

1. The-Axiom-of-Completeness

- 1.1. Initial Definition for \mathbb{R}

Note Information

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- **References:**
 - Abbott, S., Understanding Analysis

Main Content

Main Idea

\mathbb{R} is an ordered field and contains \mathbb{Q} as a subfield.

Explanation

\mathbb{R} is a field, meaning that addition and multiplication of real numbers are commutative, associative, and the distributive property holds. \mathbb{R} also has an order, meaning the following two properties hold:

1. If $x \in \mathbb{R}$ and $y \in \mathbb{R}$, then one and only one of the statements

$$x < y, \quad x = y, \quad y < x$$

is true.

2. If $x, y, z \in \mathbb{R}$, if $x < y$ and $y < z$, then $x < z$.

Finally, \mathbb{R} is a set containing \mathbb{Q} . The operations of addition and multiplication on \mathbb{Q} extend to all of \mathbb{R} in such a way that every element of \mathbb{R} has an additive inverse and every nonzero element of \mathbb{R} has a multiplicative inverse.

Review

1. Define the set of real numbers.

Links to Other Notes

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