Basic Mathematics [1]

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Chapter 1: Numbers

Section 1.1: The Integers

Section 1.2: Rules for Addition

Practice

Justify each step, using commutativity and associativity in proving the following identities.
1. $(a+b)+(c+d)=(a+d)+(b+c)$
Answer:

$$\begin{aligned}(a+b)+(c+d) &= a+b+c+d && \text{by associativity}\\ &= a+d+b+c && \text{by commutativity}\\ &= (a+d)+(b+c) && \text{by associativity}\end{aligned}$$

3. $(a-b)+(c-d)=(a+c)+(-b-d)$
Answer:

$$\begin{aligned}(a-b)+(c-d) &= a-b+c-d && \text{by associativity}\\ &= a+c-b-d && \text{by commutativity}\\ &= (a+c)+(-b-d) && \text{by associativity}\end{aligned}$$

5. $(a-b)+(c-d)=(a-d)+(c-b)$
Answer:

$$\begin{aligned}(a-b)+(c-d) &= a-b+c-d && \text{by associativity}\\ &= a-d+c-b && \text{by commutativity}\\ &= (a-d)+(c-b) && \text{by associativity}\end{aligned}$$

7. $(a-b)+(c-d)=-(b+d)-(-a-c)$
Answer:

$$\begin{aligned}(a-b)+(c-d) &= a-b+c-d && \text{by associativity}\\ &= -b-d+a+c && \text{by commutativity}\\ &= -(b+d)-(-a-c)\end{aligned}$$

References

[1] Serge Lang. *Basic Mathematics*. Addison-Wesley, 1971.