Oregon ARML PoTDs - Spring 2024

PoTD Problem 6

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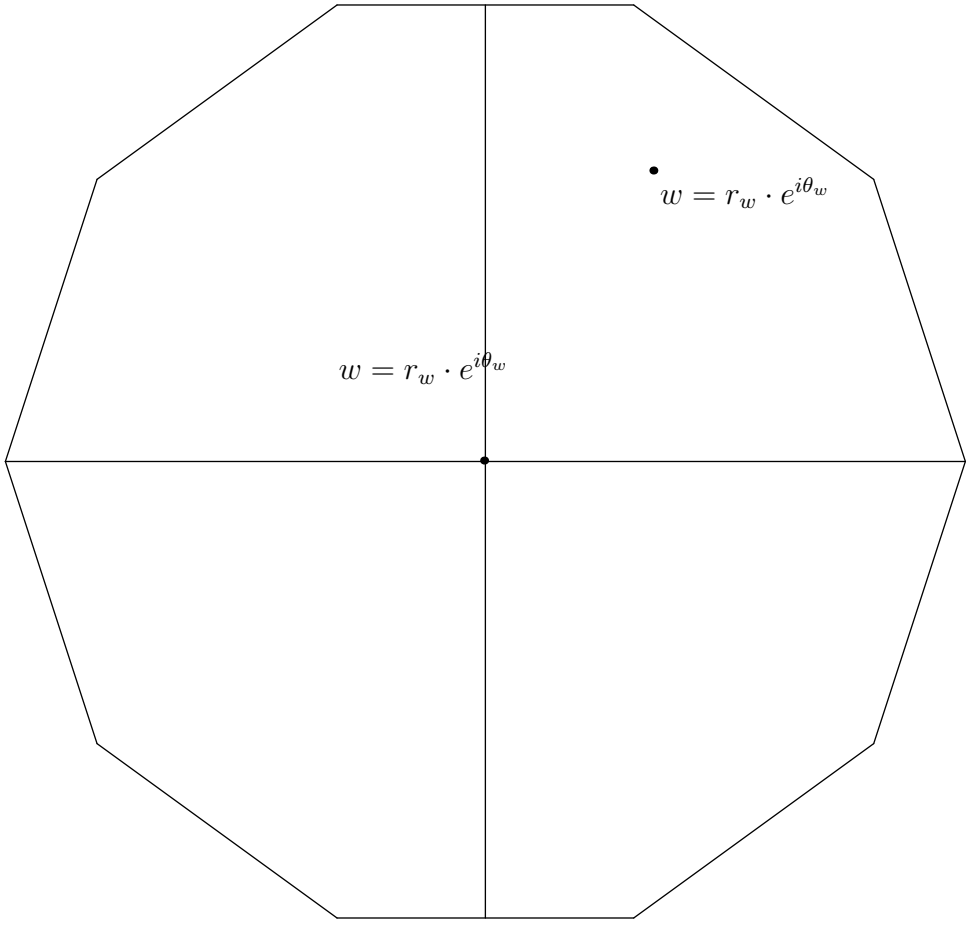
Contents

1 Problem

Let $n \geq 3$ be a positive integer. Consider the polygon formed by the nth roots of unity. If complex z, w are in this polygon, prove zw is too.

2 Modulus

Put z and w in exponential form. Let $w = r_w \cdot e^{i\theta_w}$ and $z = r_z \cdot e^{i\theta_z}$. Now, notice that the polygon is symmetric with rotation symmetry of angle $\frac{2\pi}{n}$. Thus, we only consider $\theta \pmod{\frac{2\pi}{n}}$.



3 Law Of Sines

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4 Inequalities I

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5 Inequalities II

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