$c_h[q,h]$	Artificial output function harmonic coefficient of dimensions q and h
$c_p[q,p]$	Artificial output function polynomial coefficient of dimensions q and p
$f(t[q])\,\forall q \in [1,Q]$	Artificial output parameter
H	Maximum degree of harmonic fitting
h	Artificial output function harmonic coefficient degree dimension index
i	Index of measurement instance
P	Maximum degree of polynomial fitting
p	Artificial output function polynomial coefficient degree dimension index
Q	Input parameter dimension count
q	Input paramater dimensional index
$t[q]$	Artificial input parameter of dimension q
$x[q,i]$	Measured input parameter of dimension q at instance i
$y[i]$	Measured output parameter at instance i
$\delta[q,h]$	Phase offset of harmonic fitting of dimensions q and n
$\varepsilon[i]$	Phase offset of harmonic fitting of dimensions q and n
$\omega[q]$	Fundamental frequency of harmonic fitting of dimension q

This document describes the formulation of an algorithm to select parameters for a function that is fitted to a measured data set. The family of functions that are generated by this algorithm are a linear combination of polynomial and harmonic terms. This may be considered as a hybrid of Taylor-series and Fourier-series fitting.

Assume we are given a set of measurements. We arbitrarily select one dimension of these measurements as our output parameter set, $y[i]$. The remainder are the input parameter set, $x[q,i]$.

The goal of the algorithm is to determine the coefficients of the function that best match the measure dataset.

The function has the form described in equation (1).

$$f(t[q]) = c_p[1,0] + \sum_{q=1}^{q=Q}\left(\left(\sum_{n=1}^{p=P} c_p[q,n]\cdot(t[q])^n\right) + \left(\sum_{h=1}^{h=H} c_h[q,h]\cdot\cos(h\cdot\omega[q]\cdot t[q]+\delta[q,h])\right)\right) \tag{1}$$

$c_p[1,0]$ is the static offset of the fitting function.

The error between the fitting function and the measured data is defined as in equation (2).

$$\varepsilon[i] = f(x[q,i]) - y[i] \tag{2}$$

The algorithm minimizes the net square error.

$$0 = \frac{\partial}{\partial c_h[q,h]}\left(\varepsilon^2[i]\right) = \varepsilon[i]\cdot\frac{\partial\varepsilon[i]}{\partial c_h[q,h]} \tag{3a}$$

$$0 = \frac{\partial}{\partial \delta[q,h]}\left(\varepsilon^2[i]\right) = \varepsilon[i]\cdot\frac{\partial\varepsilon[i]}{\partial \delta[q,h]} \tag{3b}$$

$$0 = \frac{\partial}{\partial \omega[q]}\left(\varepsilon^2[i]\right) = \varepsilon[i]\cdot\frac{\partial\varepsilon[i]}{\partial \omega[q]} \tag{3c}$$

$$0 = \frac{\partial}{\partial c_p[q,p]}\left(\varepsilon^2[i]\right) = \varepsilon[i]\cdot\frac{\partial\varepsilon[i]}{\partial c_p[q,p]} \tag{3d}$$

Each of the partial derivatives in equation (3) are expanded in equation (4)

$$\frac{\partial\varepsilon[i]}{\partial c_h[q,h]} = \cos(h\cdot\omega[q]\,x[q,i]+\delta[q,h]) \tag{4a}$$