Epidemic Networks in R

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Learning Objectives

1. Understand SI/SIR modeling, and the ways transmission and recovery rates affect epidemic dynamics

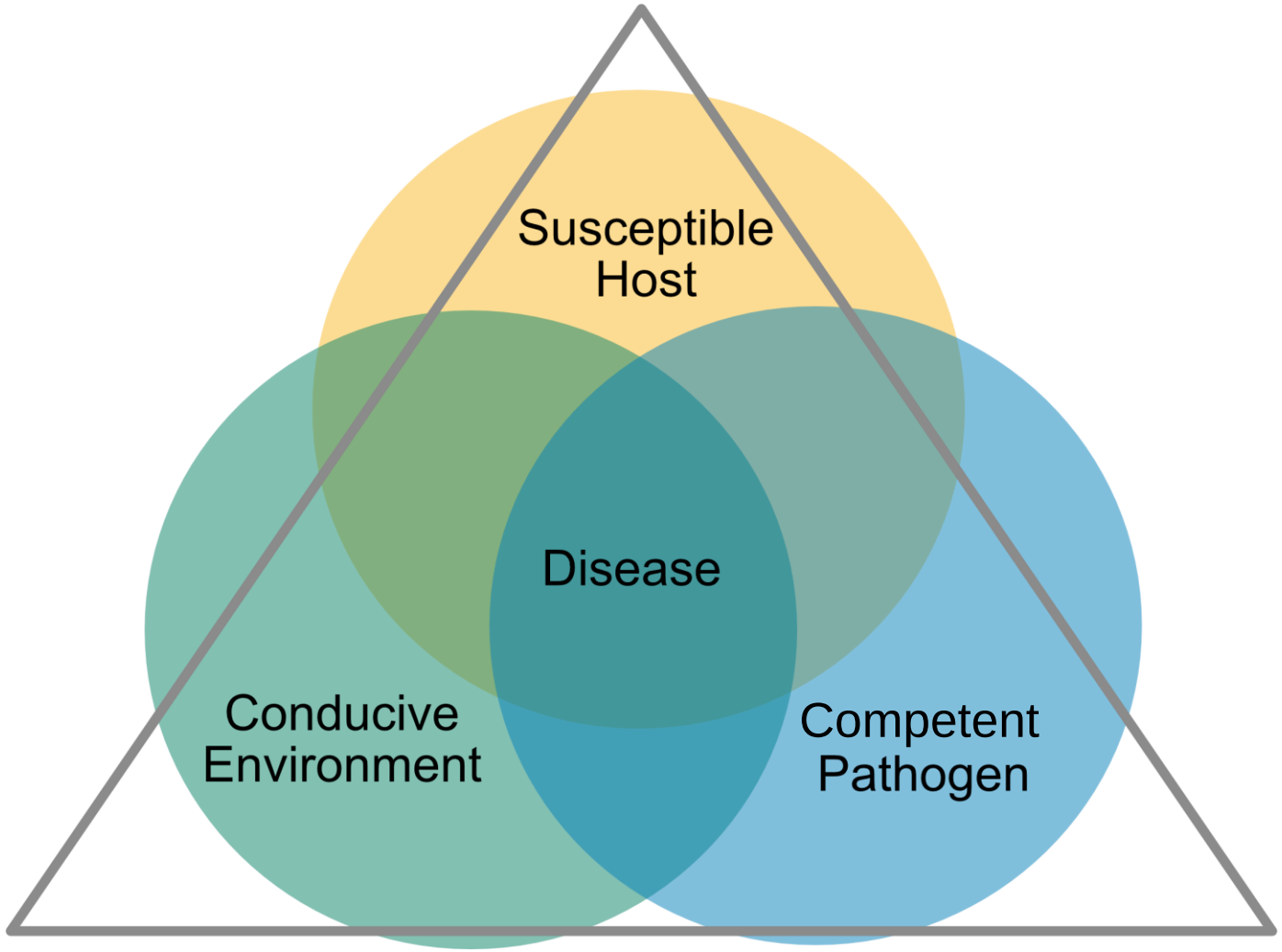
2. Describe the differences between Random, Small-world, and Scale-free networks

3. Be able to simulate epidemic expansion across a network in R

Epidemiology

The study of the occurrence, distribution, and control of a disease and a population

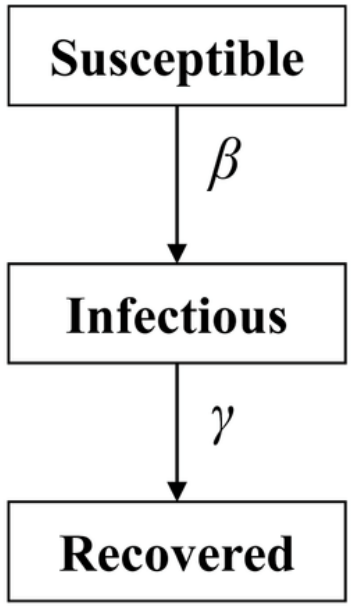
1. Susceptible Host
2. Conducive Environment
3. Pathogen Organism
4. Time
5. People



Disease Triangle

SI Modeling

SIR Modeling



$$\frac{dS}{dt} = -\beta SI$$

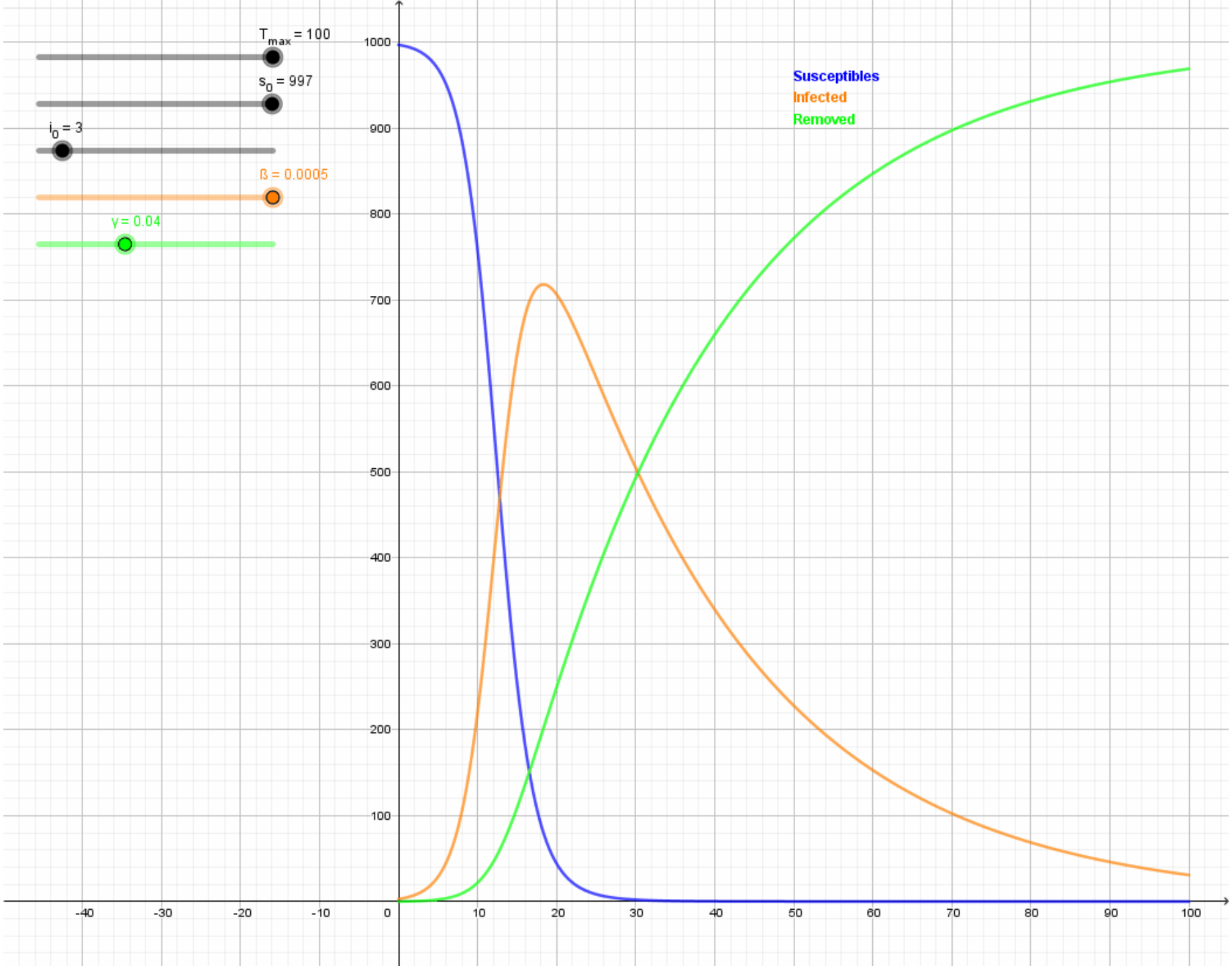
$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

S = Susceptible
I = Infected
R = Recovered
ß = Transmission rate
γ = Recovery rate

Objectives: i) Understand disease dynamics, ii) estimate key parameters, iii) predict outbreaks, iv) assess interventions

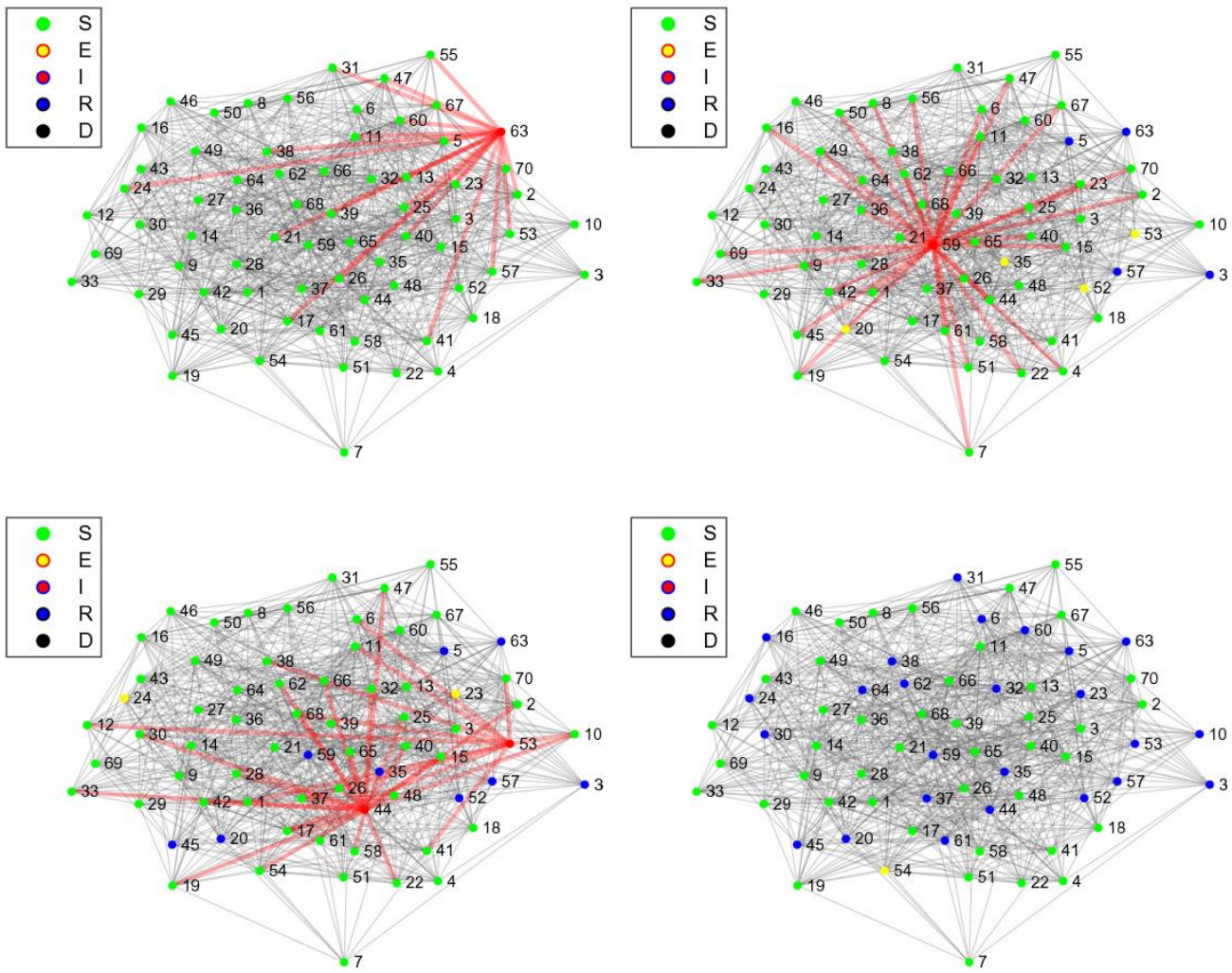
Source: (Paula Luz et al., 2010)


T_{max} = 100
s_0 = 997
i_0 = 3
ß = 0.0005
γ = 0.04
Susceptibles
Infected
Removed
1000
900
800
700
600
500
400
300
200
100
-40
-30
-20
-10
0
10
20
30
40
50
60
70
80
90
100

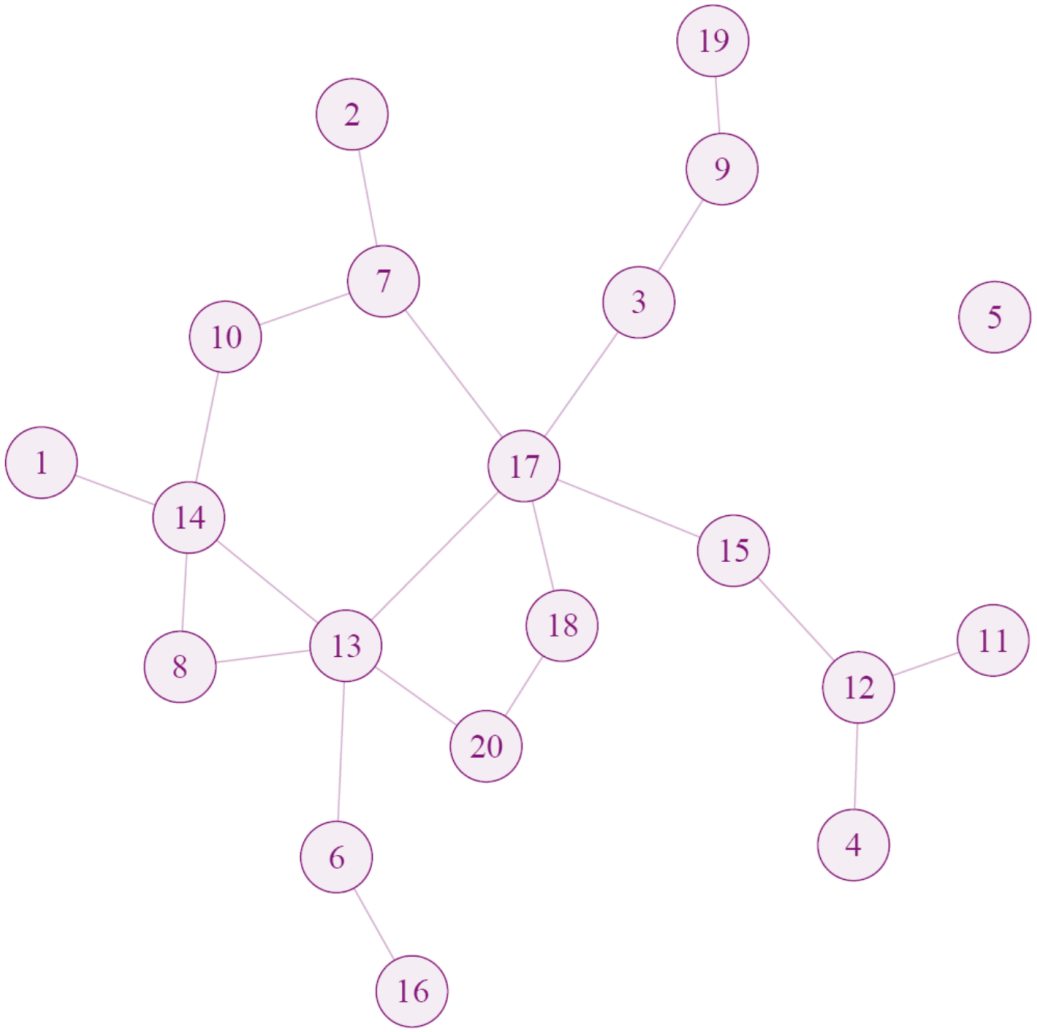
Network-Based Epidemic Models

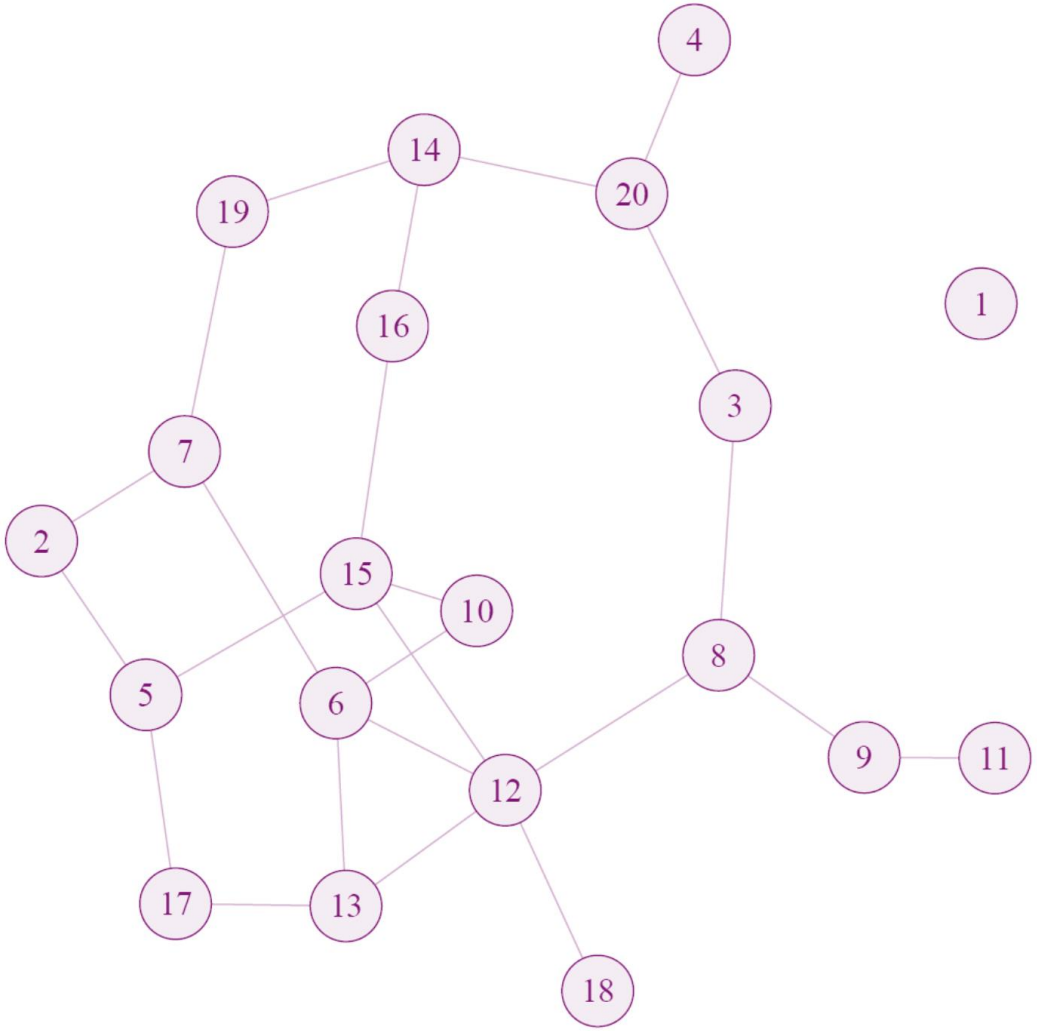
- Individuals (plants, farms, nurseries, people, etc.) are nodes
- Edges represent interactions where disease transmission may occur

- Random Networks
- Scale-Free Networks
- Small-World Networks



Source: (Abhishek Tomy et al., 2022)

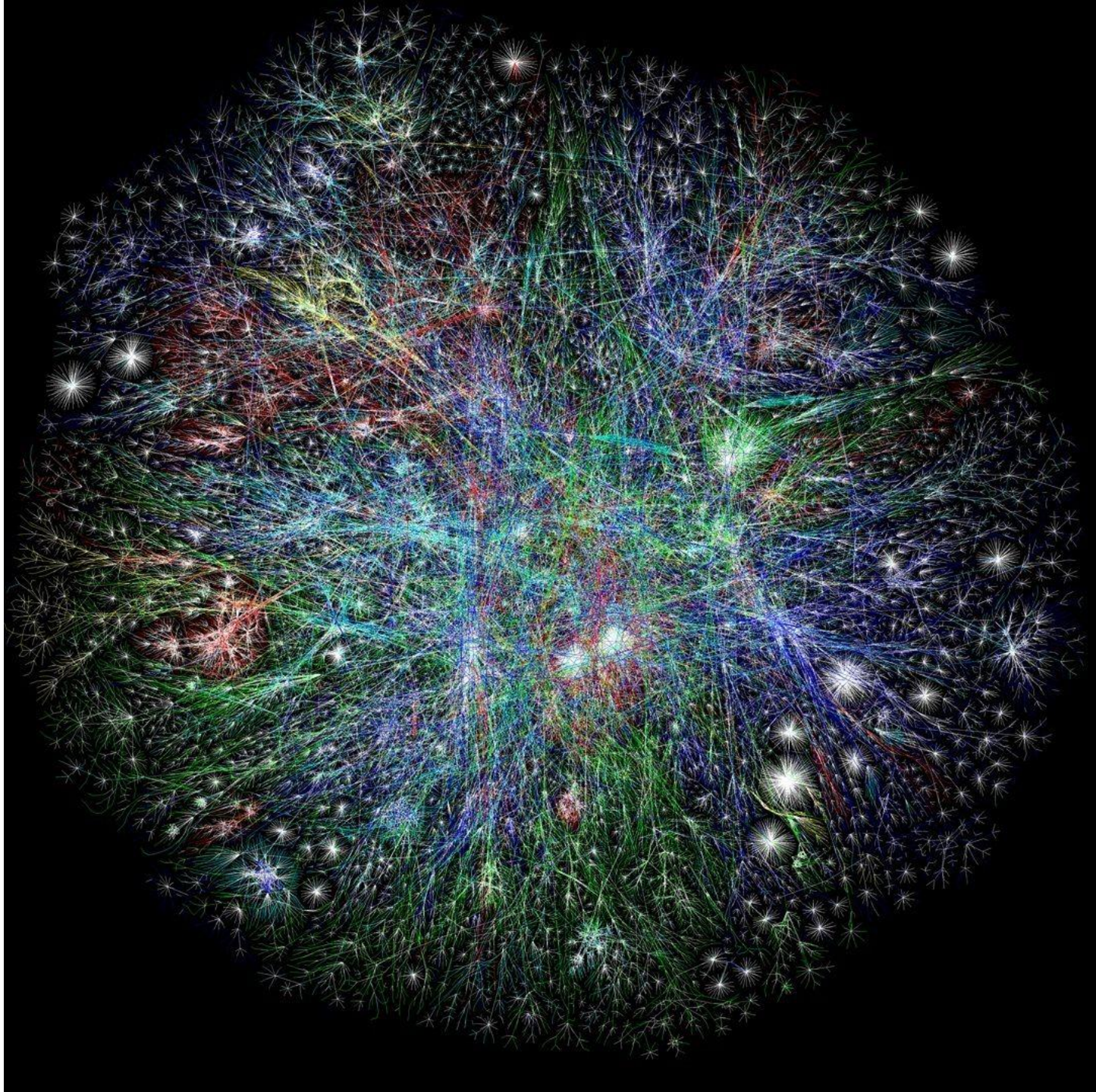




Random networks

Small-World Networks

- A mathematical graph (or network) where most nodes are neighbors, or the distance from one node to another is short
- A person is connected to any other person by six or fewer social connections
- A web page is 19 pages away from any other page

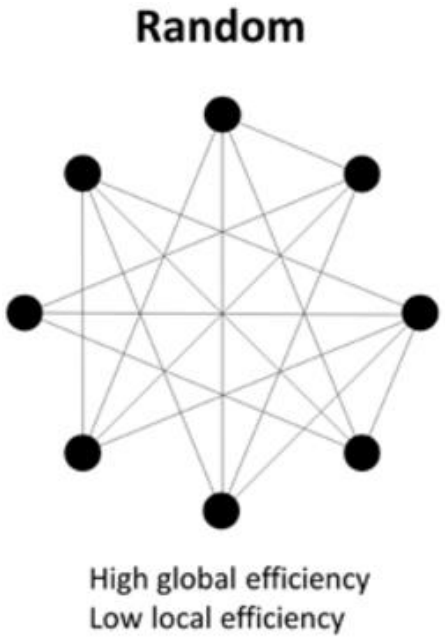


Small-World Networks

- Small-world networks form cliques, or networks with high **clustering coefficient (σ)**

$$\sigma = \frac{1}{N}\sum_{i}^{N}\frac{2 * T_i}{N_i(N_i - 1)}$$

- Where T_i is the number of triangle that include node i, N_i is the neighbors of node i, and N is the total number of nodes



$$\sigma_1 = \frac{1}{7}\sum_{i=1} \frac{2 * 2}{6\,(6-1)}$$

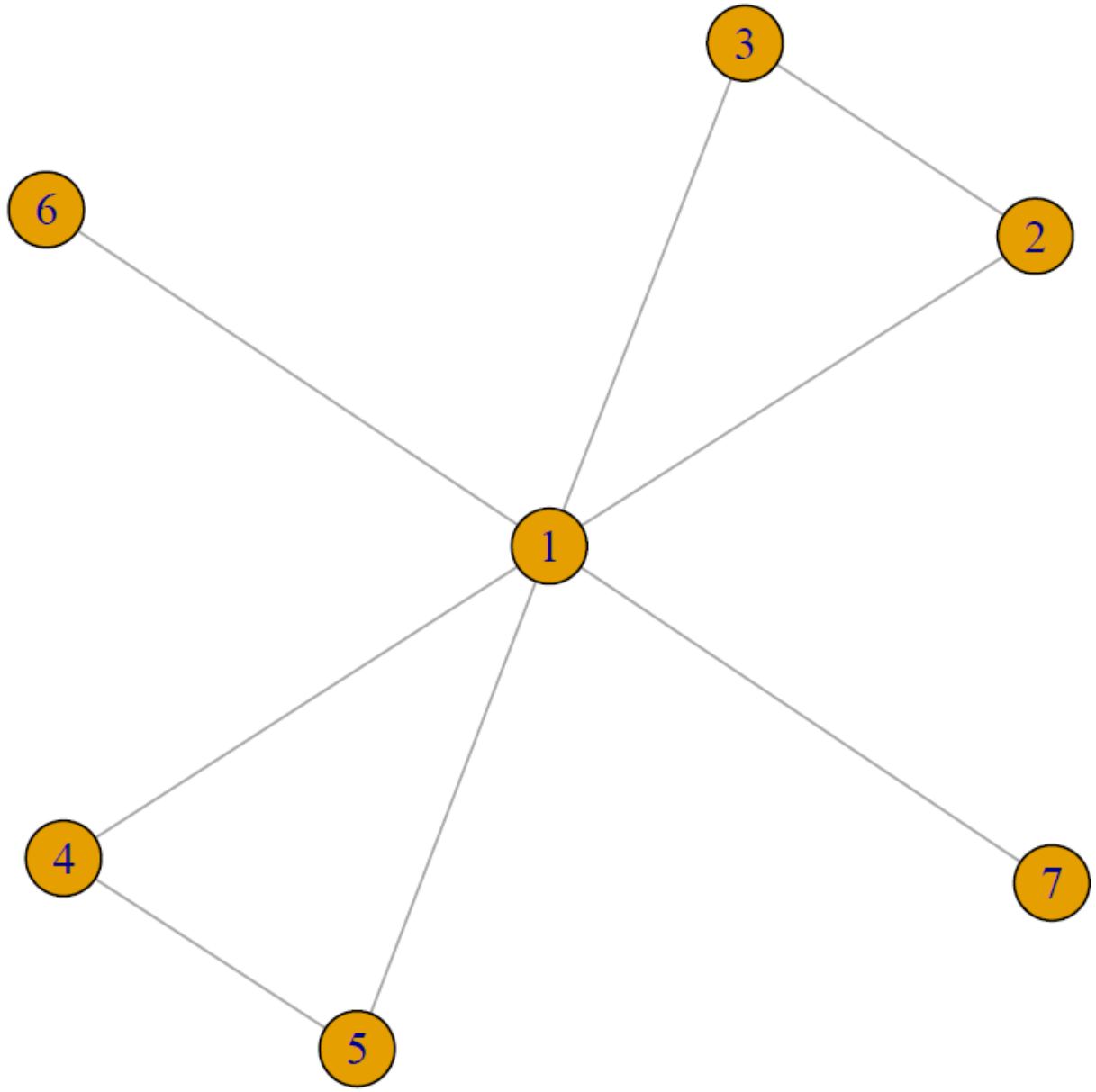
$$\sigma_2 = \frac{1}{7}\sum_{i=2} \frac{2 * 1}{2\,(2-1)}$$

...

...

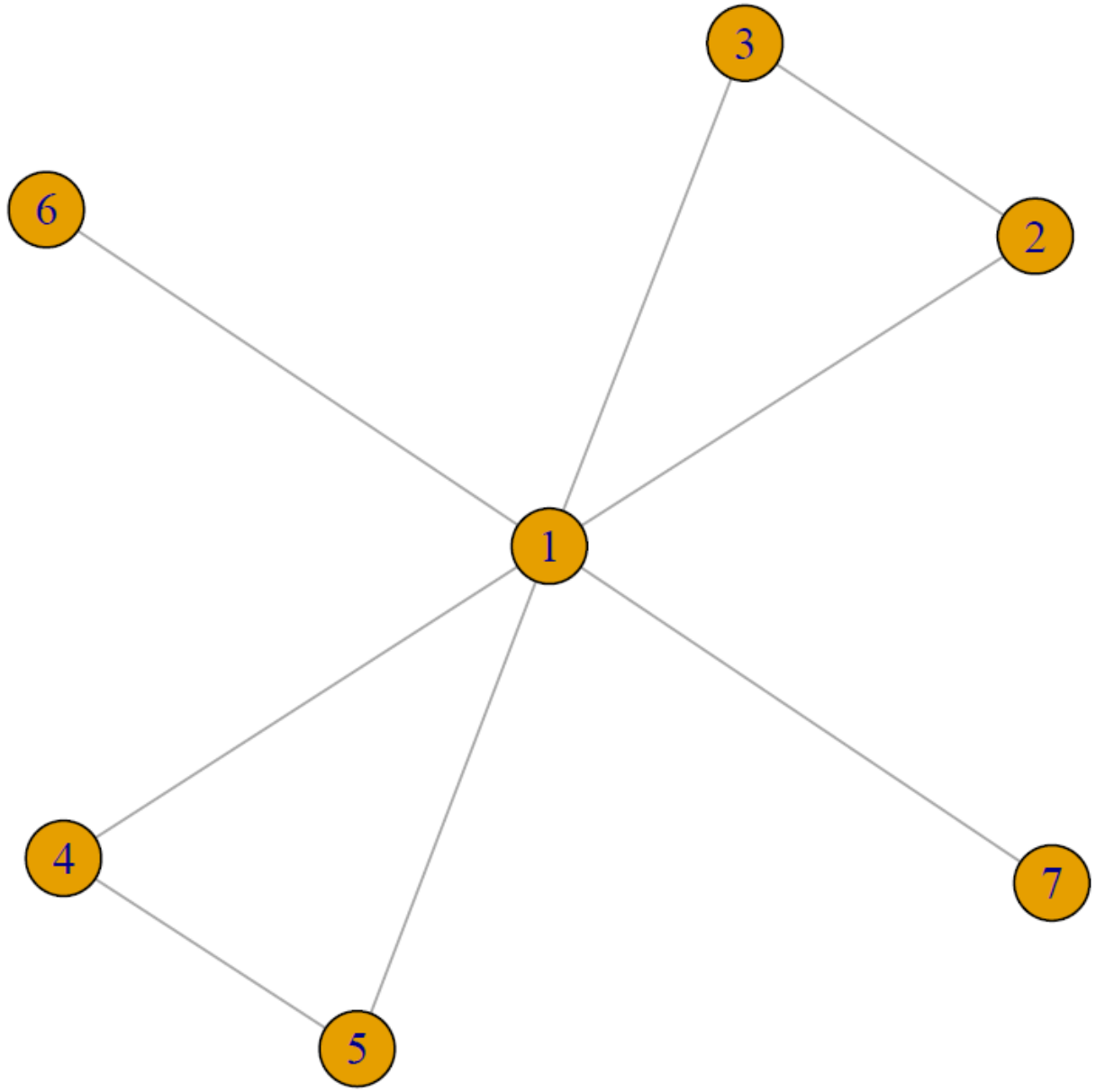
...

$$\sigma_7 = \frac{1}{7}\sum_{i=7} \frac{2 * 0}{1(1-1)}$$



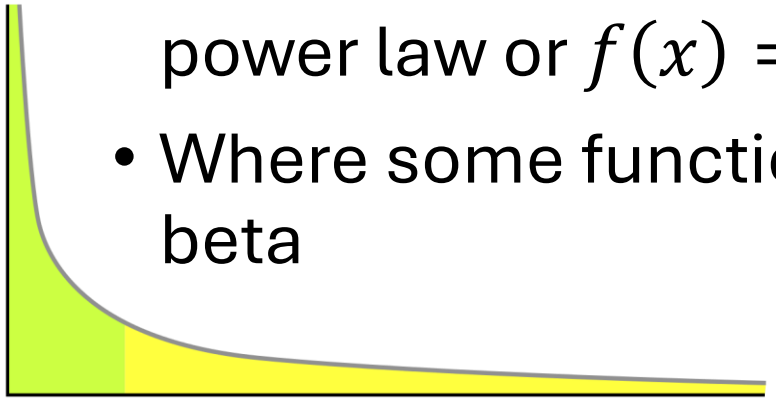
```
small_world←
graph(c(1,2,2,3,3,1,1,4,
4,5,5,1,1,6,1,7),
directed = FALSE)

(1/7)*sum(transitivity(s
mall_world,type="local")
,na.rm=TRUE)
```

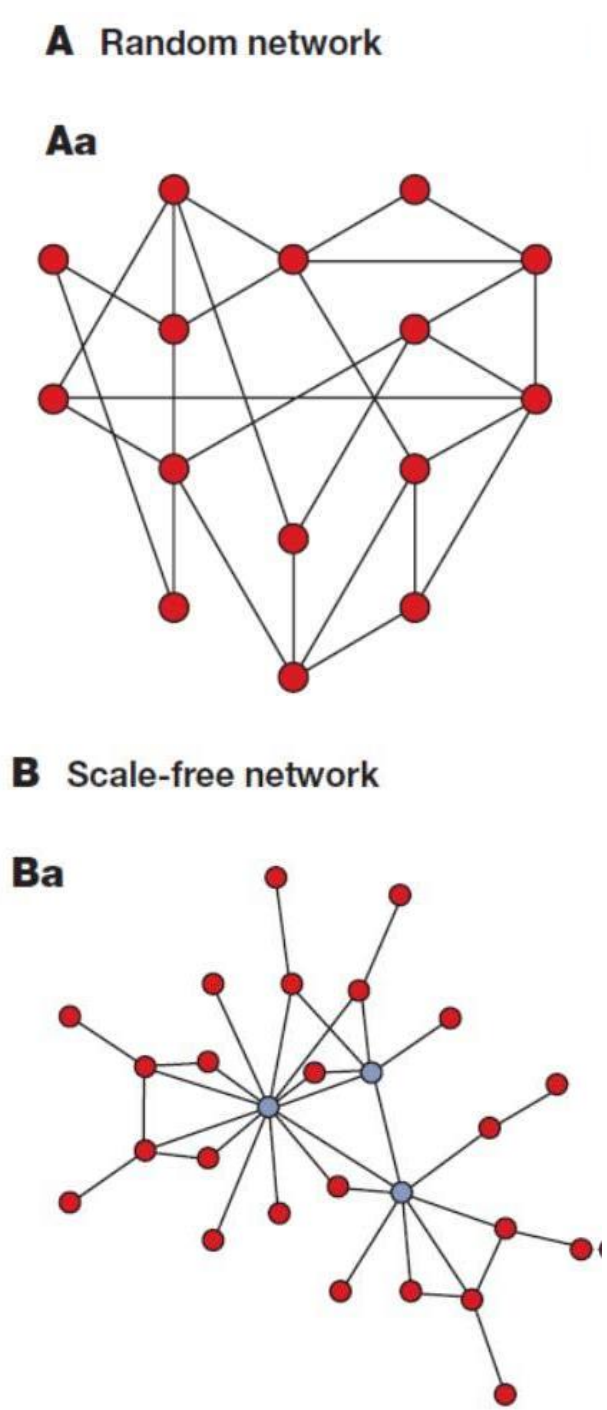


Scale-Free Networks

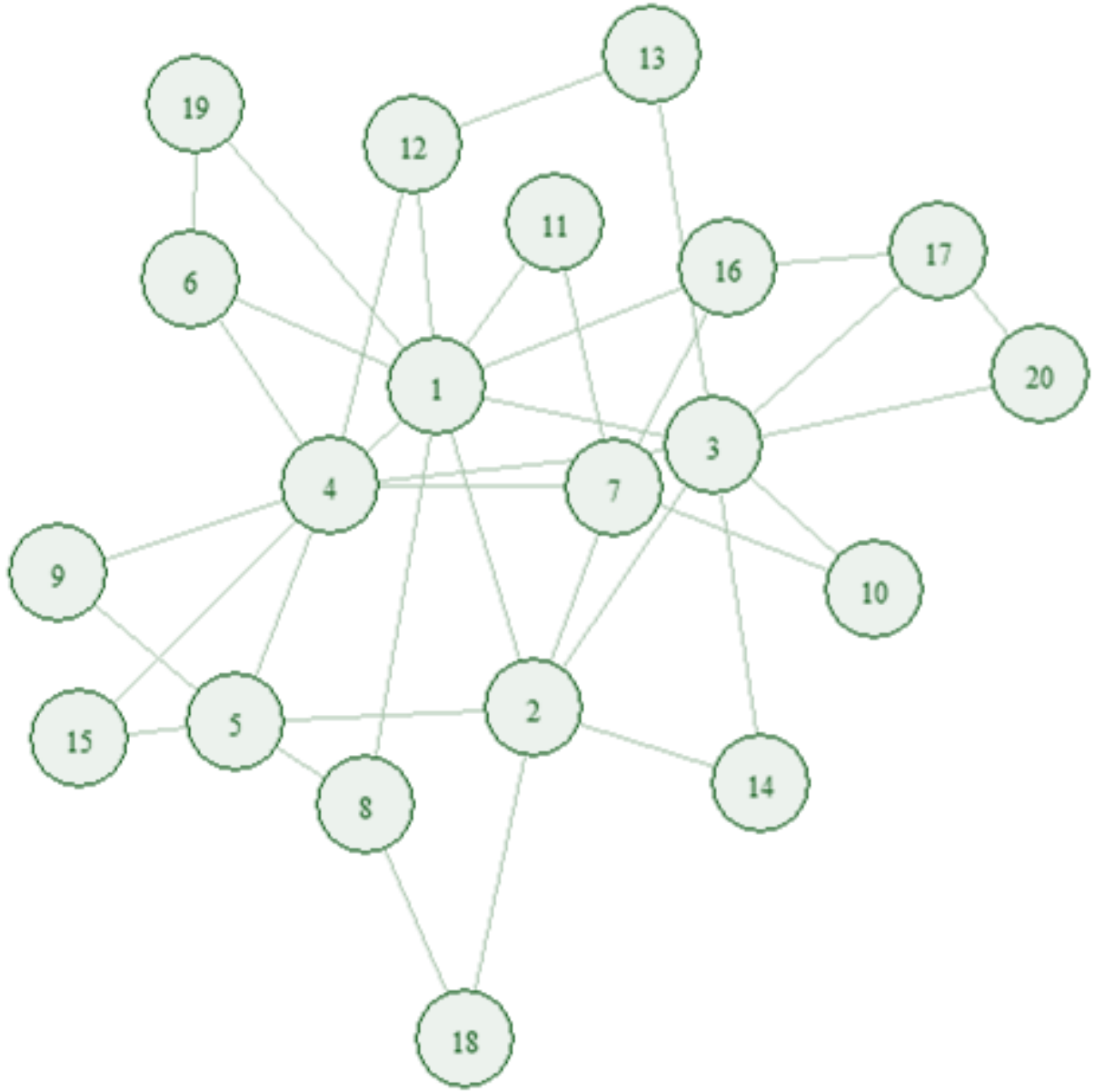
- A network whose degree distribution follows a power law or $f(x) = ax^{-\beta}$
- Where some function f(x) is scaled by some value beta

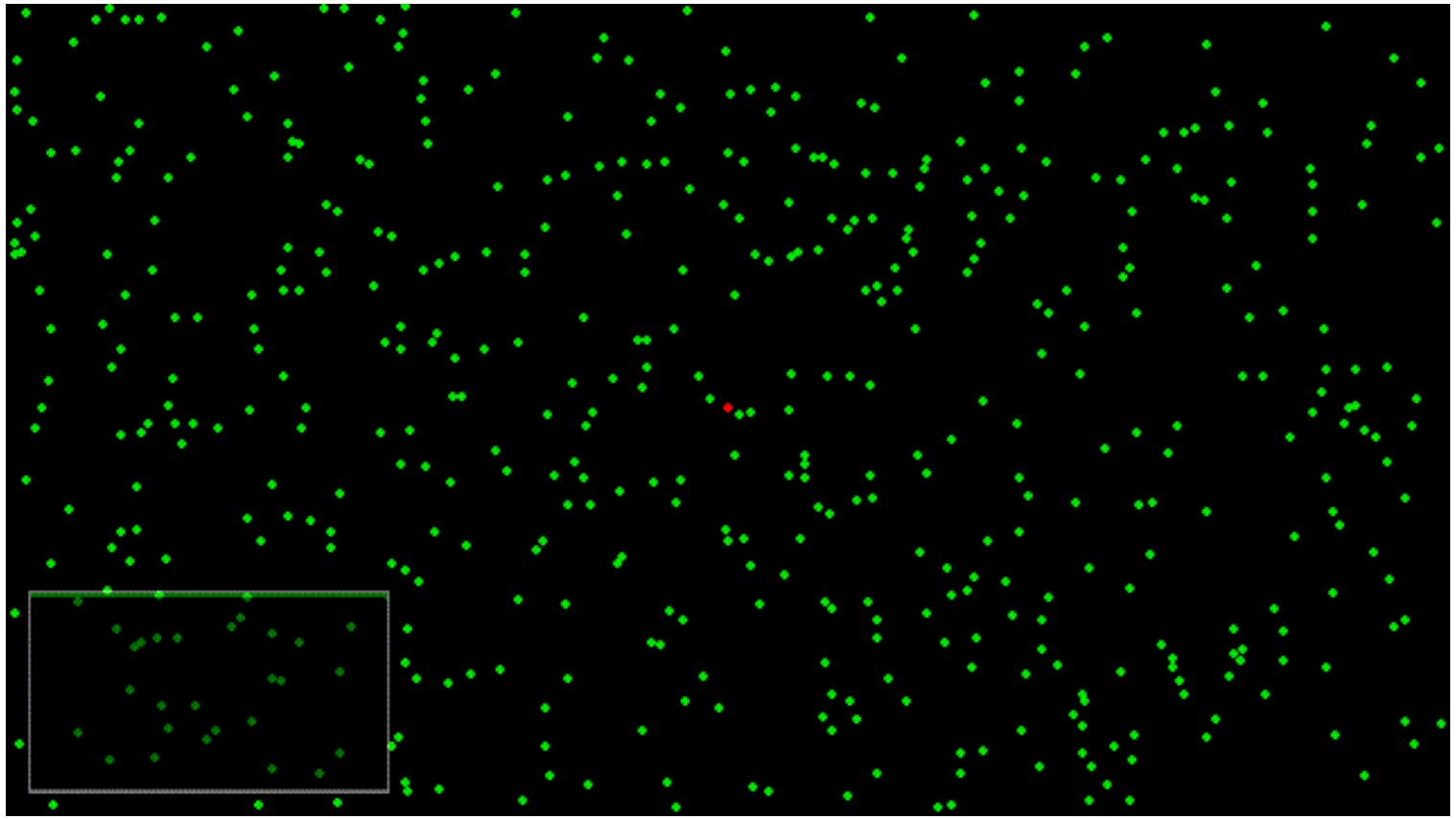


- Preferential attachment is the idea that when a node joins a network, it is more likely to form connections with nodes with high degrees
- Network “hubs”
- `hist(degree(small_world))`



```
scale_free_network ←
barabasi.game(n = 20, m
= 2, directed = FALSE)
```

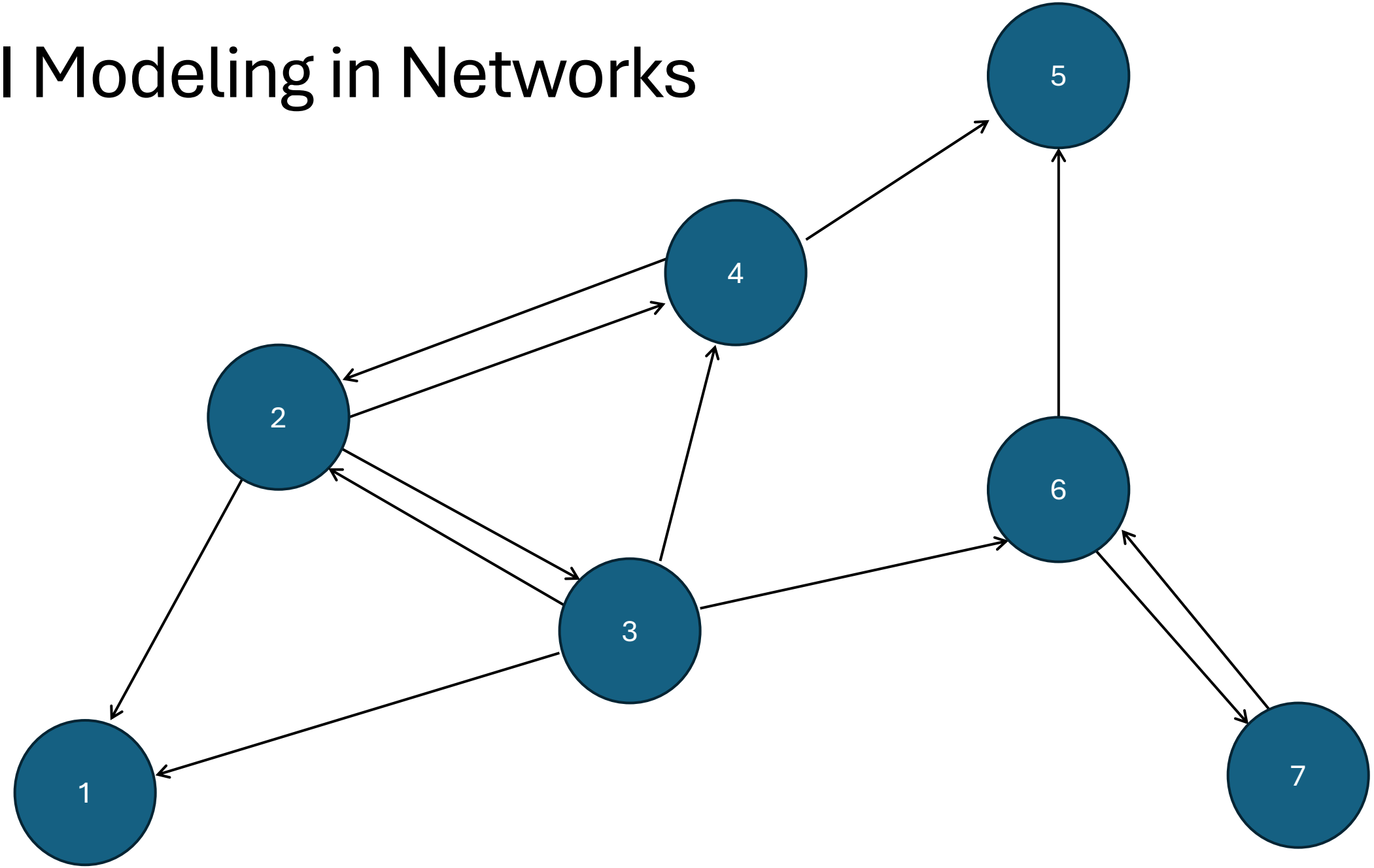




Epidemic Simulations

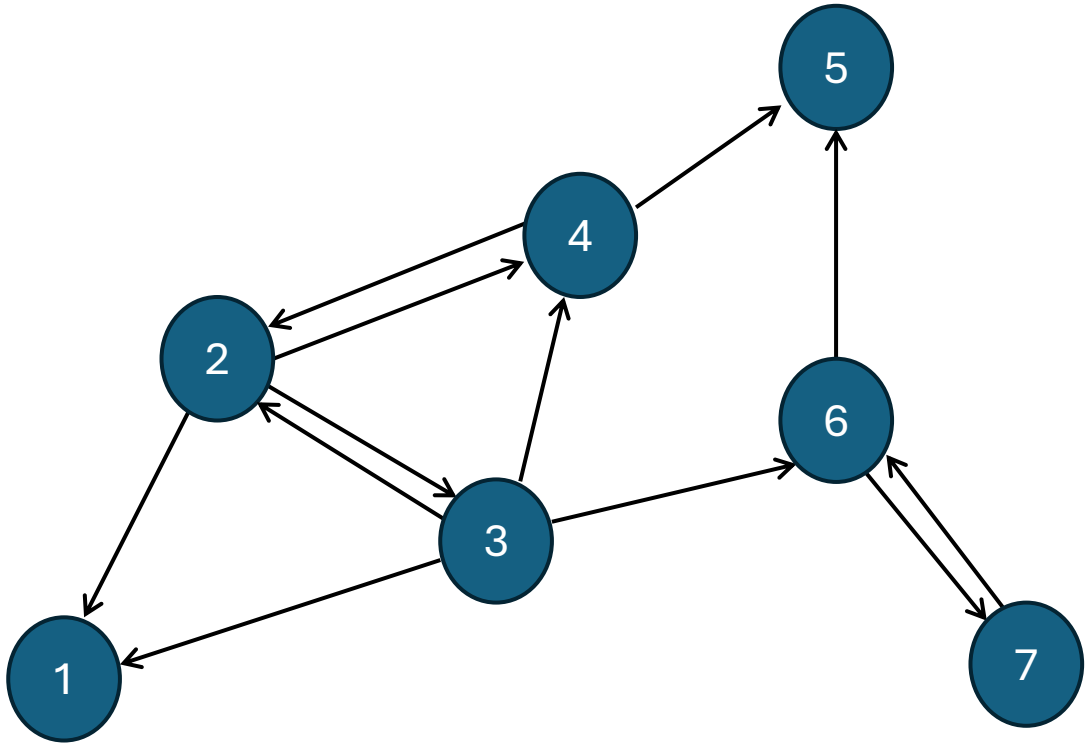
- Using available data
- Simulating scenarios
- Uncertainty quantification

SI Modeling in Networks

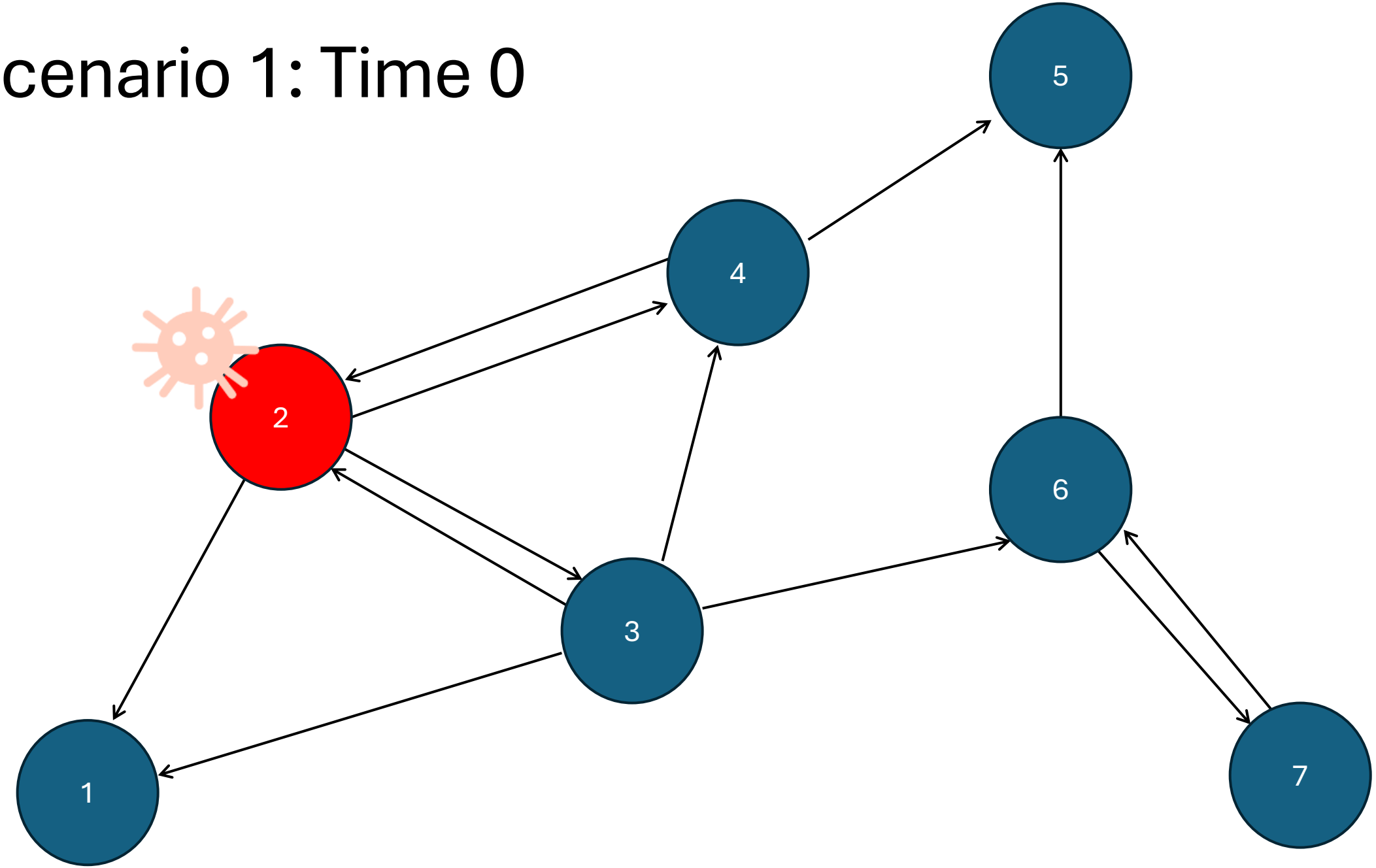


SI Modeling in Networks

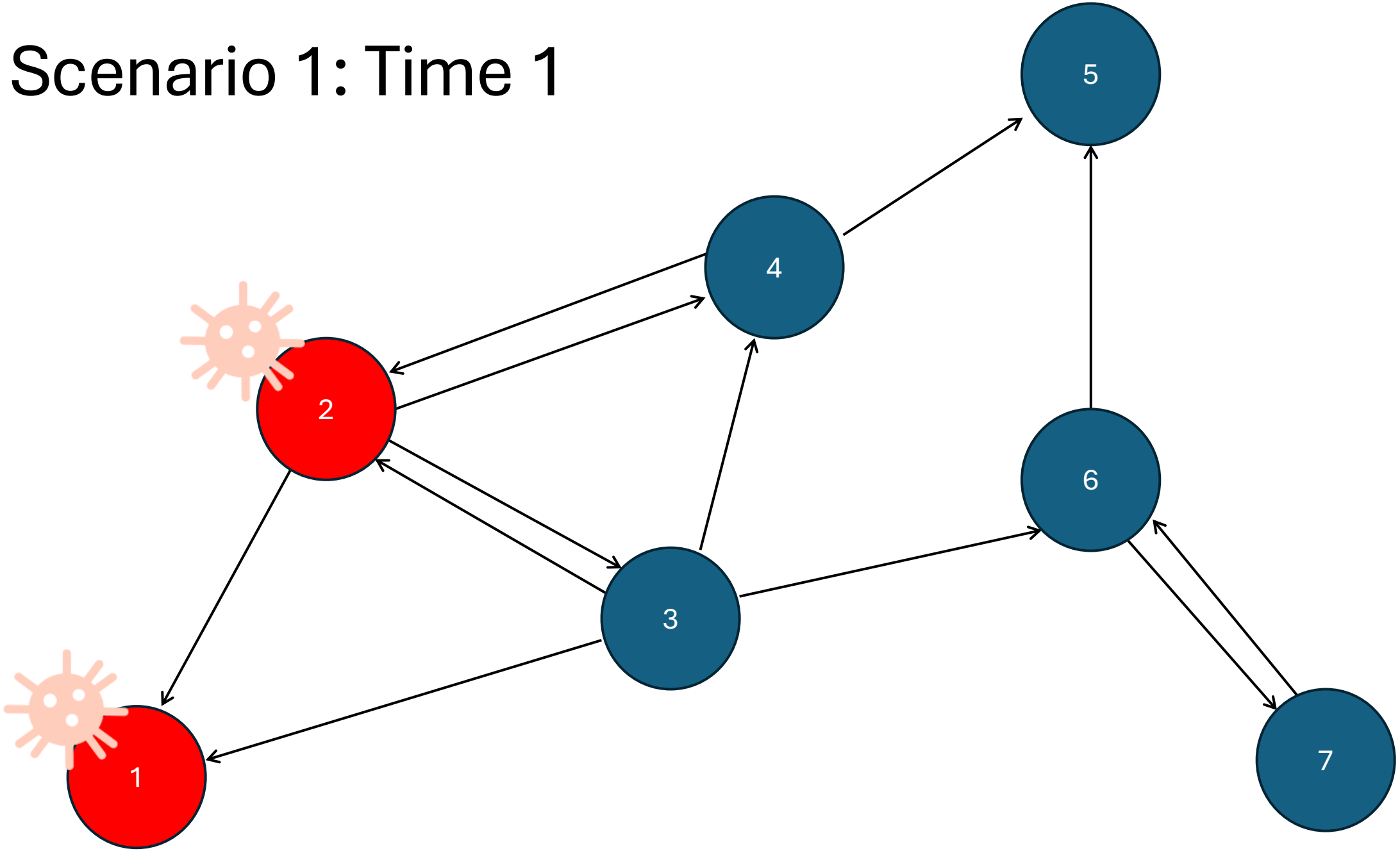
- In this network, disease moves **directionally** from one infected node to another
- **NO management exists** (there is no **Recovered** class)
- All nodes are Susceptible to the disease
- Once infected, a node remains infected



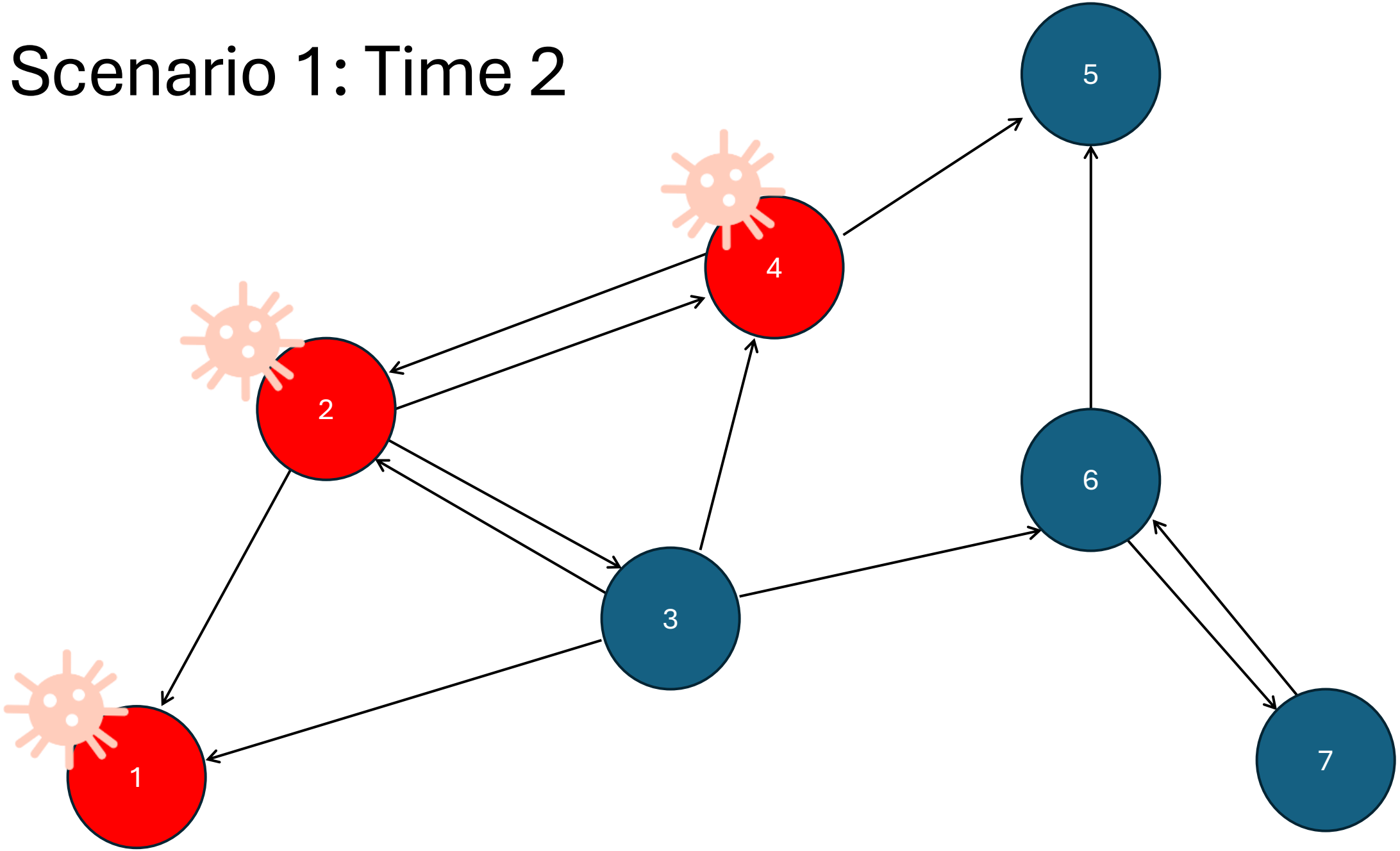
Scenario 1: Time 0



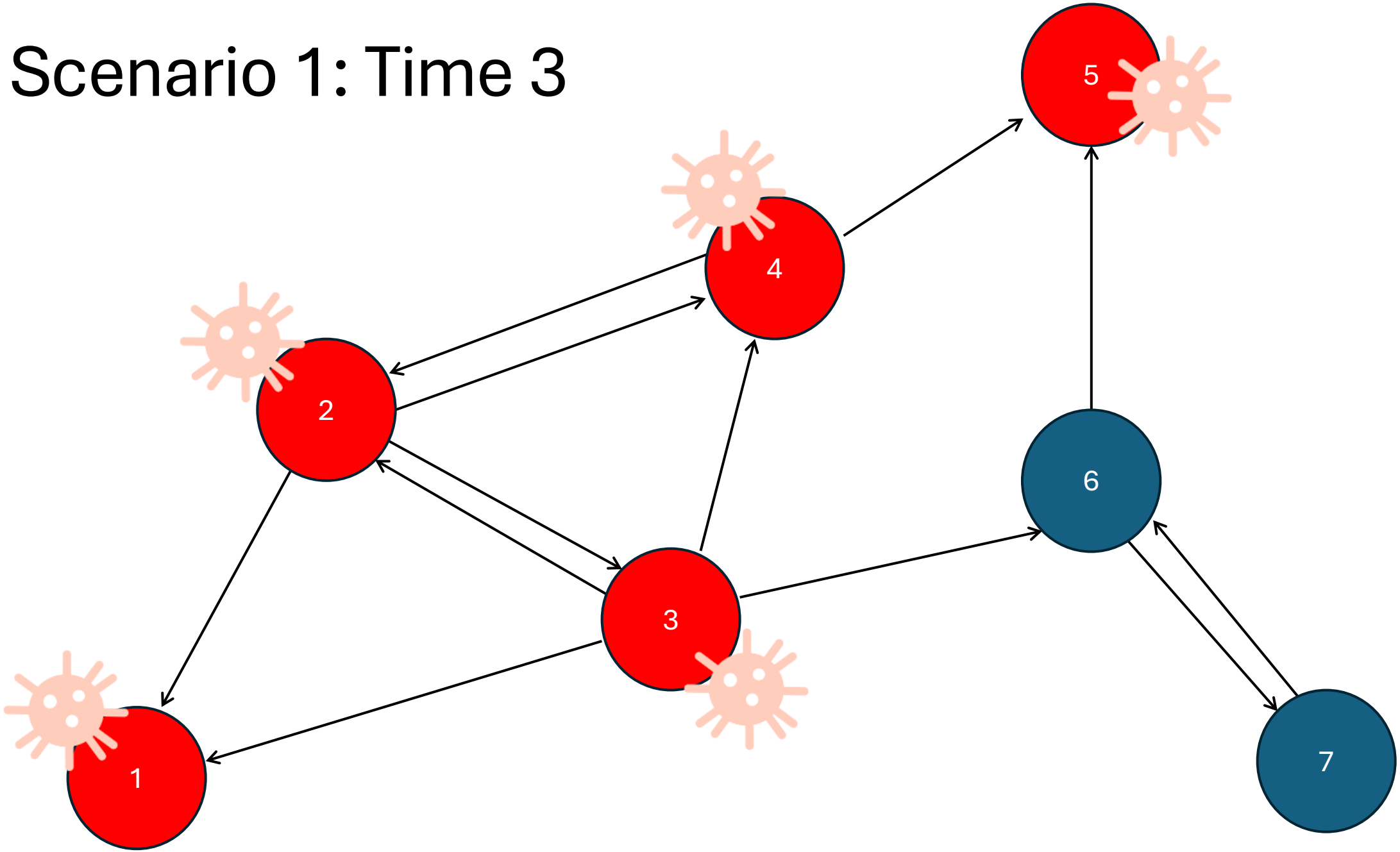
Scenario 1: Time 1



Scenario 1: Time 2

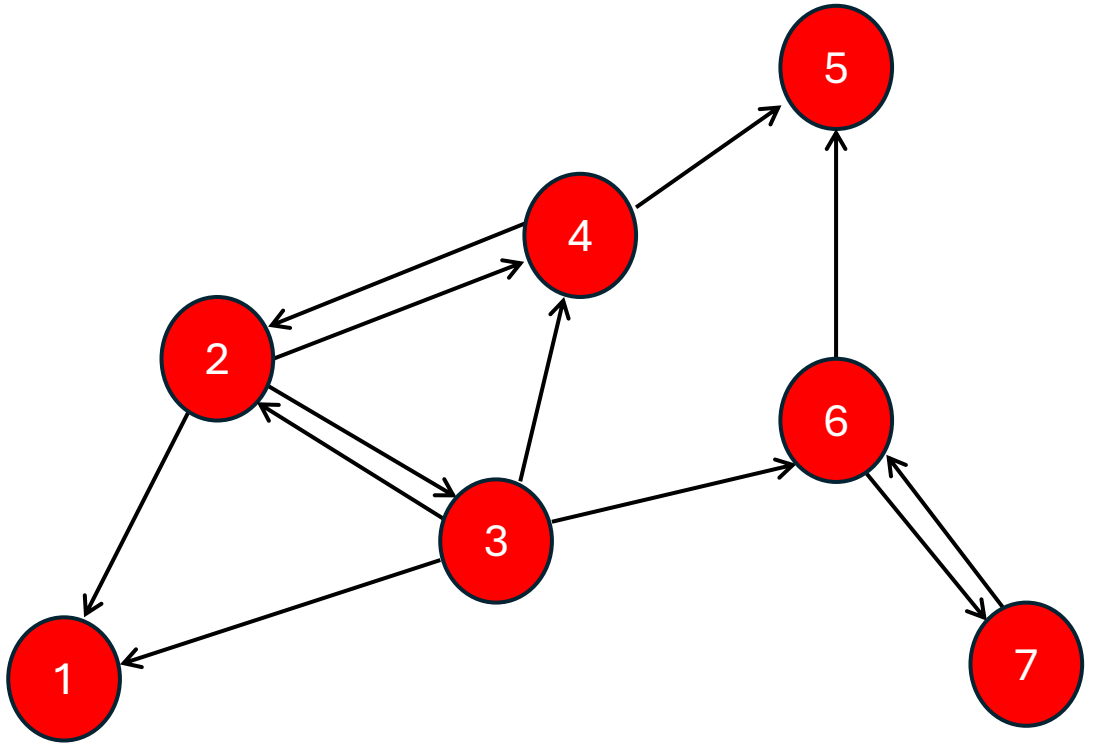


Scenario 1: Time 3



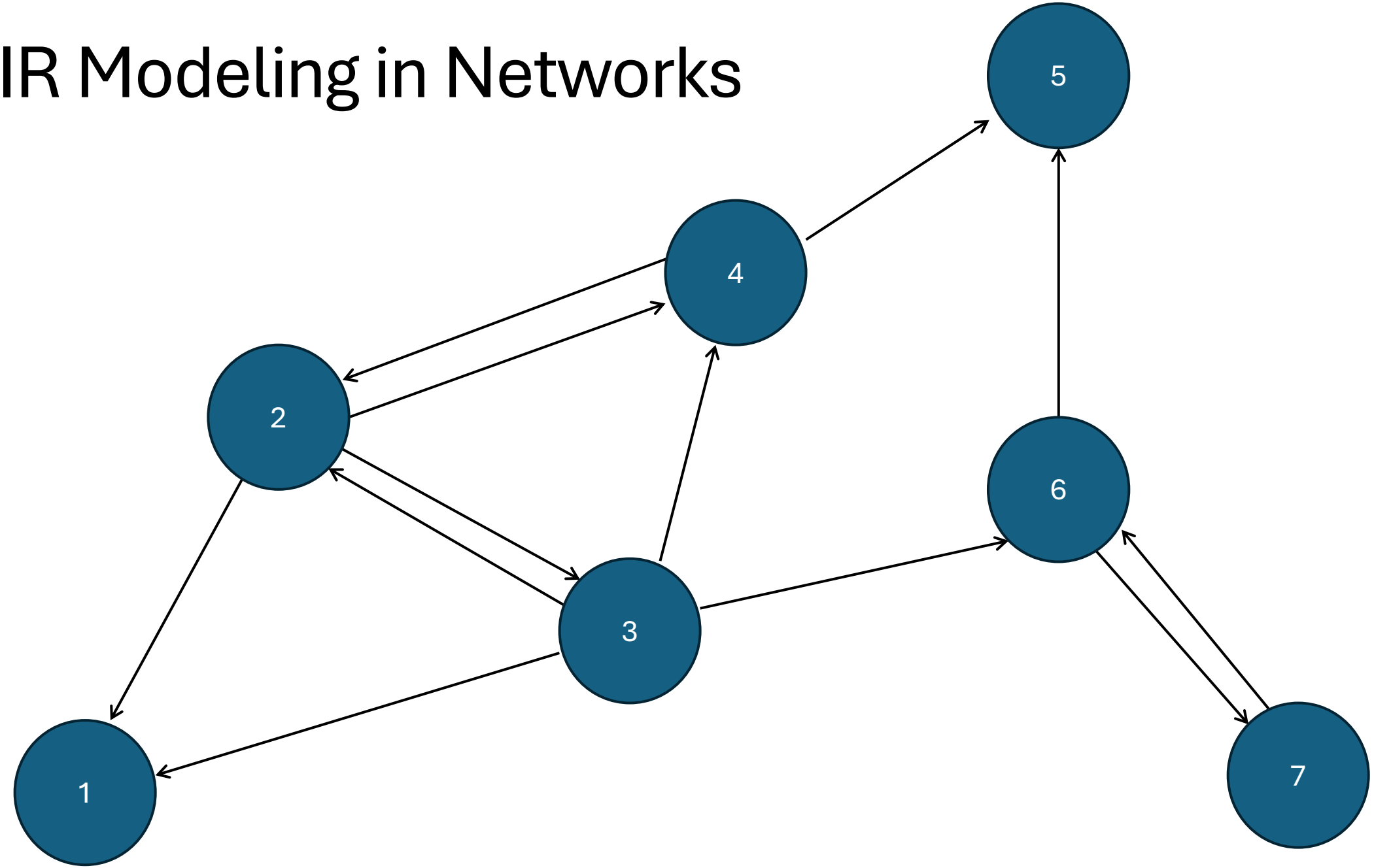
SI Modeling in Networks

- Without management, disease will move entirely throughout this network in **at most 5-time steps**
- This could be an example of a persistent soilborne disease, moved through shared planting material



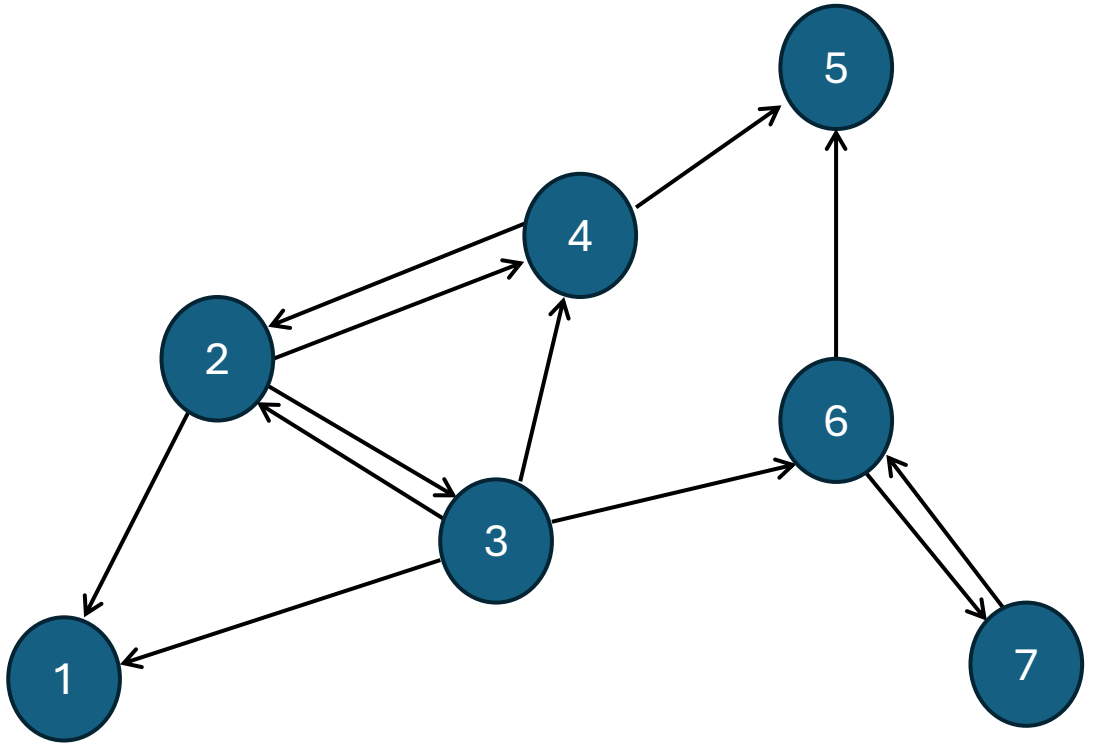
Let’s try an example of an SI model in R

SIR Modeling in Networks

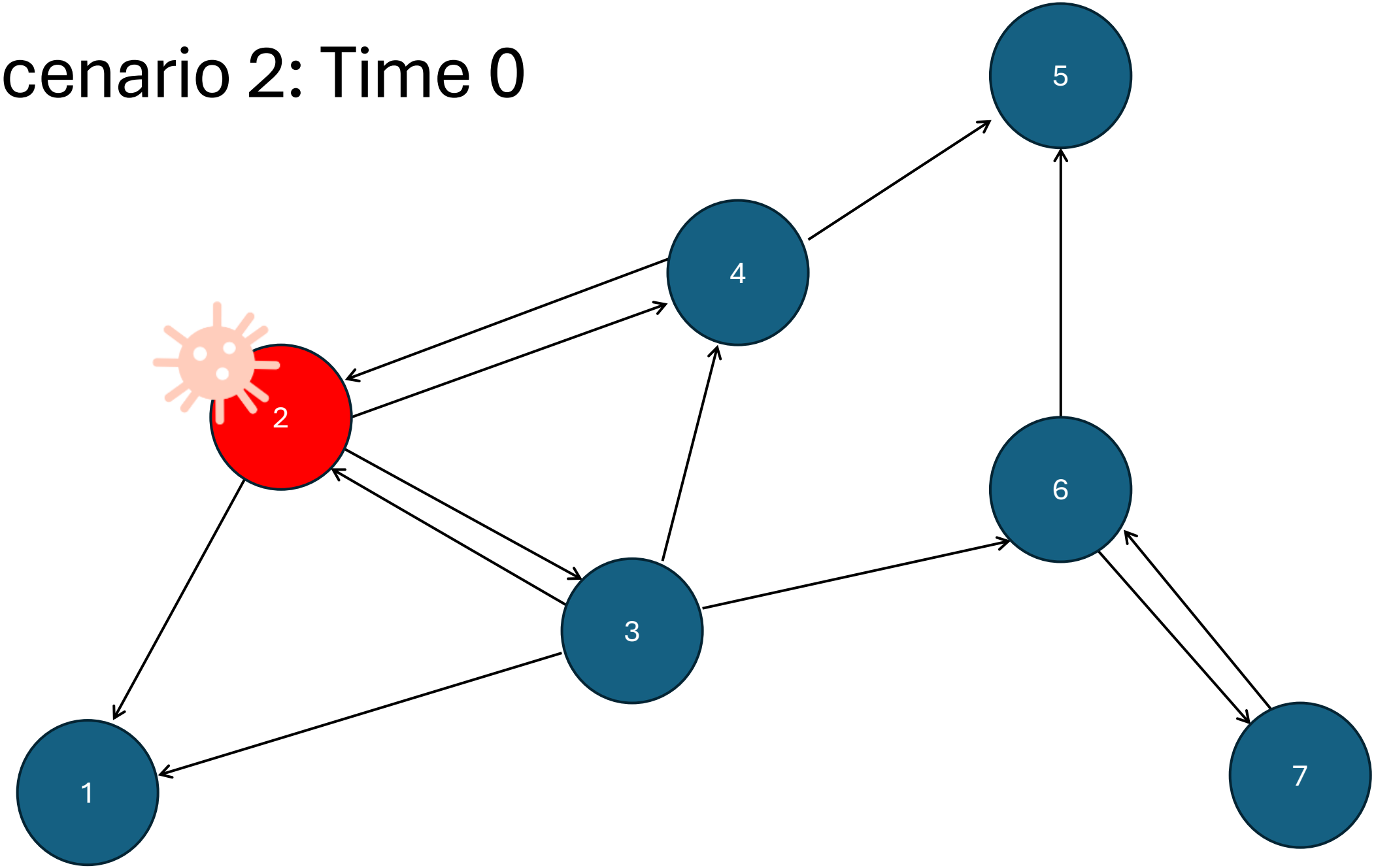


SIR Modeling in Networks

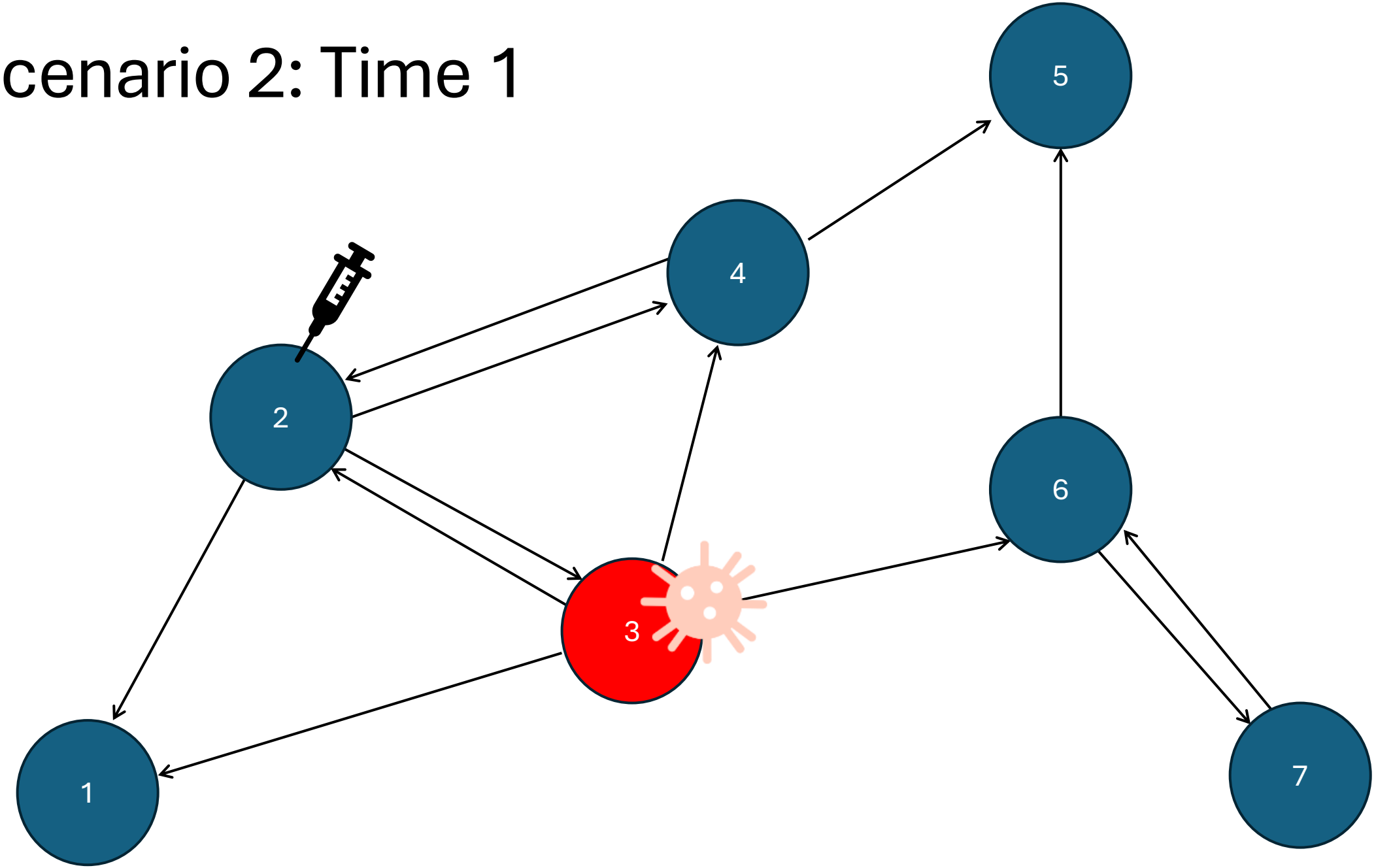
- In this network, disease moves **directionally** from one infected node to another
- **Management exists,** where nodes are diseased for at least **1 time step,** and management is **perfect** (no prob of failure)
- All nodes are Susceptible to the disease, even after prior infection



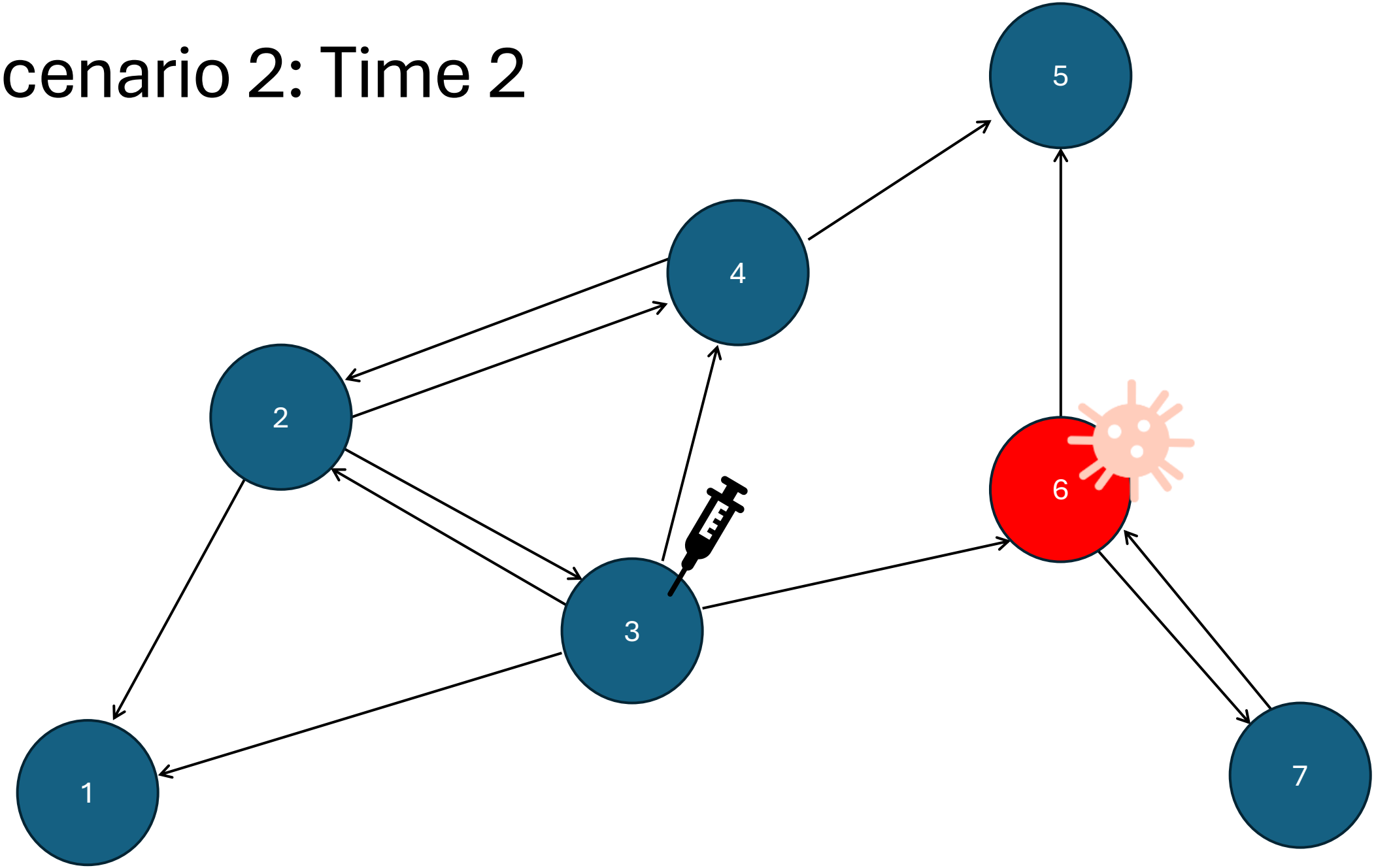
Scenario 2: Time 0



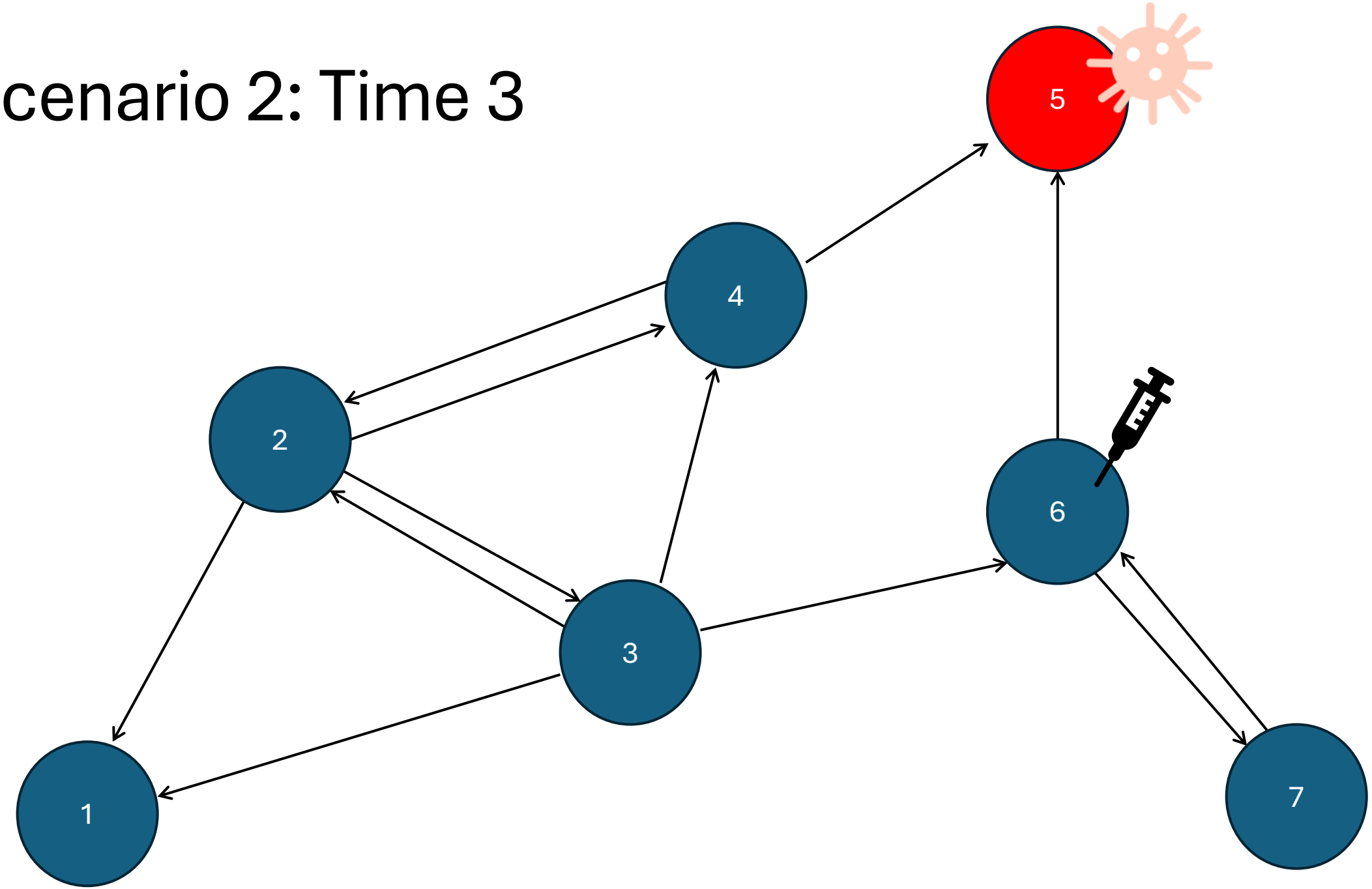
Scenario 2: Time 1



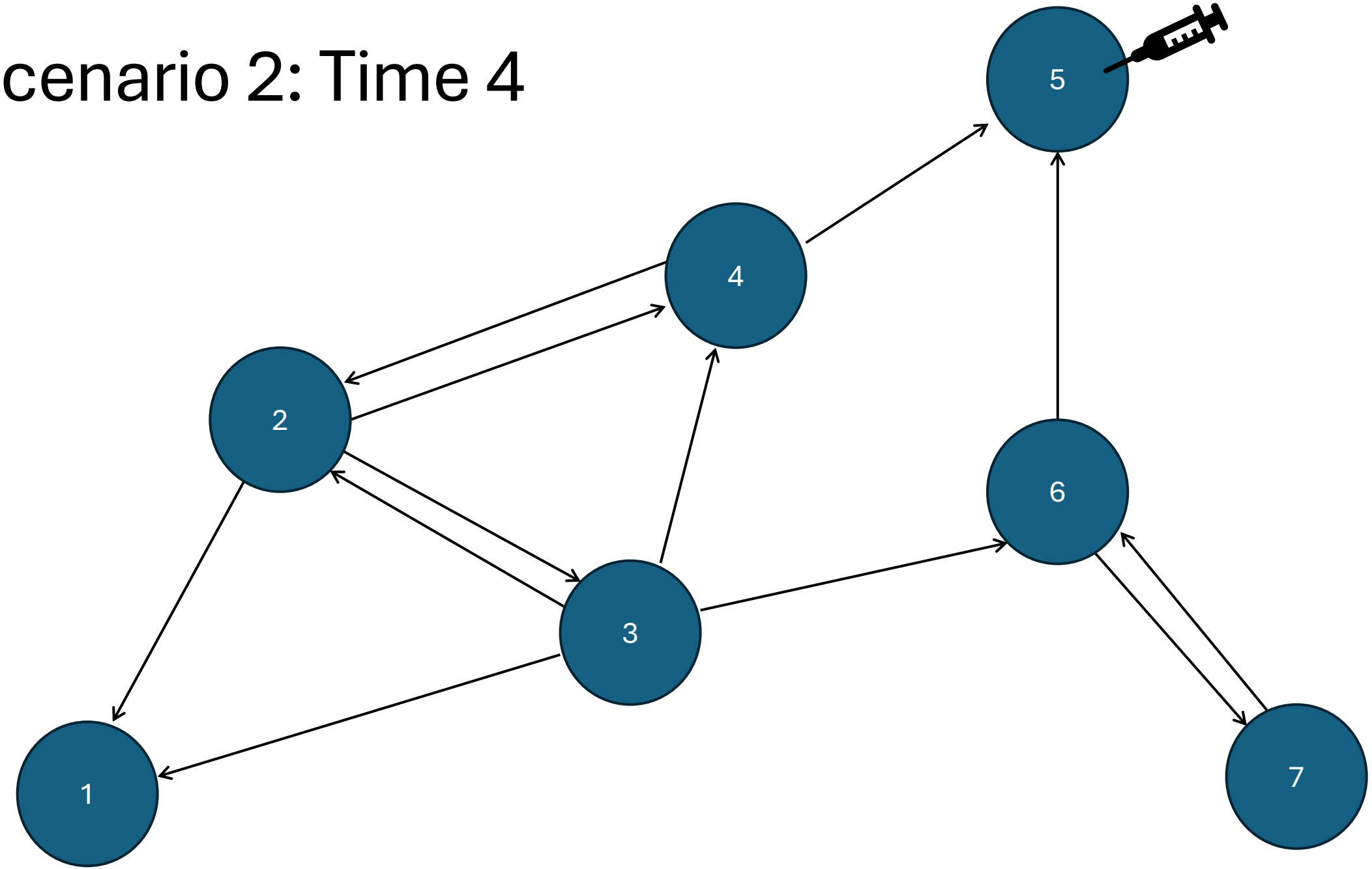
Scenario 2: Time 2



Scenario 2: Time 3



Scenario 2: Time 4



Epidemic networks

- Summary…
- …
- …

- https://create.kahoot.it/my-library/kahoots/91f2db72-0c6d-407e-91b6-2c054fc3ba3f