Supplementary File for "Minimizing the energy consumption of flexible assembly systems with tool change processes using hybrid heuristic search"

Theorem 1: h_1, h_2, h_3, and h_5 are admissible.

Proof: Let β^* denote the actual optimal schedule from (M, α) to a final vertex, i.e., $M[\beta^* > M_f$. Let $h^*(M, \alpha)$ denote the total energy consumption of β^*. Let $EW(\alpha, \beta^*)$, $EO(\alpha, \beta^*)$, $EI(\alpha, \beta^*)$, $EH(\alpha, \beta^*)$ and $EC(\alpha, \beta^*)$ denote the total energy consumption of working, occupied, idle, hold and tool change state from α to $\alpha\beta^*$, respectively. Under E_1, $h^*(M, \alpha) = EH(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*)$, while under E_2, $h^*(M, \alpha) = EW(\alpha, \beta^*) + EO(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*)$.

Under E_2, $h_1(M, \alpha) = \sum_{p\in P_o\cup P_s}\sum_{j\in \mathbf{N}_{M(p)}}(RE(\zeta(p, j), M, \alpha) + PE(p))$ is the minimum working energy consumption of all parts from (M, α) to the end, thus $h_1(M, \alpha) \le EW(\alpha, \beta^*)$ under E_2. Since $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha)$ is only part of the occupied energy, $\sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le EO(\alpha, \beta^*)$. Since $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha)$ is only part of the idle energy, $\sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) \le EI(\alpha, \beta^*)$. Since $EC(\alpha, \beta^*) \ge 0$, $h_5(M, \alpha) = h_1(M, \alpha) + \sum_{r_i\in R}\delta(r_i, M, \alpha)G(r_i, M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le EW(\alpha, \beta^*) + EO(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*) = h^*(M, \alpha)$, hence h_5 is admissible under E_2. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence h_1, h_2 and h_3 is admissible under E_2.

Since $(h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha))$ is only part of the hold energy consumption of all parts from (M, α) to the end, $h_1(M, \alpha) + \sum_{t\in\Pi}\kappa(t, M, \alpha)J(t, M, \alpha) \le EH(\alpha, \beta^*)$. $h_5(M, \alpha) \le EH(\alpha, \beta^*) + EI(\alpha, \beta^*) + EC(\alpha, \beta^*) = h^*(M, \alpha)$, hence h_5 is admissible under E_1. By the definitions of $h_1(M, \alpha)$-$h_5(M, \alpha)$, $h_2(M, \alpha) \le h_5(M, \alpha)$ and $h_1(M, \alpha) \le h_3(M, \alpha) \le h_5(M, \alpha)$, hence h_1, h_2 and h_3 is admissible under E_1. ■

Theorem 2: D^{2}WS can always yield a solution

Proof: All vertexes in D^{2}WS are generated under a DAP, which prohibit firings that lead APNS from safe markings to deadlock. Thus all markings of generated vertexes are safe. That is to say, from any markings expect for M_f, there is at least one enabled transition that can fire and lead to a safe marking.

There are two termination conditions for D^{2}WS: the final marking is reached or OPEN = ∅. If D^{2}WS terminates with the final marking reached, then D^{2}WS yields a solution. Assume the final marking has not been reached but D^{2}WS terminates with OPEN = ∅. Then, all vertexes in the current window (upon termination of the algorithm) are explored. Let (M, α) be an explored vertex with the deepest depth among all the vertexes in the current window. Since $M_f \neq M$, there must be a vertex (M_1, α_1) in the current window generated from (M, α) and $|\alpha_1| = |\alpha| + 1$. ($M_1$, α_1) will be kept in the current window unless there is a vertex at (M_1, α_1) with marking M_1 or the number of successor vertexes of (M, α) is more than *max_size*. That is a contradiction to the assumption that (M, α) is an explored vertex with the deepest depth among all vertexes in the current window. Thus, D^{2}WS can always end with the final marking. ■

TABLE IV

PARAMETERS OF SEARCH WINDOWS

Search windows	*high*	*max_vertexes*	*max_size*	*max_top*
*SW*01	3	3	3	9
*SW*02	6	4	3	9
*SW*03	5	4	2	6
*SW*04	2	2	3	9
*SW*05	4	2	3	9
*SW*06	2	2	5	15
*SW*07	3	4	2	6
*SW*08	4	2	2	6
*SW*09	4	5	4	12
*SW*10	4	4	3	9

TABLE V

SIMULATION RESULTS OF 20 INSTANCES UNDER E_1

$\rho(1), \rho(2), \rho(3), \rho(4)$	h_1		h_2		h_3		h_4		h_5		h_6	
	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG
7, 7, 5, 5	5011.6	5383.8	4975.0	5348.5	5112.4	5466.8	**4747.3**	**5069.9**	5150.1	5517.5	4829.4	5092.9
8, 8, 7, 7	6239.6	6669.1	6154.0	6543.4	6372.8	6961.7	**5917.1**	6261.7	6116.0	6775.3	6040.0	**6234.6**
7, 7, 6, 6	5499.5	5693.1	5429.5	5795.1	5514.9	5764.1	5231.3	**5495.4**	5341.6	5975.2	**5213.0**	5497.6
11, 11, 10, 10	8403.0	9095.8	8417.4	8926.0	8543.7	9213.5	8151.0	8482.2	8584.5	9261.1	**8079.3**	**8444.9**
15, 15, 8, 8	9220.1	9659.3	9128.5	9534.5	9379.3	9854.2	**8921.7**	**9190.7**	9501.0	10059.9	8972.8	9228.0
5, 5, 4, 4	3827.3	4121.3	3891.8	4152.3	3943.7	4351.6	3694.6	**3865.8**	3909.9	4385.3	**3675.4**	3899.3
7, 7, 11, 11	7662.2	7833.8	7694.7	8070.4	7768.6	8100.6	**7315.7**	**7674.1**	7717.7	8239.8	7355.6	7749.5
12, 12, 15, 15	11139.1	11858.9	11023.7	11603.7	11358.9	11892.4	10606.6	11060.3	11242.8	12069.5	**10510.3**	**10967.1**
5, 5, 13, 13	7810.6	8408.9	7857.1	8245.0	7878.1	8384.0	**7469.7**	**7782.5**	7975.6	8298.1	7568.4	7830.5
6, 6, 9, 9	6553.2	6927.7	6378.0	6769.4	6501.8	6919.9	**6093.8**	**6394.8**	6317.9	6870.8	6217.9	6432.1
4, 4, 10, 10	6175.3	6505.7	6260.6	6547.4	6125.9	6559.2	**5894.2**	**6096.8**	6272.2	6584.0	5965.1	6160.4
5, 5, 8, 8	5761.3	6013.5	5771.0	6021.9	5703.1	6076.5	5491.4	5704.2	5763.3	6135.1	**5435.1**	**5694.9**
6, 6, 12, 12	7676.0	8246.1	7812.5	8138.6	7769.0	8312.1	**7340.7**	7731.2	7717.3	8128.7	7490.3	**7703.7**
15, 15, 13, 13	11148.6	11790.0	10996.8	11491.1	11452.1	11921.4	**10673.9**	**11054.8**	11239.7	12082.8	10753.8	11062.4
9, 9, 6, 6	6168.4	6616.6	6083.4	6491.3	6293.7	6762.7	**5961.0**	6216.9	6183.0	6749.9	6040.8	**6208.0**
6, 6, 6, 6	4958.2	5442.1	5016.7	5402.5	5167.8	5683.4	**4755.3**	5131.9	4978.3	5507.9	4793.5	**5040.2**
14, 14, 13, 13	10652.9	11327.3	10820.7	11509.2	11036.9	11544.3	10326.4	10795.7	10866.8	11685.3	**10290.7**	**10682.8**
5, 5, 6, 6	4868.6	5055.6	4857.8	5086.5	4821.7	5236.0	**4465.3**	**4734.7**	4860.2	5181.7	4573.3	4766.7
15, 15, 14, 14	11727.0	12431.9	11423.5	12028.4	11972.5	12461.2	**11001.3**	**11471.9**	11641.9	12708.5	11052.7	11491.9
3, 3, 14, 14	7445.9	7677.4	7546.4	7883.9	7389.4	7874.2	**7202.1**	7638.6	7458.0	7937.6	7353.9	**7559.6**

Bold fonts means the best among the six heuristic functions.

TABLE VI

SIMULATION RESULTS OF 20 INSTANCES UNDER E_2

$\rho(1), \rho(2), \rho(3), \rho(4)$	h_1		h_2		h_3		h_4		h_5		h_6	
	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG	BST	AVG
7, 7, 5, 5	4707.1	4931.6	4694.9	4978.7	4932.8	5169.8	4723.3	4787.8	4789.0	5076.2	**4671.1**	**4744.3**
8, 8, 7, 7	5927.3	6478.7	5767.4	6145.9	5956.0	6277.8	5772.7	5897.4	5843.6	6201.7	**5709.3**	**5825.6**
7, 7, 6, 6	**5042.5**	5193.5	5061.5	5359.1	5198.2	5469.3	5070.8	5160.7	5121.3	5466.0	5044.8	**5145.3**
11, 11, 10, 10	8012.6	8378.0	8059.9	8493.4	8056.2	8542.0	7943.5	8051.0	7951.5	8463.6	**7862.6**	**7933.9**
15, 15, 8, 8	8688.7	8991.6	8750.4	9129.4	8866.6	9108.7	8681.5	8788.8	8880.6	9201.3	**8577.5**	**8719.5**
5, 5, 4, 4	3636.3	3821.7	3614.7	3836.2	3719.5	3974.8	3604.2	**3691.8**	3740.9	3951.0	**3594.6**	3692.8
7, 7, 11, 11	7198.3	7372.5	7257.1	7605.6	7321.7	7690.2	7233.2	7331.6	7409.2	7655.4	**7122.5**	**7260.4**
12, 12, 15, 15	10500.9	10786.7	10518.2	10923.2	10567.6	11091.3	10339.2	10486.9	10391.1	10838.1	**10234.6**	**10415.5**
5, 5, 13, 13	7522.1	7719.4	7438.5	7652.7	7648.6	7899.7	7446.5	7567.3	7566.9	7872.3	**7375.1**	**7513.1**
6, 6, 9, 9	6058.5	6239.1	6015.3	6334.5	6151.3	6484.9	5973.7	**6096.1**	6162.2	6419.6	**5953.9**	6121.2
4, 4, 10, 10	5964.2	6092.6	5927.0	6085.8	5971.1	6202.3	**5732.7**	5968.7	5973.7	6164.2	5814.5	**5935.9**
5, 5, 8, 8	5364.9	5554.8	5262.9	5545.4	5396.0	5675.7	5295.2	5447.3	5378.3	5670.2	**5239.1**	**5379.7**
6, 6, 12, 12	7451.2	7617.1	7291.5	7624.8	7403.7	7758.5	**7272.1**	**7423.4**	7506.8	7777.5	7334.5	7448.3
15, 15, 13, 13	10492.5	10676.1	10514.8	10843.0	10717.4	11136.9	10430.6	10550.7	10468.5	10989.0	**10424.8**	**10547.1**
9, 9, 6, 6	5863.6	5987.7	5789.1	6121.9	5943.8	6184.5	5753.7	**5827.3**	5855.6	6173.4	**5723.7**	5837.1
6, 6, 6, 6	4819.1	5047.4	4731.8	5010.8	4895.5	5218.7	4769.0	4845.8	4864.7	5104.3	**4697.0**	**4808.6**
14, 14, 13, 13	10043.5	10348.9	10257.6	10753.4	10457.4	10684.1	10082.0	**10169.2**	10313.2	10697.9	**9988.2**	10178.9
5, 5, 6, 6	4524.9	4715.6	4378.8	4667.6	4516.2	4874.2	**4373.6**	4565.8	4505.6	4782.0	4429.6	**4518.8**
15, 15, 14, 14	10980.9	11259.6	10891.6	11498.6	11227.8	11554.5	10766.2	10927.9	10760.6	11365.7	**10750.4**	**10914.8**
3, 3, 14, 14	7198.4	7369.0	7390.6	7488.6	7372.1	7532.0	**7194.9**	7441.1	7226.3	7504.1	7199.1	**7292.3**

Bold fonts means the best among the six heuristic functions.

TABLE VII

PARAMETERS OF SEARCH WINDOWS USED IN THE SECOND EXPERIMENT

	Search windows	*high*	*max_vertexes*	*max_size*	*max_top*
Group 1	*GW*01	2	2	2	6
	*GW*02	2	2	3	9
	*GW*03	2	3	3	9
	*GW*04	3	3	3	9
Group 2	*GW*05	3	3	4	12
	*GW*06	3	4	4	12
	*GW*07	4	4	4	12
	*GW*08	4	4	5	15
Group 3	*GW*09	4	5	5	15
	*GW*10	5	5	5	15
	*GW*11	5	5	6	18
	*GW*12	5	6	6	18
Group 4	*GW*13	6	6	6	18
	*GW*14	6	6	7	21
	*GW*15	6	7	7	21
	*GW*16	7	7	7	21

TABLE VIII

SCHEDULING RESULTS USING FOUR GROUPS OF SEARCH WINDOWS AND H_4 UNDER E_1

$\rho(1), \rho(2),$	*Group* 1		*Group* 2		*Group* 3		*Group* 4	
$\rho(3), \rho(4)$	BST	AVG	BST	AVG	BST	AVG	BST	AVG
7, 7, 5, 5	5003.0	5210.1	4845.5	4999.7	4899.0	4955.3	**4774.8**	**4809.1**
8, 8, 7, 7	6157.1	6362.7	5970.5	6130.8	**5858.2**	**5939.7**	5928.8	5966.7
7, 7, 6, 6	5362.3	5606.4	5322.2	5435.5	5126.7	5206.7	**5117.0**	**5145.4**
11, 11, 10, 10	8409.8	8524.5	8086.5	8441.6	**8047.9**	8246.3	8063.6	**8116.9**
15, 15, 8, 8	9018.6	9270.4	8846.0	9087.8	**8745.7**	8951.3	8810.0	**8857.7**
5, 5, 4, 4	3814.6	4047.7	3780.3	3818.2	3799.6	3811.0	**3691.0**	**3723.0**
7, 7, 11, 11	7619.9	8033.5	**7174.9**	7436.2	7229.2	**7360.3**	7190.7	7365.2
12, 12, 15, 15	10942.1	11259.7	10485.2	10806.5	**10400.8**	**10572.0**	10570.7	10663.5
5, 5, 13, 13	7590.8	8035.9	7542.9	7758.5	**7484.0**	**7503.0**	7494.1	7507.3
6, 6, 9, 9	6233.8	6610.1	6249.1	6358.7	**6109.5**	**6142.9**	6171.2	6176.7
4, 4, 10, 10	6001.9	6261.9	5915.8	6060.4	5875.3	**5881.3**	**5835.1**	5950.7
5, 5, 8, 8	5491.4	5855.6	5439.9	5549.0	5281.9	5361.4	**5257.2**	**5360.4**
6, 6, 12, 12	7493.5	7847.6	7518.6	7656.3	7315.8	**7400.3**	**7287.7**	7583.8
15, 15, 13, 13	10747.9	11163.9	10692.9	11015.7	10725.8	**10791.1**	**10632.1**	10817.8
9, 9, 6, 6	6104.4	6286.9	5965.8	6116.0	**5857.5**	5931.9	5898.8	**5917.5**
6, 6, 6, 6	5056.6	5288.2	4852.1	5021.5	**4763.5**	**4799.6**	4828.7	4910.0
14, 14, 13, 13	10523.5	10876.8	**10219.2**	10726.2	10337.8	10504.1	10302.7	**10349.1**
5, 5, 6, 6	4651.2	4943.9	4503.0	4570.4	**4425.3**	**4492.8**	4457.4	4570.8
15, 15, 14, 14	11199.7	11638.8	11278.3	11596.4	**11016.2**	11143.0	11019.6	**11040.3**
3, 3, 14, 14	7340.9	7905.7	7238.8	7392.8	**7237.5**	7258.8	7241.3	**7241.4**

Bold fonts means the best among the six heuristic functions.

TABLE VIIII

SCHEDULING RESULTS USING FOUR GROUPS OF SEARCH WINDOWS AND H_6 UNDER E_2

$\rho(1), \rho(2),$	*Group* 1		*Group* 2		*Group* 3		*Group* 4	
$\rho(3), \rho(4)$	BST	AVG	BST	AVG	BST	AVG	BST	AVG
7, 7, 5, 5	4671.1	4806.9	**4654.7**	4749.9	4676.8	**4735.7**	4717.2	4773.7
8, 8, 7, 7	5715.7	5931.5	5785.6	5817.4	**5698.5**	**5801.5**	5839.3	5857.2
7, 7, 6, 6	5044.8	5243.3	5037.0	5086.2	**5016.0**	**5059.5**	5102.4	5141.5
11, 11, 10, 10	7891.3	8079.9	7870.9	**7933.6**	**7853.1**	7935.7	7993.3	8050.8
15, 15, 8, 8	**8577.5**	8735.9	8646.5	8675.1	8645.2	**8761.0**	8643.4	8753.6
5, 5, 4, 4	**3599.1**	3777.7	3641.8	3695.6	3613.6	**3653.0**	3633.6	3668.1
7, 7, 11, 11	7153.2	7399.8	**7131.1**	**7207.7**	7147.7	7219.7	7223.2	7244.7
12, 12, 15, 15	10357.6	10581.6	**10220.6**	**10313.8**	10299.2	10367.6	10317.1	10409.4
5, 5, 13, 13	**7375.1**	7641.3	7375.4	**7440.6**	7449.6	7475.4	7496.7	7521.5
6, 6, 9, 9	6140.5	6260.2	**5944.7**	**6020.5**	6042.2	6099.9	6059.1	6100.5
4, 4, 10, 10	5994.3	6044.7	**5791.1**	5883.8	5851.6	5876.8	5807.1	**5828.8**
5, 5, 8, 8	5287.9	5533.7	**5226.5**	**5287.5**	5312.2	5356.2	5355.3	5365.6
6, 6, 12, 12	7375.5	7603.8	7263.9	7366.6	**7245.7**	**7283.1**	7322.4	7372.2
15, 15, 13, 13	10449.8	10584.9	**10359.4**	10493.8	10404.5	**10491.7**	10453.8	10523.9
9, 9, 6, 6	5780.9	5873.1	5776.4	5815.8	5744.4	**5811.1**	**5743.0**	5848.4
6, 6, 6, 6	4782.5	4951.4	4741.6	4784.4	**4717.6**	**4741.4**	4767.4	4833.8
14, 14, 13, 13	10092.3	10282.3	**10046.8**	10135.8	10057.4	**10073.4**	10149.0	10165.9
5, 5, 6, 6	4429.6	4670.2	**4327.4**	**4432.9**	4454.7	4471.8	4491.2	4499.3
15, 15, 14, 14	10819.8	10975.2	10737.9	10794.6	**10680.2**	**10764.1**	10889.0	10962.3
3, 3, 14, 14	7244.5	7369.2	**7191.9**	7278.6	7201.9	**7230.9**	7203.0	7266.3

Bold fonts means the best among the six heuristic functions.

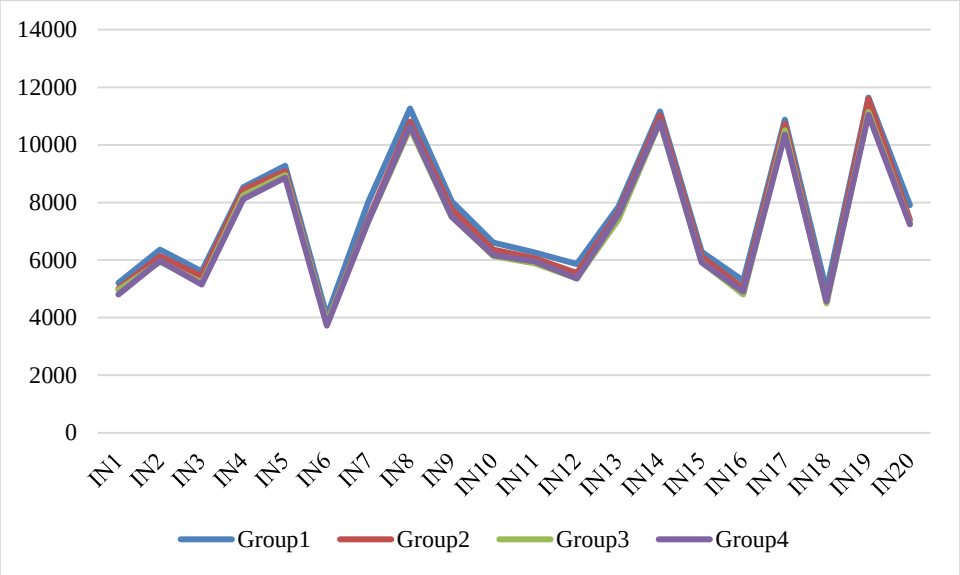

Fig. 4. Best scheduling results using four groups of search windows and h_4 under E_1.

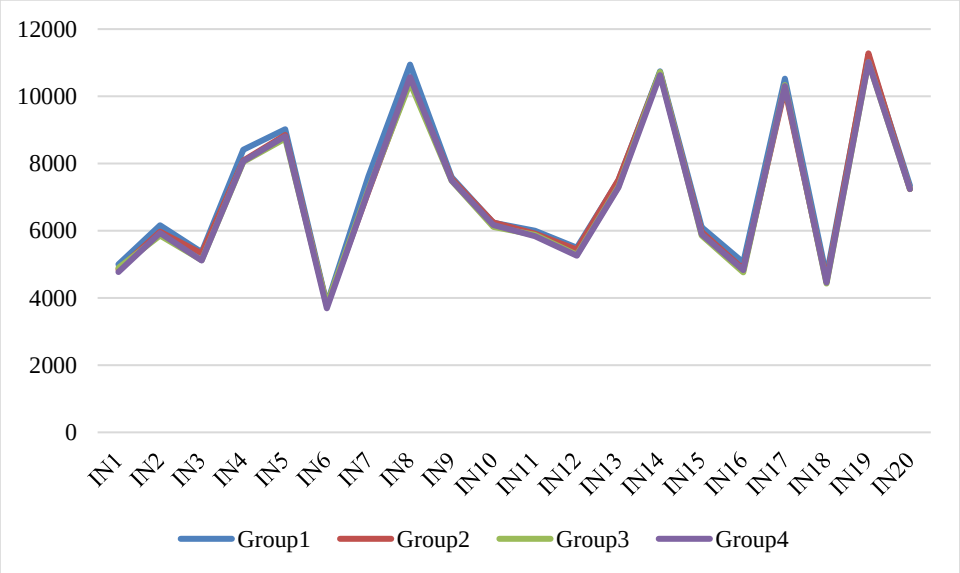

Fig. 5. Average scheduling results using four groups of search windows and h_4 under E_1.

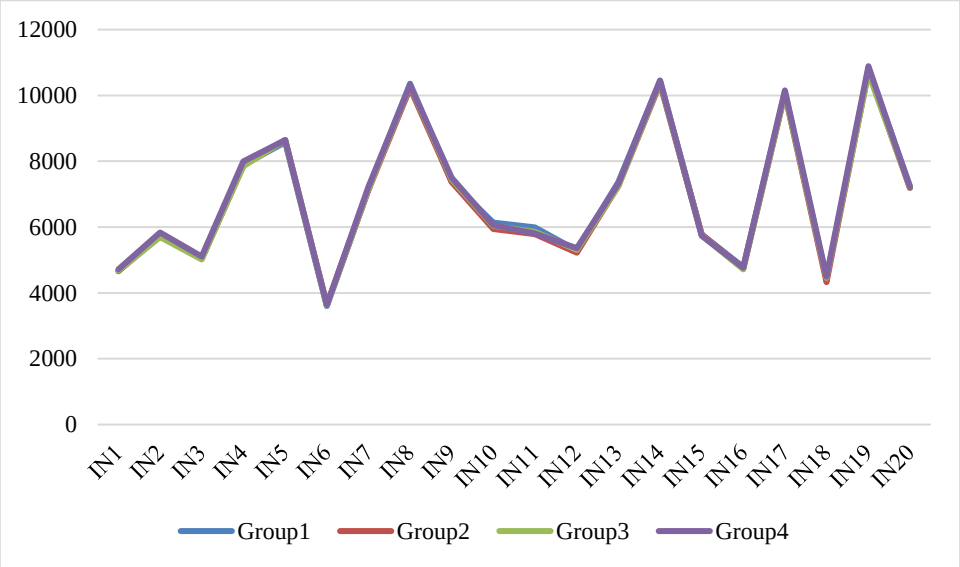

Fig. 6. Best scheduling results using four groups of search windows and h_{6s} under E_2.

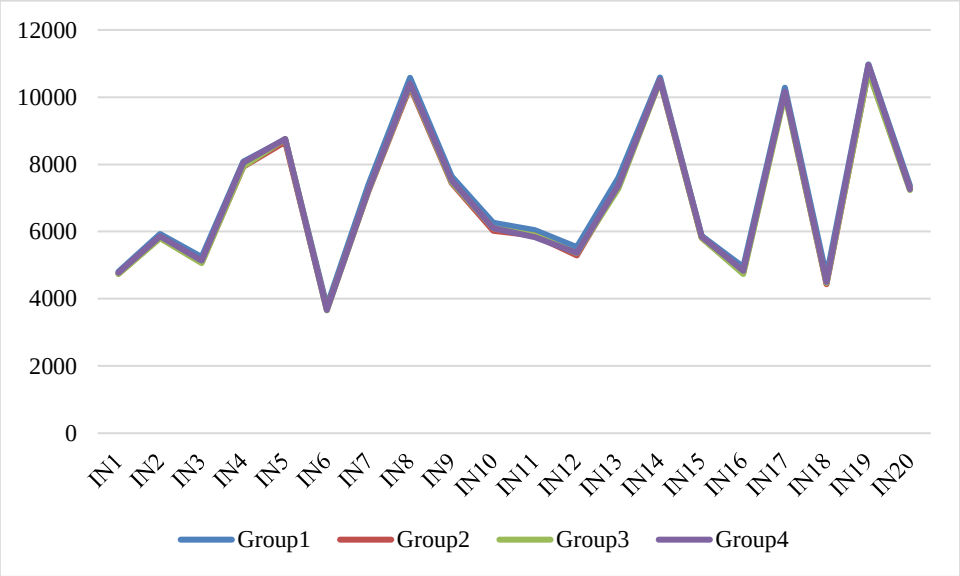

Fig. 7. Average scheduling results using four groups of search windows and h_6 under E_2.