 = 80$ and $E_P(p_{2s}) = d(p_{21}) \times e_0(r_1) + d(p_{22}) \times e_0(r_4) = 30 \times 2 + 20 \times 2 = 100$. Then $h_1(M_0, \alpha_0) = E_R(p_{1s}, 1, M_0, \alpha_0) + E_P(p_{1s}) + E_R(p_{2s}, 1, M_0, \alpha_0) + E_P(p_{2s}) = 180$.

The rest of $h_4(M_0, \alpha_0)$ under E_1 is calculated as follows. $\Pi = \{t_{12}, t_{22}, t_{13}\}$, for t_{12} and t_{22}, since $M(p_{11}) = M(p_{21}) = 0$, $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$. For assembly transition t_{13}, since $p_{12} \in {}^{(o)}t_{13}$, $p_{1s} \in I(p_{12})$ and $M(p_{1s}) = 1 > 0$, $\theta(t_{13}, M_0, \alpha_0) = 1$.

$\Pi = \{t_{12}, t_{13}, t_{22}\}$. Since $|{}^{(o)}t_{12}| = |{}^{(o)}t_{22}| = 1$ and $M_0(p_{11}) = M_0(p_{21}) = 0$, $\theta(t_{12}, M_0, \alpha_0) = \theta(t_{22}, M_0, \alpha_0) = 0$. $E_{O1}(t_{13}, M_0, \alpha_0)$ and $E_{O2}(t_{13}, M_0, \alpha_0)$ are calculated as follows. Since $M_0(r_3) > 0$, $T_R(r_3, M_0, \alpha_0) = 0$. $T_O(t_{13}, M_0, \alpha_0) = \max\{T_O(p_{12}, M_0, \alpha_0), T_O(p_{22}, M_0, \alpha_0)\} = \max\{d(p_{11}) + d(p_{12}), d(p_{21}) + d(p_{22})\} = \max\{10 + 5, 30 + 20\} = 50$. Then $E_{O1}(t_{13}, M_0, \alpha_0) = (e_0(r_2) + e_0(r_4)) \times (\max\{T_O(t_{13}, M_0, \alpha_0), T_R(r_3, M_0, \alpha_0)\} - T_O(t_{13}, M_0, \alpha_0)) = (10 + 2) \times (\max\{50, 0\} - 50) = 0$, and $E_{O2}(t_{13}, M_0, \alpha_0) = (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{12}, M_0, \alpha_0)) \times e_0(r_2) + (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{22}, M_0, \alpha_0)) \times e_0(r_4) = (50 - 15) \times 10 + (50 - 50) \times 2 = 350$. Thus $h_4(M_0, \alpha_0) = h_1(M_0, \alpha_0) + \theta(t_{13}, M_0, \alpha_0)(E_{O1}(t_{13}, M_0, \alpha_0) + E_{O2}(t_{13}, M_0, \alpha_0)) = 180 + 1 \times (0 + 350) = 530$ under E_1.

Similarly, under E_2, $E_{O2}(t_{13}, M_0, \alpha_0) = (T_O(t_{13}, M_0, \alpha_0) - T_O(p_{12}, M_0, \alpha_0)) \times e_1(r_2) + (T_O(t_{13}, M_0, \alpha_0) -$

$T_O(p_{22}, M_0, \alpha_0)) \times e_1(r_4) = (50 - 15) \times 5 + (50 - 50) \times 1 = 175$, hence $h_4(M_0, \alpha_0) = 180 + 1 \times (0 + 175) = 355$.

Let's calculate $h_6(M_0, \alpha_0)$ under E_1 and E_2. Since ${}^{(o)}(r_1^\bullet) = \varnothing$, $\delta(r_1, M_0, \alpha_0) = 0$. Since ${}^{(o)}(r_2^\bullet) = \{p_{11}\}$, ${}^{(o)}(r_4^\bullet) = \{p_{21}\}$, ${}^{(o)}(r_3^\bullet) = \{p_{12}, p_{22}\}$, and $M(p_{11}) = M(p_{12}) = M(p_{21}) = M(p_{22}) = 0$, $\delta(r_2, M_0, \alpha_0) = \delta(r_3, M_0, \alpha_0) = \delta(r_4, M_0, \alpha_0) = 0$. Then $h_2(M_0, \alpha_0) = h_1(M_0, \alpha_0)$ under both E_1 and E_2. $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) = 530$ under E_1 and $h_6(M_0, \alpha_0) = 355$ under E_2.

$$\boldsymbol{T}(\alpha) = \begin{pmatrix} 35 & 50 & 60 & 70 \\ 0 & 35 & 60 & \end{pmatrix}$$

The energy consumption of a feasible schedule $\alpha = t_{21}t_{22}t_{11}t_{12}t_{13}t_{14}$ is calculated as follows. It can be checked that $M_0[\alpha > M_F$. To calculate the energy consumption during α, matrix $\boldsymbol{T}(\alpha)$ is generated. Let q_{ij} denote the j-th operation for q_i, then the Gantt chart of α is shown in Fig. 2.

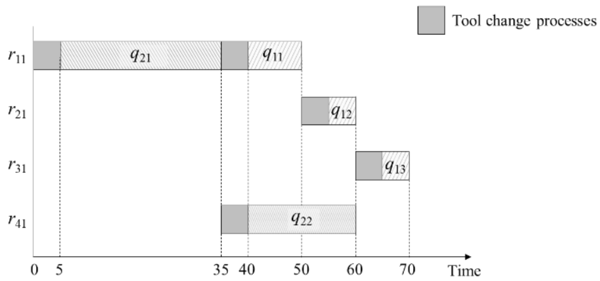

Fig. 2. Gantt chart of schedule α

Under E_1, $E_H(\alpha) = e_0(r_1)T_H(\alpha, p_{11}) + e_0(r_2)T_H(\alpha, p_{12}) + e_0(r_3)T_H(\alpha, p_{13}) + e_0(r_1)T_H(\alpha, p_{21}) + e_0(r_4)T_H(\alpha, p_{22}) = 2 \times 10 + 10 \times 5 + 2 \times 5 + 2 \times 30 + 2 \times 20 = 180$. From Fig. 2, it is easy to find that the idle time of r_1, r_2, r_3 and r_4 are 20, 60, 60 and 45, respectively. Thus, $E_I(\alpha) = 20 \times 0.2 + 60 \times 1 + 60 \times 0.2 + 45 \times 0.2 = 85$. Tool change processes occur before q_{11}, q_{12}, q_{13}, q_{21} and q_{22}, thus $E_C(\alpha) = 50$. Then $E_1(\alpha) = E_H(\alpha) + E_I(\alpha) + E_C(\alpha) = 180 + 85 + 50 = 315$. Since there is no occupied time during α under E_2, $E_2(\alpha) = E_1(\alpha) = 315$.

Let $h_a^*(M_0, \alpha_0)$ and $h_b^*(M_0, \alpha_0)$ be the actual optimal total energy consumption from (M_0, α_0) to M_F under E_1 and E_2. For a feasible solution α, $E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$ and $E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$. Since $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_1(\alpha) \geq h_a^*(M_0, \alpha_0)$ under E_1 and $h_6(M_0, \alpha_0) = h_4(M_0, \alpha_0) > E_2(\alpha) \geq h_b^*(M_0, \alpha_0)$ under E_2, h_4 and h_6 are not admissible under both E_1 and E_2.

D²WS

Input: An APNS(N, M_0) and *high*, *max_vertexes*, *max_size*, *max_top*;

Output: a feasible transition sequence α;

```
OPEN = {(M_0, α_0)}; CLOSED = ∅; bottom_depth = 0; top_depth = high; /* initialization */
while (OPEN ≠ ∅) do
    if (φ'(bottom_depth) = 0)
        top_depth++; bottom_depth++;
    end if
    select a vertex (M, α) from OPEN with minimum f(M, α) and |α| < top_depth;
    OPEN = OPEN \ {(M, α)}; CLOSED = CLOSED ∪ {(M, α)};
    sort elements in Ξ(M, C) in ascending order according to their A(t, M, α);
    υ(α) = 0; /* υ(α) counts the number of successor vertexes of (M, α)*/
    while (Ξ(M, C) ≠ ∅) do
        let t ∈ Ξ(M, C) be the first transition in Ξ(M, C);
        Ξ(M, C) = Ξ(M, C) \ {t};
        let M[t > M_1, and α_1 = αt;
        if (M_1 = M_F)
            return α_1;
        end if
        if (∃ a node (M_1, α_2) in OPEN satisfying B(M_1, α_1) ≤ B(M_1, α_2))
            OPEN = (OPEN \ {(M_1, α_2)}) ∪ {(M_1, α_1)};
        else if (∄a node with M_1 in OPEN ∪ CLOSED or ∃ a node (M_1, α_2) in CLOSED satisfying B(M_1, α_1) < B(M_1, α_2))
            if (φ(|α_1|) < max_size or f(M_1, α_1) < f̄(α_1))
                OPEN = OPEN ∪ {(M_1, α_1)}; υ(α)++;
            end if
            if (φ'(top_depth) ≥ max_top)
                discard vertexes at depth bottom_depth;
                top_depth++; bottom_depth++;
            end if
            if (υ(α) ≥ max_vertexes), break;
            end if
        end if-else if
    end while
end while
```

Theorem 2: D^{2}WS can always output a solution

Proof: All vertexes in D^{2}WS are generated under a DAP, which prohibit transition firings that lead APNS from safe markings to deadlock. Thus all markings of generated vertexes are safe. That is to say, from any markings except for M_F, there is at least one enabled transition that can fire and lead to a safe marking.

There are two termination conditions for D^{2}WS: the final marking is reached or $OPEN = \varnothing$. If D^{2}WS terminates with the final marking reached, then D^{2}WS yields a solution. Assume the final marking has not been reached but D^{2}WS terminates with $OPEN = \varnothing$. Then, all vertexes in the current window (upon termination of the algorithm) are explored. Let (M, α) be an explored vertex with the deepest depth among all the vertexes in the current window. Since $M_F \neq M$, there must be a vertex (M_1, α_1) in the current window generated from (M, α) and $|\alpha_1| = |\alpha| + 1$. (M_1, α_1) will be kept in the current window unless there is a vertex at the depth of $|\alpha_1|$ with marking M_1 or the number of successor vertexes of (M, α) is more than *max_size*. That is a contradiction to the assumption that (M, α) is an explored vertex with the deepest depth among all vertexes in the current window. Thus, D^{2}WS can always end with the final marking. ■

The parameters of the search windows are randomly distributed in the range of [2, 5] for *max_vertexes*, [2, 7] for *high*, and [2, 5] for *max_size*. *max_top* is always 3 times as big as *max_size*. The parameters of search windows used in the first experiment are shown in II

TABLE II

PARAMETERS OF SEARCH WINDOWS.

Search windows	*high*	*max_vertexes*	*max_size*	*max_top*
*SW*01	3	3	3	9
*SW*02	6	4	3	9
*SW*03	5	4	2	6
*SW*04	2	2	3	9
*SW*05	4	2	3	9
*SW*06	2	2	5	15
*SW*07	3	4	2	6
*SW*08	4	2	2	6
*SW*09	4	5	4	12
*SW*10	4	4	3	9

TABLE III

PARAMETERS OF SEARCH WINDOWS USED IN THE SECOND EXPERIMENT.

	Search windows	*high*	*max_vertexes*	*max_size*	*max_top*
Group 1	*GW*01	2	2	2	6
	*GW*02	2	2	3	9
	*GW*03	2	3	3	9
	*GW*04	3	3	3	9
Group 2	*GW*05	3	3	4	12
	*GW*06	3	4	4	12
	*GW*07	4	4	4	12
	*GW*08	4	4	5	15
Group 3	*GW*09	4	5	5	15
	*GW*10	5	5	5	15
	*GW*11	5	5	6	18
	*GW*12	5	6	6	18
Group 4	*GW*13	6	6	6	18
	*GW*14	6	6	7	21
	*GW*15	6	7	7	21
	*GW*16	7	7	7	21

TABLE IV

RESULTS OF NONPARAMETRIC FRIEDMAN'S TESTS (SIGNIFICANCE LEVEL = 0.05).

Data for nonparametric Friedman's tests (significance level = 0.05)	*p*-value
The average results found by six heuristic functions under E_1	0.000
The best results found by six heuristic functions under E_1	0.000
The average results found by six heuristic functions under E_1	0.000
The best results found by six heuristic functions under E_2	0.000
The average results found by h_4 under E_1 using four groups of search windows	0.000
The best results found by h_4 under E_1 using four groups of search windows	0.000
The average results found by h_6 under E_2 using four groups of search windows	0.000
The best results found by h_6 under E_2 using four groups of search windows	0.000

The multiple comparisons method used in this paper can be found in (M. Hollander and D. A. Wolfe, *Nonparametric Statistical Methods*. New York, NY, USA: Wiley, 1973.)

TABLE V

MULTIPLE COMPARISON RESULTS USING THE AVERAGE SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_1.

h_i	h_j	*p*-value
1	2	0.713
1	3	0.026
1	4	0.000
1	5	0.001
1	6	0.000
2	3	0.000
2	4	0.000
2	5	0.000
2	6	0.000
3	4	0.000
3	5	0.920
3	6	0.000
4	5	0.000
4	6	1.000
5	6	0.000

TABLE VI

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_1.

h_i	h_j	*p*-value
1	2	0.990
1	3	0.023
1	4	0.000
1	5	0.777
1	6	0.000
2	3	0.003
2	4	0.000
2	5	0.398
2	6	0.000
3	4	0.000
3	5	0.425
3	6	0.000
4	5	0.000
4	6	0.724
5	6	0.000

TABLE VII

MULTIPLE COMPARISON RESULTS USING THE AVERAGE SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_2.

h_i	h_j	p-value
1	2	0.065
1	3	0.000
1	4	0.000
1	5	0.000
1	6	0.000
2	3	0.000
2	4	0.000
2	5	0.182
2	6	0.000
3	4	0.000
3	5	0.277
3	6	0.000
4	5	0.000
4	6	0.686
5	6	0.000

TABLE VIII

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY SIX HEURISTIC FUNCTIONS UNDER E_2.

h_i	h_j	p-value
1	2	0.944
1	3	0.000
1	4	0.006
1	5	0.547
1	6	0.000
2	3	0.000
2	4	0.079
2	5	0.110
2	6	0.000
3	4	0.000
3	5	0.003
3	6	0.000
4	5	0.000
4	6	0.548
5	6	0.000

TABLE IX

MULTIPLE COMPARISON RESULTS USING AVERAGE SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_4 UNDER E_1.

Group i	Group j	p-value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	0.000
2	4	0.000
3	4	0.986

TABLE X

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_4 UNDER E_1.

Group *i*	Group *j*	*p*-value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	0.027
2	4	0.032
3	4	1.000

TABLE XI

MULTIPLE COMPARISON RESULTS USING AVERAGE SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_6 UNDER E_2.

Group *i*	Group *j*	*p*-value
1	2	0.000
1	3	0.000
1	4	0.000
2	3	1.000
2	4	0.004
3	4	0.004

TABLE XII

MULTIPLE COMPARISON RESULTS USING THE BEST SOLUTIONS FOUND BY FOUR GROUPS OF SEARCH WINDOWS AND h_6 UNDER E_2.

Group *i*	Group *j*	*p*-value
1	2	0.005
1	3	0.107
1	4	0.467
2	3	0.644
2	4	0.000
3	4	0.002