

Differential Equations Week **12**

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May 7, 2025

Problem 1

Solve the following differential equation:

$$\begin{cases} P' &= P(c - r(N)) \\ Q' &= Pr(N) \\ N &= P + Q \end{cases} \quad (1.1)$$

for $r(N) = b(1 + \log N)$.

Solution 1.1 — Note that $Q(t)$ is very annoying, let's turn this into an p, N differential equation system.

$$\begin{cases} P' &= P(c - r(N)) \\ N' &= cP \end{cases} . \quad (1.2)$$

Now we solve the phase plane differential equation,

$$\frac{dP}{dN} = \frac{c - br(N)}{c} \quad (1.3)$$

$$P = N - \frac{1}{c} \int_0^k r(t) dt + \alpha. \quad (1.4)$$