Differential Equations Week **12**

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Problem 1

Solve the following differential equation:

$$\begin{cases} P' &= P(c - r(N)) \\ Q' &= Pr(N) \\ N &= P + Q \end{cases} \tag{1.1}$$

for $r(N) = b(1 + \log N)$.

Solution 1.1 — Note that $Q(t)$ is very annoying, let's turn this into an p, N differential equation system.

$$\begin{cases} P' &= P(c - r(N)) \\ N' &= cP \end{cases}. \tag{1.2}$$

Now we solve the phase plane differential equation,

$$\frac{dP}{dN} = \frac{c - br(N)}{c} \tag{1.3}$$

$$P = N - \frac{1}{c}\int_0^k r(t)\,dt + \alpha. \tag{1.4}$$