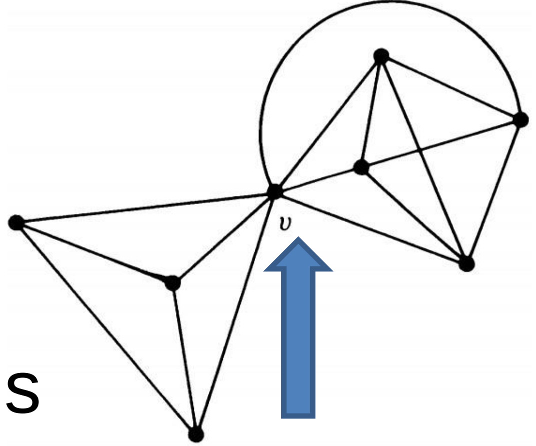
Connectivity, Separability & Isomorphism

Connected and Disconnected Graphs

A graph is **connected** if one can move from each vertex of the graph to every other vertex of the graph *along edges of the graph*. If not, the graph is **disconnected**. The connected pieces of a graph are called the **components** of the graph.

Connectivity & Separability



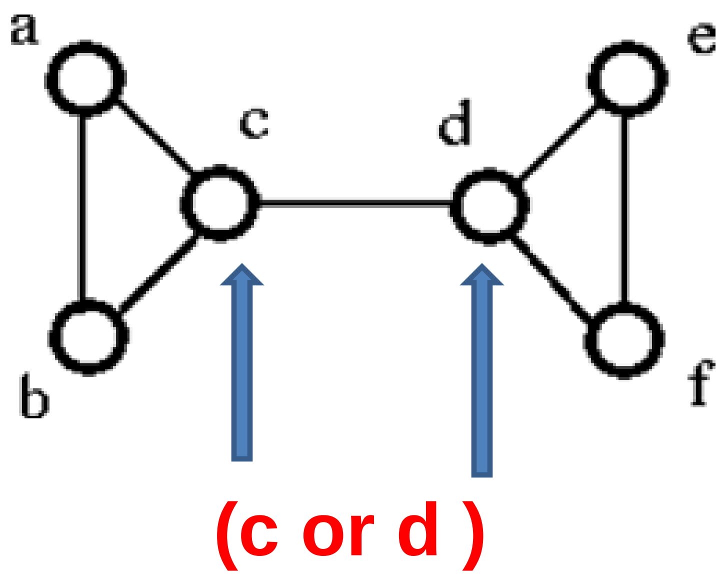
- The **Edge connectivity- λ(G):** of a connected graph G is the minimum number of edges whose removal makes G disconnected. When λ(G) ≥ k, then graph G is said to be **k-edge-connected**.
- The **Vertex connectivity- μ(G):** of a connected graph G is the minimum number of vertices whose removal makes G disconnected or reduces to a trivial graph.

Separable graph is having vertex connectivity to be one

- Theorem – A vertex *v* in a connected graph *G* is a **cut-vertex** iff there exist two vertices *x* and *y* in *G* such that every path between *x* and *y* passes through *v*

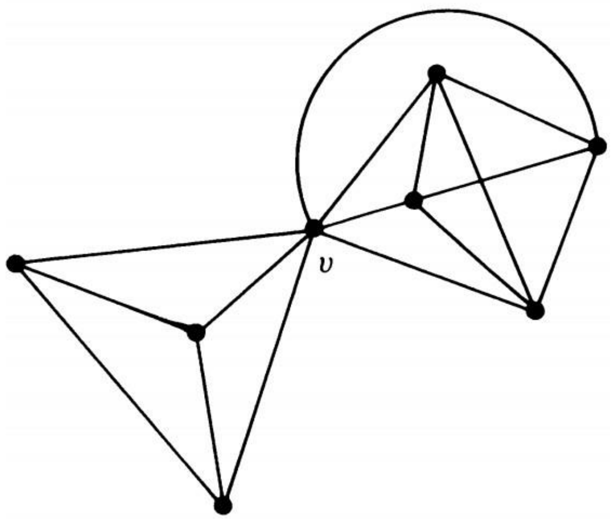
Vertex Connectivity

1-connected graph: by removing one minimum vertex the connected graph becomes disconnected graph.



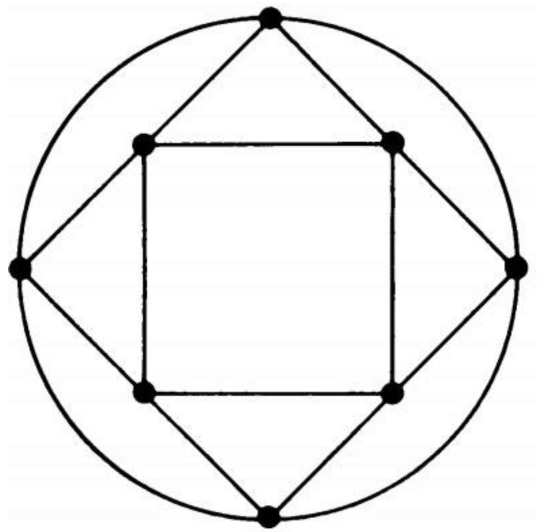
Connectivity & Separability

Graph with n=8, e=16



Edge Connectivity = 3

Vertex Connectivity = 1



Edge Connectivity = 4

Vertex Connectivity = 4

Connectivity & Separability

Theorem – The edge connectivity of a graph *G* cannot exceed the degree of the vertex with the smallest degree in *G*

Theorem – The vertex connectivity of any graph *G* can never exceed the edge connectivity of *G*

Connectivity & Separability

- Theorem – The maximum vertex connectivity one can achieve with a graph *G* of *n* vertices and *e* edges($e \geq (n-1)$) is the integral part of the number $2e/n$; that is, *floor(2e/n)*

n

m-connected graph

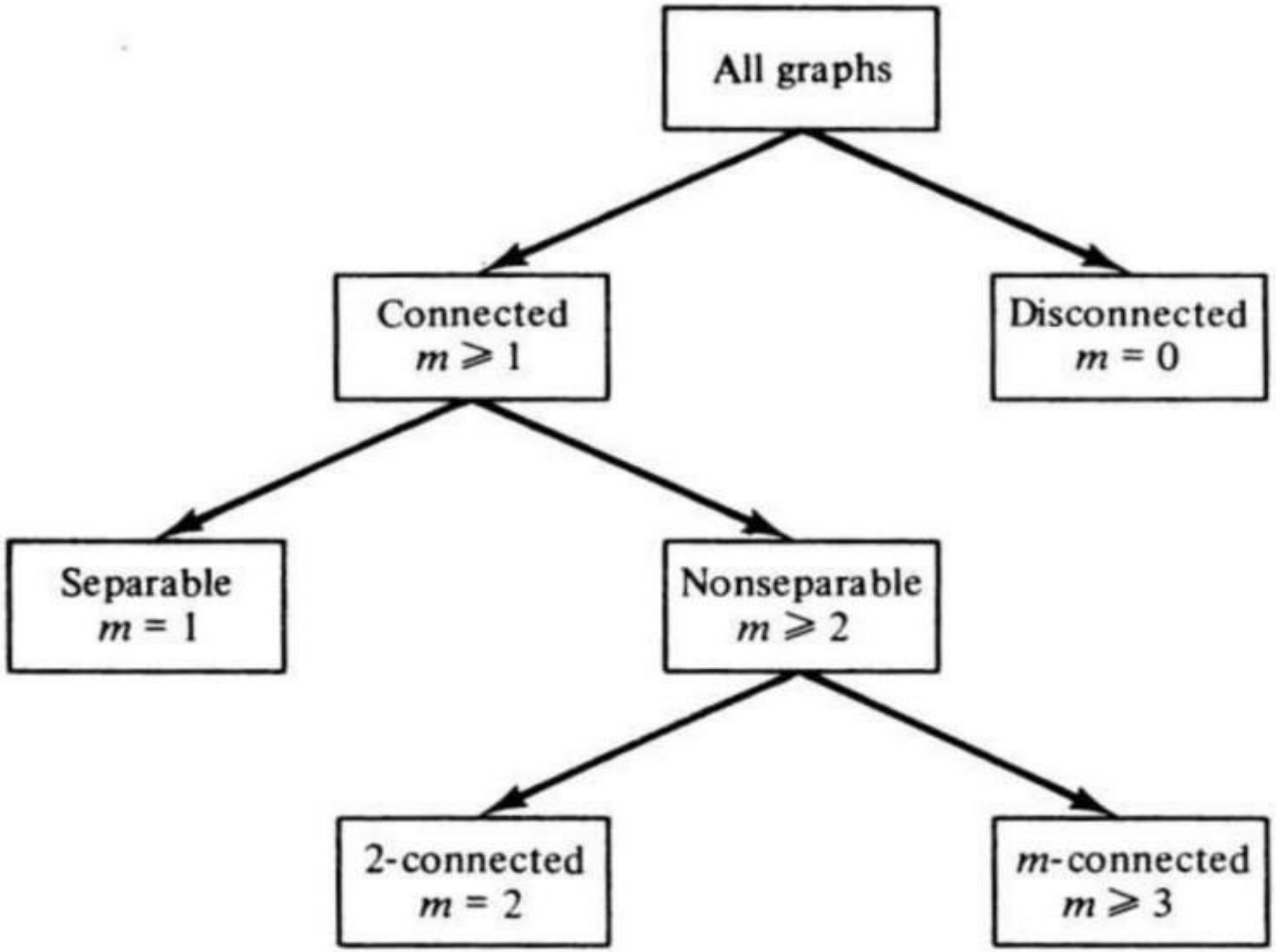
- *m-connected:* if the vertex connectivity of *G* is *m*
- *1-connected* graph is a separable graph

Theorem: A connected graph *G* is *m*-connected iff every pair of vertices in *G* is joined by *m* or more paths that do not intersect, and at least one pair of vertices is joined by exactly m nonintersecting paths.

Theorem: The edge connectivity of a graph *G* is m; iff every pair of vertices in *G* is joined by m or more edge-disjoint paths.

(i.e., paths that may intersect, but have no edges in common), and at least one pair of vertices is joined by exactly m edge-disjoint paths.

Summary

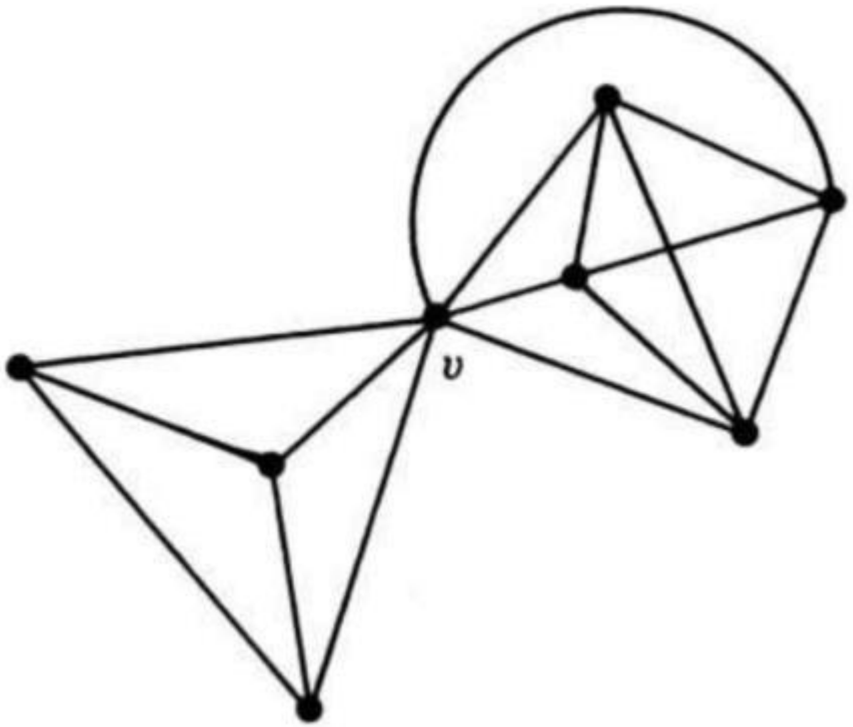

All graphs
Connected
$m \geqslant 1$
Disconnected
$m = 0$
Separable
$m = 1$
Nonseparable
$m \geqslant 2$
2-connected
$m = 2$
m-connected
$m \geqslant 3$

1-Isomorphism

- Blocks: largest nonseparable subgraphs in a graph

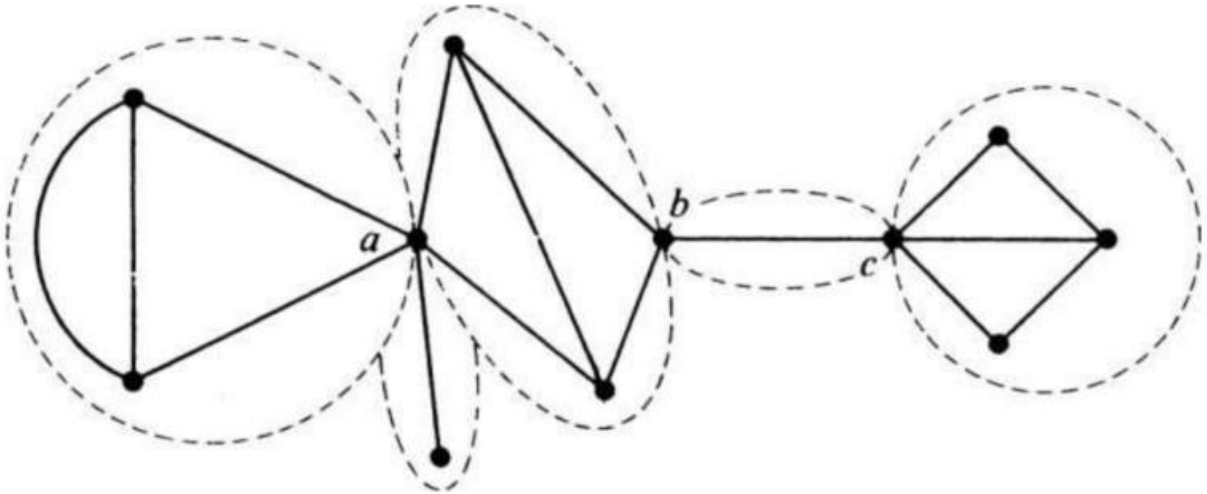
* Not to be confused with component

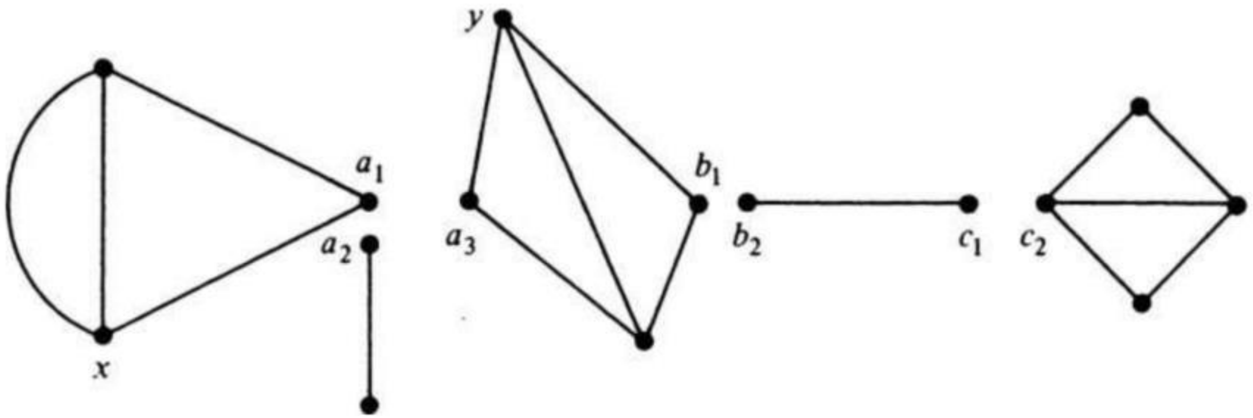
- If vertex v is removed this graph has two blocks



1-Isomorphism

- *Operation-1*: “Split” a cut-vertex into two vertices to produce two disjoint subgraphs
- Two graphs *G*1 and *G*2 are said to be *1-isomorphic* if they become isomorphic to each other under repeated application of the *Operation-1*.





1-Isomorphism

- Theorem - If $G1$ and $G2$ are two 1-isomorphic graphs, the rank of $G1$ equals the rank of $G2$ and the nullity of $G1$ equals the nullity of $G2$
- *Note: Split* operation increases the number of vertices by 1; Increases number of components by 1

→ Rank remains invariant

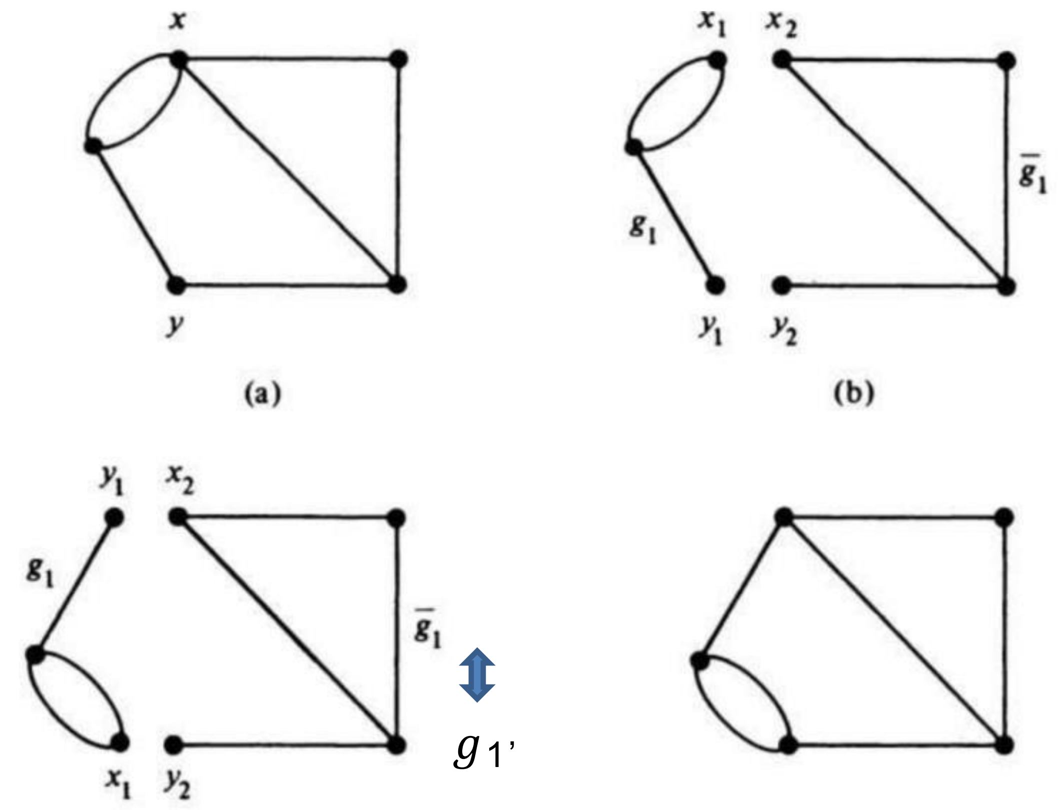
→ Nullity = number of edges - Rank

→ No edges are destroyed or no new edges are created

2-Isomorphism

Operation 2:

- "Split" the vertex x into x_1 and x_2 and the vertex y into y_1 and y_2 such that G is split into g_1 and $g_{1'}$
- Let vertices x_1 and y_1 go with g_1 and x_2 and y_2 with $g_{1'}$
- Now rejoin the graphs g_1 and $g_{1'}$ by merging x_1 with y_2 and x_2 wit y_1



2-Isomorphism

- Two graphs are said to be *2-isomorphic* if they become isomorphic after undergoing *operation 1* or *operation 2*, or both operations any number of times