Module 1

Random Variable

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Basic :

1) Random experiment :-

⇒ If the outcome of experiment is not unique Then it is called random experiment

2) sample space

⇒ The collection of all outcomes is called sample space. It is denoted by S or Ω

3) Sample points :-

⇒ outcome of random experiment is called sample points.

Generally sample points are descriptive in nature which are described by words or symbols.

eg: i) Tossing of coin, then sample space is $\{H, T\}$

ii) In selecting the suit of card from pack of playing cards, S = {diamond, Heart, club, spade}.

However, in practice it is easier to deal with numerical outcomes. In turn we associate real number with each outcome. For that we need a random variable. Whenever we do this, we are dealing with fun^c whose domain is the sample space Ω & whose range is the set of real numbers.

4] Random variable : -

Consider the experiment of ~~coin~~ tossing of 2 coins simultaneous. Then $\Omega = \{HH, HT, TH, TT\}$

Suppose if we are interested in finding the no. of heads in outcomes then we define a fun^c to measure the no. of heads in outcome. Let this fun^c be x.

$x(HH) = 2$, $x(HT) = 1$, $x(TH) = 1$, $x(TT) = 0$. ie values of x are $0, 1, 2$.

Def^n :-

Let S be sample space corresponding to random experiment. A fun^c $X : \Omega \to R$ (where R is real line) is called r.v. ie r.v is a fun^c that assigns a real no. $x(\omega)$ to each element $\omega \in \Omega$.

Remark :-

(I) If $\Omega = \{-4, 0, 4\}$, $x(\omega) = \sqrt{\omega}$

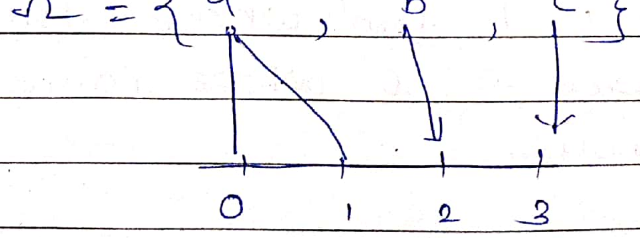
$x(-4) = \sqrt{-4}$ is imaginary no.

$x(-4) \notin R$ so x is not real valued fun^c

$\therefore$ x is not r.v

(II) If $\Omega = \{a, b, c\}$ & $x : \Omega \to A$ as follows

$\Omega = \{a, b, c\}$



$x(a) = 0$,
$x(b) = 2$
$x(c) = 3$
$x(a) = 1$

It is not fcn^c.

x is not r.v.

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(iii) R.V are denoted ~~by~~ capital letters etc. & their values by lower case letters.

~~Types of~~
Types of r.v
i) Discrete r.v
ii) Continuous r.v.

Discrete r.v:—

A r.v X is said to be discrete if it takes finite or countably infinite no. of values.
ie The possible values of X are isolated but need not be only int. values.
ie If $\Omega = \{w_1, w_2 \ldots w_n\}$ is sample space Then for discrete r.v. X, it is possible to assume values
$X(w_1) = x_1, \quad X(w_2) = x_2 \ldots \quad X(w_n) = x_n$ which are isolated.

eg:—
i) No. of students present in class
ii) No. of days of rainfalls in sangli city.
iii) No. of attempts required to pass the exam.
iv) No. of leaves taken by employee for period of one year.

Illustrations:
i) A lot contains 10 items of which 4 are defective. A random sample of 5 is taken from lot. If X is no. of defective articles in sample Then $X = \{0, 1, 2, 3, 4\}$
ii) A fair coin is tossed until head appears for 1st time. Let X be no. of trials needed Then
$X = \{1, 2, 3 \ldots\}$ infinite.

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Continuous r.v:

A r.v x is said to be continuous if it can take all possible values betⁿ certain limits ie uncountable.

In other words continue type of variables which involve measuring rather than counting. They are described by interval or union of interval

eg: age, height, weight, tempʳ, life of electric component etc.

Probability mass funᶜ (pmf)

Let x be discrete r.v defined on sample space Ω. suppose $\{x_1, x_2 \dots x_n\}$ be range set of x. With each of x_i we assign a number $p_i = P[x = x_i]$ called probability of x_i such that

(I) $p_i \geq 0 \quad i = 1, 2 \dots n$

(II) $\sum_{i=1}^{n} p_i = 1$

The funᶜ P defined above is called pmf of x.

Probability distribution:

The table containing the values of x along with probabilities given by pmf is called prob. distribution of r.v x.

x	x_1	x_2	$\dots$	x_n	Total
$P(x = x_i)$	p_1	p_2	$\dots$	p_n	1

(5)

Ex:-

1. A fair coin is tossed 3 times. Find prob. distribution of no. of heads.

→

Here the random expt is tossing of coin 3 times & x be r.v which denotes no. of heads observed.

values of x are 0, 1, 2, 3.

since $S = \{HHH, HHT, HTT, TTT, TTH, THH, THT, HTH\}$

$$x = \begin{cases} 0 & \text{iff outcome is } \{TTT\} \\ 1 & \text{iff outcome is } \{HTT\} \text{ or } \{TTH\} \text{ or } \{THT\} \\ 2 & \text{iff outcome is } \{HHT\} \text{ or } \{THH\} \text{ or } \{HTH\} \\ 3 & \text{iff outcome is } \{TTT\} \end{cases}$$

Therefore

$P(X=0) = P(TTT) = 1/8$

$P(X=1) = P(\{HTT\} \text{ or } \{TTH\} \text{ or } \{THT\})$

$= 3/8$

$P(X=2) = 3/8$

$P(X=3) = 1/8$

X	0	1	2	3	Total
$P(X)$	1/8	3/8	3/8	1/8	1

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(2) A die is thrown once. The no. on uppermost face is noted. Find prob. distribution.

→ $S = \{1, 2, 3, 4, 5, 6\}$

x — no. of points appearing on uppermost face

Values of x are 1, 2, 3, 4, 5, 6

x	1	2	3	4	5	6	
$P(X)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$\boxed{1}$

(3) Three cards are drawn at random successively with replacement from well shuffled pack of cards. Getting a card of diamond is termed as success. Obtain prob. distribution of no. of successes.

→ $x \to$ no. of successes in 3 cards drawn.

possible values of x are 0, 1, 2, 3.

$$P(X=0) = \frac{39}{52} \times \frac{39}{52} \times \frac{39}{52} = \frac{27}{64}$$

$$P(X=1) = \frac{13 \times 39 \times 39}{52 \times 52 \times 52} + \frac{39 \times 13 \times 39}{52 \times 52 \times 52} + \frac{39 \times 39 \times 13}{52 \times 52 \times 52} = \frac{27}{64}$$

$$P(X=2) = \frac{13 \times 13 \times 39}{52 \times 52 \times 52} + \frac{13 \times 39 \times 13}{52 \times 52 \times 52} + \frac{39 \times 13 \times 13}{52 \times 52 \times 52} = \frac{9}{64}$$

$$P(X=3) = \frac{13 \times 13 \times 13}{52 \times 52 \times 52} = \frac{1}{64}$$

x	0	1	2	3	
$P(X)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$	$\boxed{1}$

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$P(X=3) = \frac{13 \times 13 \times 13}{52 \times 52 \times 52} = \frac{1}{64}$

x	0	1	2	3
$P(x)$	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

4] Two fair dice are thrown. Find the prob. distribution of the sum of points appearing on uppermost faces.

H.W

5] A discrete r.v X has the following distribution

X	-2	-1	0	1	2	3
$P(X=x)$	0.1	k	0.2	$2k$	0.3	$3k$

find (i) k

ii) $P(X \geq 2)$ & $P(-2 < X < 2)$

iii) find mine value of c such that $P(X \leq c) > 0.4$

→ $\sum_{x=-2}^{3} P(X=x) = 1$

$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$

$\Rightarrow \boxed{k = 1/15} = 0.066$

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$P(X \geq 2) = P(X=2) + P(X=3)$

$= 1/2$

$P(-2 < X < 2) = P(X=-1) + P(X=0) + P(X=1)$

$= \frac{2}{5}$

For finding min. value of c we use trial error method

Take $c = 0$

$P(X \leq 0) = 0.1 + \frac{1}{15} + 0.2 = 0.36 < 0.5$

Take $c = 1$

$P(X \leq 1) = 0.36 + \frac{2}{15} = 0.\cancel{t}5 \not> 0.4$

$\therefore$ Min value of $c = 1$

(6) verify whether following fun^c can be regarded as pmf for given values of x

$$P(x) = \begin{cases} \frac{3x+2}{24} & x = 1, 2, 3 \\ 0 & o.w \end{cases}$$

→ For $P(x)$ to be pmf we must have

$P(x) \geq 0 \quad \forall x$

$\sum P(x) = 1$

Here

$P(x)$ is obviously non -ve for $x = 1, 2, 3$.

$\sum P(x) = 1$.

$\therefore P(x)$ is pmf

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Probability density function: -(pdf)

For a continuous r.v X, the funᶜ f(x) satisfying the following is known as pdf

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

iii) $P(a<x<b) = \int_{a}^{b} f(x)\,dx$ = area under $f(x)$ betⁿ ordinates $x=a$ & $x=b$

Remark :-

(1) $P(a<x<b) = P(a\leq x<b) = P(a<x\leq b)$
$= P(a\leq x\leq b)$

ie inclusion or non inclusion of end points does not change prob. which is not case in discrete distribution.

\# $\Gamma n = \int_0^{\infty} e^{-x} x^{n-1}\,dx$, $\int_0^{\infty} e^{-kx} x^{n-1}\,dx = \frac{\Gamma n}{k^n}$ $\boxed{\Gamma n = (n-1)! \quad \Gamma n+1 = n\Gamma n}$

(1) verify whether the following funᶜ can be considered as valid pdf

$$f(x) = \begin{cases} \frac{3x(2-x)}{4} & 0\leq x\leq 2 \\ 0 & o.w \end{cases}$$

→ Here $f(x) \geq 0 \quad \forall x$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 \frac{3x(2-x)}{4}\,dx$$

$= 1$

2. Are the following pdf? Justify.

i) $f(x) = \lambda e^{-\lambda x}$ $x \geq 0, \lambda > 0$

ii)

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 4-2x & 1 < x < 2 \\ 0 & o.w \end{cases}$$

→ (i) $f(x)$ is pdf

ii) $\cancel{\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx} = \cancel{\int_{-\infty}^{\infty} f(x)\,dx}$

$f(x) \geq 0$ for $0<x<1$ & $1<x<2$

& $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx$

~~iterative~~

$+ \int_2^{\infty} f(x)\,dx$

$= \int_0^1 2x\,dx + \int_1^2 (4-2x)\,dx$

$= 2 \neq 1$

$f(x)$ is not p.d.f

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8) If a random variable x has density fun^c

$$f(x) = \begin{cases} 1/4 & -2 < x < 2 \\ 0 & o.w \end{cases}$$

Find (i) $P(x < 1)$

(ii) $P(|x| > 1)$

(iii) $P(2x+3 > 5]$

(iv) $P(-1 < x < 1 \mid x > 0)$

→ $P(x < 1) = \int_{-\infty}^{1} f(x)\,dx$

$$= \int_{-2}^{1} \frac{1}{4}\,dx = 3/4$$

ii) $P(|x| > 1) = P(x > 1 \text{ or } -x > 1)$

$$= P(x > 1) + P(x < -1)]$$

$$= \int_{1}^{\infty} f(x)\,dx + \int_{-\infty}^{-1} f(x)\,dx$$

$$= \int_{1}^{2} \frac{1}{4}\,dx + \int_{-2}^{-1} \frac{1}{4}\,dx = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

iii) $P(2x+3 > 5) = P(2x > 2)$

$$= P(x > 1)$$

$$= 1 - P(x \le 1) \quad \text{or} \quad \int_{1}^{\infty} f(x)\,dx$$

$$= 1 - 3/4 = 1/4$$

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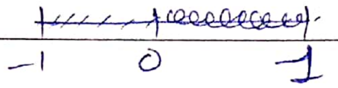
$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

iv) $P(-1 < x < 1 \mid x > 0) = \dfrac{P[(-1 < x < 1) \cap (x > 0)]}{P(x > 0)}$



$$= \frac{P(0 < x < 1)}{P(x > 0)} = \frac{\int_0^1 \frac{1}{4}\,dx}{\int_0^2 \frac{1}{4}\,dx}$$

$$= \frac{1/4}{1/2} = \frac{1}{2}$$

(4) The daily consumption of electric power (in millions of kW hrs) is r.v having pdf

$$f(x) = \begin{cases} \frac{1}{9}\, x\, e^{-x/3} & x > 0 \\ 0 & x \le 0 \end{cases}$$

If total production is 12 million kW hrs. Determine the prob. that there is powercut (shortage) on any day

→ If daily consumption exceeds 12 million kW hrs then there is power cut.

ie we have to find $P(x > 12)$

Consider $P(x \le 12) = \int_0^{12} f(x)\,dx = \int_0^{12} \frac{1}{9}\, x e^{-x/3}\,dx$

$$= \frac{1}{9}\left[(x)\left(\frac{e^{-x/3}}{-1/3}\right) - (1)\left(\frac{e^{-x/3}}{1/9}\right)\right]_0^{12}$$

$$= \frac{1}{9}\left[\left(-36e^{-4} - 9e^{-4}\right) - (0 - 9)\right] = \frac{1}{9}\left[-45e^{-4} + 9\right]$$

$$= 1 - 5e^{-4}$$

we have $P(x > 12) = 1 - P(x \le 12)$

$$= 1 - (1 - 5e^{-4}) = 5e^{-4} = 0.09157$$

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5 suppose a continuous r.v x has pdf

$$f(x) = \begin{cases} K(1-x^2) & 0<x<1 \\ 0 & o.w \end{cases}$$

a) Find K (ii) $P(0.1<x<0.2)$

c) $P(x>0.5)$

~~Using dist fun^c~~, determine

(i) $P(\cancel{x<0.3})$

→

As $f(x)$ is pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$K\int_0^1 (1-x^2)\,dx = 1$$

$$K\left[x - \frac{x^3}{3}\right]_0^1 = 1$$

$$K\left(1-\frac{1}{3}\right) = 1 \qquad \boxed{K = 3/2}$$

(ii) $$P(0.1<x<0.2) = \int_{0.1}^{0.2} K(1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.1}^{0.2}$$

$$= 0.1465$$

(iii) $$P(x>0.5) = \int_{0.5}^{\infty} \frac{3}{2}(1-x^2)\,dx = \int_{0.5}^{1} \frac{3}{2}(1-x^2)\,dx$$

$$= 0.3125$$

(14)

cumulative distribution fun^c (cdf) OR
distributive function (d.f)

a] For discrete r.v :-
Let X be discrete r.v taking values $x_1, x_2, \ldots x_n$. Then cdf is defined as

$$F(x) = P[X \leq x]$$

$$= \sum_{j=1}^{n} P_j$$

and it is tabulated as

X	x_1	x_2	x_3	----	x_n	Total
$P[X=x]$	P_1	P_2	P_3		P_n	1
$F(x)$	P_1	P_1+P_2	$P_1+P_2+P_3$	...	$P_1+P_2+\cdots+P_n = \sum_{j=1}^{n} P_j$	

Remark:-

(1) $P[X > x] = 1 - P[X \leq x]$

$P[X > x] = 1 - F(x)$

$$F(x_i) = \sum_{j=1}^{i} p_j$$

$$F(x_{i-1}) = \sum_{j=1}^{i-1} p_j$$

$$F(x_i) - F(x_{i-1}) = \sum_{j=1}^{i} p_j - \sum_{j=1}^{i-1} p_j$$

$$= p_i$$

$$= P[X = x_i]$$

ie successive differences of cdf gives probability distribution of x.

eg:- Let Probability dist. of x be

x	0	1	2	3	Total
$P(x)$	0.2	0.3	0.3	0.2	1

Then distribution func. of x is

x	0	1	2	3
$F(x)$	0.2	0.5	0.8	1

(ii) $F(x)$ is defined for each & every value of x

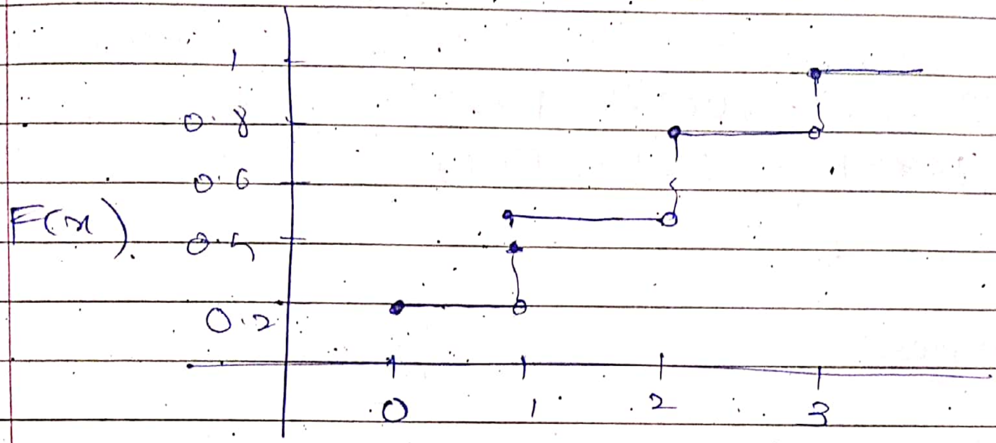
(iii) $0 \leq F(x) \leq 1$ (why?)

(iv) The graph of distribution func looks like step so it is also known as step func.

($F(x)$ remains constant betn two consecutive values of x & takes jumps at each value of X)

eg:

X	0	1	2	3
$F(x)$	0.2	0.5	0.8	1
$p(x)$	0.2	0.3	0.3	0.2



$F(-\infty) = 0$, $\quad F(\infty) = 1$ $\qquad F(-\infty) = \lim_{x \to -\infty} F(x) = 0$

If a & b real nos where $a<b$

v) $P(a < x < b) = p(x \in (a,b))$

$= \;\; a \; (\longrightarrow) \; b$

$= P(x \le b) - P(x \le a) - P(x = b)$

$= F(b) - F(a) - P(b)$

vi) $P(a \le x \le b) = P(x \le b) - P(x \le a) + P(x = a)$

$F(b) - F(a) + P(a)$

vii) $P(a < x \le b) = P(x \le b) - P(x \le a)$

$= F(b) - F(a)$

$P(a \le x < b) = p(x \le b) - P(x \le a) - P(x=b) + P(x=a)$

$= F(b) - F(a) - P(b) + P(a)$

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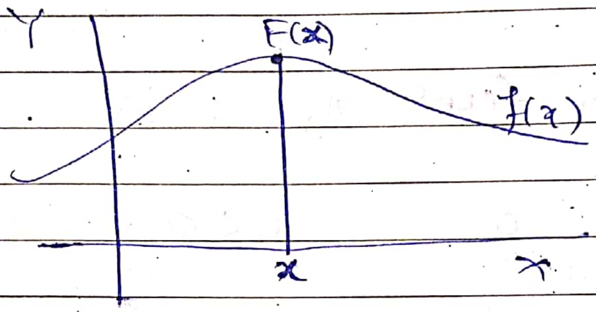
(b) cdf of continuous r.v

We have described distribution func. for discrete r.v. sim'y we can define d.f for conti. r.v. by replacing summation by int.

Geometrically it means area under curve bdd by x line & ~~line~~ below line $X = x$ where x is pt. at which dist func is sought.



Def^n. –

Let X be cont. r.v. with pdf $f(x)$ then

$$F(x) = P[X \le x]$$

$$= \int_{-\infty}^{x} f(t)\,dt$$

Remark : –

Let $f(x)$ & $F(x)$ denote pdf & d.f for conti. r.v resp.

(I) $F(x)$ is defined for every real no. x

(II) $0 \le F(x) \le 1$

(III) Derivative of $F(x)$ exist for all x & $F'(x) = \frac{d}{dx} F(x) = f(x)$

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In other words the cdf for continuous r-v is found by int. pdf. & pdf of continuous r-v can be obtained by diffentiati cdf. (This is from fundamental Thm of calculus). This is relation betn cdf & pdf.

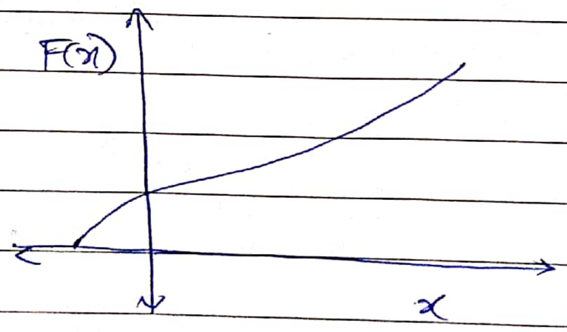
(iv) $F(-\infty) = \lim_{x \to -\infty} F(x)$

$= \lim_{x \to -\infty} \int_{-\infty}^{x} f(t)\,dt = 0$

$F(\infty) = \lim_{x \to \infty} F(x)$

$= \lim_{x \to \infty} \int_{-\infty}^{x} f(t)\,dt = 1$ (As $f(x)$ is pdf)

v) The graph of distribution func of continuous r-v is smooth unbroken curve.



vi) $P(a < x < b) = P(a \le x < b) = P(a < x \le b)$

$= P(a \le x \le b) = F(b) - F(a)$.

(for continuous r.v inclusion or non inclusion doesn't affect on prob.)

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(#) Following is distribution fun^c of discrete r v

x	-4	-3	-2	-1	0	1	2	3
$F(X)$	0.09	0.21	0.35	0.53	0.69	0.82	0.92	1

Find prob. dist. of x

(ii) $P(|x| \geq 2)$ (iii) $P[|x| \leq 1]$

iv) $P[x \geq 2 \mid x > 0]$ (v) $P(x > -2)$

→ Using

$$P(x_i) = F(x_i) - F(x_{i-1})$$

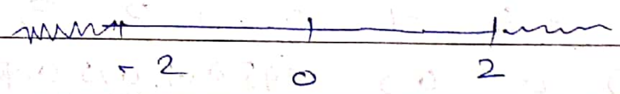
ie $P_1 = F_1$ ie $P(-4) = F(-4) = 0.09$

x	-4	-3	-2	-1	0	1	2	3
$P(x)$	0.09	0.12	0.14	0.18	0.16	0.13	0.10	0.08

$P(-3) = F(-3) - F(-4)$
$= 0.21 - 0.09 = 0.12$

$P(-2) = F(-2) - F(-3) = 0.35 - 0.21 = 0.14$

(ii) $P[|x| > 2) = P(x > 2 \text{ or } -x > 2)$
$= P(x > 2 \text{ or } x < -2)$



$= P(x=3) + P(x=-3) + P(x=-4)$
$= 0.29$

(iii) $P(|x| \leq 1) = P(x \leq 1 \text{ or } -x \leq 1)$
$= P(x \leq 1 \text{ or } x \geq -1)$
$= P(-1 \leq x \leq 1) = 0.18 + 0.16 + 0.13$
$= 0.47.$

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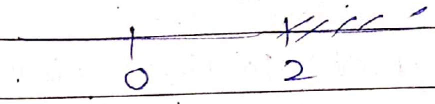
(iv) $P[X \geq 2 \mid X > 0] = \dfrac{P[X \geq 2 \text{ and } X > 0]}{P(X > 0)}$

$=$ 

$= \dfrac{P(X=2) + P(X=3)}{P(X=1) + P(X=2) + P(X=3)}$

$= \dfrac{0.10 + 0.08}{0.13 + 0.10 + 0.08}$

$= 0.58064$

(v) $P(X > -2) = P(-1) + P(0) + P(1) + P(2) + P(3)$

$= 0.18 + 0.16 + 0.13 + 0.10 + 0.8$

$= 0.65$

or

$P(X > -2) = 1 - P(X \leq -2)$

$= 1 - F(-2)$

$= 1 - 0.35 = 0.65$

3. The following is cdf of discrete r.v

X	-3	-1	0	1	2	3	5	8
F(X)	0.1	0.3	0.45	0.65	0.75	0.90	0.95	1

i) find pmf of X

ii) $P(0 < X < 2)$ (iii) $P(1 \leq X \leq 3)$

iv) $P(-3 < X \leq 2)$ (v) $P(-1 \leq X < 1)$

vi) $P(X = \text{even})$ (vii) $P(X > 2)$

(viii) $P(X \geq 3)$ (ix) $P(X = -3 \mid X < 0)$

(x) $P(X \leq 3 \mid X > 0)$

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→ we know that

$\therefore P(x_i) = F(x_i) - F(x_{i-1})$

~~P(-3) = F(-1) - F(-3)~~

$P(-3) = F(-3) = 0.1$

$P(-1) = F(-1) - F(-3)$

$= 0.3 - 0.1 = 0.2$

x	-3	-1	0	1	2	3	5	8
P(x)	0.1	0.2	0.15	0.20	0.10	0.15	0.05	0.05

(ii) $P(0 < x < 2) = P(x=1) = 0.2$

OR

$P(0 < x < 2) = F(2) - F(0) - P(2)$

$= 0.75 - 0.45 - 0.1 = 0.2$

(iii) $P(1 \le x \le 3) = 0.2 + 0.1 + 0.15 = 0.45$

OR

$= F(3) - F(1) + P(1)$

$= 0.90 - 0.65 + 0.20 = .45$

(iv) $P(-3 < x \le 2) = 0.2 + 0.15 + 0.20 + 0.10$

$= 0.65$

OR

$P(-3 < x \le 2) = F(2) - F(-3)$

$= 0.75 - 0.1 = 0.65$

(v) $P(-1 \le x < 1) = 0.2 + 0.15 = 0.35$

or

$= F(1) - F(-1) - P(1) + P(-1)$

$= 0.65 - 0.3 - 0.2 + 0.2 = 0.35$

(vi) $P(X = \text{even}) = P(0) + P(2) + P(8)$

$= 0.3$

(vii) $P(X > 2) = P(3) + P(5) + P(8)$

$= 0.15 + 0.05 + 0.05$

$= 0.25$

OR

$= 1 - F(2)$

$= 0.25$

(viii) $P(X \geq 3) = 0.25$

OR

$= 1 - P(X < 3)$

$= 1 - [F(3) - P(3)]$

$= 1 - [0.90 - 0.15]$

$= 0.25$

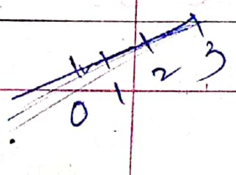
(ix) $P(X = -3 \mid X < 0)$

$= \dfrac{P[X = -3 \cap X < 0]}{P(X < 0)}$

$= \dfrac{P(X = -3)}{P(X < 0)}$

$= \dfrac{0.1}{0.3} = 1/3$

(x) $P(X \leq 3 \mid X > 0) = \dfrac{P[(X \leq 3) \cap (X > 0)]}{P(X > 0)}$



$= \dfrac{\cancel{P(X>0)}\; P(0 < X \leq 3)}{P(X > 0)}$

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$$= \frac{0.3 + 0.15}{0.55} = \frac{0.45}{0.55} = 0.8182$$

4] Let X be discrete r.v taking 3 values 10, 20, 30 with prob. $3c^2$, $4c - 10c^2$, $5c - 1$

(i) Find c (ii) obtain distribution of X

(iii) Plot dist. function obtain in (ii) above.

→ since

$$\sum p_i = 1.$$

$$3c^2 + 4c - 10c^2 + 5c - 1 = 1$$

$$-7c^2 + 9c - 1 = 1$$

$$7c^2 - 9c + 2 = 0$$

$$7c^2 - 7c - 2c + 2 = 0$$

$$7c(c-1) - 2(c-1) = 0$$

$$c = 1, \quad c = 2/7$$

But for $c = 1$, $P(x = 10) = 3c^2 = 3 > 1$ not possible

$\therefore$ $c = 1$ is not possible

$\therefore$ $\boxed{c = 2/7}$

(ii) Dist. func:

	X	10	20	30
$\frac{8}{7} - \frac{40}{49}$	$P(X)$	$\frac{12}{49}$	$\frac{16}{49}$	$\frac{3}{7}$
$\frac{10}{7} - 1$	$F(X)$	$\frac{12}{49}$	$\frac{28}{49}$	1

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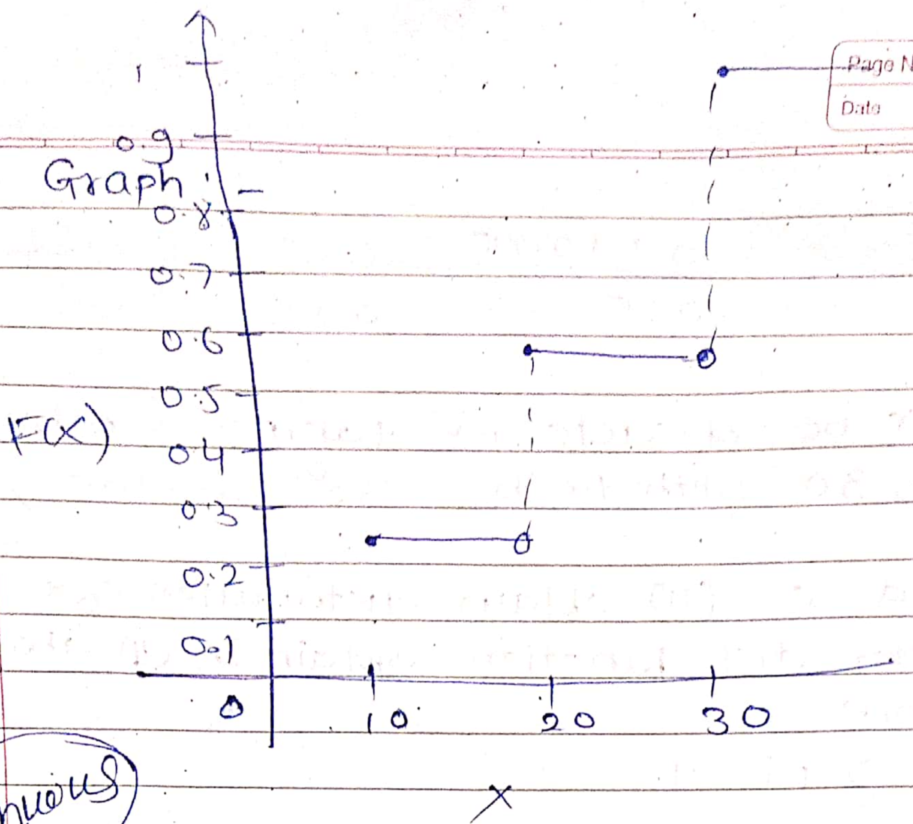
Graph :



Continuous

5. If the r.v X has pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1) & -1 < x < 1 \\ 0 & \text{o.w} \end{cases}$$

(I) Find distribution func of X

(II) and hence find $P\left[-\frac{1}{2} < X < \frac{1}{3}\right]$

(I) For case $-1 < x < 1$

$$F(x) = P[X \le x]$$

$$= \int_{-\infty}^{x} f(t)\,dt$$

$$= \int_{-1}^{x} \frac{1}{2}(t+1)\,dt = \frac{1}{2}\left[\frac{t^2}{2} + t\right]_{-1}^{x}$$

$$= \frac{1}{2}\left\{\left(\frac{x^2}{2} + x\right) - \left(\frac{1}{2} - 1\right)\right]$$

for

for

2

→

*

$$= \frac{1}{2}\left\{ \frac{x^2}{2} + x + \frac{1}{2} \right\} = \frac{x^2+2x+1}{4} \quad -1 < x < 1$$

case (II) For $x < -1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{-1} \underset{0}{\underset{\uparrow}{f(x)}}\,dx = 0$$

case (III) for $x > 1$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{-1}$$

$$= \int_{-\infty}^{-1} \underset{0}{\underset{\uparrow}{f(x)}}\,dx + \int_{-1}^{1} f(x)\,dx + \int_{1}^{x} \underset{0}{\underset{\uparrow}{f(x)}}\,dx.$$

$$= \int_{-1}^{1} \frac{x+1}{2}\,dx = \frac{1}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right)\right]$$

$$= 1$$

$$\therefore F(x) = \begin{cases} 0, & x < -1 \\ \dfrac{x^2+2x+1}{4}, & -1 < x < 1 \\ 1 & x > 1 \end{cases}$$

Now,

$$P\left[-\tfrac{1}{2} < x < \tfrac{1}{3}\right] = F\left(\tfrac{1}{3}\right) - F\left(-\tfrac{1}{2}\right)$$

$$= \left[\frac{\left(\frac{1}{3}\right)^2 + \frac{2}{3} + 1}{4}\right] - \left[\frac{\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) + 1}{4}\right]$$

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(26) The dif of r.v is

$$F(x) = \begin{cases} 0 & x < 0 \\ 2x^2 & 0 \le x \le 1/2 \\ 4x - 2x^2 - 1 & 1/2 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Find pdf of x

→ As we know that

$$f(x) = \frac{d}{dx} F(x)$$

$$\therefore f(x) = \begin{cases} 0 & x < 0,\ x > 1 \\ 4x & 0 \le x \le 1/2 \\ 4 - 4x & 1/2 \le x \le 1 \end{cases}$$

suppose r.v takes values −2, −1, 0, 2 with resp. prob. $\frac{k+2}{10}$, $\frac{2k-3}{10}$, $\frac{3k-4}{10}$, $\frac{k+1}{10}$.

Find prob. dist. & dif of x.

→ As $\sum p_i = 1$

$$\Rightarrow \frac{k+2}{10} + \frac{2k-3}{10} + \frac{3k-4}{10} + \frac{k+1}{10} = 1$$

$$\Rightarrow \underline{\underline{k = 2}}$$

x	−2	−1	0	2
$P(x)$	4/10	1/10	2/10	3/10
$F(x)$	4/10	5/10	7/10	10/10 = 1

(27)

A projectile is fired at a target. The distn from the pt. of impact to centre of target (in m.) is r.v x with pdf

$$f(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & \text{o.w} \end{cases}$$

find.

i) $P(x < 0.4)$ (ii) Dist. fun^c of x.

→

i) $P(x < 0.4) = \int_0^{0.4} 6x(1-x)\,dx$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{0.4}$$

$$= 6\left[\frac{0.16}{2} - \frac{0.64}{3}\right]$$

$$= 0.35202$$

ii) Dist. fun^c of x.

case (i) if $x < 0$

$$F(x) = P[X \le x]$$

$$= \int_{-\infty}^{0} f(t)\,dt = 0$$

Case (ii) : $0 < x < 1$

$$F(x) = P[X \le x]$$

$$= \int_0^x f(t)\,dt$$

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$$= \int_0^x 6t(1-t)\,dt$$

$$= 6\left[\frac{t^2}{2} - \frac{t^3}{3}\right]_0^x$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]$$

$$= 3x^2 - 2x^3 \qquad 0 < x < 1$$

case (iii)

If $x > 1$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$= \int_{-\infty}^{0} f(t)\,dt + \int_0^1 f(t)\,dt + \int_1^x f(t)\,dt$$

$$= \int_0^1 6t(1-t)\,dt + \int_1^0 \cancel{6t(1-t)}\,dt$$

$$= 6\left[\frac{t^2}{2} - \frac{t^3}{3}\right]_0^1 + \cancel{6\left[\frac{t^2}{2} - \frac{t^3}{3}\right]_1^x}$$

$\frac{1}{2} - \frac{1}{3}$

$$= 6\left(\frac{1}{6}\right) + \cancel{6\left[\left(\frac{x^2}{2} - \frac{x^3}{3}\right) - \right]} \quad \underline{1}$$

Dist:

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 3x^2 - 2x^3 & 0 < x < 1 \\ 1 & x > 1 \end{cases}$$

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Bivariate Random Variable:-

So far we have learnt what is mean by a discrete r.v, probability distribution. Throughout we considered a single characteristic say X.

eg:- the no. of heads (X) occured when 3 coins are tossed, sum of two nos (X) on uppermost faces when two dices thrown etc. Hence setup was univariate concering single ~~However in~~ random variable.

However in many situation we are interested in two characteristic say X & Y simultaneously. ie there are two r.v to be observed at the same time.

In other words X & Y are two r.v's defined on same sample space Ω

eg:-

(I) Let X denote the no. of boys born in a hospital in one week & Y ~~that~~ of girls born in the same hospital in same week. Then X & Y are r.v both taking values 0, 1, 2....

(II) X may be sale of toothpaste of brand A while Y may be sale of —"— of B at certain shop.

(III) X may be age of student & Y may be his IQ. etc

These situations demand handling two variables at a time. Hence we combine them into an ordered pair (X, Y) & called it as bivariate or two dim. r.v on Ω

If no. of possible values (X, Y) is finite then (X, Y) is called bivariate discrete r.v

Note that (X, Y) is discrete iff X & Y both are discrete. Hence if X takes $x_1, x_2 \dots x_n$ & Y takes $y_1, y_2 \dots y_n$ Then range space of (X, Y) is

$$R_{(X,Y)} = \left\{ (x_i, y_j) \; ; \; \begin{matrix} i = 1, 2 \dots m \\ j = 1, 2 \dots n \end{matrix} \right\}$$

It contain $m \times n$ points.

Joint probability mass func^c of (X, Y) :-

Let (X, Y) be discrete bivariate r.v defined on Ω. Let range of (X, Y) be

$$R_{(X,Y)} = \left\{ (x_i, y_j) \; \middle| \; \begin{matrix} i = 1, 2 \dots m \\ j = 1, 2 \dots n \end{matrix} \right.$$

For each $(x_i, y_j) \in R_{(X,Y)}$ we define $P(x_i, y_j) = P_{ij}$ as follows.

$$P_{ij} = P(x_i, y_j) = P[X = x_i, Y = y_j]$$
$$i = 1, 2 \dots m$$
$$j = 1, 2 \dots n$$

satisfying the following condition

i) $P_{ij} \geq 0 \quad \forall i, j$

ii) $\sum_{i=1}^{m} \sum_{j=1}^{n} P_{ij} = 1$

The above func^c P is called joint pmf of (X, Y)

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The set $\{(x_i, y_j, P_{ij}) ;\ i=1,\dots m,\ j=1,2\dots n\}$ is called joint prob. distribution of (X, Y). It can be represented as

$X \backslash Y$	y_1	y_2	...	y_j	...	y_n	Total
x_1	P_{11}	P_{12}		P_{1j}		P_{1n}	$P_{1\cdot}$
x_2	P_{21}	P_{22}				P_{2n}	$P_{2\cdot}$
⋮							
x_i	P_{i1}	P_{i2}		P_{ij}		P_{in}	$P_{i\cdot}$
x_m	P_{m1}	P_{m2}				P_{mn}	$P_{m\cdot}$
Total	$P_{\cdot 1}$	$P_{\cdot 2}$		$P_{\cdot j}$		$P_{\cdot n}$	1

Thus

$$P_{ij} = P[X = x_i \cap Y = y_j] \quad i = 1,2\dots m,\ j = 1,2\dots n.$$

i^{th} row totals $= P_{i\cdot} = \sum_{j=1}^{n} P_{ij} = P_{i1} + P_{i2} + \dots + P_{in}$

j^{th} column total $= P_{\cdot j} = \sum_{i=1}^{m} P_{ij} = P_{1j} + P_{2j} + \dots + P_{mj}$

The row totals $P_{1\cdot}, P_{2\cdot}, \dots, P_{m\cdot}$ represent marginal probabilities of X while column totals $P_{\cdot 1}, P_{\cdot 2}, \dots, P_{\cdot n}$ represent marginal prob. of Y. Note $\sum_{i=1}^{n} P_{i\cdot} = \sum_{j=1}^{m} P_{\cdot j} = \sum_{i=1}^{m}\sum_{j=1}^{n} P_{ij} = 1$

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Marginal distr of X is

X	x_1	x_2	...	x_i	...	x_m	Total
$P(X=x_i)$	$P_{1\cdot}$	$P_{2\cdot}$	...	$P_{i\cdot}$	...	$P_{m\cdot}$	1

Marginal distr of Y is

Y	y_1	y_2	...	y_n	Total
$P(Y=y_j)$	$P_{\cdot 1}$	$P_{\cdot 2}$	...	$P_{\cdot n}$	1.

Joint distribution function of two dimensional discrete r.v :

The idea of joint distr function (distribution funn) in respect of bivariate distribution is the same as that of distribution func of univariate distributions.

Defn :-

Let (X, Y) be two dimensional discrete random variable defined on Ω. The joint cumulative distribution funC $F(x, y)$ is defined as

$$F(x,y) = P[X \le x, Y \le y], \quad x, y \in R$$

* Properties :-

1) The distribution funC $F(x,y)$ is non decreasing in each of variables i.e

(a) If $y_1 \le y_2$, $F(x, y_1) \le F(x, y_2)$

(b) If $x_1 \le x_2$, $F(x_1, y) \le F(x_2, y)$

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ii) $0 \le F(x,y) \le 1$

iii) $\lim_{\substack{x \to -\infty \\ y \to -\infty}} F(x,y) = 0$, $\lim_{\substack{x \to \infty \\ y \to \infty}} F(x,y) = 1$

iv) let a, b, c, d be any real nos. with $a<b$, $c<d$ then

$$P[a<X\le b,\ c<Y\le d] = F(b,d) + F(a,c) - F(b,c) - F(a,d)$$

v] $F(x,\infty)$ = dist^r fun^c of x alone

$F(\infty,y)$ = —u— Y alone

(1) Two fruits are to be selected at random & without replacement from 4 mangoes, 2 oranges & 3 apples. Let X & Y denote resp. of the mangoes & oranges selected. Obtain the joint prob. dist of (X, Y) and evaluate distribution or fun $x=1$ and $y=0$

→

Possible values of X are 0, 1, 2
Y are 0, 1, 2

The total no. of ways of selecting two fruits from group of 9 fruits is $^9C_2 = \frac{9!}{2!\,7!} = \frac{9\times 8}{2} = 3$

X \ Y	0	1	2	Total
0	3/36	6/36	1/36	10/36
1	12/36	8/36	0	20/36
2	6/36	0	0	6/36
Total	21/36	14/36	1/36	1

since $P(X=0, Y=0) = \frac{^3C_2}{^9C_2} = \frac{3}{36}$

$P(X=0, Y=1) = \frac{^2C_1 \cdot {^3C_1}}{^9C_2} = \frac{6}{36}$

$P(X=0, Y=2) = \frac{^2C_2}{^9C_2} = \frac{1}{36}$

$P(X=1, Y=0) = \frac{^4C_1 \cdot {^3C_1}}{^9C_2} = \frac{12}{36}$

$P(X=1, Y=1) = \frac{^4C_1 \cdot {^2C_1}}{^9C_2} = \frac{8}{36}$

runs - Take full seq^n & cut it at each point where there is transition.

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$P(X=1, Y=2) = 0$ As we have to select only two fruits at random

$P(X=2, Y=0) = \frac{4C_2}{9C_2} = \frac{6}{36}$

$P(X=2, Y=1) = 0$

$P(X=2, Y=2) = 0$

(ii) $F(1,0) = P[X \le 1, Y \le 0]$

$= P[(0,0) \cup (1,0)]$

$= \frac{3}{36} + \frac{12}{36} = \frac{15}{36}$

(2) Let a fair coin be tossed 3 times. If X & Y denote no. of tails & the no. of runs resp. obtain joint prob. distribution of X & Y.

→

	sample pt.	HHH	HHT	HTT	TTT	TTH	THH	HTH	THT
value of X	No. of tails	0	1	2	3	2	1	1	2
value of Y	No. of runs	1	2	2	1	2	2	3	3

X \ Y	1	2	3	
0	1/8	0	0	1/8
1	0	2/8	1/8	3/8
2	0	2/8	1/8	3/8
3	1/8	0	0	1/8

As $P(X=0, Y=1) = \frac{1}{8}$, $P(X=0, Y=3) = 0$

$P(X=0, Y=2) = 0$, $P(X=1, Y=1) = 0$

$P(X=1, Y=2) = P(HHT \text{ or } THH) = 2/8$
$P(X=2, Y=2) = P(HTT \text{ or } TTH) = 2/8$ etc

(3) An urn contain 3 tickets numbered 1, 2, 3 & two tickets are drawn in succession. If x is the no. on the first ticket drawn. & Y is second ——

find prob. dist of (X, Y).

→ Here we solve problem in two ways :—
(i). A ticket is replaced after draw (with replacement selection method)
(ii) A ticket is not replaced after draw. (Without replacement selection method)

(i). Possible value of $X = 1, 2, 3$
$Y = 1, 2, 3$

X \ Y	1	2	3
1	1/9	1/9	1/9
2	1/9	1/9	1/9
3	1/9	1/9	1/9

Here,

$P(X=1, Y=1) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$, $P(X=1, Y=2) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$ and so on

(ii) possible values of $X = 1, 2, 3$
$Y = 1, 2, 3$

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$X \backslash Y$	1	2	3
1	0	1/6	1/6
2	1/6	0	1/6
3	1/6	1/6	0

since,

$P(X=1, Y=1) = 0$ (since no replacement)

$P(X=1, Y=2) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$P(X=1, Y=3) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$P(X=2, Y=1) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$P(X=2, Y=2) = 0$ ($\because$ since no replacement)

$P(X=3, Y=3) = 0$

$P(X=2, Y=3) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$P(X=3, Y=1) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$P(X=3, Y=2) = \frac{1}{6}$

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4) Two beads are selected at random without replacement from a bowl containing 4 blue, 1 red, 2 black beads. let X — no. of red beads drawn.

Y – no. of black beads drawn.

(i) Find joint pmf of (X, Y)

(ii) obtain marginal pmfs of X & Y

(iii) $P(X<Y)$.

→ Possible value of X are $0, 1$

Y are $0, 1, 2$

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4 blue . 1 red . 2 black.

$P(X=0, Y=0) = P(\text{neither red nor black})$

$= P(\text{blue})$

$= \dfrac{4C_2}{7C_2} = \dfrac{2}{7}$

$P(X=0, Y=1) = P(1\text{ black}, 1\text{ blue})$

$= \dfrac{4C_1 \cdot 2C_1}{7C_2} = \dfrac{8}{21}$

$P(X=0, Y=2) = P(2\text{ black}) = \dfrac{2C_2}{7C_2} = \dfrac{1}{21}$

$P(X=1, Y=0) = P(1\text{ red}, 1\text{ blue})$

$= \dfrac{4C_1 \cdot 1C_1}{7C_2} = \dfrac{4}{21}$

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(39) $P(X=1, Y=1) = \frac{{}^1C_1 \cdot {}^2C_1}{{}^7C_2} = \frac{2}{21}$

$P(X=1, Y=2) = \frac{{}^1C_1 \cdot {}^2C_2}{{}^7C_2} = 0$ as only two beads are selected.

X \ Y	0	1	2	Total
0	2/7	8/21	1/21	15/21
1	4/21	2/21	0	6/21
Total	10/21	10/21	1/21	1

(ii) Marginal prob. dist. are

X	0	1
P(X)	15/21	6/21

Y	0	1	2
P(Y)	10/21	10/21	1/21

(iii) $P(X<Y) = P(0,1) + P(0,2) + P(1,2)$
$= \frac{8}{21} + \frac{1}{21} + 0 = \frac{9}{21} = \frac{3}{7}$

5] A committee of two persons is formed by selecting them at random & without replacement from group of 10 persons of whom 2 are mathematician, 4 are statisticians & 4 are engineers. Let X & Y denote the no. of mathematicians & statisticians resp. on the committee (i) Obtain joint prob. dist of X & Y (ii) F(1,1) & F(0,2).

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→ The possible values of x are 0, 1, 2
Y are 0, 1, 2.

Thus all possible ordered pairs are
(0,0), (0,1), (0,2), (1,0), (1,1), (1,2),
(2,0) (2,1), (2,2).

$P(X=0, Y=1) = P[1 \text{ engineer}, 1 \text{ statis}]$

$= \frac{{}^4C_1 \cdot {}^4C_1}{{}^{10}C_2} = \frac{16}{45}$.

$P(X=0, Y=0) = P(\text{engineer})$

$= \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45}$

$P(X=0, Y=2) = \frac{{}^4C_2}{{}^{10}C_2} = \frac{6}{45}$.

$P(X=1, Y=0) = \frac{{}^2C_1 \cdot {}^4C_1}{{}^{10}C_2} = \frac{8}{45}$

$P(X=1, Y=1) = \frac{{}^2C_1 \cdot {}^4C_1}{{}^{10}C_2} = \frac{8}{45}$.

$P(X=1, Y=2) = 0$ as the committee has only two persons. So there are no sample pt. corresponding to the events.

$P(X=2, Y=0) = \frac{{}^2C_2 \cdot {}^4C_0}{{}^{10}C_2} = \frac{1}{45}$

$P(X=2, Y=1) = 0$.

$P(X=2, Y=2) = 0$

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ie Joint prob. dist. of X, & Y as follow

X \ Y	0	1	2	Total
0	6/45	16/45	6/45	28/45
1	8/45	8/45	0	16/45
2	1/45	0	0	1/45
Total	15/45	24/45	6/45	1

(ii) $F(1,1) = P[X \le 1, Y \le 1]$

$$= P[(0,0) \cup (0,1) \cup (1,0) \cup (1,1)]$$

$$= \frac{6}{45} + \frac{16}{45} + \frac{8}{45} + \frac{8}{45} = \frac{38}{45}$$

$F(0,2) = P[X \le 0, Y \le 2]$

$$= P[(0,0) \cup (0,1) \cup (0,2)]$$

$$= \frac{6}{45} + \frac{16}{45} + \frac{6}{45} = \frac{28}{45}$$

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(42)

(6) Two fair dice are thrown. Let X denote the absolute difference betn two scores & Y denote maxm of two scores.
1) obtain joint prob. dist of (X, Y)
(ii) Also obtain marginal distribution of {X & Y}.

→

Here

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

~~Her~~ The possible values of X = 0, 1, 2, 3, 4, 5
— " — of Y = 1, 2, 3, 4, 5, 6

X \ Y	1	2	3	4	5	6	Total
0	1/36	1/36	1/36	1/36	1/36	1/36	6/36
1	0	2/36	2/36	2/36	2/36	2/36	10/36
2	0	0	2/36	2/36	2/36	2/36	8/36
3	0	0	0	2/36	2/36	2/36	6/36
4	0	0	0	0	2/36	2/36	4/36
5	0	0	0	0	0	2/36	2/36
Total	1/36	3/36	5/36	7/36	9/36	11/36	1

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As,

$P(\underset{\text{diff}}{X=0}, \underset{\max^m}{Y=1}) = P(1,1) = 1/36$

$P(X=0, Y=2) = P(2,2) = 1/36$

$P(X=0, Y=3) = P(3,3) = 1/36$

$P(X=0, Y=4) = P(4,4) = 1/36$

$P(X=0, Y=5) = P(5,5) = 1/36$

$P(X=0, Y=6) = P(6,6) = 1/36$

$P(\underset{\text{diff}}{X=1}, \underset{\max}{Y=2}) = P[(1,2) \cup (2,1)] = 2/36$

$P(X=1, Y=1)$ = ie prob. that the diff. betn two nos. 1 & max^m 1 This is not possible

$\therefore P(X=1, Y=1) = 0$

$P(X=1, Y=3) = P[(2,3) \cup (3,2)] = 2/36$

$P(X=1, Y=4) = P[(3,4) \cup (4,3)] = 2/36$

$P(X=1, Y=5) = P[(4,5) \cup (5,4)] = 2/36$

$P(X=1, Y=6) = P[(5,6) \cup (6,5)] = 2/36$

How,

$P(X=2, Y=1)$ ie prob. that diff. is 2 & max^m 1 which is not possible $\Rightarrow P(X=2, Y=1) = 0$

and so on —

(ii) Marginal dist of X

X	0	1	2	3	4	5	Total
P(X)	6/36	10/36	8/36	6/36	4/36	2/36	1

Marginal dist. of Y

Y	1	2	3	4	5	6	Total
P(Y)	1/36	3/36	5/36	7/36	9/36	11/36	1