Note Information

- **ID:** 202501120904
- **Timestamp:** Sunday 12th January, 2025 15:11
- **Tags:** Tutoring, Chhean, Session-1
- **References:**
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Main Content

Main Idea

Suppose a particle is moving on the x-axis in a simple harmonic motion. Its velocity, in meters per second, at time t, for $0 \leq t \leq 100$ seconds, is given by $v(t) = -\frac{5}{3}\sin(\frac{t}{3})$. The total distance traveled by the particle in the time interval $0 \leq t \leq 21\pi$ seconds is 70 meters.

Explanation

The velocity of the particle is modeled by $v(t) = -\frac{5}{3}\sin(\frac{t}{3})$. The total distance the particle travels in the time interval $0 \leq t \leq 21\pi$ is equal to $\int_0^{21\pi} |v(t)|dt$, where $|v(t)| = \frac{5}{3}|\sin(\frac{t}{3})|$. Since $\sin(\frac{t}{3}) = 0$ when $t = 3n\pi$ for all integers n, the velocity function maintains its sign throughout the interval $[3n\pi, 3(n+1)\pi]$. The period for the velocity function is 6π, thus twice the aforementioned interval is equal to the full period. This relationship can be modeled through the following expressions:

$$\begin{aligned}\frac{5}{3}\int_0^{6\pi} |\sin(\frac{t}{3})|dt &= \frac{5}{3}\cdot 2\int_0^{3\pi} \sin(\frac{t}{3})dt \\ &= \frac{5}{3}\cdot 2[-3\cos(\frac{t}{3})]_0^(3\pi) \\ &= \frac{5}{3}\cdot 2[6] \\ &= 20\end{aligned}$$

The interval from 0 to 21π is equal to 3.5 periods. Therefore, the total distance traveled by the particle is equal to

$$3 \cdot 20 + \frac{5}{3}\cdot 6 = 70 \text{ meters}$$

Review

1.

Links to Other Notes

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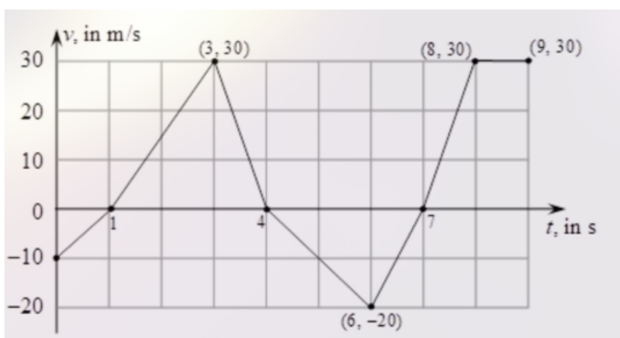
Main Content

Main Idea
Suppose a particle is moving on the x-axis between the time $t = 0$ seconds and $t = 9$ seconds. Its initial position at $t = 0$ seconds is $x(0) = 2$ meters. The velocity-time graph of the motion is shown below.



On the x-axis, the abscissa of the farthest point to the right of the origin that the particle reaches over the time interval $0 \leq t \leq 9$ seconds is 42 meters. **Explanation**
By looking at the graph, we notice that the velocity is positive on the intervals $1 \leq t \leq 4$ and $7 \leq t \leq 9$. The velocity is negative on the intervals $0 \leq t \leq 1$ and $4 \leq t \leq 7$. The area under the curve between $t = 1$ and $t = 4$ is calculated by finding the area of the triangle with base 3 and height 30:

$$A_1 = \frac{1}{2} \cdot 3 \cdot 30 = 45$$

The area under the curve for the other three intervals are computed in a similar way below:

$$A_2 = \frac{1}{2} \cdot 1 \cdot 30 + 1 \cdot 30 = 30$$
$$A_3 = \frac{1}{2} \cdot 1 \cdot 10 = 5$$
$$A_4 = \frac{1}{2} \cdot 3 \cdot 20 = 30$$

Using the areas above, we can compute $x(t)$ at key times, using the fact that $x(t) = x(0) + \int_0^t v(t)dt$:
At $t = 1$:

$$x(1) = x(0) - A_3 = 2 - 5 = -3$$

At $t = 4$:

$$x(4) = x(1) + A_1 = -3 + 45 = 42$$

At $t = 7$:

$$x(7) = x(4) - A_4 = 42 - 30 = 12$$

At $t = 9$:

$$x(9) = x(7) - A_2 = 12 + 30 = 42$$

The particle is farthest to the right at $t = 4$ and $t = 9$, where:

$$x_{max} = 42 \text{ meters}$$

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