NUMERICAL FINANCE PROJECT

PRICING OF BERMUDAN BASKET OPTIONS

Lecturer : Kaiza AMOUH

Deadline : 5^{th} **May 2024**

SCOPE : The goal of this project is to price a Bermudan Basket Option using Monte Carlo simulation, and combine several variance reduction methods in order to reach a minimal simulation variance. The underlying diffusion is assumed to be a multi-dimensional Black-Scholes diffusion

The project can be implemented either in C++ or Python

- A main program must define and control the parameters to use, then call different Classes to perform the simulation
- The project's architecture is free, and its design will particularly be appreciated.
- The program must be fully commented and parameters must be checked for consistency.

In addition to your program, you must also produce a User Guide PDF file that :

- Explains the modifiable variables in the main function
- Illustrates your results for different sets of inputs through graphs and tables
- Details your analysis and comments

DELIVERY : This project must be sent by 5^{th} **May 2024** at **23:59** to the following address : kaiza.amouh@gmail.com

The sent email MUST contain the complete names of all team members. You can either attach your program and user guide, or send a **WeTransfer** link if your files are too heavy.

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1. Given n correlated assets $S^1, S^2, ..., S^n$, and some (possibly negative) weights $\alpha_1, \alpha_2, ..., \alpha_n$, a *European* Basket Call option pays the following payoff only at maturity T.

$$\left(\sum_{i=1}^{n} \alpha_i S_T^i - K\right)^+ \tag{1}$$

(a) Perform a Monte Carlo Simulation using basic Pseudo-Random numbers, without implementing any variance reduction method.

(b) Show the gain in variance and the gain in required number of simulations to enter a given confidence interval. You should cumulatively include the following variance reduction methods:

(i) Quasi-Random numbers
(ii) Static Control Variate
(iii) Antithetic Random variables

2. A *Bermudan* option with exercise dates $t_0 = 0, t_1, ..., t_N = T$ pays upon the (random) exercise date τ, the amount

$$\left(\sum_{i=1}^{n} \alpha_i S_\tau^i - K\right)^+ \tag{2}$$

(a) Using basic Pseudo-Random numbers without any variance reduction, implement the *Longstaff-Schwarz* algorithm to price this option.

(b) Combining all the above-mentioned variance reduction methods, show the gain in variance and the gain in required number of simulations to enter a given confidence interval

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Simulation of a correlated Gaussian Vector
Let $X \sim \mathcal{N}(\mu, \Sigma)$ be a *Gaussian Vector*. If Σ is a diagonal matrix, one can independently simulate each $\mathcal{N}(0,1)$ component of the vector.
If Σ is not diagonal, components X_i must be simulated as linear combinations of $Y_i \sim \mathcal{N}(0,1)$

$$X = \mu + BY, \quad where\ B\ is\ a\ square\ matrix\ satisfying\ \ C = BB^t \tag{3}$$

- If Σ is invertible, its Cholesky decomposition generates a lower triangular matrix B
- If not, since Σ is always positive semidefinite, one can use the diagonalization process and find

$$\Sigma = ODO^t \quad where\ O\ is\ an\ Orthogonal\ matrix\ and\ D\ Diagonal \tag{4}$$

One can then choose $B = OD^{\frac{1}{2}}$