CMSE 820 HW4

This HW is due on Oct 6st at 11:59 pm.

Question 1: For a norm $\|\cdot\|$ on $\mathbb{R}^{p\times n}$, the associated dual norm, denoted as $\|\cdot\|^*$, is defined

$$\|G\|^* = \max_{\|B\|\leq 1} \mathrm{Tr}(B^T G).$$

a. Prove that the dual norm associated with spectral norm is the nuclear norm. Here the spectral norm of matrix $A \in \mathbb{R}^{p\times n}$ is $\|A\| = \sigma_1(A)$, where $\sigma_1(A)$ is the laregest singular value of A. (Hint: you may need to use the von neumann's inequality)

b. Prove that the dual norm associated with nuclear norm is the spectral norm.

c. Prove that the nuclear norm is a convex function.

Question 2: For the matrix completion problem, assume the data point $x \in \mathbb{R}^p$ (with the top M entries are unobserved) is not precise and has some noise. Given we know the subspace S in term of $\mu \in \mathbb{R}^p$ and $U_d\mathbb{R}^{p\times d}$, we can compute y and x_U through the following optimization problem:

$$\min_{y,x_U} \|x - \mu - Uy\|^2.$$

Derive the closed-form solution for y and x_U.

Question 3: Exercise 3.7 (Properties of the $\ell_{2,1}$ Norm) from the book "Generalized Principal Component Analysis" which you can find in the Reference folder on D2L.