1. Limits
    1.1. Motivation for Limits
    1.2. Moving Closer and Closer
    1.3. One-sided Limits

Note Information

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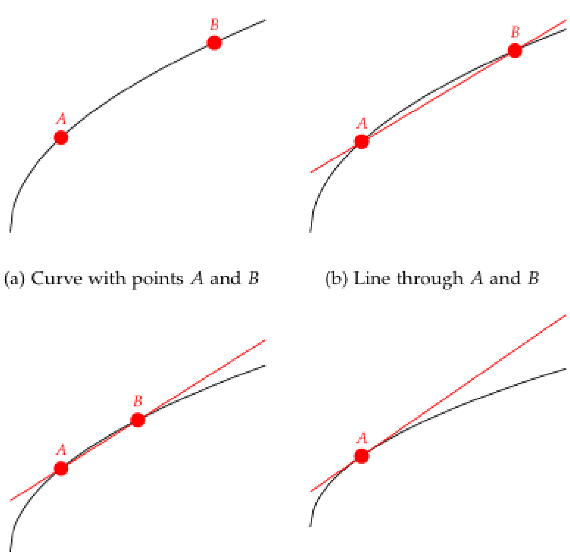
- **References:**
  - Calculus I: Single Variable Calculus

Main Content

Main Idea
Limits are foundational to the study of derivatives and integrals, the two main concepts in Calculus.

Explanation
Imagine a curve, with two points labeled A and B (a). Let there be a line connecting the two points (b). As B approaches A (c), the line becomes tangent to the curve at point A (d). The slope of this tangent line represents the derivative at A.



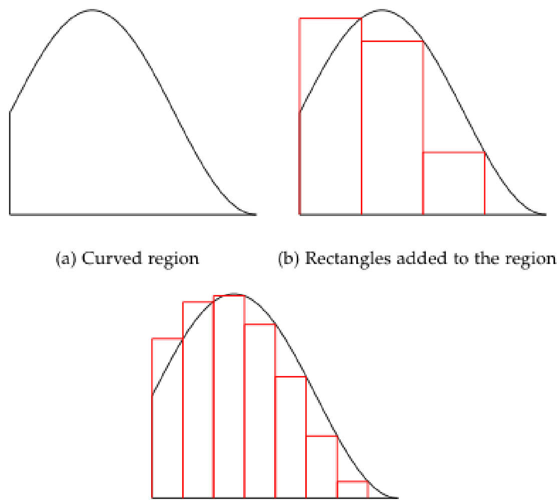
(a) Curve with points A and B

(b) Line through A and B

(c) A approaching B

(d) Tangent line of the curve at point A

Now, imagine a curvy region, and suppose we would like to measure its area (a). Let us start by filling the region with rectangles, since the area of rectangles is easy to measure (b). As the width of each rectangle gets smaller, the total area of the rectangles gets closer to the area of the curvy region (c). Thus, the integral, which measures the area of a curvy region, is the limit of the total area of the rectangles as the width approaches 0.



(a) Curved region (b) Rectangles added to the region

(c) More rectangles added to the region

Review

1. Describe the motivation behind studying limits.

Links to Other Notes

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Main Content

Main Idea
Let $f(x) = \frac{\sqrt{3-5x+x^2+x^3}}{x-1}$. As x approaches 1 from the left, $f(x)$ approaches -2.

Explanation
To determine what $f(x)$ approaches as x approaches 1 from the left, we must plug values into $f(x)$ that are slightly less than 1 and determine its output. For example, when $x = 0.9$, $f(x) = -1.97$. When $x = 0.99$, $f(x) = -1.99$. Finally, when $x = 0.999$, $f(x) = -2.00$.

Review

1. What does $f(x)$ approach as x approaches 1 from the right? Draw the graph of $f(x)$.

Links to Other Notes

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Main Content

Main Idea

The left-sided limit is defined as the value $f(x)$ approaches as x approaches a from the left:

$$\lim_{x \to a^-} f(x).$$

The right-sided limit is defined as the value $f(x)$ approaches as x approaches a from the right:

$$\lim_{x \to a^+} f(x).$$

Explanation

Let $g(x) = \frac{x}{\tan(2x)}$. To find $\lim_{x \to 0^-} g(x)$, we must plug values slightly less than 0 into $g(x)$:

$$\begin{aligned} g(-0.1) &= 0.493 \\ g(-0.01) &= 0.499 \\ g(-0.001) &= 0.500 \end{aligned}$$

Thus, $\lim_{x \to 0^-} g(x) = 0.50$.

Review

1. Define the left-sided limit.
2. Define the right-sided limit.
3. Let $h(x) = \frac{|x| + \sin x}{x^2}$. Find $\lim_{x \to 0^+} h(x)$.
4. Let $j(x) = \sin(13/x)$. Find $\lim_{x \to 0^+} j(x)$.

Links to Other Notes

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- Moving Closer and Closer

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