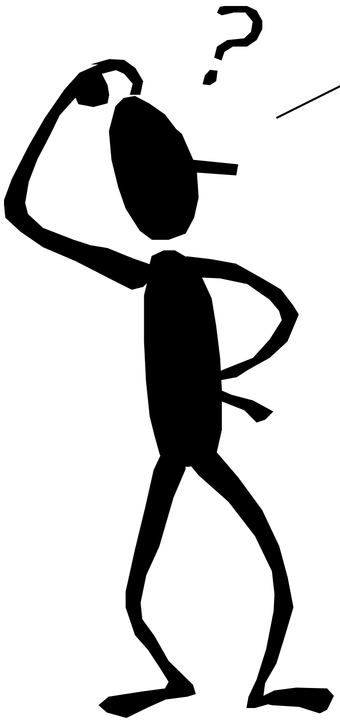
Do The Math!

A Guide to Common FRC Engineering Calculations

FRC 2016 Palmetto Kick Off

Center for Advanced Technical Studies

Do the Math!



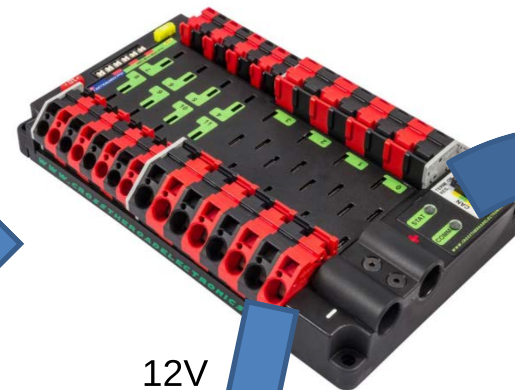
Math?!?
Why do I need that?
Let's build stuff!


SCOTTY, I NEED
MORE POWER!


I'M GIVING ALL WE'VE GOT
CAPTAIN...
ANY MORE AND WE'LL BROWN
OUT!!!!

Power Source --- Power Users

Battery
12ish Volts



12V
Power Supply

Supply	roboRIO Brownout Behavior
4.5V to 6.3V	• Processor, AI, AO: **<u>Running</u>** • PWM, Relays, CAN: **<u>STOPPED</u>** • <u>Must get back to 7.5V</u>
< 4.5V	• All functions stop • <u>Must rise above 4.65V to Re-boot</u>

$$V = I \times R$$

Tools needed

Calculate

Algebra

Trigonometry

Geometry

Verify

Do The Math!
Topics

- Concepts / Terms:
  - Force / Weight / Mass / Work / Energy / Power / Efficiency
- Example 1: Lift power requirements
- Spreadsheet overview / Gear / Sprockets
- Motor curves
- Example 1: Linear device spreadsheet calculator
- Intake Mechanisms and Conveyors
- Rotating Arms
- Drive trains
- Leverage

Force / Weight / Mass / Torque / Work / Energy / Power / Efficiency

CONCEPTS

Concepts: Force / Weight / Mass

Force

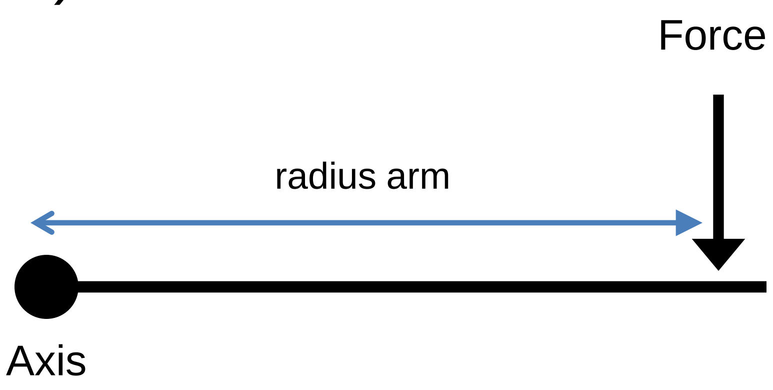
- Force = mass * acceleration
- 1 N (newton) = 1 kg–m / sec ^{2}
- Acceleration due to earth's gravity:
  - 9.8 m / sec ^{2}

Weight / Mass

- 1 kg (kilogram) "weighs" 9.8 N on earth
- 1 lb_f = 4.45 N
- 2.2 lb_m = 1 kg

Concepts: Torque

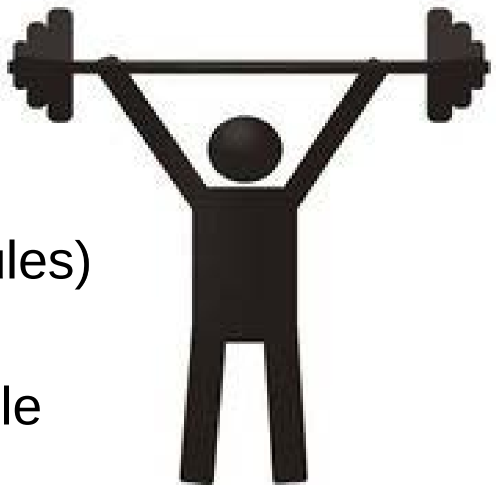
- Tendency of force to rotate an object around an axis or fulcrum ("twisting force")
- Torque = Force x radius $\tau = F \bullet R$
- English units: ft-lbs (foot pounds)
- SI units: N-m (newton meters)



Concepts: Work / Energy

Mechanical Work

- Work = Force x Distance
  - English units: ft-lbs
  - SI units: Nm (Newton-Meters) or J (Joules)
    - 1 Nm of work = 1 Joule
  - Rotating systems Work = torque x angle traveled in radians



Energy is required to do Work

- Units are equivalent (work=energy)

Concepts: Power

Power = Energy used per unit time

- Energy is “how much” work
- Power is “how fast” work can be done
  - Increase power to do work faster

- Power = Work / Time
  - English units: HP (horsepower) = 550 ft-lbs/sec
  - SI Units: W (watt) = 1 Nm / sec or 1 Joule / sec
  - Conversion 1 HP = 746 W
  - Rotating system
    - Power = torque x rotational speed in radians / sec

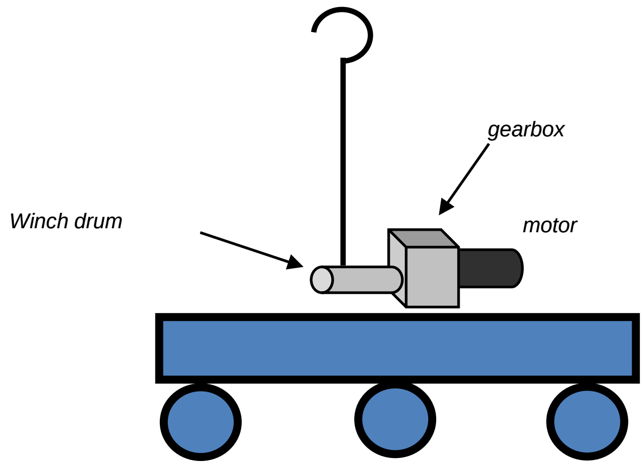
Power requirements / Concept: Efficiency

EXAMPLE 1: LIFT A ROBOT

Example 1: Lift a robot

The end game rules provide for bonus points if you lift your robot off the ground by hanging on a bar. Our game strategy is to lift our robot off the floor two feet in 2.5 seconds for maximum points.

Our concept design…



- How much power is required?
  - Calculate power in watts
  - Assume an ideal system with no inefficiencies
  - Assume robot, bumpers and battery weigh 140 lbs

Concept Application: Lift a robot

Solution:

- Power = Work / time
- Time = given specification of 2.5 seconds.
- How much Work is required?
  - Work = Force x Distance
  - Assume the robot is lifted vertically
  - How much force? We need enough force to overcome robot weight (force of gravity)
    - Newtons = 140 lbs * 4.45 N / lb = 623 N
  - How much distance?
    - Assume the robot is lifted vertically
    - Distance = 2 ft * 0.3 m / ft = 0.6 m
  - Work = 623N * 0.6m = 373.8Nm
- Calculate Power
  - Power = Work / time
  - Power = 373.8 Nm / 2.5 sec
  - Power = 149.52 Nm/sec = ***149.52 Watts***

Concept: Efficiency

No system is perfect. Some energy or power put in a system is lost through imperfections. System losses occur at every step from the energy source to the energy use.

Definition of Efficiency

System efficiency is defined as --- Power output / Power input (expressed as a percentage)

Examples of system losses:

- Friction in gears or sprockets
- Rolling friction of a wheel in contact with the ground
- Air drag of a car or airplane
- Electrical resistance in wiring and circuits

Conservation of Energy

Energy is neither created or destroyed. Total energy in an isolated system remains constant.

Question:

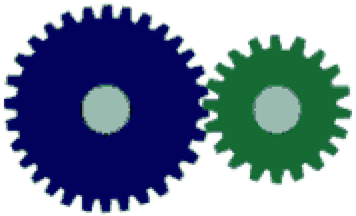
•If no system is 100% efficient AND energy must be conserved, where do energy losses go?

Spreadsheet Overview / Gears / Sprockets

SPREADSHEET REVIEW

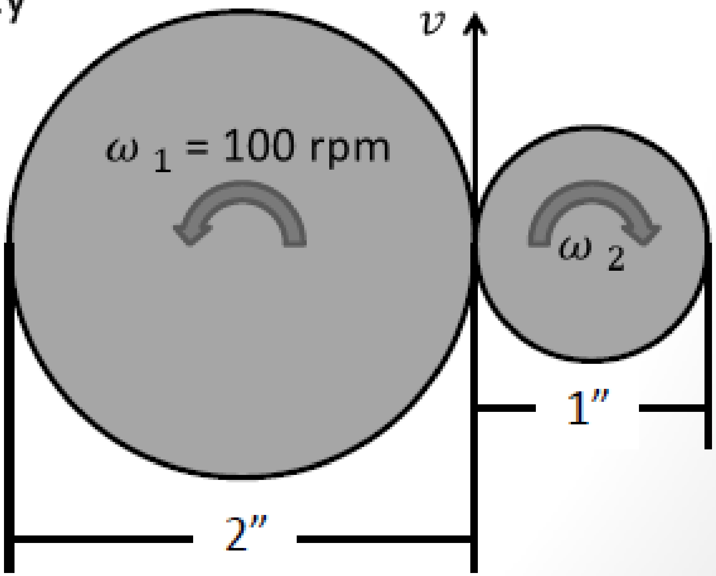
Mechanical Advantage: Gears

- Transmit power from motor to mechanical load
- Primary use:
  - Increase “mechanical advantage”
  - Change high speed / low torque motor output to low speed / high torque load



Mechanical Advantage: Gears

- Gear to increase or decrease speed (angular velocity) and torque
- Speed is inversely dependant on torque
  - As speed increases, torque decreases (and vise versa)
- Meshing gears may have different angular velocities, but they will always have the same linear velocity



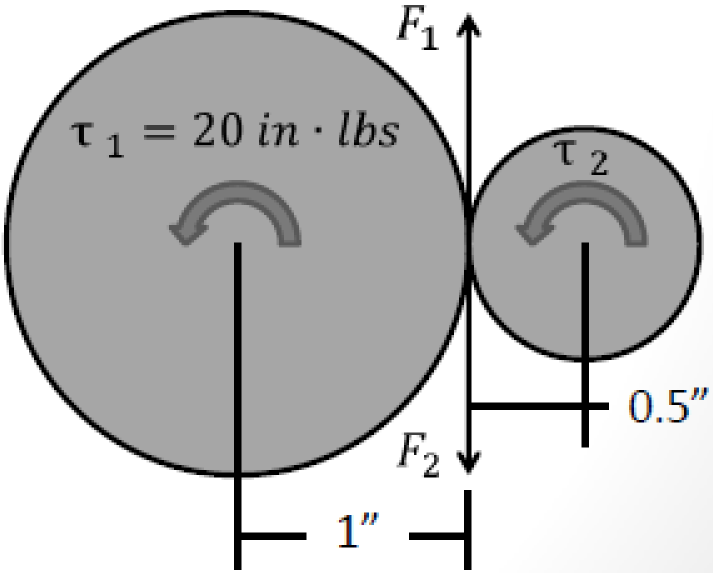
- $v_1 = v_2 = v$
- $\omega_1 \cdot \pi \cdot D_1 = \omega_2 \cdot \pi \cdot D_2$
- $\boxed{\frac{\omega_1}{\omega_2} = \frac{D_2}{D_1}}$
- $\omega_2 = \omega_1 \cdot \frac{D_1}{D_2}$
- $\omega_2 = 100(rpm) \cdot \frac{2''}{1''} = 200rpm$

Source: Simbotics Resources: www.simbotics.org

Mechanical Advantage: Gears

- Meshing gears may have different torques, but when they are static (motor is stalled) they will have an equal, but opposite force

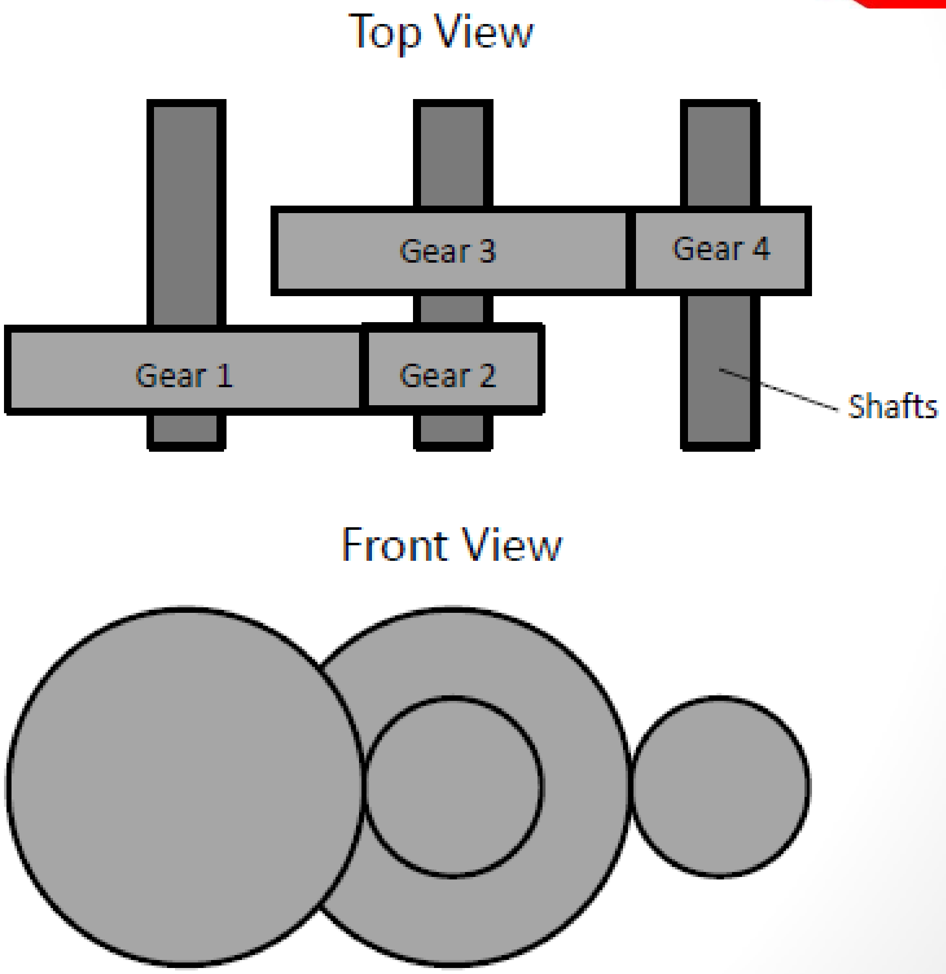
- $F_1 = F_2$
- $\frac{\tau_1}{d_1} = \frac{\tau_2}{d_2}$
- $\boxed{\frac{\tau_1}{\tau_2} = \frac{d_1}{d_2}}$
- $\tau_2 = \frac{d_2}{d_1} \cdot \tau_1$
- $\tau_2 = \frac{0.5"}{1"} \cdot 20\ in \cdot lbs = 10\ in \cdot lbs$



Source: Simbotics Resources: www.simbotics.org

Mechanical Advantage: Gears

- Gears fixed on the same shaft (2 and 3) have the same angular velocity
- Therefore
  - $\omega_1 \cdot \frac{D_1}{D_2} = \omega_2$
  - $\omega_3 = \omega_4 \cdot \frac{D_4}{D_3}$
  - $\omega_2 = \omega_3$
  - $\omega_1 \cdot \frac{D_1}{D_2} = \omega_4 \cdot \frac{D_4}{D_3}$
  - $\frac{\omega_1}{\omega_4} = \frac{D_2}{D_1} \cdot \frac{D_4}{D_3}$
- Gear ratios from each stage are multiplied



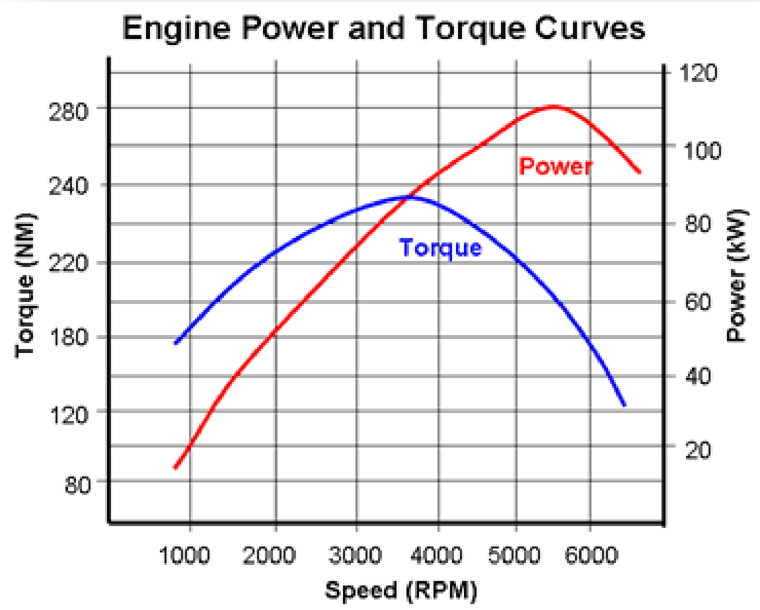
Source: Simbotics Resources: www.simbotics.org

DC Motor Performance Curves / FRC Motors

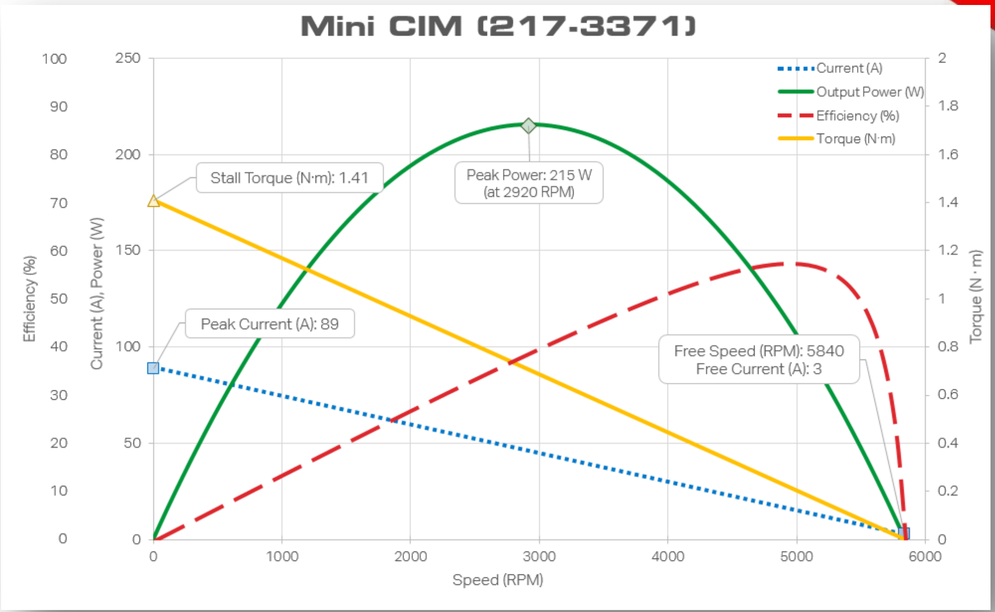
MOTOR CURVES

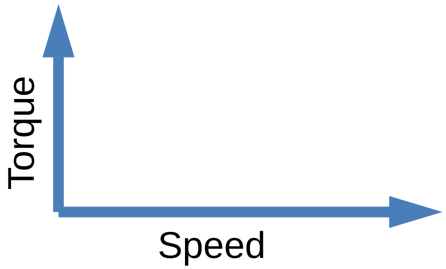
Motor Curves

Automotive

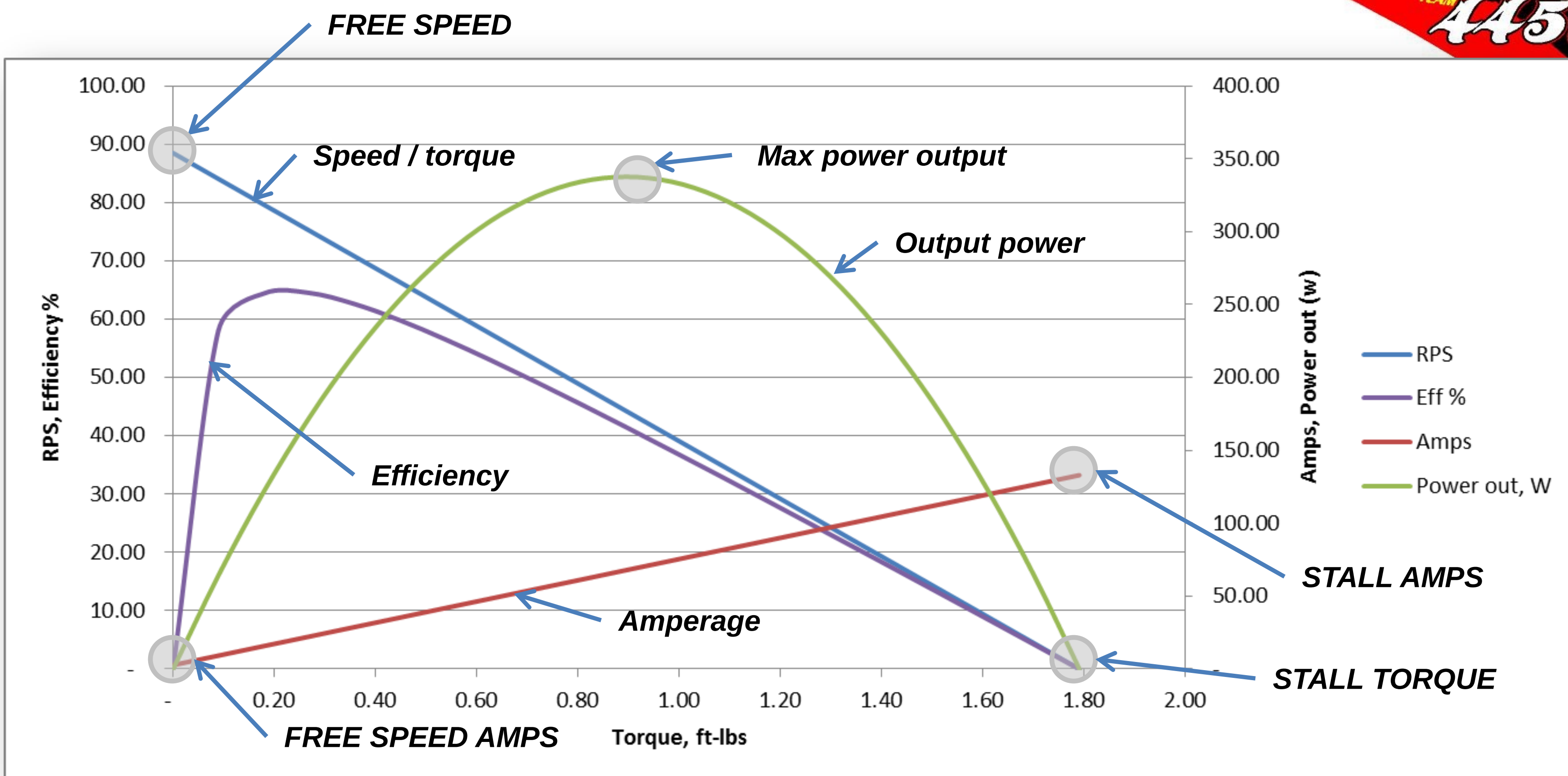


DC Motor





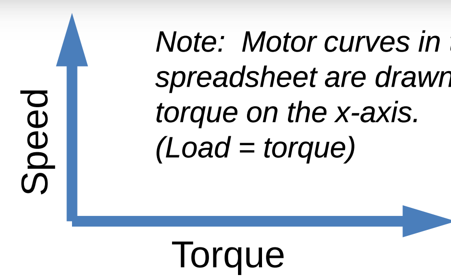
DC Motor Performance Curve



CIM Motor at 12Vdc

Key motor stats

- Free speed
- Free speed amps
- Stall torque
- Stall torque amps
- Max power
- Reference voltage



Note: Motor curves in the spreadsheet are drawn with torque on the x-axis. (Load = torque)

DC Motor “Thermal Mass”

CIM (enclosed motor)
- Free speed 5310 rpm
- Max power 340 watts
- Weight 2.8 lbs
- Power weight ratio *121watts / lb*

Vex 775Pro (open case motor)
- Free speed 18,700 rpm
- Max power 347 watts
- Weight 0.8 lbs
- Power weight ratio *434 watts / lb*

Calculate with

EXAMPLE 1: LIFT A ROBOT (SPREADSHEET VERSION)

Example 1: Lift a robot
Linear Device Motor Selector

Goal

- Lift 140 lb robot 2 feet in 2.5 seconds
- Calculated power requirement 150W

Linear Device Motor Selector

TOC

AndyMark / AM 9015 (am-0912)

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Input System Characteristics

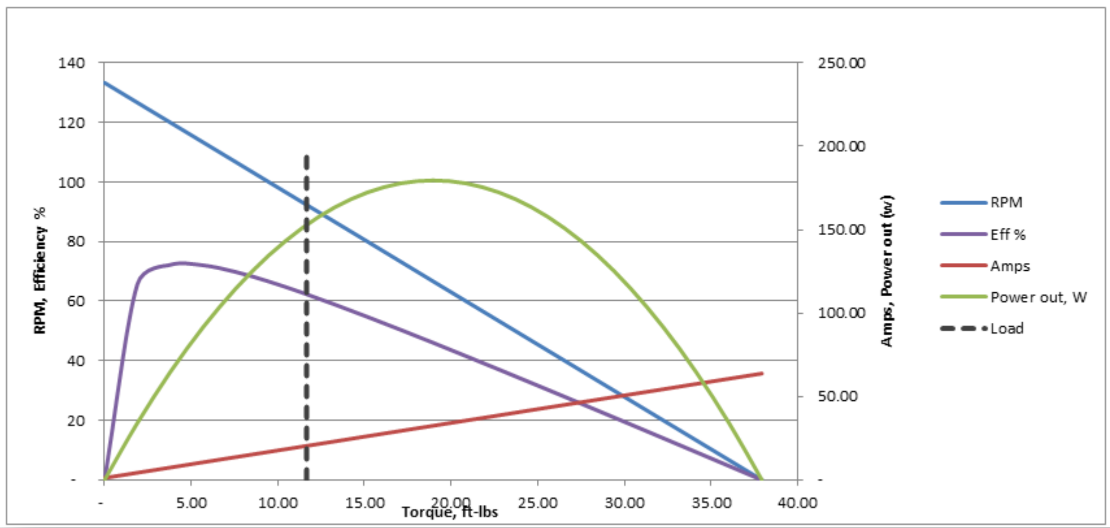
# of Motor & Gear - sprocket ratio				
# of motors	1			
Reduction	120.0	to 1		
Efficiency	100%			
Free speed	133	rpm	13.96	rad/s
Stall Amps	63.8	Free Amps	1.2	
Stall Torque	37.9	ft-lbs	51	N-m

Loaded Mechanism Outputs

Amperage at load	20.5	A
Stall load	455	lbs
RPM at load	92	rpm
Travel rate	9.67	in / sec
Time to travel 24 in	2.48	sec
Travel rate	0.81	ft / sec

LOAD				
Force needed	140.0	lbs		
Pulley radius	1.00	in		
Needed torque	11.7	ft-lbs		
Applied voltage	12.0	V		
Travel Distance	24.0	in	2.51	rad/s

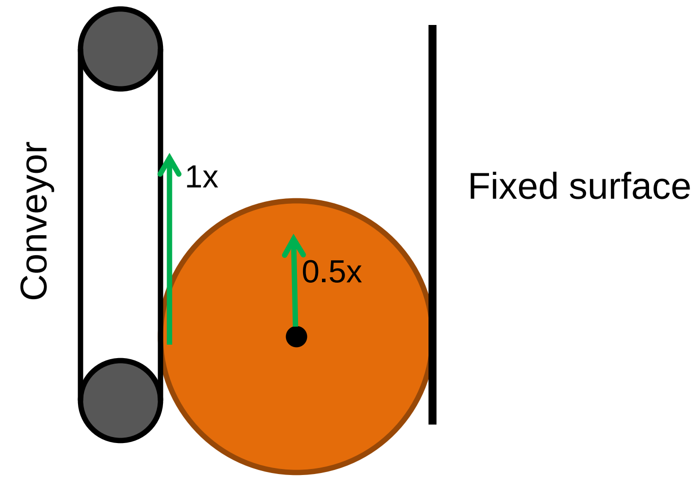
		pitch	
	# of teeth	dia	radius
25 Chain	25	1.995	0.998
35 Chain	17	2.041	1.021



Other linear device examples

INTAKE MECHANISMS AND CONVEYERS

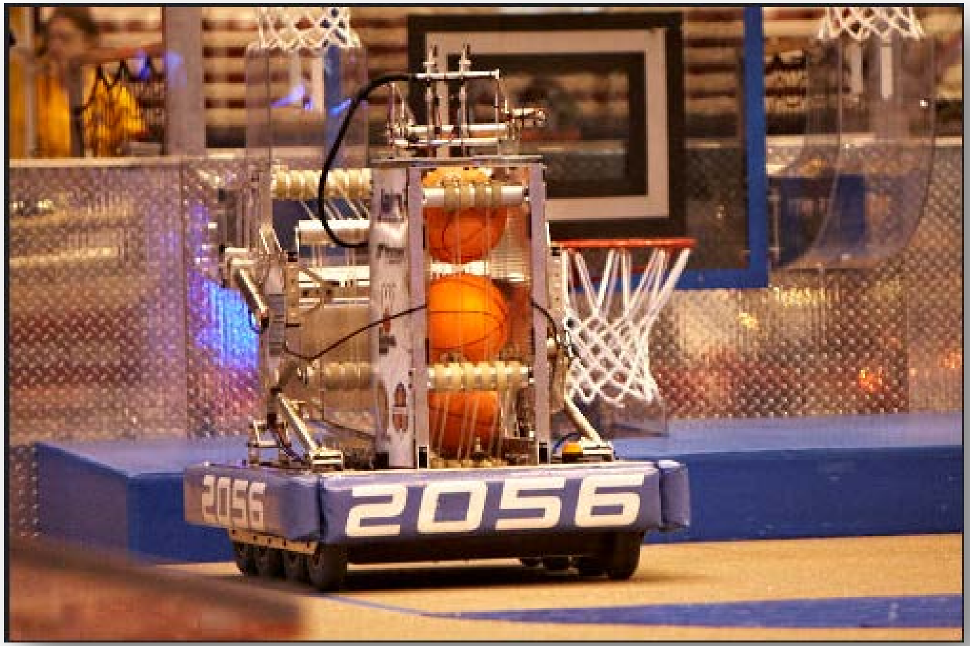
Intake / Conveyor
One side fixed – One side moving

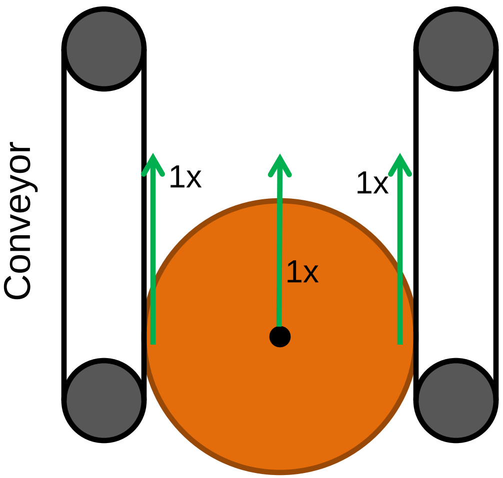


Game object moves at
½ the surface speed of the conveyor.

<u>NOTE:</u> Spreadsheet calculates conveyor speeds, not game object speeds for this case. Also, conveyor is only handling half the game object weight as well.

Intake / Conveyor
Both sides moving



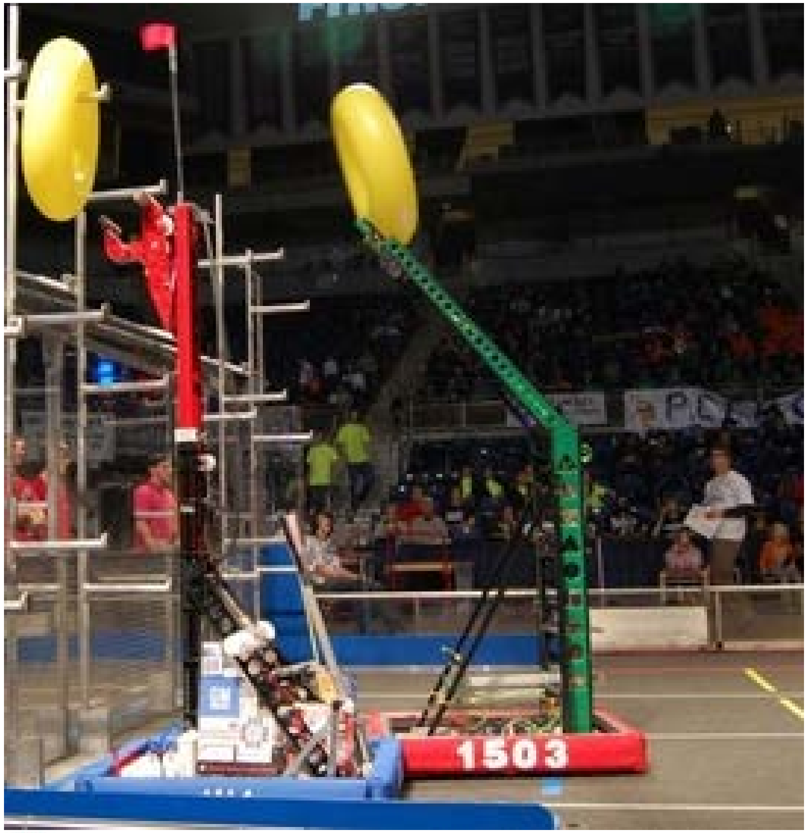


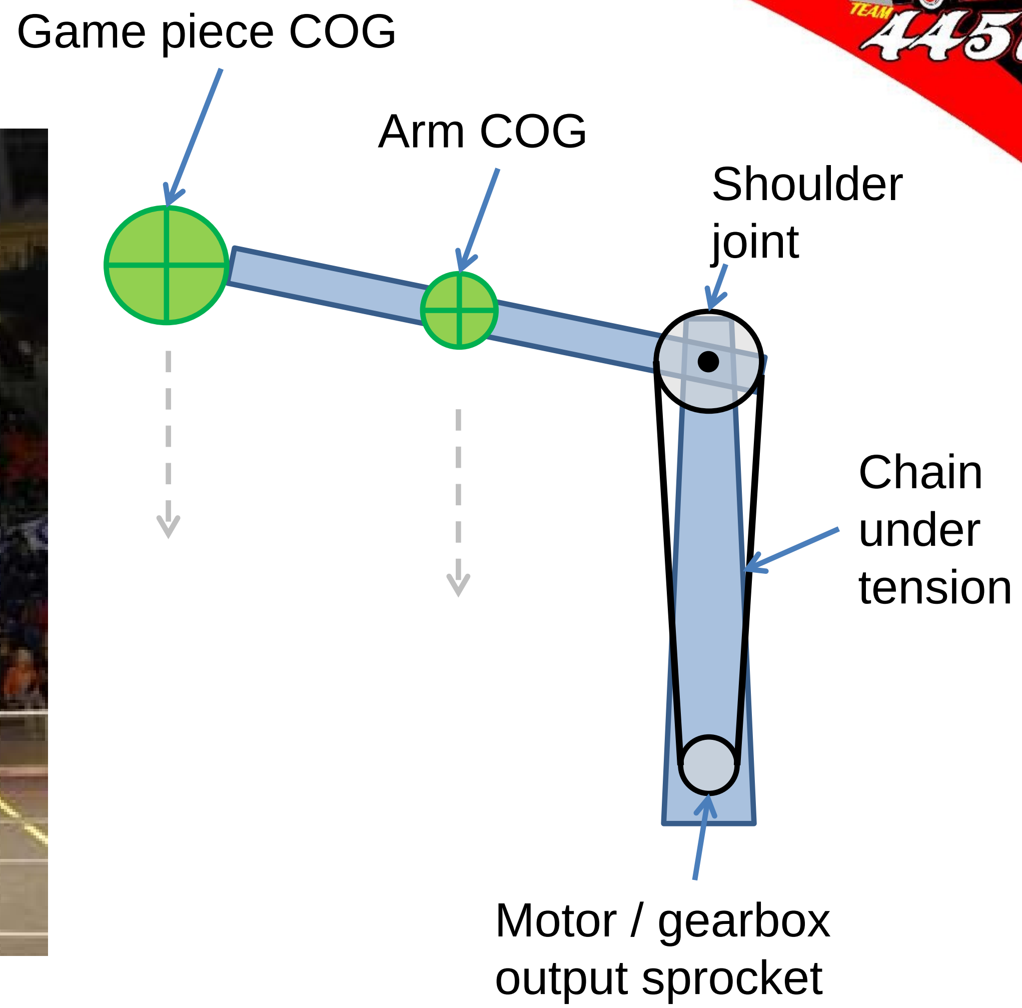
Both conveyors and the game object move at the same speed.

Load calculator and motor selector

ROTATING ARMS

Rotating Arm Loads





Design considerations

DRIVE TRAINS

Design Considerations

- Traction limited (will the wheels slip?) and hill climbing traction
  - Drive train forces tab
- Speed and pushing force
  - JVN DT Single Speed and JVN DT Two Speed tabs
- Turning and acceleration (speed to distance)
  - JeeseK Model tab

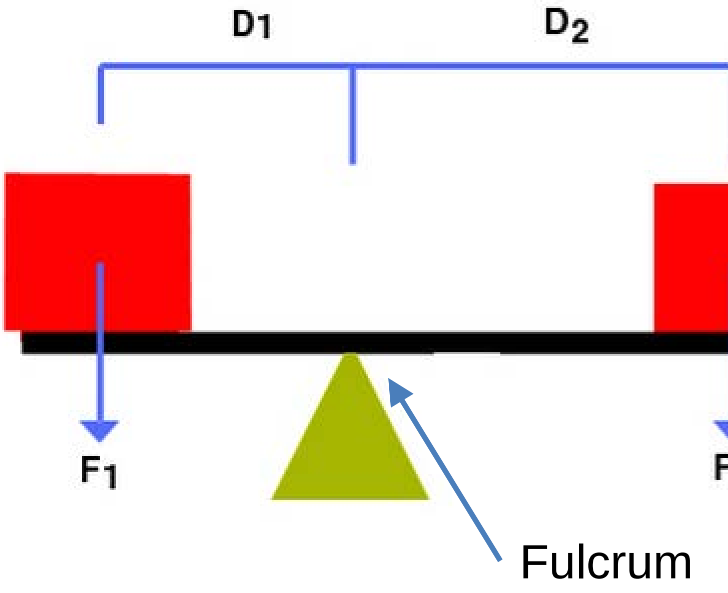
Design considerations

LEVERAGE

Lever

- Force times distance around a pivot is a "moment"**.
- Lever --- moments are balanced around the pivot (a.k.a. fulcrum)
- F1 x D1 = F2 x D2

- 5 lbs x 4 ft = __ lbs x 1 ft

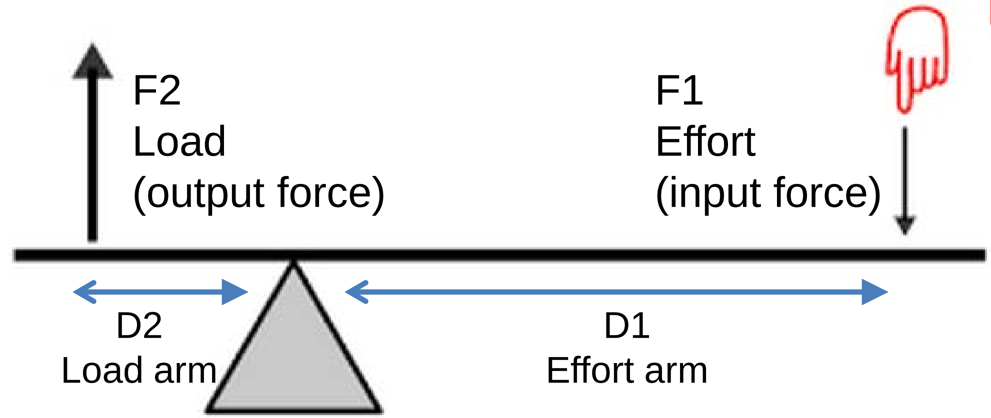


**There are other types of moments. Moment of inertia. Moment of area.

Force that causes rotation around a pivot is also known as "torque"

Mechanical Advantage

- F1 x D1 = F2 x D2
- F2 = F1 x (D1/D2)



<u>Mechanical Advantage</u> = Effort arm length ÷ Load arm length

F2 = 5 lbs x (4 ft / 1 ft) = 20 lbs

4 to 1 mechanical advantage
(shown as a ratio --- 4:1)

Lever Classes

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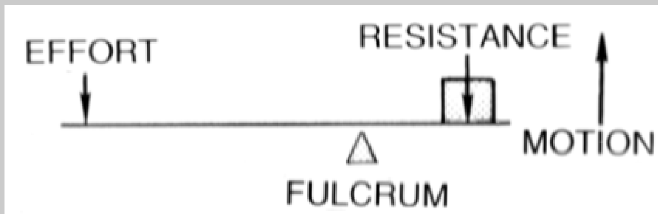
<tr><th></th><th>Diagram</th><th>Example</th></tr>
<tr><td>Class 1</td><td>
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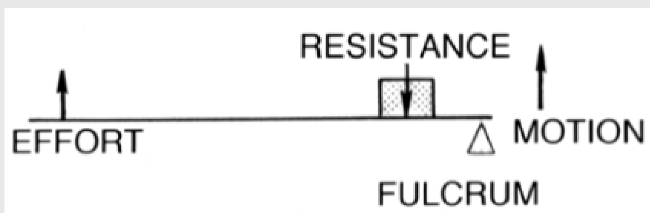
<tr><td>Class 2</td><td>
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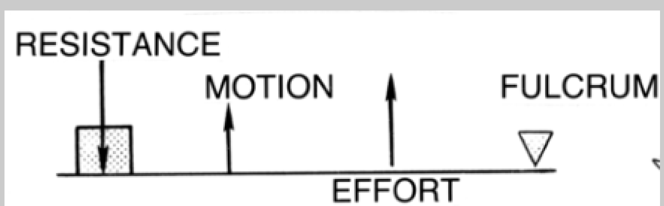
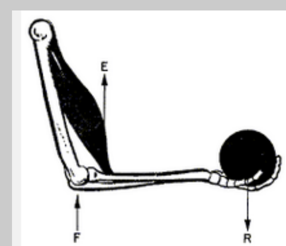
<tr><td>Class 3</td><td>
</td><td>
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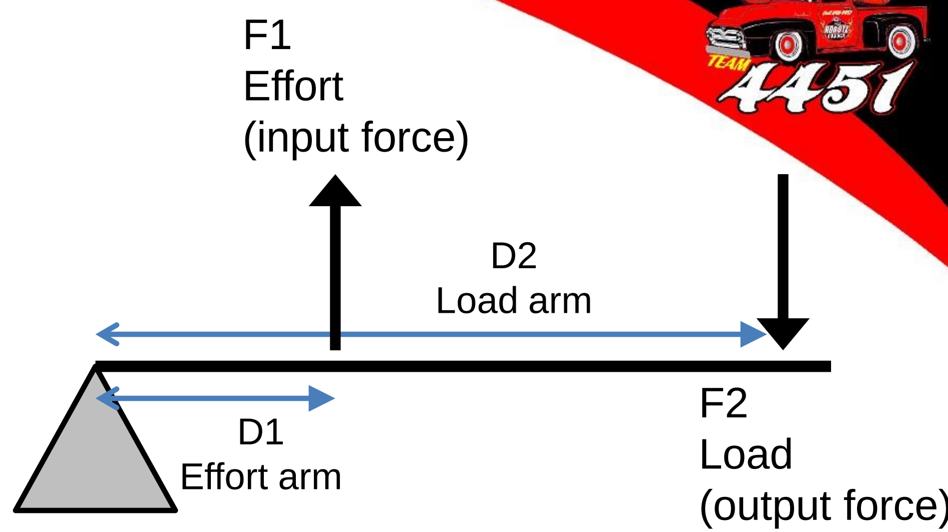
Name that class!

Mechanical Advantage
Class 3 Lever



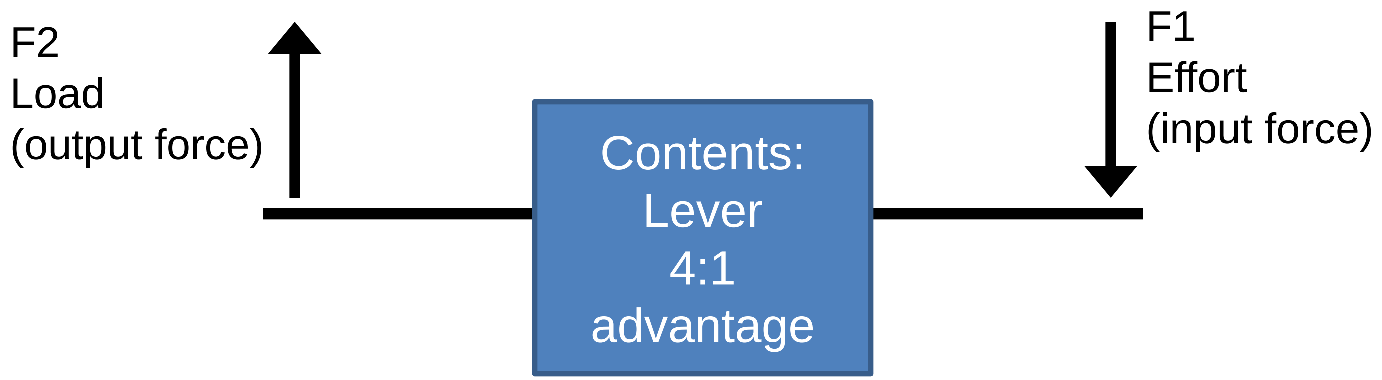
- $F1 \times D1 = F2 \times D2$
- $F2 = F1 \times \underbrace{(D1/D2)}$

<u>Mechanical Advantage</u> = Effort arm length ÷ Load arm length

F2 = 5 lbs x (1 ft / 4 ft) = 1.25 lbs

0.25 to 1 mechanical advantage
(less than 1 --- "disadvantage")

Magic box?



Lever device can increase output force. What do I lose?

DISTANCE

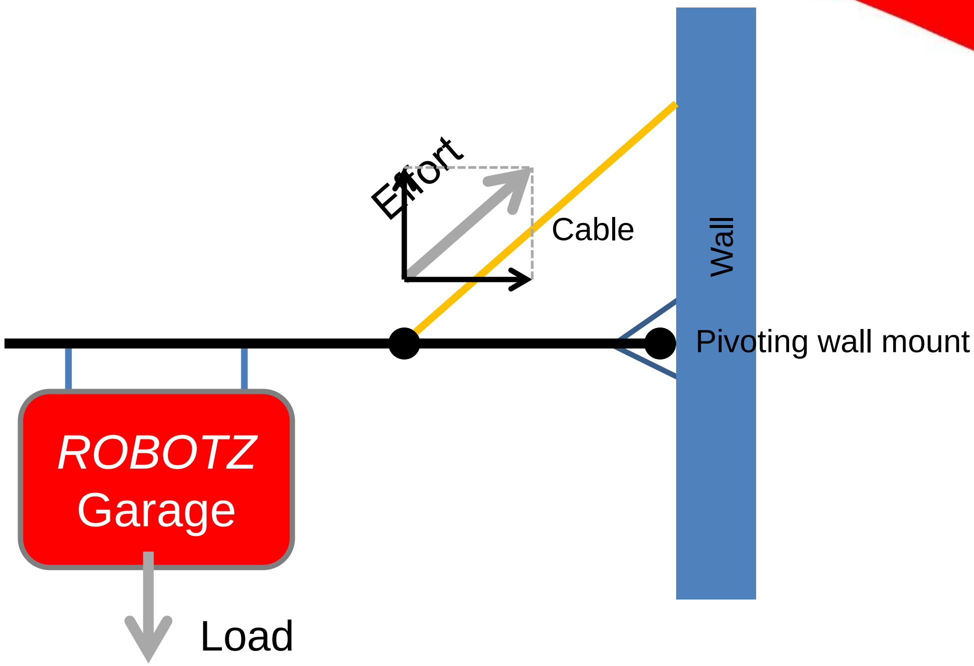
Engineering principle:

- Conservation of energy
- Conservation of power

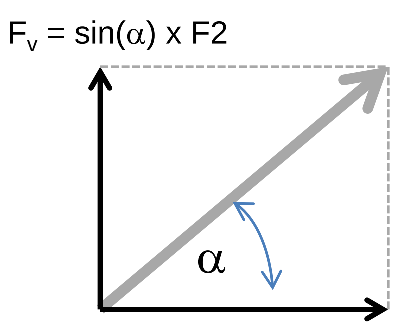
Force1 x Distance 1 =
Force 2 x Distance 2

Distance traveled, not distance from fulcrum

Vectors



- Only force perpendicular to pivot is used for leverage



F2 = Cable Tension