1)(a) $C = SN(d_1) - Ke^{-rT} N(d_2)$
where $d_1 = \dfrac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

Trial 1 : $\sigma = 0.2$

$d_1 = \dfrac{\ln\left(\frac{38}{35}\right) + \left(0.06 + \frac{0.04}{2}\right)\frac{1}{3}}{0.2\sqrt{1/3}} \simeq 0.896,\ d_2 \simeq 0.781$

$N(d_1) \simeq 0.815,\ N(d_2) = 0.782$

$C = 38(0.815) - 35e^{-0.06(1/3)}(0.782) \simeq 30.97 - 26.86$

$C \simeq 4.11$

Trial 2: $\sigma = 0.3$

$d_1 = 0.691,\ d_2 = 0.518$

$N(d_1) = 0.756,\ N(d_2) = 0.698$

$C = 38(0.756) - 35e^{-0.06/3}(0.698) \simeq 28.73 - 23.90 = 4.83$

Trial 3 : $\sigma = 0.25$

$d_1 = 0.774,\ d_2 = 0.641,\ N(d_1) = 0.780,\ N(d_2) = 0.739$

$C = 38(0.780) - 35e^{-0.06(1/3)}(0.739) = 29.64 - 25.37 = 4.27$

As it is very close, try $\sigma = 0.23$ gives $C = 4.06$
so, by interpolation, σ is approximately 0.228.

(b). $P = C + Ke^{-rT} - S = 4.20 + 35e^{-0.02} - 38 \simeq 4.20 + 34.31 - 38 = 0.51$
with $\sigma = 0.28,\ d_1 = 0.725,\ d_2 = 0.563,\ N(-d_1) = 0.235,\ N(-d_2) = 0.287$
$P = 35e^{-0.02}(0.287) - 38(0.235) = 10.03 - 8.93 = 1.10.$

(c). Expected revenue, S = 38 million, strike (cost to launch) = 35 million, option value = 4.20 million, expiry = 4 months, volatility = 0.28
If the firm launches now, it pays \$35M for an expected revenue = \$3M. But with uncertainty, the value of waiting is \$4.2M so, the option is to wait more than immediate NPV gain because if expected revenue drops, it avoids loss while if it goes up, firm gains more upside.

Assignment 4
Binomial Model

Telco Dt.: Pg.:

Soln 2: A) $S_0 = 100$, $K = 105$ $T = 10$ days

Let α be the number of upward steps

$$S_T = S_0 + (\alpha) - (10-\alpha) = S_0 + 2\alpha - 10$$

for the option to end ITM, $S_T > K$

$$\Rightarrow \cancel{S_0} + 2\alpha - 10 > \cancel{S_0} + 5$$
$$\alpha > 7.5$$
$$\Rightarrow \alpha = 8, 9 \text{ or } 10$$

$$P(\alpha = 8) = {}^{10}C_8 \left(\tfrac{1}{2}\right)^{10}, \quad P(\alpha = 9) = {}^{10}C_9 \left(\tfrac{1}{2}\right)^{10}$$

$$P(\alpha = 10) = {}^{10}C_{10} \left(\tfrac{1}{2}\right)^{10}$$

$$\text{Total probability} = \frac{\left({}^{10}C_2 + {}^{10}C_1 + 1\right)}{2^{10}} = \frac{56}{2^{10}}$$

b) Expected Payoff $= E[\max(S_T - K), 0]$
when $S_T - K > 0$ option is ITM.

$$P(ITM) = \frac{56}{2^{10}} \Rightarrow P(ATM \text{ or } OTM) = 1 - \frac{56}{2^{10}}$$

$$\Rightarrow E[\max(S_T - K, 0)] = {}^{10}C_2 \left(\tfrac{1}{2}\right)^{10} \cdot 1$$

$$+ {}^{10}C_1 \left(\tfrac{1}{2}\right)^{10} \cdot 3 + {}^{10}C_0 \left(\tfrac{1}{2}\right)^{10} \cdot 5 + 0$$

$$= \frac{45}{2^{10}} + \frac{30}{2^{10}} + \frac{5}{2^{10}} = \frac{80}{2^{10}} = \frac{10}{128}$$

telco Dt.:
Pg.:

Fair value of the option = Expected payoff

$= \frac{80}{1024} = \frac{10}{128} = \frac{5}{64}$

B) Continuous Normal distribution Model

$E[|X|] = \sigma\sqrt{\frac{2}{\pi}} = 1$

& expected absolute daily move is 1 (given)

$\Rightarrow \sigma = \sqrt{\frac{\pi}{2}} \approx 1.25$

10 days standard deviation $= \sqrt{10} \times \sqrt{\frac{\pi}{2}} = \sqrt{5\pi} \approx 3.96$

b) expected payoff $= E[\max(S_T - K), 0]$

$= \int_{S>K} (S-K) \times P(S_T = S)\, dS$

where P is the Pdf of S.

$\Rightarrow E[\max(S_T - K), 0] = \int_K^{\infty} (s-K) f_{S_T}(s)\, ds$

daily returns $X_i \sim N(0, 10\sigma^2)$, $\sigma = \sqrt{\frac{\pi}{2}}$

$\Rightarrow S = S_0 + \sum_{i=1}^{10} X_i$

$\Rightarrow S \sim N(S_0, 10\sigma^2)$

$\Rightarrow f_{S_T}(S) = \frac{1}{\sqrt{2\pi(10\sigma^2)}} e^{-\frac{(x-S_0)^2}{2(10)\sigma^2}}$

normalizing $S(t)$ & substituting $z = \frac{S - S_0}{\sigma}$

seeb

$$f_{S_T}(s) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$\Rightarrow \text{expected payoff} = \int_{\frac{K-S_0}{\sigma}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \sigma \, dz \, (S_0+\sigma z - K)$$

$\sigma = \sqrt{5\pi}$

$$\Rightarrow \int_{\frac{K-S_0}{\sigma}}^{\infty} ((S_0+\sigma z) - K) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$$

$$= (S_0 - K)\int_{\frac{K-S_0}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} + \sigma \int_{\frac{K-S_0}{\sigma}}^{\infty} z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$$

Let

$d = \frac{K - S_0}{\sigma}$

$$\Rightarrow E[\max(S_T - K), 0] = (S_0 - K)(1 - \Phi(d)) + \sigma \phi(d)$$

where $\Phi(d) = \int_d^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$ & $\phi(d) = \frac{1}{\sqrt{2\pi}} e^{-d^2/2}$

Normal cumulative distribution function

This integral is evaluated in python

c) Uniform Distribution Model

a) Let the support be $[-x, x]$

Pdf $f(x) = \frac{1}{2x}$

$$\Rightarrow E[|x|] = \int_{-x}^{x} |t| \cdot \frac{1}{2x} \, dt = \frac{1}{x}\int_0^x |t| \, dt = \frac{x}{2}$$

Telco Dt.: Pg.:

Since $E[|X|] = 1$

$\Rightarrow X = 2$

$\Rightarrow$ support is $[-2, 2]$

b) Binomial distribution is discrete
Normal distribution has continuous bell curve.

in uniform distribution, the step size has more dispersion & can take any value in $[2,2]$

$\Rightarrow$ Terminal price will have a more triangular distribution

c) simulating the fair price of the call option under this model.

Contributions: Gavrit: 2,3
Jayaraman: 1.