

Summary: Proof that Properly Anticipated Prices Fluctuate Randomly

If there existed absolute knowledge of a future price change then that future price change would already be reflected in the market according to the efficient markets hypothesis as presented by Eugene Fama. This is a foundational belief to claim that price changes exhibit a random walk with no bias. Assuming that no market participant can predict the unforeseeable future would there exist any systematic pattern to how prices change after-the-fact? Does mathematical probability reflect empirical price changes in the market? In his paper "Proof that Proper Anticipated Prices Fluctuate Randomly," Paul Samuelson seeks to explore the mathematical derivation of price changes built on a base model of stochastic price.

In the initial specification of the model Samuelson finds that, on average, there would be no overall drift in prices. This model specification implies that all future knowledge is discounted into the current price $Y(T,t)$, however it does not imply independence from $Y(T+1, t-1)$. Therefore, the price change overtime would not exhibit a Brownian walk, which means it will not wander indefinitely in a positive or negative direction. Rather, price fluctuations in futures markets over a period of time will reconvene to the central anticipated price.

The model is then expanded to include a positive percentage yield that would offset the opportunity cost of a risk free interest rate that would outperform the fair game process. Under this new specification of the model the functional form of X does not change the martingale properties of the resulting Y sequence. Therefore, the expected value of the change in Y would be equal to zero. However, this does not specify that the median value of the change in Y is equal to zero, meaning at any given sub-interval it is possible to see a positive or negative overall trend.

Samuelson concludes that the modeled proof does not prove perfect competition in markets, nor that randomness in price fluctuations has a good or bad market externality. In addition, the probability distributions are not explicitly explained and may empirically be found and applied to obtain a different result. Likewise, the mathematical proof does not seek to claim pareto-optimality or define a representative individual.