Simulation 7

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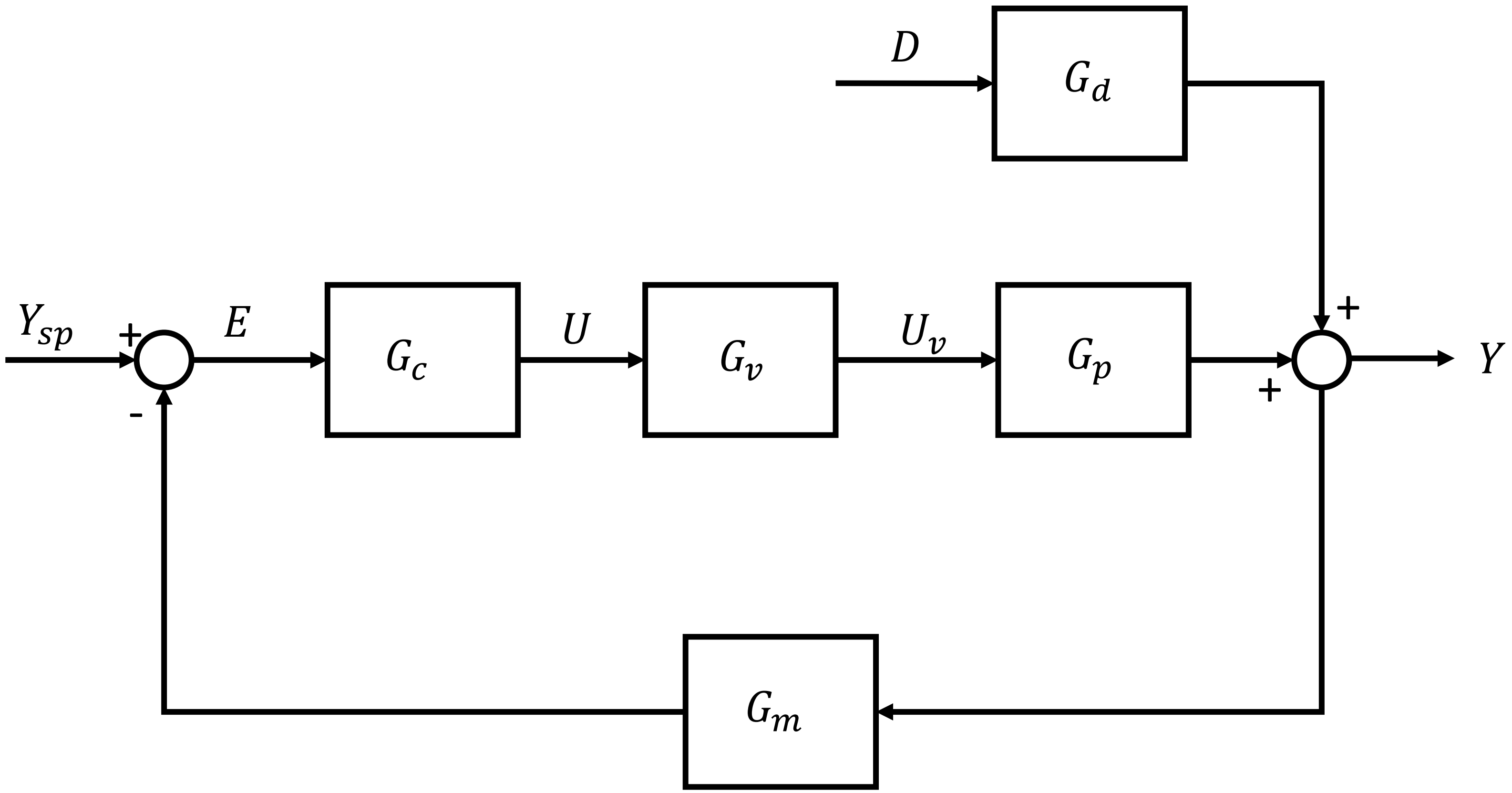
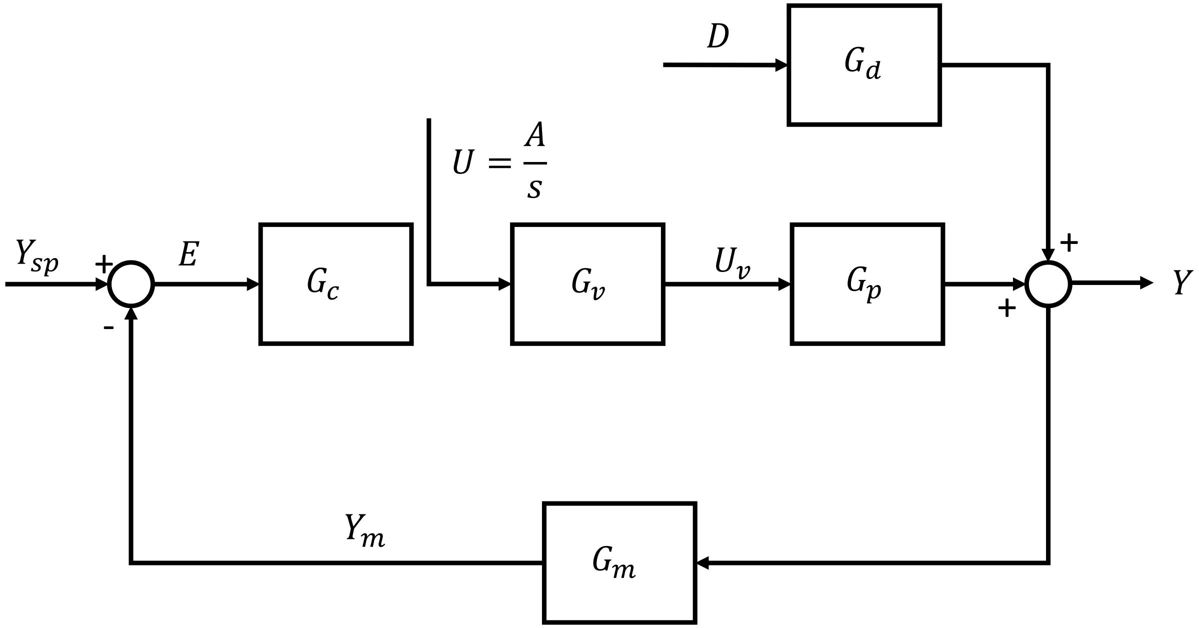
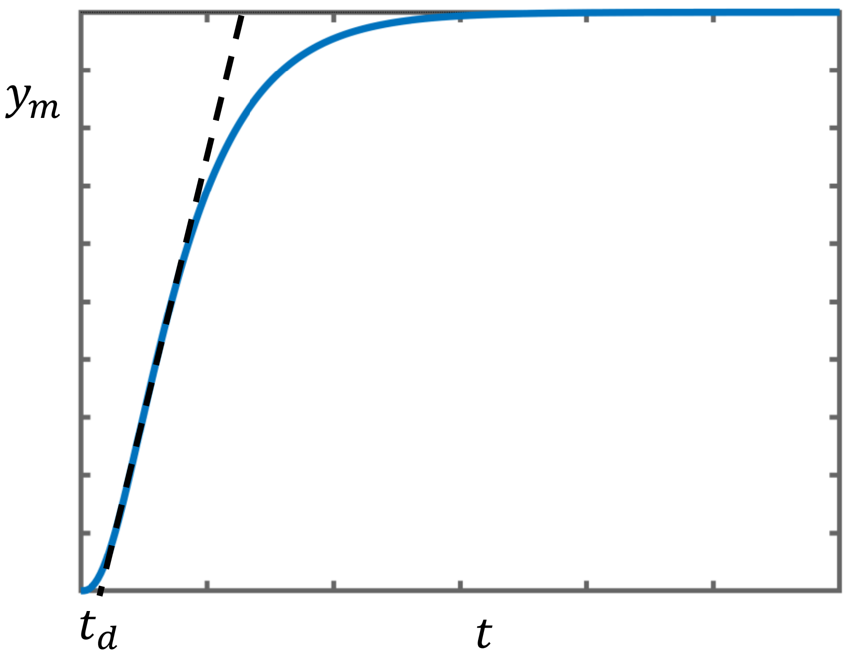
Closed-loop Block Diagram



G_c: controller
G_p: plant
G_d: disturbance
G_v: valve
G_m: measurement device

Process reaction curve method



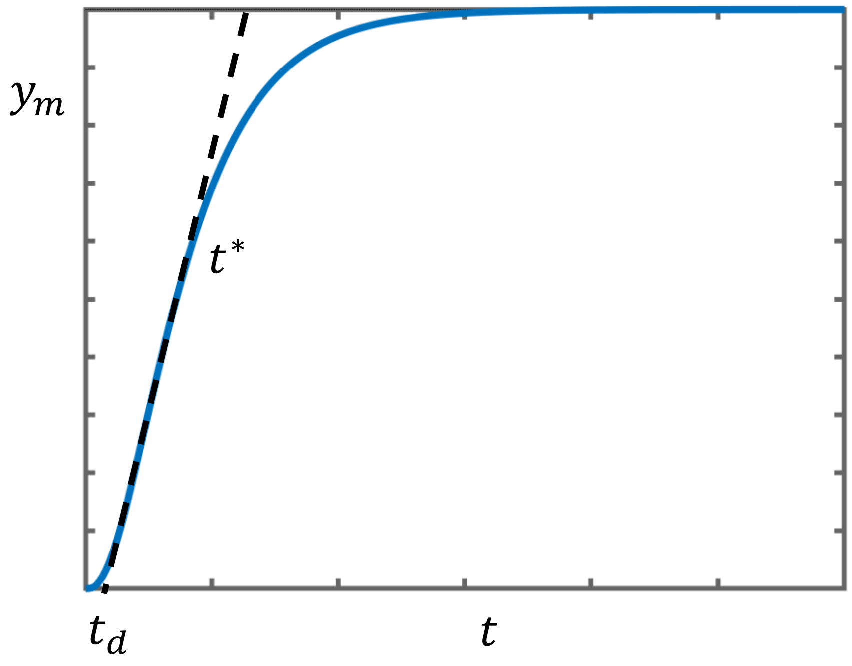


- Process reaction curve method developed by Cohen and Coon
- Based on the observation that response of most processes have sigmoid shape to step change in input
- $G_{PRC} = \frac{Y_m}{U} = G_v G_p G_m \approx \frac{Ke^{-t_d s}}{\tau s+1}$
- $K = \frac{output\ at\ steady\ state}{input\ at\ steady\ state} = \frac{B}{A}, \tau = \frac{B}{M}$
- M is the slope of the sigmoidal response at inflection point
- t_d: time elapsed until the system responded
- Using Matlab function `step` plots the step response

Finding Inflection Point

Partial fraction expansion

- Use `residue` function in Matlab to find partial fraction expansion of G_{PRC}
- Usage of `residue` function
  - $G = \frac{b(s)}{a(s)} = \frac{-4s+8}{s^2+6s+8}$
  - `b=[-4 8];`
  - `a=[1 6 8];`
  - `[r, p] = residue(b, a);`
  - $\frac{b(s)}{a(s)} = \frac{r(1)}{s-p(1)} + \frac{r(2)}{s-p(2)} + \cdots$



Inverse Laplace

- $y = r(1)e^{p(1)t} + r(2)e^{p(2)t} + \cdots$

Finding inflection point

- $\frac{dy}{dt} = r(1)p(1)e^{p(1)t} + r(2)p(2)e^{p(2)t} + \cdots$
- $\frac{d^2y}{dt^2} = r(1)p(1)^2e^{p(1)t} + r(2)p(2)^2e^{p(2)t} + \cdots = 0$

Finding equation of the line

- $y^l - y(t^*) = \frac{dy}{dt}\Big|_{t^*}(t - t^*)$

Cohen-Coon Controller Settings

- P Controller
  - $K_c = \frac{1}{K}\frac{\tau}{t_d}\left(1 + \frac{t_d}{3\tau}\right)$
- PI Controller
  - $K_c = \frac{1}{K}\frac{\tau}{t_d}\left(0.9 + \frac{t_d}{12\tau}\right)$
  - $\tau_I = \frac{t_d\left(30+\frac{3t_d}{\tau}\right)}{\left(9+\frac{20t_d}{\tau}\right)}$
- PID Controller
  - $K_c = \frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3} + \frac{t_d}{4\tau}\right)$
  - $\tau_I = \frac{t_d\left(32+\frac{6t_d}{\tau}\right)}{\left(13+\frac{8t_d}{\tau}\right)}$
  - $\tau_D = t_d\frac{4}{11+\frac{2t_d}{\tau}}$

Bode plot

- $G_{ol} = G_c G_v G_p G_m$
- Three options
  - Use of Matlab functions – `zpk, bode` (specify G_{ol})
  - `controlSystemDesigner` (specify $G_v G_p G_m$)
  - Substitute $s = j\omega$ in G_{ol}, find analytical function of $|G_{ol}(j\omega)|$ and $\phi(G_{ol}(j\omega))$