Note Information

- **ID:** `202501120904`
- **Timestamp:** `Sunday 12th January, 2025 17:25`
- **Tags:** `Tutoring, Chhean, Session-1`
- **References:**
  -

Main Content

Main Idea

Suppose a particle is moving on the x-axis in a simple harmonic motion. Its velocity, in meters per second, at time t, for $0 \leq t \leq 100$ seconds, is given by $v(t) = -\frac{5}{3}\sin(\frac{t}{3})$. The total distance traveled by the particle in the time interval $0 \leq t \leq 21\pi$ seconds is 70 meters.

Explanation

The velocity of the particle is modeled by $v(t) = -\frac{5}{3}\sin(\frac{t}{3})$. The total distance the particle travels in the time interval $0 \leq t \leq 21\pi$ is equal to $\int_0^{21\pi} |v(t)|dt$, where $|v(t)| = \frac{5}{3}|\sin(\frac{t}{3})|$. Since $\sin(\frac{t}{3}) = 0$ when $t = 3n\pi$ for all integers n, the velocity function maintains its sign throughout the interval $[3n\pi, 3(n+1)\pi]$. The period for the velocity function is 6π, thus twice the aforementioned interval is equal to the full period. This relationship can be modeled through the following expressions:

$$\begin{aligned}
\frac{5}{3}\int_0^{6\pi} |\sin(\frac{t}{3})|dt &= \frac{5}{3} \cdot 2\int_0^{3\pi} \sin(\frac{t}{3})dt \\
&= \frac{5}{3} \cdot 2[-3\cos(\frac{t}{3})]_0^(3\pi) \\
&= \frac{5}{3} \cdot 2[6] \\
&= 20
\end{aligned}$$

The interval from 0 to 21π is equal to 3.5 periods. Therefore, the total distance traveled by the particle is equal to

$$3 \cdot 20 + \frac{5}{3} \cdot 6 = 70 \text{ meters}$$

Review

1.

Links to Other Notes

-

Table of Contents

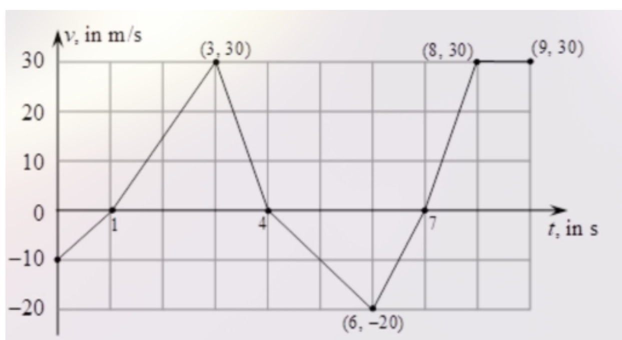
Note Information

- **ID:** `202501121340`
- **Timestamp:** `Sunday 12th January, 2025 17:25`
- **Tags:** `Tutoring, Chhean, Session-1`
- **References:**
  -

Main Content

Main Idea

Suppose a particle is moving on the x-axis between the time $t = 0$ seconds and $t = 9$ seconds. Its initial position at $t = 0$ seconds is $x(0) = 2$ meters. The velocity-time graph of the motion is shown below.



On the x-axis, the abscissa of the farthest point to the right of the origin that the particle reaches over the time interval $0 \leq t \leq 9$ seconds is 42 meters.

Explanation

By looking at the graph, we notice that the velocity is positive on the intervals $1 \leq t \leq 4$ and $7 \leq t \leq 9$. The velocity is negative on the intervals $0 \leq t \leq 1$ and $4 \leq t \leq 7$. The area under the curve between $t = 1$ and $t = 4$ is calculated by finding the area of the triangle with base 3 and height 30:

$$A_1 = \frac{1}{2} \cdot 3 \cdot 30 = 45$$

The area under the curve for the other three intervals are computed in a similar way below:

$$A_2 = \frac{1}{2} \cdot 1 \cdot 30 + 1 \cdot 30 = 30$$
$$A_3 = \frac{1}{2} \cdot 1 \cdot 10 = 5$$
$$A_4 = \frac{1}{2} \cdot 3 \cdot 20 = 30$$

Using the areas above, we can compute $x(t)$ at key times, using the fact that $x(t) = x(0) + \int_0^t v(t)dt$:
At $t = 1$:

$$x(1) = x(0) - A_3 = 2 - 5 = -3$$

At $t = 4$:

$$x(4) = x(1) + A_1 = -3 + 45 = 42$$

At $t = 7$:

$$x(7) = x(4) - A_4 = 42 - 30 = 12$$

At $t = 9$:

$$x(9) = x(7) - A_2 = 12 + 30 = 42$$

The particle is farthest to the right at $t = 4$ and $t = 9$, where:

$$x_{max} = 42 \text{ meters}$$

Review

1.

Links to Other Notes

-

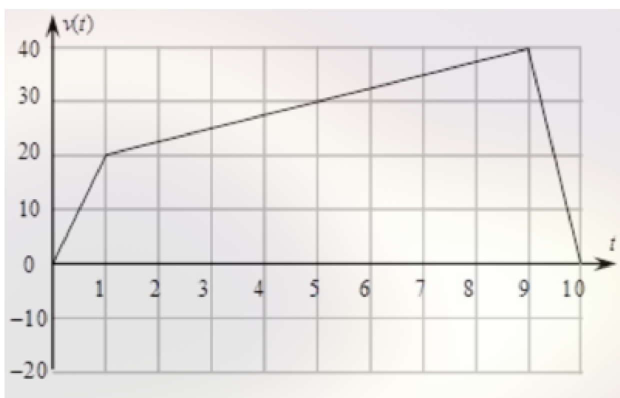
Table of Contents

Note Information

- **ID:** `202501121522`
- **Timestamp:** `Sunday 12th January, 2025 17:25`
- **Tags:** `Tutoring, Chhean, Session-1`
- **References:**
  -

Main Content

Main Idea
The velocity-time graph of a particle moving along a straight line is shown below. Time t is measured in minutes, and the velocity $v(t)$ is measured in meters per minute.



If the average acceleration of the particle in the time interval $1 \leq t \leq 9$ minutes is k m/min^2, the value of k is .
Explanation
The average acceleartion can be calculated using the following formula:

$$k = \frac{\delta v}{\delta t}$$

According to the graph, $\delta v = v(9) - v(1) = 40 - 20 = 20$. Thus,

$$k = \frac{20}{8} = \frac{5}{2} = 2.5 \text{ m / min}^2$$

Review

1.

Links to Other Notes

-

Table of Contents

Note Information

- **ID:** `202501121636`
- **Timestamp:** `Sunday 12th January, 2025 17:25`
- **Tags:** `Tutoring, Chhean, Session-1`
- **References:**
    -

Main Content

Main Idea
Suppose a cylindrical reservoir of radius 4 meters is being filled with water. The rate at which the level of water is increasing is given by $u(t) = 8 - 2\sin(\frac{\pi t}{18})$ meter per minute. The rate at which the volume of water in the reservoir is increasing at $t = 9$ minutes is π cubic meters per minute.

Explanation
The volume of water in a cylindrical reservoir is given by:

$$V = \pi r^2 h,$$

where $r = 4$ and h is the water level in the reservoir. The rate of change of the volume is related to the rate of change of the water level by:

$$\begin{aligned}\frac{dV}{dt} &= \pi r^2 \frac{dh}{dt} \\ &= 16\pi(8 - 2\sin(\frac{\pi t}{18}))\end{aligned}$$

At $t = 9$:

$$\begin{aligned}\frac{dV}{dt} &= 16\pi(8 - 2\sin(\frac{\pi(9)}{18})) \\ &= 16\pi(6) \\ &= 96\pi \text{ cubic meters per minute}\end{aligned}$$

Review

1.

Links to Other Notes

-

Table of Contents

Note Information

- **ID:** 202501121712
- **Timestamp:** Sunday 12$^{\text{th}}$ January, 2025 17:25
- **Tags:** Tutoring, Chhean, Session-1
- **References:**
  –

Main Content

Main Idea
Suppose, for $0 \leq t \leq 20$ hours, water is being pumped into a reservoir at the rate of $R(t) = \frac{1}{3}t^2 - t + 2$ cubic meters per hour and removed from the reservoir at the rate of $r(t) = 2t + 14$ cubic meters per hour. If the amount of water in the reservoir is increasing during the time interval $(a, 20)$ and decreasing elsewhere, the value of a is 12.

Explanation
To find the time interval in which the amount of water in the reservoir is increasing, we set up the following inequality:

$$R(t) - r(t) > 0.$$

We only care about the critical point in which the net rate of change switches sign, thus we need to solve:

$$R(t) - r(t) = 0.$$

The net rate of change is equal to $\frac{1}{3}t^2 - 3t - 12$. Setting this expression equal to 0 and solving gives us:

$$\begin{aligned} \frac{1}{3}t^2 - 3t - 12 &= 0 \\ t^2 - 9t - 36 &= 0 \\ (t-12)(t+3) &= 0 \\ t &= 12, -3 \end{aligned}$$

Since $t = -3$ is not within the inteval $0 \leq t \leq 20$, $a = 12$.

Review

1.

Links to Other Notes

-

Table of Contents

- TOC