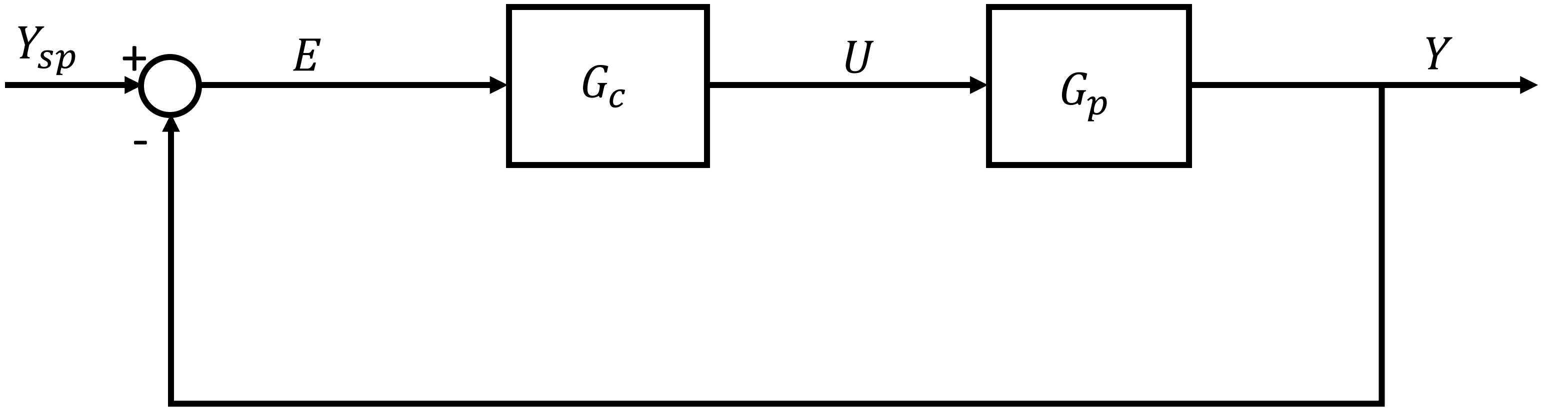
Simulation 6

Ishan Bajaj

Root Locus with P Controller



$$Y = G_{cl} Y_{sp}$$

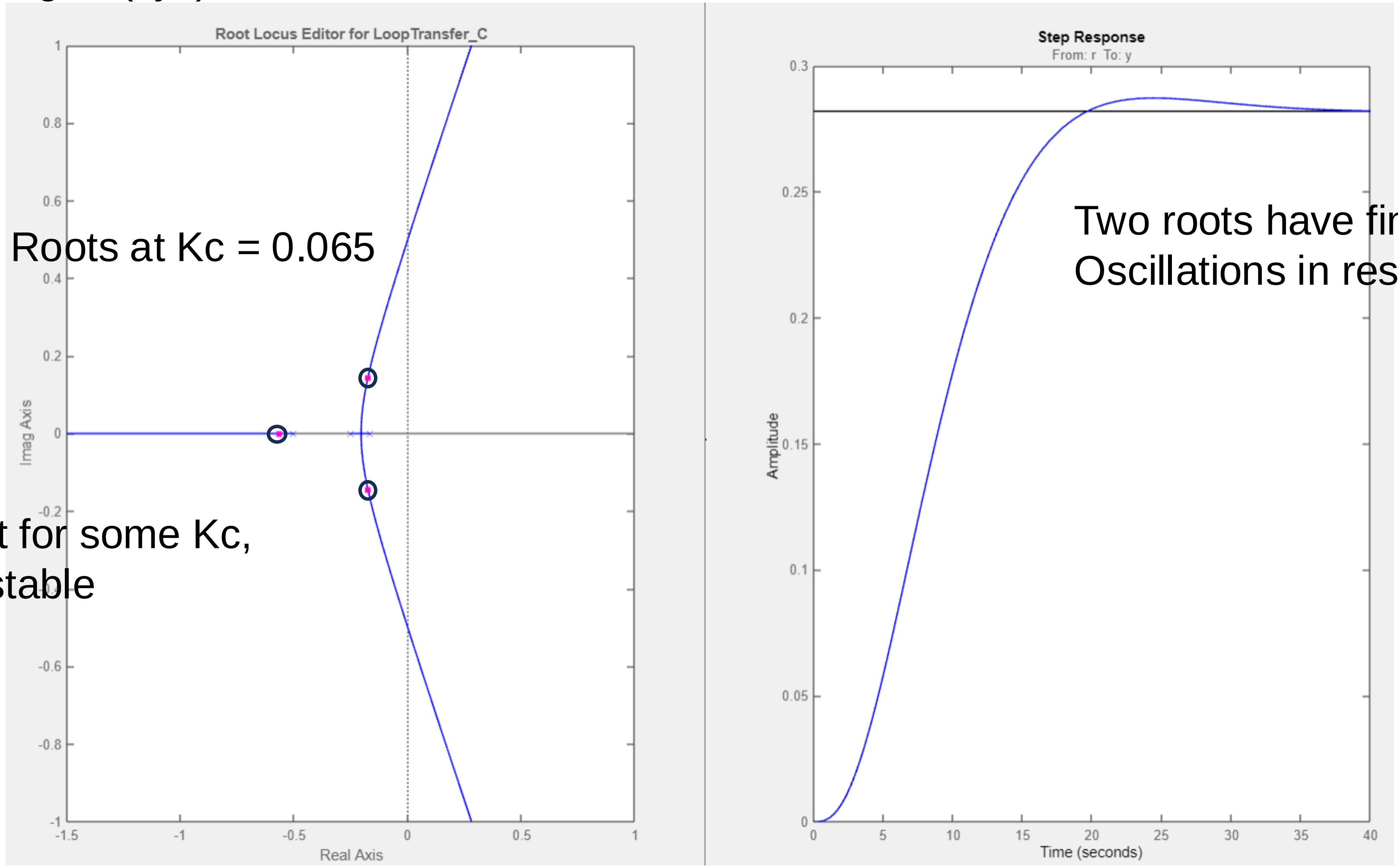
$$G_{cl} = \frac{G_c G_p}{1 + G_c G_p}$$

$$G_p = \frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}$$

- Assume proportional controller, $G_c = K_c$
- Denominator polynomial (characteristic equation), $1 + G_c G_p = (\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1) + K_p K_c$
- Root locus is a plot of roots of denominator polynomial equation as some parameter is varied
- In other words, it is a plot of poles of the closed-loop TF as some parameter is varied
- For this case, it is a degree 3 polynomial, so we will get 3 roots at a fixed Kc

Root Locus with P Controller

- Kp = 6; tau1 = 2; tau2 = 4; tau3 = 6;
- sys = tf([Kp], [tau1*tau2*tau3 (tau1*tau2 + tau2*tau3 + tau1*tau3) (tau1+tau2+tau3) 1]);
- controlSystemDesigner(sys)

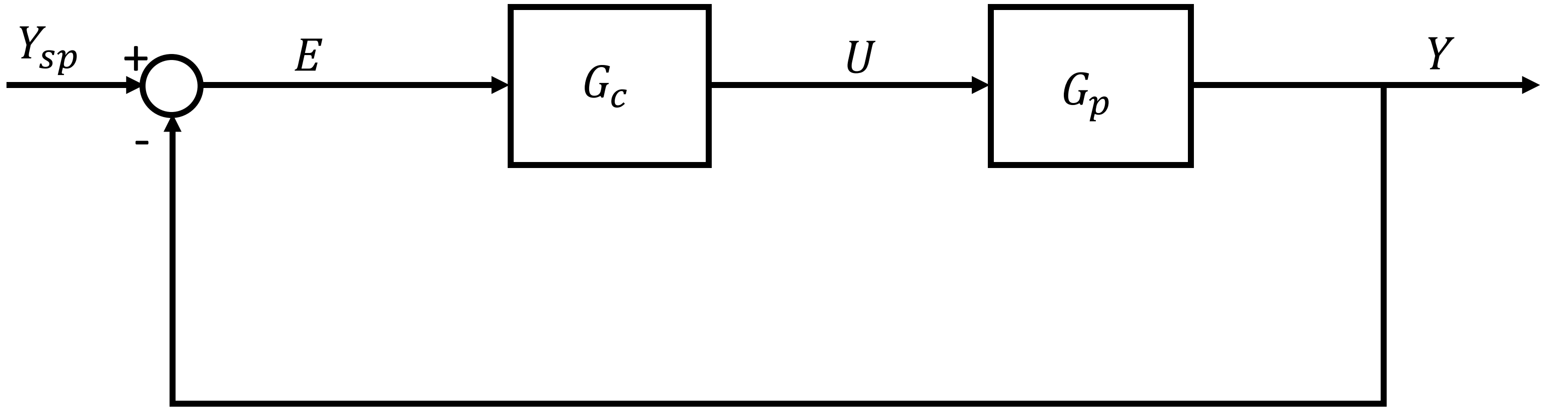


Roots at Kc = 0.065

Two roots have finite imaginary part
Oscillations in response

Root locus indicate that for some Kc,
response becomes unstable

Root Locus with PI Controller



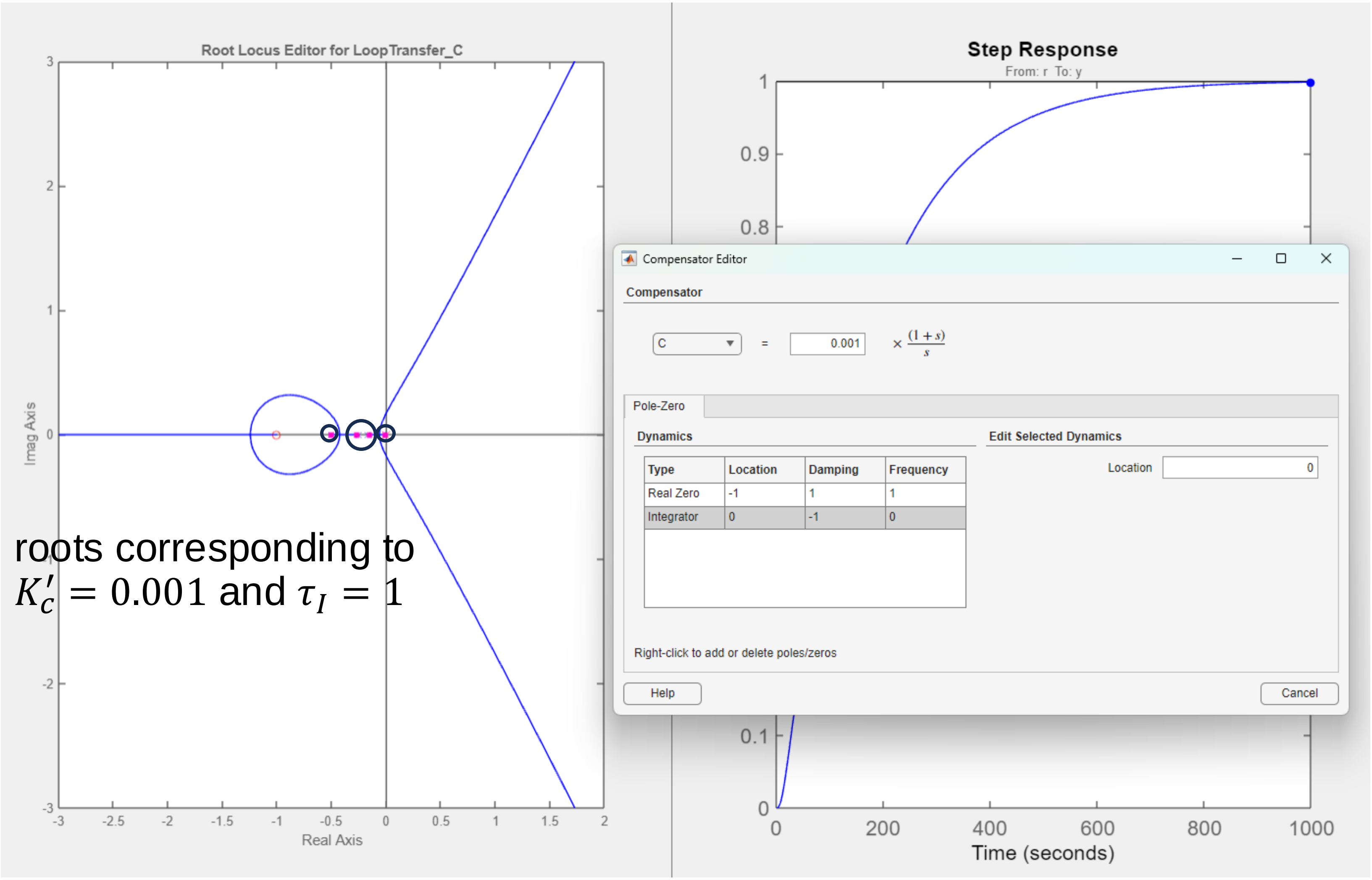
- Assume PI Controller, $G_c = K_c\left(1 + \frac{1}{\tau_I s}\right) = K_c\frac{(\tau_I s+1)}{\tau_I s} = K_c'\frac{(\tau_I s+1)}{s}$
- Denominator polynomial (characteristic equation), $1 + G_c G_p = s(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1) + K_p K_c'(\tau_I s + 1)$
- Degree 4 polynomial

PI Controller in Control System Designer

- Right click on root locus plot
- Select “Edit compensator”
- Compensator ↔ controller

$$G_c = K_c' \frac{(\tau_I s + 1)}{s}$$

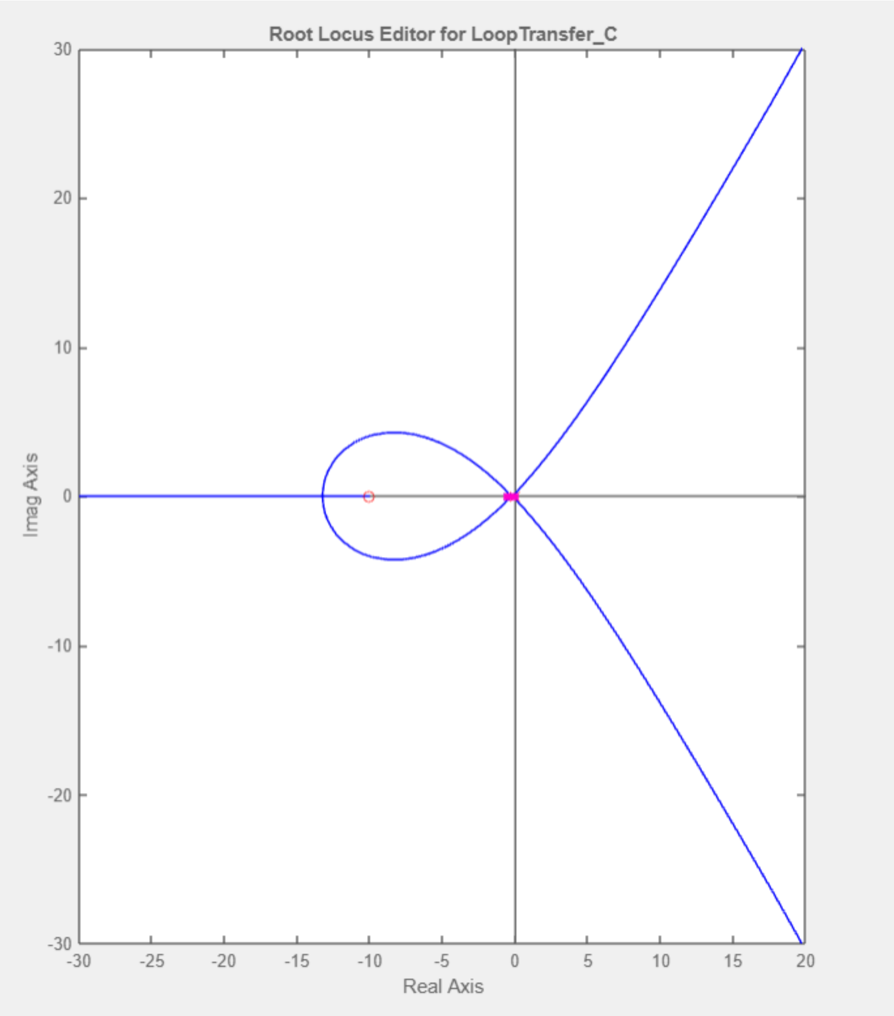
- Root locus based on changing K_c'
- Comment on chosen K_c' and τ_I values
- Roots close to right-half plane
- Small change in **plant** parameters can lead to instability
- Estimated parameters have uncertainty

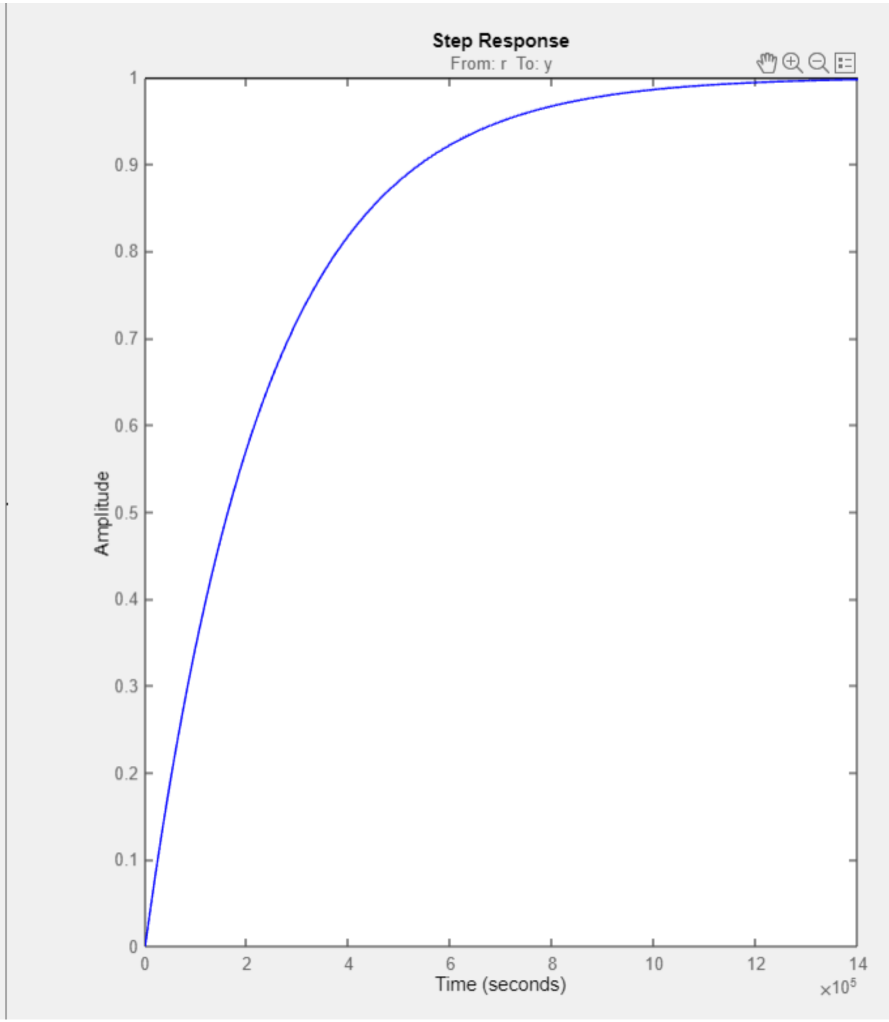


roots corresponding to
$K_c' = 0.001$ and $\tau_I = 1$

PI Controller in Control System Designer

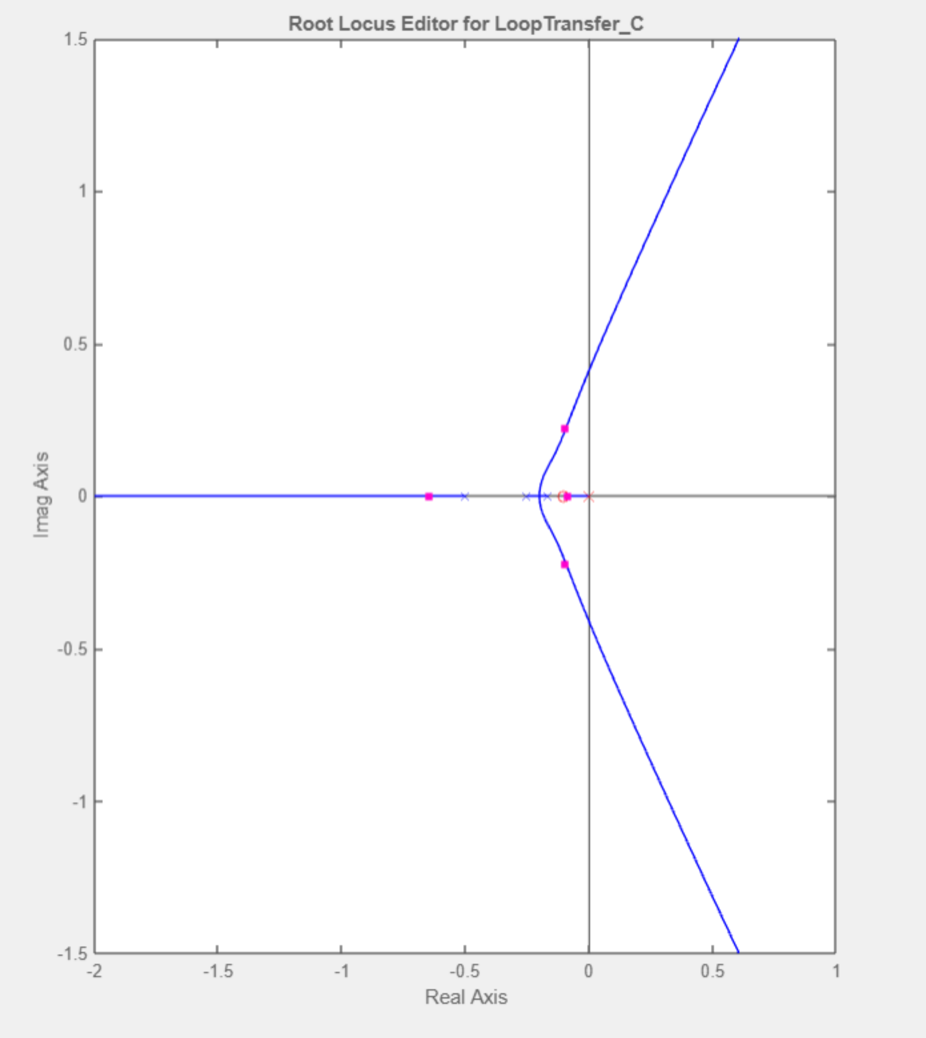
- Decrease $\tau_I = 0.1$, note that integral controller can make system unstable/close to unstable
- No offset

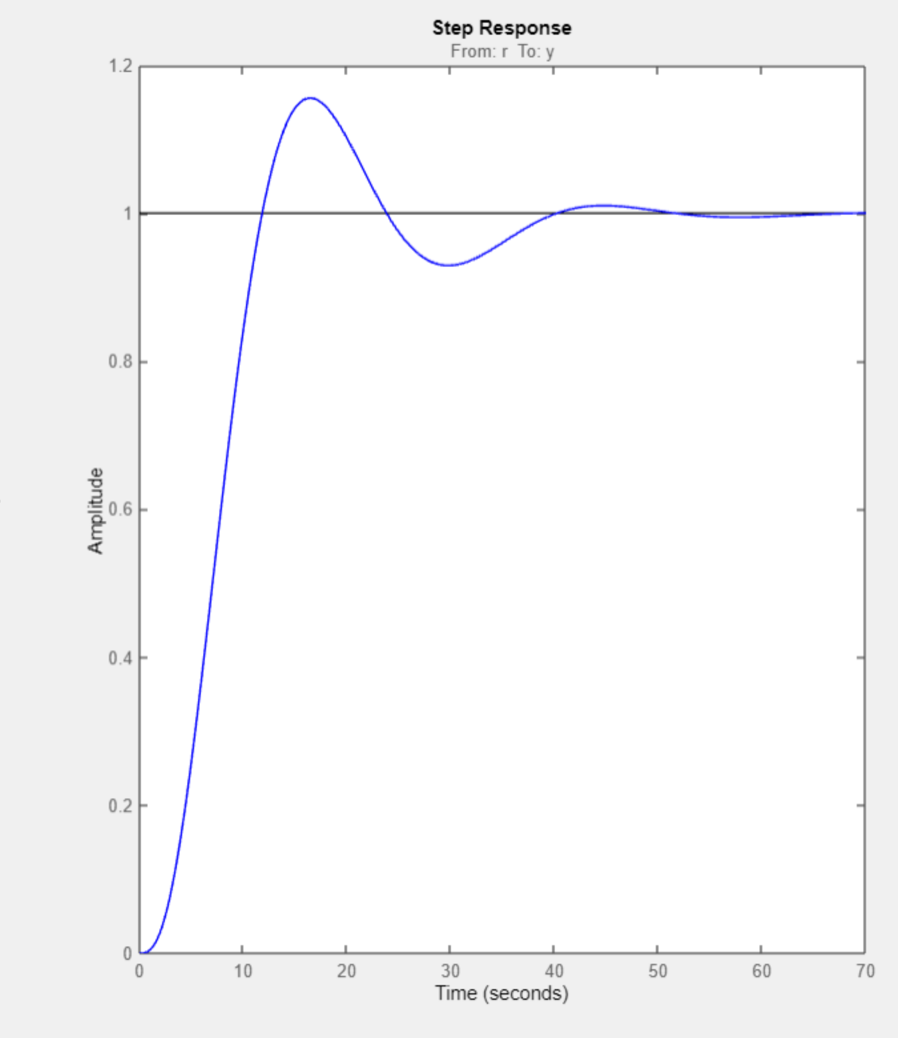




PI Controller in Control System Designer

- Increase $\tau_I = 10$





Tuning P/PI/PID