How To Fool People

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What kind of utility function rewards this behaviour?
You need to be desprate. There needs to be a sharp cutoff where you're willing to go for it. So what is this utility function?

Prerequisites (see previous presentation)

Definition (Relative Risk)

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Most common utility functions: $\log x$, x

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General Math Pt 1: Seperable Equation For $u'(x)^2$

Say we have a risk function $r(x)$. Note that

$$r(x) = \frac{xu''(x)}{u'(x)} \tag{1}$$

$$((u'(x))^2)' = 2u'(x)u''(x) \tag{2}$$

$$u''(x) = \frac{(u'(x)^2)'}{2u'(x)} \tag{3}$$

$$\frac{2r(x)}{x} = \frac{((u'(x))^2)'}{(u'(x))^2} \tag{4}$$

General Math Pt 2: General Equation For $u'(x)$

Now solve:

$$\int \frac{2r(x)}{x}\,dx = \log\left|(u'(x))^2\right| + C \tag{5}$$

$$u'(x) = Ae^{\int \frac{r(x)}{x}\,dx} \tag{6}$$

$$u(x) = A\int e^{\int \frac{r(x)}{x}\,dx}\,dx + \tag{7}$$

$$\cong \int e^{\int \frac{r(x)}{x}\,dx} \tag{8}$$

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$$
\begin{aligned}
u(x) &= \int e^{\int \frac{r(x)}{x}\,dx}\,dx && (9)\\
&= \int e^{\int \frac{x^2/a-1}{x}\,dx}\,dx && (10)\\
&= \int e^{\frac{x^2}{2a}-\log x}\,dx && (11)\\
&\cong \int_1^t \frac{1}{t} e^{\frac{t^2}{2a}}\,dt && (12)
\end{aligned}
$$

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$$\int_1^c \frac{1}{x} e^{\frac{x^2}{2a}} \, dx < p \int_1^m \frac{1}{x} e^{\frac{x^2}{2a}} \, dx \tag{13}$$

$$k \cdot \frac{1}{c} \int_1^c \frac{1}{x} e^{\frac{x^2}{2a}} \, dx < \frac{1}{m} \int_1^m \frac{1}{x} e^{\frac{x^2}{2a}} \, dx. \tag{14}$$

Irrational Decsions

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From last time, we know that whenever the second derivative of a utility function rises above zero, people start making irrational decisions. Let's take a look at this one, then.

$$u'(x) = \frac{1}{x} e^{\frac{x^2}{2a}} \tag{15}$$

$$u''(x) = -\frac{1}{x^2} e^{\frac{x^2}{2a}} + \frac{x}{a} e^{\frac{x^2}{2a}} \tag{16}$$

$$\frac{1}{x^2} = \frac{x}{a} \tag{17}$$

$$x = \sqrt[3]{a}. \tag{18}$$

This finally gives meaning to a: its units are $\$^3$, and its cube root is a thershold for irrational decisions. Above $\sqrt[3]{a}$, we get too enticed by the possibility of money and make poor decisions.

Questions

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Thanks for listening!