Time-Series Homework

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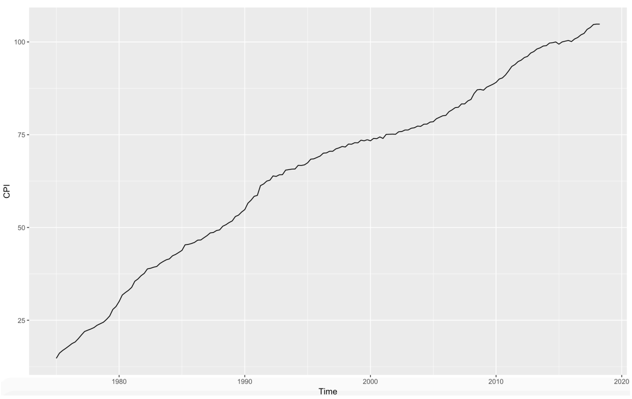
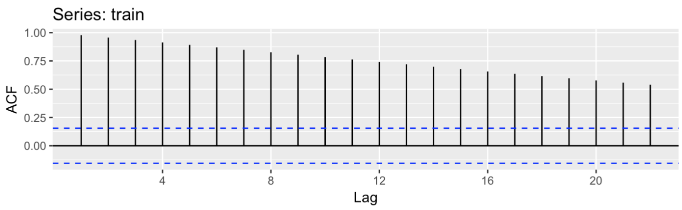
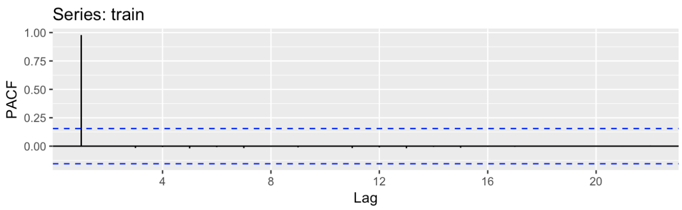
We have taken quarterly data of CPI, GDP and government bond yield spread between 10-year and 3-month government bonds of the United Kingdom from 1975 to 2018 from investing.com and fred.stlouisfed.org. We begin with univariate forecasting, continue with multivariate forecasting and structural analysis.

Univariate Forecasting

For univariate case we decided to forecast CPI. The train of data consists of observations from 1975 to 2015, while test consists of data from 2015 to second quarter of 2018. From the plot of the graph we see that the variance is stabilised during the considered time-frame and therefore we don't need to make Box-Cox transformations.

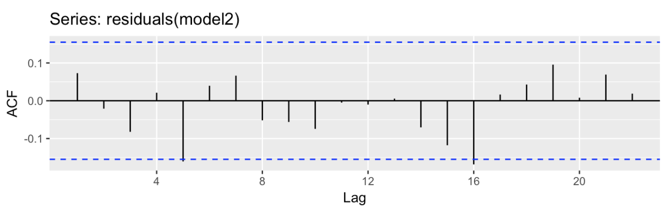


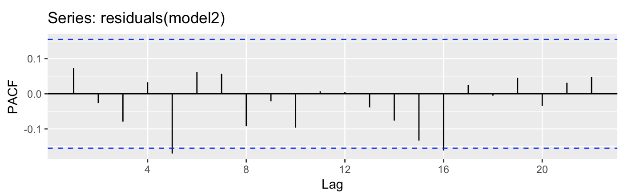




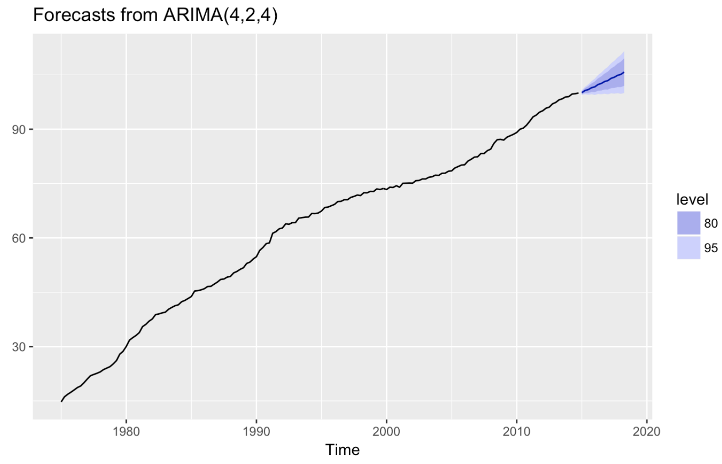
The plot of data clearly indicates that data is non-stationary. ACF of data is slowly decreasing, so we can see that there is a sign of unit root in our series. The PACF also indicates unit root. Augmented Dickey-Fuller test suggests that for our initial time series null hypothesis of non stationarity can not be rejected. We take first difference and we get that for the new series the null hypothesis of non stationarity is rejected towards alternative hypothesis of stationarity at any viable level. We build our model using default function of auto.arima. Auto ARIMA suggests that our data is second order difference stationary. It computes that ARIMA(0,2,1) is the optimal choice for our data. ACF and PACF of residuals suggest that residuals are not White Noise. Box-Ljung test indicates that null hypothesis is rejected and there are no White Noise residuals in our model. In order to check other models, we look at different set of parameters choosing the ones with lowest AIC.

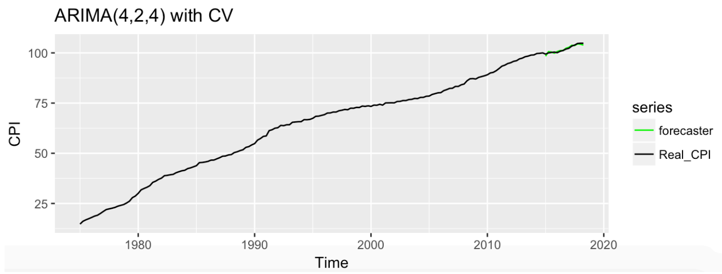
We get that best model is ARIMA(4,2,4), and use it for our further analysis. We also try models ARIMA(4,1,4) and ARIMA(3,2,4) that have AIC close to the best, maybe in test data this models will be better. For ARIMA(4,2,4) we plot ACF and PACF of residuals.





Box-Ljung test indicates that null hypothesis of White Noise residuals can't be rejected and we can move further in our analysis. We build forecast from 2015 and draw confidence intervals for them.





The intervals are big, so we check residuals for normality with tests of Jarque-Bera and Shapiro and get that our residuals are not normal. We also build forecasts with other optimal models and for their case get that residuals are not normal as well. RMSE on test equals to 1 and is higher than on train (0.3). Our model is overfitted, but overall it works good, because of the range of our target variable.

In order to make our model even better we perform cross validation with window 1. Obviously, the results of cross validation are much better because for every period we have the value of the recurrent period. RMSE of CV forecasts is equal to 0.53, so our original opinion was correct. Our final best model is ARIMA(4,2,4) with cross validation.

Multivariate Forecasting

Our Vector Autoregression Model is built to forecast UK GDP, CPI and bond yield spread. Our approach is justified by some theoretical and empirical works that study relationships between GDP and bond yield spreads and GDP and inflation. There are studies reveal that in well-developed financial markets there is a positive statistical significance between the economic growth and the yield spread (Moffatt and Zang, 2012). In addition, it is necessary to consider an effect of CPI on both GDP since the UK implemented inflation targeting in 1992. Finally, GDP can influence inflation (when demand increases, prices grow) and bond yields (through rising expectations about future rates).

We split data to training and test in order to check the quality of our forecasts. We assume that predicting for more that 2 years is unreasonable since too many outside shocks can happen during this period. Thus, our training sample covers the period 1975-2016 Q1, and our test sample begins in 2016 Q2 and finishes in 2018 Q1.
We implement a majority view that all data in VAR must be stationary (Enders, 2015) and take the first difference from all time series (according to ADF test). Therefore, we model differences of GDP, CPI and yield spreads in VAR approach.

ADF test results (p-value)	Initial time-series	First difference	Second difference
GDP	Non-stationary (0.45)	Stationary (0.01)	Stationary (0.01)
CPI	Non-stationary (0.47)	Stationary at 6% (0.054)	Stationary (0.01)
Bond yield spread	Stationary at 5% (0.04)	Stationary (0.01)	Stationary (0.01)

Lag selection procedure gives the following results shown in the table. Since we know that AIC usually overestimates the appropriate number of lags, we start with building VAR(4). To carry out a test for serially correlated errors we use Breusch-Godfrey statistic test and asymptotic Portmanteau statistic since our sample is large enough (165 observations).

AIC(n)	HQ(n)	SC(n)	FPE(n)
5	4	4	5

Then we also test VAR(3) to check whether a more parsimonious model does not have autocorrelated errors.

	Asymptotic Portmanteau p-value	Breusch-Godfrey p-value	Jarque-Bera test result
VAR(4)	0.1	<0.01	Errors are not normal
VAR(3)	<0.01	<0.01	Errors are not normal

In terms of errors correlation, VAR(3) is superior to VAR(4), however, we will study both models as Portmanteau test p-value for VAR(4) is on the conventional boundary. Errors in both models are not normal according to Jarque-Bera test, thus confidence intervals in both models will not be reliable.

Then we build forecasts for both training and test sets using VAR(4) and VAR(3). We seek choosing the model that has better predictive ability and is as parsimonious as possible. We use mean absolute scaled error (MASE) as a quality measure to eliminate scale problems when comparing forecasts for variables of different scale. As a benchmark forecast we use naive forecast for difference of yield spreads and seasonal naive forecast for differences of GDP and CPI since they are affected by some cyclical changes during the year.

MASE	GDP difference		CPI difference		Yield Spread difference	
	Training set	Test set	Training set	Test set	Training set	Test set
VAR(4)	0.5840402	0.4685961	0.6115291	0.2550752	0.8581425	1.5321411
VAR(3)	0.5801015	0.2793518	0.6393092	0.3312091	1.110668	1.194001
Naive	-	-	-	-	1.8146961	0.8352001
Seasonal naive	1	0.47	1	0.3889721	-	-

We understand that our forecast for difference of yield spread is worse than a naive forecast. However, it can be explained by inertia of yields during our test period because yields have been significantly low and stable since the Financial Crisis, making spread difference less volatile. Comparing VAR(4) and VAR(3), GDP forecast of VAR(3) is superior to that of VAR(4), and the forecast for CPI of both models are quite comparable. In addition, in test sample VAR(3) forecasts yield spreads better, thus, as we want to choose the best forecasting parsimonious model, we select VAR(3) as the best for forecasting this system and use it in structural analysis.

Multivariate structural analysis

We assume that the following chain of effects in the given quarter exists:
Bond yield spread difference responds to changes in differences of GDP and CPI, CPI difference responds to changes in GDP difference, GDP difference does not respond to CPI and yield spread changes. This is a rather common assumption in many economic researches, but in our model bond yield spread mimics a role of an interest rate. We consider prices in the UK sticky, thus the difference of CPI is not affected by changes in yield spread difference in the same quarter, but it is surely dependent on current economic activity. Bond yield spread difference is easily affected as it is determined by well-developed UK financial market, and difference of GDP is causally prior to other variables in the system.

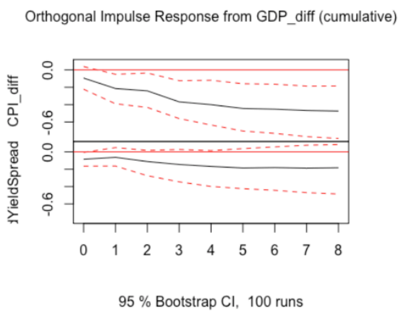
IRF

It is rather difficult to interpret effects of differences on differences but we will try to make this as clear as possible. Also we admit that we have probably captured the true data-generating process, which was impossible without differencing.

Each shock has to be interpreted in the following way: "If the variable X grew by z units in the previous period, then in the following quarter in will increase by z+1 units".

1. Influence of shocks in GDP difference.

IRF demonstrates that shock in GDP difference increases GDP difference in the first quarter by around 1500, and the cumulative effect is a growth in GDP improvement by roughly 3000 during 8 quarters. The increasing growth of GDP in absolute measure can be an indicator of good economic conditions, thus it improves GDP in the following quarters.



One unit shock in GDP difference leads to decrease in CPI difference by 0.6 during 8 quarters. This can be a result of inflation targeting policy of the UK that would try to cool down price growth during the economic boom. Negative cumulative response of spread difference might be due to an increase in short-term rates due to high demand in the debt market.

2. Influence of shocks in CPI difference.

Sharp growth in CPI increase rate leads to a greater rise in CPI in the next quarter. However, absolute growth in other quarters is almost unaffected which can be a sign of efficient inflation targeting policy. Effect of CPI shock on spread movement is rather marginal in the first 2 quarters, but then increases short-term yields and diminishes spread absolute change by 0.2. The central bank barely can control 3-month inflation acceleration which leads to raise in short-term rates, but long-term rates remain almost unaffected. CPI difference a little bit decreases GDP difference, but then GDP pace grows. This effect can be explained by inflation targeting policy in the UK.

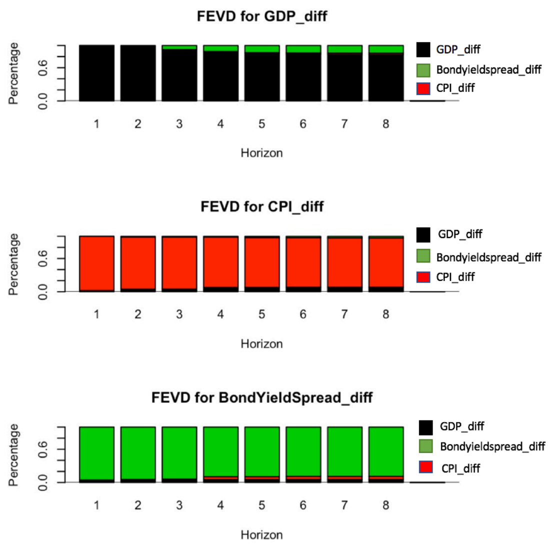
3. Influence of shocks in government bond yield spread difference.

In this case spread difference increases considerably, perhaps, because of growing expectations about long-term rates implied in yield curve.

This shock leads to a small drop in CPI difference path, but in 8 quarters CPI difference cumulative change is about 0.2. Perhaps, an increase in spread dynamics can be associated with higher long-term rates, higher expectations about GDP and thus higher inflation expectations and inflation itself.

This shock almost surely has a negative effect on GDP difference according to our SVAR. This is a typical behaviour as economy struggles when spreads widen due to some uncertainty and increase in relative costs of long-term investments.

FEVD



In long-run (8 quarters) forecast error of difference between quarterly GDPs is significantly affected by shocks in difference between quarterly spreads (14% of error). Obviously, shock in a debt market affects both short-term and long-term investment, therefore, this effect is noticeable. CPI has a minor influence since the UK controls inflation.

Forecast of difference between quarterly CPIs is barely affected by bond market since, again, inflation targeting policy is used. CPI is affected by GDP shocks, but this effect only accelerates in short-term when inflation targeting is difficult.

Forecast of difference between quarterly yield spreads in long-run is evenly affected by GDP and CPI. Thus, market equally considers information about shocks in GDP and price level.

Influence at h = 8	Forecasted Value at h = 8		
	GDP_diff	CPI_diff	Spread_diff
GDP_diff	0.836	0.08	0.054
CPI_diff	0.025	0.89	0.056
Spread_diff	0.138	0.028	0.88