

Oregon ARML PoTDs - Spring 2024

PoTD Problem 6

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1 Problem

For what natural numbers $n > 2$ is $\phi(n)$ prime?

2 Coprime

Remember: $\phi(n)$ is multiplicative. So, if p is a prime greater than 3, and k divides p :

$$\phi(pn) \mid \phi(k) \tag{1}$$

$$\phi(pn) = (p-1) \cdot \phi(n), \tag{2}$$

so $\phi(n) = 1$, and $n = 1$ or 2 ! So, only numbers of the form 2^n , or p , or $2p$ with p prime could be valid.

3 Testing

If p is a prime greater than 5, since p is odd, $p - 1$ is not prime, so $\phi(p)$ and $\phi(2p)$ are not prime. For 2^n , we have $\phi(2^n) = 2^{n-1}$ which isn't prime for $n \geq 3$. So, $p = 3$ for p and $2p$ and $k = 2$ for 2^k are the only contenders left. These all work:

x	$\phi(x)$
3	2
4	2
6	2