

1. The-Axiom-of-Completeness
 - 1.1. Initial Definition for \mathbb{R}
 - 1.2. Axiom of Completeness
 - 1.3. Upper and Lower Bounds

Note Information

- **ID:** 202501180703
- **Timestamp:** Saturday 18th January, 2025 07:41
- **Tags:** Mathematics, Analysis-I-Abbott, The-Axiom-of-Completeness
- **References:**
 - Abbott, S., Understanding Analysis

Main Content

Main Idea

\mathbb{R} is an ordered field and contains \mathbb{Q} as a subfield.

Explanation

\mathbb{R} is a field, meaning that addition and multiplication of real numbers are commutative, associative, and the distributive property holds. \mathbb{R} also has an order, meaning the following two properties hold:

1. If $x \in \mathbb{R}$ and $y \in \mathbb{R}$, then one and only one of the statements

$$x < y, \quad x = y, \quad y < x$$

is true.

2. If $x, y, z \in \mathbb{R}$, if $x < y$ and $y < z$, then $x < z$.

Finally, \mathbb{R} is a set containing \mathbb{Q} . The operations of addition and multiplication on \mathbb{Q} extend to all of \mathbb{R} in such a way that every element of \mathbb{R} has an additive inverse and every nonzero element of \mathbb{R} has a multiplicative inverse.

Review

1. Define the set of real numbers.

Links to Other Notes

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Main Content

Main Idea

Every nonempty set of real numbers that is bounded above has a least upper bound.

Explanation

Review

1. Define the Axiom of Completeness.

Links to Other Notes

- Initial Definition for \mathbb{R}

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Main Content

Main Idea

A set $A \subset \mathbb{R}$ is bounded above if there exists a number $b \in \mathbb{R}$ such that $a \leq b$ for all $a \in A$. The number b is called an upper bound for A . Likewise, the set A is bounded below if there exists a lower bound $l \in \mathbb{R}$ such that $l \leq a$ for every $a \in A$.

Explanation

Review

1. Define upper and lower bounds.

Links to Other Notes

- Initial Definition for \mathbb{R}
- Axiom of Completeness

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