Differential Equations Week **7**

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March 27, 2025

Problem 1 (Problem 1, generalized)

Find extra particular solutions to the differential equation

$$y'' - 2xy' + \lambda y = 0. \tag{1.0}$$

for $\lambda = 4, 6$.

Solution 1.1 — Consider $4x^{\lambda/2}$. We get, plugging in to the differential operator:

$$\lambda\left(\lambda - 2\right)x^{\lambda/2-2}. \tag{1.1}$$

For $\lambda = 4$, this gives the constant 8 needed whixh means we just need to subtract two, giving $2x^2 - 1$ asour first solution.
At 6, we get $24x$, and try kx to subtract $6x$ giving $2x^3 - 3x$ for this equation.
Reduction of order time! For $\lambda = 4, 6$ in top, bottom respectivly: (use eqn. 8 in the textbook)

$$\boxed{(2x^2 - 1)\int \frac{e^{x^2}}{(2x^2-1)^2}dx} \tag{1.2}$$

$$\boxed{(3x^2 - 2x)\int \frac{e^{x^2}}{(3x^2-2x)^2}dx} \tag{1.3}$$

$$\tag{1.4}$$

Problem 2 (Problem 2)

Solve the differential equation

$$(x^2 - 1)y'' - 2xy' + 2y = 0. \tag{2.1}$$

Solution 2.1 — Immediately try xf. We get:

$$(x^2 - 1)(f' + xf'') - 2x(f + f'x) + 2fx = 0 \tag{2.2}$$
$$x^2 f' - f' + x^3 f'' - xf'' - 2fx + 2f'x^2 + 2fx = 0 \tag{2.3}$$
$$(x^3 - x)f'' + 3f'(x^2 - 1) = 0 \tag{2.4}$$
$$3f' = xf'' \tag{2.5}$$
$$f' = x^3 \tag{2.6}$$
$$f \equiv x^4 \tag{2.7}$$
$$y(x) = c_1 x + c_2 x^4. \tag{2.8}$$