Linear Algebra 1 Exam 1 6/12/3

Name: **Signature:**
Show your work.
The best five questions will count.
Twenty points per question.

Question 1

Consider the linear system in the variables (x, y, z, t, u), given by the following matrix, in echelon form:

$$\left(\begin{array}{ccccc|c} 1 & 2 & -1 & 3 & 1 & 2 \\ 0 & 1 & 1 & 3 & -1 & 4 \\ 0 & 0 & 0 & 1 & 2 & 3 \end{array}\right)$$

- Reduce the system to reduced echelon form and give the general solution.
- Write your answer in the form of the sum of a particular solution and the general solution of the associated homogeneous system.
- How many free variables are there?
- Which are the pivot variables?
- Is there a solution with $y = z = t = 1$ and if so is what is the solution and is it unique?

Question 2

Consider the following linear system:

$$\begin{aligned} x - 3y + 2z &= 8 \\ 3x - 8y - 5z &= 11 \\ 2x - 4y - 18z &= -10 \end{aligned}$$

- Find the general solution of the system.
- Give a geometric interpretation of the system and your solution.
- Is there a solution of the system with $x = 0$?
  If so, find it.

Question 3

Everyplace.com has three levels of employee, levels A, B and C.

- Last year level A employees each received $10,000$ stock options, level B employees each recieved $5,000$ stock options and level C employees $2,500$ stock options.
- Bonuses for a record year were paid out at $\$20,000$ for levels A and B and $\$10,000$ for level C.
- Base salaries were $\$120,000$ for level A, $\$80,000$ for level B and $\$50,000$ for level C.
- Last year a total of $300,000$ stock options were given out, total bonuses of $\$1,000,000$ and total base salaries of $\$5,000,000$.

How many employees does Everyplace.com have?

Question 4

Give an example of each of the following or explain why no such example can exist:

- An inconsistent linear system in three variables, with a coefficient matrix of rank two.
- A consistent linear system with three equations and two unknowns, with a coefficient matrix of rank one.
- A consistent linear system with three equations and two unknowns, with a coefficient matrix of rank larger than one.
- A linear system of two equations in three unknowns, with an invertible coefficient matrix.
- A linear system in three variables, whose geometrical interpretation is three planes intersecting in a line.

Question 5

Let A be the following matrix:

$$\begin{vmatrix} 1 & 3 \\ -2 & -8 \end{vmatrix}$$

- Compute the matrices A^2, AA^T and A^{-1}.
- Find numbers p and q, such that $A^2 = pA + qI$, where I is the 2×2 identity matrix.
- Write A and A^T as a product of elementary matrices.
- Let $B = A - tI$, where t is a scalar.
  For which values of t is B not invertible?

Question 6

Calculate the following determinant (using suitable properties of the determinant to simplify the calculation):

$$\det \begin{vmatrix} x & y & z & 1 \\ 1 & -2 & 3 & 1 \\ 2 & -3 & 1 & 1 \\ 4 & -6 & 3 & 1 \end{vmatrix}$$

Also give the geometrical interpretation of the vanishing of the determinant.

Question 7

Let A and B be the following matrices:

$$A = \begin{vmatrix} 1 & 2 & -2 \\ 3 & 1 & 0 \\ 2 & 3 & -3 \end{vmatrix} \qquad B = \begin{vmatrix} 2 & 3 \\ 1 & 0 \\ 3 & -1 \end{vmatrix}$$

By row reducing a suitable matrix, solve the equation $AX = B$.
Is the matrix A invertible?
Explain your answer.