STA 527: Homework 2

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```
library(tidyverse)
```

```
Warning: package 'tidyverse' was built under R version 4.2.3

Warning: package 'tibble' was built under R version 4.2.3

Warning: package 'tidyr' was built under R version 4.2.3

Warning: package 'readr' was built under R version 4.2.3

Warning: package 'purrr' was built under R version 4.2.3

Warning: package 'dplyr' was built under R version 4.2.3

Warning: package 'stringr' was built under R version 4.2.3

Warning: package 'forcats' was built under R version 4.2.3

Warning: package 'lubridate' was built under R version 4.2.3

-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.1     v readr     2.1.4
v forcats   1.0.0     v stringr   1.5.0
v ggplot2   3.4.2     v tibble    3.2.1
v lubridate 1.9.2     v tidyr     1.3.0
v purrr     1.0.1
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to beco
```

Problem 1: Theory

##a

$$
\begin{aligned}
E(L(Y,\hat{Y})|X=x) &= E((Y-\hat{Y})^2|X=x) && (1)\\
&= E(Y^2|X=x) - 2E(Y\hat{Y}|X=x) + E(\hat{Y}^2) && (2)\\
&= Var(Y|X=x) + E(Y|X=x)^2 - 2E(Y|X=x)\hat{Y} + \hat{Y}^2 && (3)\\
&= Var(Y|X=x) + E(Y-\hat{Y}|X=x)^2 \implies && (4)\\
\frac{d}{d\hat{Y}}E(L(Y,\hat{Y})) &= -2(E(Y|X=x) - \hat{Y}) && (5)\\
& && (6)
\end{aligned}
$$

Since the expected risk is a convex function, it has a unique local maximum. Thus, setting the above derivative equal to zero, we get that $g^*(x) = E(Y|X=x)$ ##b

##c

Problem 2: Methodology and Case Study

##a

##b

##c

##d

##e

##f

Problem 3: Simulations

##a

##b