PoTD Problem 39

Garud Shah

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Notice that

$$2\cos\left(\frac{\pi}{4}\right) = \sqrt{2} \tag{1}$$

$$\cos\left(x - \frac{\pi}{4}\right) \leq 1, \tag{2}$$

so

$$2\cos\left(\frac{\pi}{4}\right)\cos\left(x - \frac{\pi}{4}\right) \leq \sqrt{2}. \tag{3}$$

So, by product-to-sum, we have

$$\cos x + \cos\left(x - \frac{\pi}{2}\right) \leq \sqrt{2}. \tag{4}$$

So, since $\frac{\pi}{2} > \sqrt{2}$ and $\sin x = \cos\left(x - \frac{\pi}{2}\right)$,

$$\frac{\pi}{2} > \cos x + \sin x. \tag{5}$$

So, $\sin x > \frac{\pi}{2} - \cos x$, and now take cosines, giving

$$\boxed{\cos\sin x > \sin\cos x}. \tag{6}$$