

1. The-Axiom-of-Completeness
 - 1.1. Initial Definition for \mathbb{R}
 - 1.2. Axiom of Completeness
 - 1.3. Upper and Lower Bounds
 - 1.4. Supremum and Infimum
 - 1.5. Maximum and Minimum

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- **References:**
 - Abbott, S., Understanding Analysis

Main Content

Main Idea

\mathbb{R} is an ordered field and contains \mathbb{Q} as a subfield.

Explanation

\mathbb{R} is a field, meaning that addition and multiplication of real numbers are commutative, associative, and the distributive property holds. \mathbb{R} also has an order, meaning the following two properties hold:

1. If $x \in \mathbb{R}$ and $y \in \mathbb{R}$, then one and only one of the statements

$$x < y, \quad x = y, \quad y < x$$

is true.

2. If $x, y, z \in \mathbb{R}$, if $x < y$ and $y < z$, then $x < z$.

Finally, \mathbb{R} is a set containing \mathbb{Q} . The operations of addition and multiplication on \mathbb{Q} extend to all of \mathbb{R} in such a way that every element of \mathbb{R} has an additive inverse and every nonzero element of \mathbb{R} has a multiplicative inverse.

Review

1. Define the set of real numbers.

Links to Other Notes

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Main Content

Main Idea

Every nonempty set of real numbers that is bounded above has a least upper bound.

Explanation

Review

1. Define the Axiom of Completeness.

Links to Other Notes

- Initial Definition for \mathbb{R}

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Main Content

Main Idea

A set $A \subset \mathbb{R}$ is bounded above if there exists a number $b \in \mathbb{R}$ such that $a \leq b$ for all $a \in A$. The number b is called an upper bound for A . Likewise, the set A is bounded below if there exists a lower bound $l \in \mathbb{R}$ such that $l \leq a$ for every $a \in A$.

Explanation

Review

1. Define upper and lower bounds.

Links to Other Notes

- Initial Definition for \mathbb{R}
- Axiom of Completeness

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Main Content

Main Idea

A real number s is the least upper bound for a set $A \subset \mathbb{R}$ if it meets the following two criteria:

1. s is an upper bound for A ;
2. if b is any upper bound for A , then $s \leq b$.

Explanation

The least upper bound is frequently called the supremum of the set A , denoted $s = \sup A$.

Review

1. Define the supremum of a set.
2. Define the infimum, or the greatest lower bound, of a set.
3. Are least upper bounds unique? Explain.
4. Let

$$A = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\} = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \dots \right\}.$$

What is $\sup A$ and $\inf A$?

Links to Other Notes

- Initial Definition for \mathbb{R}
- Axiom of Completeness
- Upper and Lower Bounds

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Main Content

Main Idea

A real number a_0 is a maximum of the set A if a_0 is an element of A and $a_0 \geq a$ for all $a \in A$.

Explanation

The supremum can exist and not be a maximum, but when a maximum exists, then it is also the supremum.

Review

1. Define maximum.
2. Define minimum.
3. Consider the open interval

$$(0, 2) = \{x \in \mathbb{R} : 0 < x < 2\},$$

and the closed interval

$$[0, 2] = \{x \in \mathbb{R} : 0 \leq x \leq 2\}.$$

What are the maximums of the two sets? What are the supremums?

Links to Other Notes

- Initial Definition for \mathbb{R}
- Axiom of Completeness
- Upper and Lower Bounds
- Supremum and Infimum

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