Convert to clause form:

Convert the following statement to clause form:

$$\forall x[B(x) \rightarrow (\exists y [Q(x,y) \wedge \neg P(y)]$$
$$\wedge \neg \exists y [Q(x,y) \wedge Q(y,x)]$$
$$\wedge \forall y [\neg B(y) \rightarrow \neg E(x,y)])]$$

1- Eliminate the implication ($\rightarrow$)

$$E1 \rightarrow E2 \quad = \quad \neg E1 \vee E2$$

$$\forall x[\neg B(x) \vee (\exists y [Q(x,y) \wedge \neg P(y)]$$
$$\wedge \neg \exists y [Q(x,y) \wedge Q(y,x)]$$
$$\wedge \forall y [\neg(\neg B(y)) \vee \neg E(x,y)])]$$

2- Move the negation down to the atomic formulas (by using the following rules)

- $\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q$
- $\neg(P \vee Q) \quad \equiv \quad \neg P \wedge \neg Q$
- $\neg(\neg(P)) \quad \equiv \quad P$
- $\neg \forall x (P (x)) \quad \equiv \quad \exists x (\neg P (x))$
- $\neg \exists x (P (x)) \quad \equiv \quad \forall x (\neg P (x))$

$$\forall x[\neg B(x) \vee (\exists y [Q(x,y) \wedge \neg P(y)]$$
$$\wedge \forall y [\neg Q(x,y) \vee \neg Q(y,x)]$$
$$\wedge \forall y [B(y) \vee \neg E(x,y)])]$$

3- Purge existential quantifiers

The function that is eliminate the existential are called " Skolem function"

$\forall x [\neg B(x) \vee ([Q(x , f (x)) \wedge \neg P(f (x))]$

$\wedge \forall y [\neg Q(x,y) \vee \neg Q(y,x)]$
$\wedge \forall y [B(y) \vee \neg E(x,y)])]$

4- Rename variables, as necessary, so that no two variables are the same.

$\forall x [\neg B(x) \vee ([Q(x , f (x)) \wedge \neg P(f (x))]$

$\wedge \forall y [\neg Q(x,y) \vee \neg Q(y,x)]$

$\wedge \forall z [B(z) \vee \neg E(x,z)])]$

5- Move the Universal quantifiers to the left of the statement.

$\forall x \ \forall y \ \forall z [\neg B(x) \vee ([Q(x , f (x)) \wedge \neg P(f (x))]$

$\wedge [\neg Q(x,y) \vee \neg Q(y,x)]$

$\wedge [B(z) \vee \neg E(x,z)])]$

6- Move the disjunction down to the literals, using distributive laws

$\mathbf{E1 \vee (E2 \wedge E3 \wedge E4 \wedge \ldots)} \quad \equiv \quad (E1 \vee E2) \wedge (E1 \vee E3) \wedge \ldots.$
$\mathbf{E1 \wedge (E2 \vee E3 \vee E4 \vee \ldots)} \quad \equiv \quad (E1 \wedge E2) \vee (E1 \wedge E3) \vee \ldots.$

$\forall x \ \forall y \ \forall z [(\neg B(x) \vee (Q(x , f (x)) \wedge \neg P(f (x))))$
$\wedge [\neg B(x) \vee \neg Q(x,y) \vee \neg Q(y,x)]$
$\wedge [\neg B(x) \vee B(z) \vee \neg E(x,z)]]$

$\forall x \ \forall y \ \forall z [(\neg B(x) \vee (Q(x , f (x))$
$\wedge (\neg B(x) \vee \neg P(f (x)))$
$\wedge (\neg B(x) \vee \neg Q(x,y) \vee \neg Q(y,x))$
$\wedge (\neg B(x) \vee B(z) \vee \neg E(x,z))]$

7- Eliminate the conjunctions

$\forall x \;[\; \neg B(x) \lor (\; Q(x , f (x) \;]$

$\forall x \;[\neg B(x) \lor \neg P(f (x)) \;]$

$\forall x \; \forall y \;[\; \neg B(x) \lor \neg Q(x,y) \lor \neg Q(y,x) \;]$

$\forall x \; \forall z \;[\neg B(x) \lor B(z) \lor \neg E(x,z) \;]$

8- Rename all the variables, as necessary, so that no two variables are the same.

$\forall x \;[\; \neg B(x) \lor (\; Q(x , f (x) \;]$

$\forall w \;[\neg B(w) \lor \neg P(f (w)) \;]$

$\forall u \; \forall y \;[\neg B(u) \lor \neg Q(u,y) \lor \neg Q(y,u) \;]$

$\forall a \; \forall z \;[\neg B(a) \lor B(z) \lor \neg E(a,z) \;]$

9- Purg the universal quntifiers.

$\neg B(x) \lor (\; Q(x , f (x)$

$\neg B(w) \lor \neg P(f (w))$

$\neg B(u) \lor \neg Q(u,y) \lor \neg Q(y,u)$

$\neg B(a) \lor B(z) \lor \neg E(a,z))$

Resolution Theorem Proving

Resolution is a technique for proving theorems in the propositional or predicate calculus that has been a part of AI problem-solving.

Resolution describes a way of finding contradictions in a database of clauses with minimum use of *substitution.* Resolution refutation proves a theorem by negating the statement to be proved and adding this negated goal to the set of axioms that are known (have been assumed) to be true. It then uses the resolution rule of inference to show that this leads to a contradiction. Once the theorem prover shows that the negated goal is inconsistent with the given set of axioms, it follows that the original goal must be consistent. This proves the theorem. .

Resolution refutation proofs involve the following steps:

1. Put the premises or axioms into *clause form* .
2. Add the negation of what is to be proved, in clause form, to the set of axioms.
3. *Resolve* these clauses together, producing new clauses that logically follow from them.
4. Produce a contradiction by generating the empty clause.
5. The substitutions used to produce the empty clause are those under which the opposite of the negated goal is true.

The following example illustrates the use of resolution theorem for reasoning with propositional logic.

Unification of Predicates

Two predicates P (t1,t2,…, tn) and Q (s1, s2,…, sn) can be unified if terms ti can be replaced by si or vice-versa. Loves (mary, Y) and Loves (X, Father-of (X)) , for instance, can be unified by the substitution S ={ mary / X , Father-of (mary) / Y }.

Conditions of Unification:

1- Both the predicates to be unified should have an equal number of terms.

2- Neither ti nor si can be a negation operator, or predicate or functions of different variables, or if ti = term belonging to si or if si = term belonging to ti then unification is not possible.

Example 1 : Consider the following knowledge base:

1. ***The-humidity-is-high*** v ***the-sky-is-cloudy***.
2. If ***the-sky-is-cloudy*** then ***it-will-rain***
3. If ***the-humidity-is-high*** then ***it-is-hot***.
4. ***it-is-not-hot***

and the goal : ***it-will-rain***. Prove by resolution theorem that the goal is derivable from the knowledge base.

***Proof**:* Let us first denote the above clauses by the following symbols.

p = ***the-humidity-is-high***, q = ***the-sky-is-cloudy***, r = ***it-will-rain***, s = ***it-is-hot***. The Conjuctive Normal Form (CNF) from the above clauses thus become :

1. p v q

2. ¬ q v r

3. ¬ p v s

4. ¬ s

and the negated goal = ¬ r. Set S thus includes all these 5 clauses. Now by resolution algorithm, we construct the solution by a tree. Since it terminates with a null clause, the goal is proved.

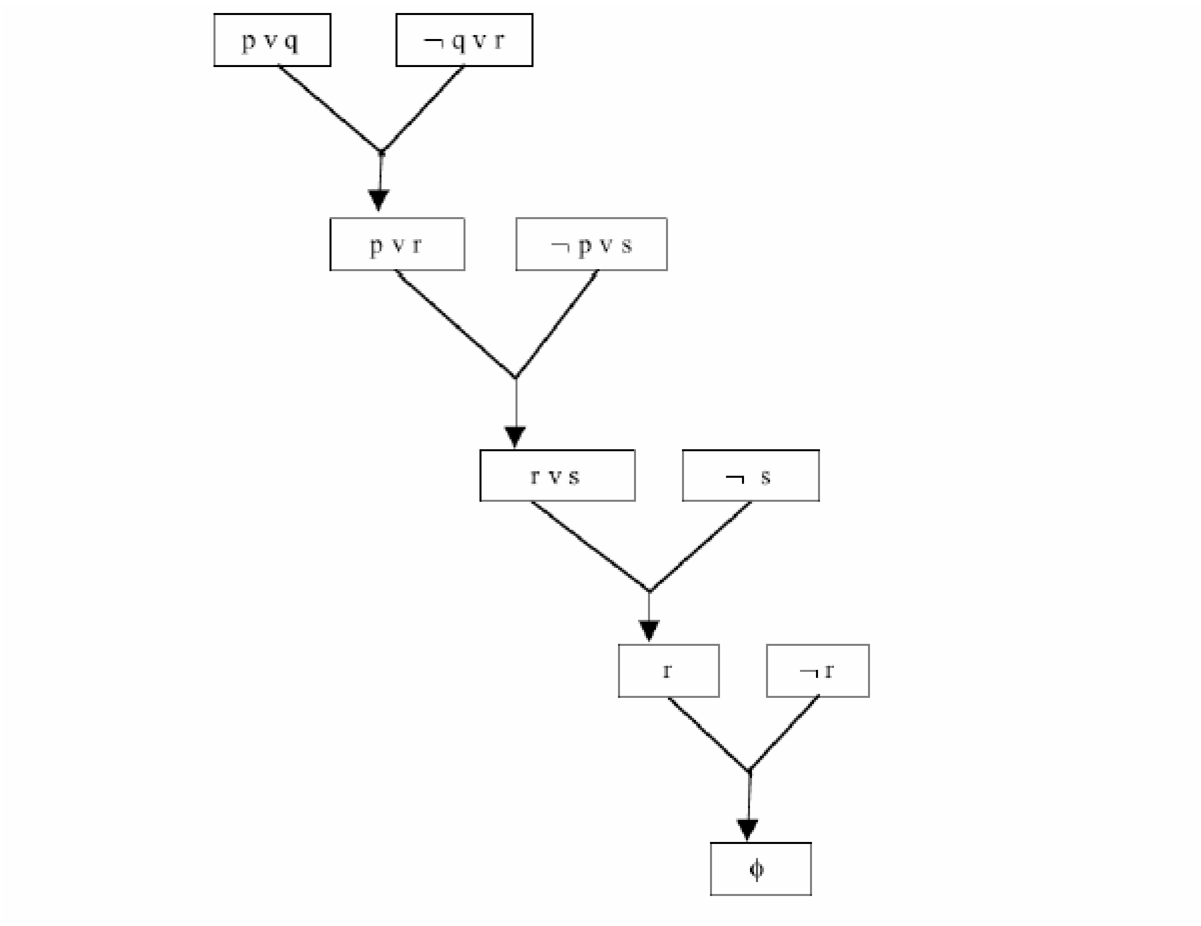

Fig. 1: The resolution tree to prove that it-will-rain.

Example 2:

"All people who are not poor and are smart are hippy. Those people who' read are not stupid. John can read are is wealthy. Happy people have exciting lives. Can anyone be found with an exciting life?"

Sol.:

a) First change the sentences to predicate form:

We assume $\forall X$ (smart (X) $\equiv$ stupid (X) and $\forall Y$ (wealthy (Y) $\equiv$ poor (Y)),

and get:

$\forall X(\neg \text{Poor}\ (X) \wedge smart\ (X) \Rightarrow happy(X))$

$\forall Y(read(Y) \Rightarrow smart\ (Y))$

read (john) $\wedge \neg$ poor (john)

$\forall Z$ (happy (Z) $\Rightarrow$ exciting (Z)

The negation of the conclusion is:

$\neg \exists W$ (exciting (W))

b) These predicate calculus expressions for the happy life problem are transformed into the following clauses:

poor (X) $\vee \neg$ smart (X) $\vee$ happy (X)

$\neg$ read (Y) $\vee$ smart(Y)

read (john)

$\neg$ poor (john)

$\neg$ happy (Z) $\vee$ exciting (Z)

$\neg$ exciting (W)

The resolution refutation for this example is found in Figure 2.

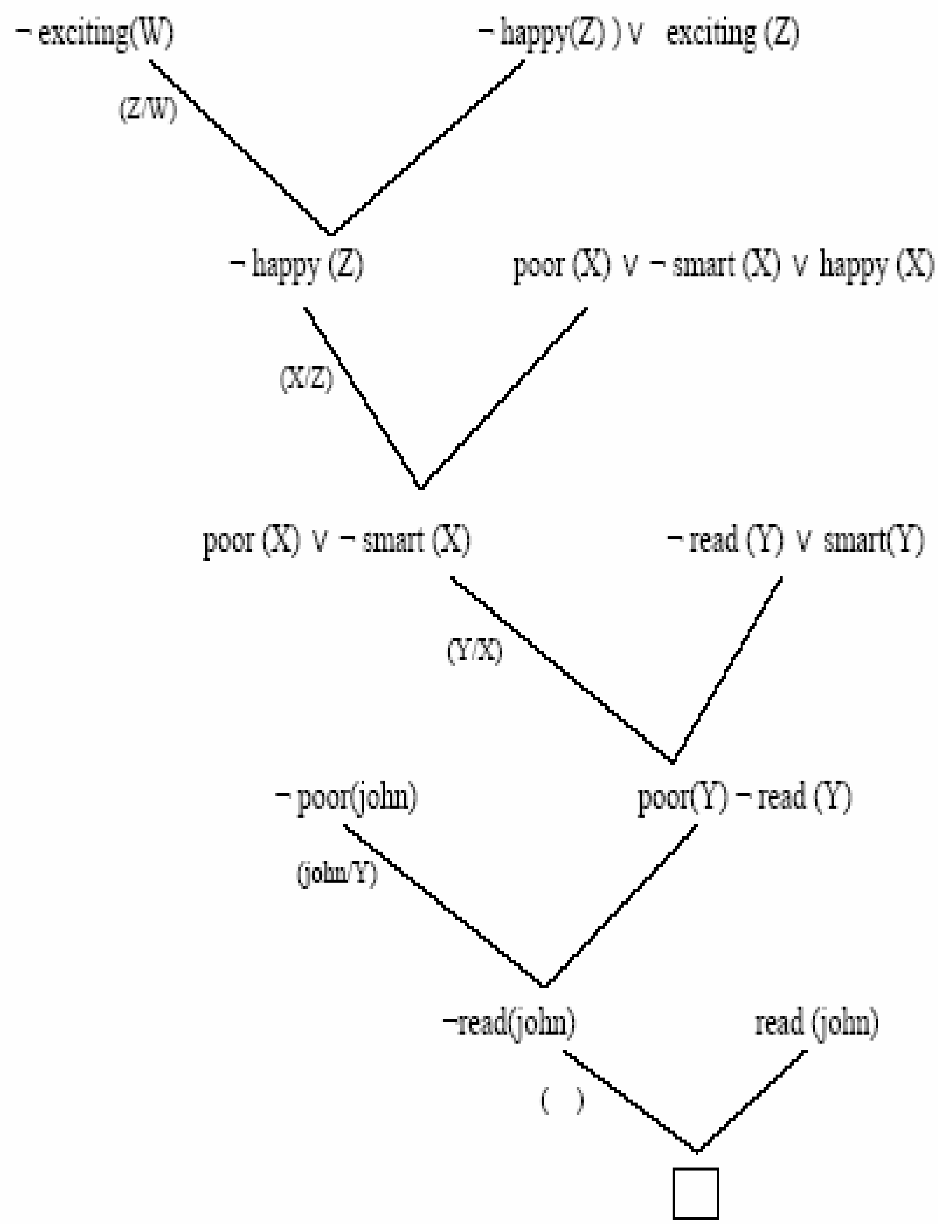

Figure (2): Resolution prove for the "exciting life" problem.