Note Information

- **ID:** 202501120904
- **Timestamp:** Monday 13th January, 2025 08:19
- **Tags:** Tutoring, Chhean, Session-1
- **References:**
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Main Content

Main Idea

Suppose a particle is moving on the x-axis in a simple harmonic motion. Its velocity, in meters per second, at time t, for $0 \leq t \leq 100$ seconds, is given by $v(t) = -\frac{5}{3}\sin(\frac{t}{3})$. The total distance traveled by the particle in the time interval $0 \leq t \leq 21\pi$ seconds is 70 meters.

Explanation

The velocity of the particle is modeled by $v(t) = -\frac{5}{3}\sin(\frac{t}{3})$. The total distance the particle travels in the time interval $0 \leq t \leq 21\pi$ is equal to $\int_0^{21\pi} |v(t)|dt$, where $|v(t)| = \frac{5}{3}|\sin(\frac{t}{3})|$. Since $\sin(\frac{t}{3}) = 0$ when $t = 3n\pi$ for all integers n, the velocity function maintains its sign throughout the interval $[3n\pi, 3(n+1)\pi]$. The period for the velocity function is 6π, thus twice the aforementioned interval is equal to the full period. This relationship can be modeled through the following expressions:

$$\begin{aligned}
\frac{5}{3}\int_0^{6\pi} |\sin(\frac{t}{3})|dt &= \frac{5}{3} \cdot 2\int_0^{3\pi} \sin(\frac{t}{3})dt \\
&= \frac{5}{3} \cdot 2[-3\cos(\frac{t}{3})]_0^(3\pi) \\
&= \frac{5}{3} \cdot 2[6] \\
&= 20
\end{aligned}$$

The interval from 0 to 21π is equal to 3.5 periods. Therefore, the total distance traveled by the particle is equal to

$$3 \cdot 20 + \frac{5}{3} \cdot 6 = 70 \text{ meters}$$

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- **ID:** `202501121340`
- **Timestamp:** `Monday 13th January, 2025 08:19`
- **Tags:** `Tutoring, Chhean, Session-1`
- **References:**
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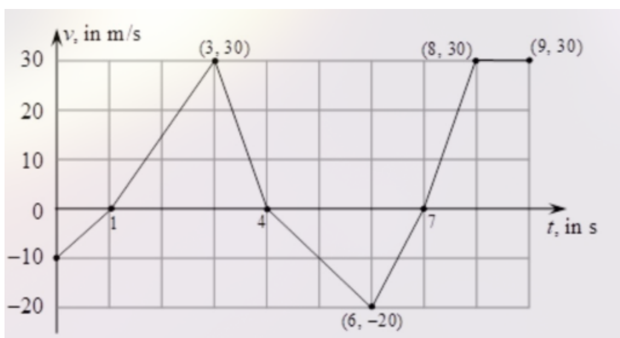
Main Content

Main Idea

Suppose a particle is moving on the x-axis between the time $t = 0$ seconds and $t = 9$ seconds. Its initial position at $t = 0$ seconds is $x(0) = 2$ meters. The velocity-time graph of the motion is shown below.



On the x-axis, the abscissa of the farthest point to the right of the origin that the particle reaches over the time interval $0 \le t \le 9$ seconds is 42 meters.

Explanation

By looking at the graph, we notice that the velocity is positive on the intervals $1 \le t \le 4$ and $7 \le t \le 9$. The velocity is negative on the intervals $0 \le t \le 1$ and $4 \le t \le 7$. The area under the curve between $t = 1$ and $t = 4$ is calculated by finding the area of the triangle with base 3 and height 30:

$$A_1 = \frac{1}{2} \cdot 3 \cdot 30 = 45$$

The area under the curve for the other three intervals are computed in a similar way below:

$$A_2 = \frac{1}{2} \cdot 1 \cdot 30 + 1 \cdot 30 = 30$$
$$A_3 = \frac{1}{2} \cdot 1 \cdot 10 = 5$$
$$A_4 = \frac{1}{2} \cdot 3 \cdot 20 = 30$$

Using the areas above, we can compute $x(t)$ at key times, using the fact that $x(t) = x(0) + \int_0^t v(t)dt$:
At $t = 1$:

$$x(1) = x(0) - A_3 = 2 - 5 = -3$$

At $t = 4$:

$$x(4) = x(1) + A_1 = -3 + 45 = 42$$

At $t = 7$:

$$x(7) = x(4) - A_4 = 42 - 30 = 12$$

At $t = 9$:

$$x(9) = x(7) - A_2 = 12 + 30 = 42$$

The particle is farthest to the right at $t = 4$ and $t = 9$, where:

$$x_{max} = 42 \text{ meters}$$

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- **ID:** `202501121522`
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- **References:**
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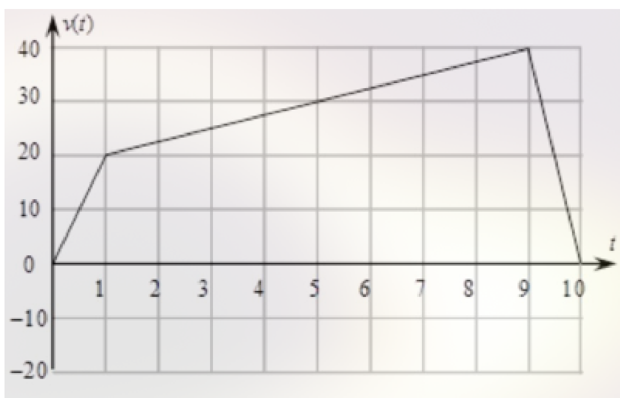
Main Content

Main Idea
The velocity-time graph of a particle moving along a straight line is shown below. Time t is measured in minutes, and the velocity $v(t)$ is measured in meters per minute.



If the average acceleration of the particle in the time interval $1 \leq t \leq 9$ minutes is k m/min^2, the value of k is .
Explanation
The average acceleartion can be calculated using the following formula:

$$k = \frac{\delta v}{\delta t}$$

According to the graph, $\delta v = v(9) - v(1) = 40 - 20 = 20$. Thus,

$$k = \frac{20}{8} = \frac{5}{2} = 2.5 \text{ m / min}^2$$

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- **ID:** 202501121636
- **Timestamp:** Monday 13^{th} January, 2025 08:19
- **Tags:** Tutoring, Chhean, Session-1
- **References:**
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Main Content

Main Idea
Suppose a cylindrical reservoir of radius 4 meters is being filled with water. The rate at which the level of water is increasing is given by $u(t) = 8 - 2\sin(\frac{\pi t}{18})$ meter per minute. The rate at which the volume of water in the reservoir is increasing at $t = 9$ minutes is π cubic meters per minute.

Explanation
The volume of water in a cylindrical reservoir is given by:

$$V = \pi r^2 h,$$

where $r = 4$ and h is the water level in the reservoir. The rate of change of the volume is related to the rate of change of the water level by:

$$\begin{aligned}\frac{dV}{dt} &= \pi r^2 \frac{dh}{dt} \\ &= 16\pi(8 - 2\sin(\frac{\pi t}{18}))\end{aligned}$$

At $t = 9$:

$$\begin{aligned}\frac{dV}{dt} &= 16\pi(8 - 2\sin(\frac{\pi(9)}{18})) \\ &= 16\pi(6) \\ &= 96\pi \text{ cubic meters per minute}\end{aligned}$$

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- **ID:** 202501121712
- **Timestamp:** Monday 13th January, 2025 08:19
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- **References:**
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Main Content

Main Idea
Suppose, for $0 \leq t \leq 20$ hours, water is being pumped into a reservoir at the rate of $R(t) = \frac{1}{3}t^2 - t + 2$ cubic meters per hour and removed from the reservoir at the rate of $r(t) = 2t + 14$ cubic meters per hour. If the amount of water in the reservoir is increasing during the time interval $(a, 20)$ and decreasing elsewhere, the value of a is 12.

Explanation
To find the time interval in which the amount of water in the reservoir is increasing, we set up the following inequality:

$$R(t) - r(t) > 0.$$

We only care about the critical point in which the net rate of change switches sign, thus we need to solve:

$$R(t) - r(t) = 0.$$

The net rate of change is equal to $\frac{1}{3}t^2 - 3t - 12$. Setting this expression equal to 0 and solving gives us:

$$\begin{aligned}
\frac{1}{3}t^2 - 3t - 12 &= 0 \\
t^2 - 9t - 36 &= 0 \\
(t-12)(t+3) &= 0 \\
t &= 12, -3
\end{aligned}$$

Since $t = -3$ is not within the inteval $0 \leq t \leq 20$, $a = 12$.

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- **ID:** 202501121828
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- **References:**
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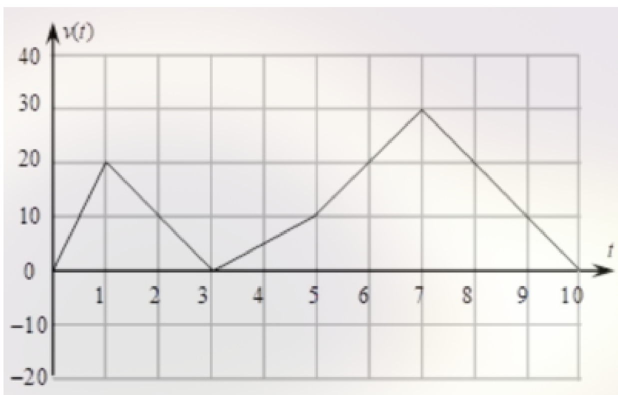
Main Content

Main Idea

The velocity-time graph of a particle moving on a straight line is shown below. Time t is measured in seconds, and the velocity $v(t)$ is measured in meters per second.



If at $t = 0$ seconds, the position of the particle is at 20 meters, then at $t = 10$ seconds, its position is at 145 meters.

Explanation

The velocity-time graph can be divided into 5 sections, comprised of 4 triangles and 1 rectangle. To begin, calculate the areas of these 5 sections:

$$A_1 = \frac{1}{2} \cdot 3 \cdot 20 = 30$$
$$A_2 = \frac{1}{2} \cdot 2 \cdot 10 = 10$$
$$A_3 = \frac{1}{2} \cdot 4 \cdot 20 = 40$$
$$A_4 = \frac{1}{2} \cdot 1 \cdot 10 = 5$$
$$A_5 = 4 \cdot 10 = 40$$

The total distance can be modeled by the following expression:

$$x(0) + \int_0^t v(t)dt.$$

Given that $x(0) = 20$ and $\int_0^t v(t)dt = A_1 + A_2 + A_3 + A_4 + A_5 = 125$, at $t = 10$:

$$x(10) = 20 + 125 = 145 \text{ meters.}$$

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- **ID:** 202501121941
- **Timestamp:** Monday 13th January, 2025 08:19
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Main Content

Main Idea
Suppose a chocolate factory opens daily at 9:00 am. The factory processes milk at the rate $p(t) = t - \frac{9}{64}t^2$ cubic meters per hour for $0 \leq t \leq 8$ hours, where $t = 0$ corresponds to 9:00 am. On a given day, the factory receives milk at the rate

$$q(t) = \begin{cases} 3 \text{ for } 0 \leq t \leq 4 \\ 0 \text{ for } 4 < t \leq 8 \end{cases} \quad \text{cubic meters per hour.}$$

If on the given day, the factory starts with 32 cubic meters of unprocessed milk in stock, then by the end of the day, the factory would be left with cubic meters of unprocessed milk.

Explanation
To find the total milk received, we integrate $q(t)$ over the interval $[0, 8]$:

$$\begin{aligned} \int_0^8 q(t)dt &= \int_0^4 3dt + \int_4^8 0dt \\ &= [3t]_0^4 + 0 \\ &= 12 \end{aligned}$$

Next, we find the total milk processed by integrating $p(t)$ over the interval $[0, 8]$:

$$\begin{aligned} \int_0^8 p(t)dt &= \int_0^8 tdt - \int_0^8 \frac{9}{64}t^2dt \\ &= [\frac{t^2}{2}]_0^8 - [\frac{3t^3}{64}]_0^8 \\ &= 32 - 24 \\ &= 8 \end{aligned}$$

Finally, we compute the net change and add it to the initial stock:

$$12 - 8 = 4$$

$$32 + 4 = 36 \text{ cubic meters}$$

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