1. The-Axiom-of-Completeness
    1.1. Initial Definition for R
    1.2. Axiom of Completeness
    1.3. Upper and Lower Bounds
    1.4. Supremum and Infimum
    1.5. Maximum and Minimum
    1.6. Q and the Axiom of Completeness

Note Information

- **ID:** `202501180703`
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- **References:**
  - Abbott, S., Understanding Analysis

Main Content

Main Idea
R is an ordered field and contains Q as a subfield.

Explanation
R is a field, meaning that addition and multiplication of real numbers are commutative, associative, and the distributive property holds. R also has an order, meaning the following two properties hold:

1. If $x \in R$ and $y \in R$, then one and only one of the statements
$$x < y, \qquad x = y, \qquad y < x$$
is true.
2. If $x, y, z \in R$, if $x < y$ and $y < z$, then $x < z$.

Finally, R is a set containing Q. The operations of addition and multiplication on Q extend to all of R in such a way that every element of R has an additive inverse and every nonzero element of R has a multiplicative inverse.

Review

1. Define the set of real numbers.

Links to Other Notes

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Note Information

- **ID:** `202501180727`
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- **References:**
  - Abbott, S., Understanding Analysis

Main Content

Main Idea
Every nonempty set of real numbers that is bounded above has a least upper bound.

Explanation

Review

1. Define the Axiom of Completeness.

Links to Other Notes

- Initial Definition for R

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Note Information

- **ID:** `202501180734`
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- **References:**
  - Abbott, S., Understanding Analysis

Main Content

Main Idea
A set $A \subset R$ is bounded above if there exists a number $b \in R$ such that $a \leq b$ for all $a \in A$. The number b is called an upper bound for A. Likewise, the set A is bounded below if there exists a lower bound $l \in R$ such that $l \leq a$ for every $a \in A$.

Explanation

Review

1. Define upper and lower bounds.

Links to Other Notes

- Initial Definition for R
- Axiom of Completeness

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Note Information

- **ID:** `202501180743`
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- **References:**
  - Abbott, S., Understanding Analysis

Main Content

Main Idea

A real number s is the least upper bound for a set $A \subset R$ if it meets the following two criteria:

1. s is an upper bound for A;
2. if b is any upper bound for A, then $s \leq b$.

Explanation

The least upper bound is frequently called the supremum of the set A, denoted $s = \sup A$.

Review

1. Define the supremum of a set.
2. Define the infimum, or the greatest lower bound, of a set.
3. Are least upper bounds unique? Explain.
4. Let
$$A = \{\frac{1}{n} : n \in N\} = \{1, \frac{1}{2}, \frac{1}{3}, ...\}.$$
What is $\sup A$ and $\inf A$?

Links to Other Notes

- Initial Definition for R
- Axiom of Completeness
- Upper and Lower Bounds

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Note Information

- **ID:** `202501181241`
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- **References:**
  - Abbott, S., Understanding Analysis

Main Content

Main Idea
A real number a_0 is a maximum of the set A if a_0 is an element of A and $a_0 \geq a$ for all $a \in A$.

Explanation
The supremum can exist and not be a maximum, but when a maximum exists, then it is also the supremum.

Review

1. Define maximum.
2. Define minimum.
3. Consider the open interval
$$(0, 2) = \{x \in R : 0 < x < 2\},$$
and the closed interval
$$[0, 2] = \{x \in R : 0 \leq x \leq 2\}.$$
What are the maximums of the two sets? What are the supremums?

Links to Other Notes

- Initial Definition for R
- Axiom of Completeness
- Upper and Lower Bounds
- Supremum and Infimum

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Note Information

- **ID:** `202501181257`
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- **References:**
  - Abbott, S., Understanding Analysis

Main Content

Main Idea
The Axiom of Completeness is not a valid statement about Q.

Explanation
Consider the set

$$S = \{r \in Q : r^2 < 2\}.$$

This set is certainly bounded above, however, when we search for the least upper bound, we can always find a smaller supremum. For example, we might try $b = 2$, $b = 3/2$, $b = 142/100$, $b = 1415/1000$, and so on.

Review

1. Is the Axiom of Completeness a valid statement about Q? Explain.
2. Does the set

   $$S = \{r \in Q : r^2 < 2\}$$

   have a supremum under R?

Links to Other Notes

- Initial Definition for R
- Axiom of Completeness
- Upper and Lower Bounds
- Supremum and Infimum
- Maximum and Minimum

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