Indefinite Integrals: JEE Maths

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1. Let a, b, c be positive real numbers. Let

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b-c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b-c)}{ab}}$$

Then $\tan\theta$ = ...

2. The numerical value of

$$\tan\{2\tan^{-1}(\frac{1}{5}) - \frac{\pi}{4}\}$$

is equal to .....

3. The greater of the two angles

$$A = 2\tan^{-1}(2\sqrt{2}-1)$$

$$B = 3\sin^{-1}(\frac{1}{3}) + \sin^{-1}(\frac{3}{5})$$

is.....

MCQ's with One Correct Answer

4. The value of $\tan^{-1}[(\cos^{-1}\frac{4}{5}) + \tan^{-1}(\frac{2}{3})]$ is
   a) $\frac{6}{17}$
   b) $\frac{7}{16}$
   c) $\frac{16}{7}$
   d) none of these

5. If we consider only the principle values of the inverse trigonometric functions then the value of $\tan(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}})$ is
   a) $\frac{\sqrt{29}}{3}$
   b) $\frac{29}{3}$
   c) $\frac{\sqrt{3}}{29}$
   d) $\frac{3}{29}$

6. The number of real solutions of

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

   a) zero
   b) one
   c) two
   d) infinite

7. If

$$\sin^{-1}(x - \frac{x^2}{2} + \frac{x^3}{4}.....) + \cos^{-1}(x^2 - \frac{x^4}{2} + \frac{x^6}{4}....) = \frac{\pi}{2}$$

for $0 < |x| < \sqrt{2}$, then x equals
   a) 1/2
   b) 1
   c) -1/2
   d) -1

8. The value of x for which

$$\sin(\cot^{-1}(1+x) = \cos(\tan^{-1}x))$$

is
   a) 1/2
   b) 1
   c) 0
   d) -1/2

9. If $0 < x < 1$, then

$$\sqrt{1+x^2}[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\} - 1]^{1/2} =$$

   a) $\frac{x}{1+x^2}$
   b) x
   c) $x\sqrt{1+x^2}$
   d) $\sqrt{1+x^2}$

10. The value of

$$\cot(\sum_{n=1}^{23}\cot^{-1}(1 + \sum_{k=1}^{n}2k)) =$$

   a) $\frac{23}{25}$
   b) $\frac{25}{23}$
   c) $\frac{23}{24}$
   d) $\frac{24}{23}$

MCQs with One or More than One Correct

11. The principal value of $\sin^{-1}(sin\frac{2\pi}{3})$ is
   a) $-\frac{2\pi}{3}$
   b) $\frac{2\pi}{3}$
   c) $\frac{4\pi}{3}$
   d) none of these

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12. If $\alpha = 3\sin^{-1}(\frac{6}{11})$ and $\beta = 3\cos^{-1}(\frac{4}{9})$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are)
    a) $\cos\beta > 0$
    b) $\sin\beta < 0$
    c) $\cos(\alpha+\beta) > 0$
    d) $\cos\alpha < 0$
13. For non-negative integers n, let
$$f(n) = \frac{\sum_{k=0}^{n}\sin(\frac{k+1}{n+2}\pi)\sin(\frac{k+2}{n+2}\pi)}{\sum_{k=0}^{n}\sin^2(\frac{k+1}{n+2}\pi)}$$
    Assuming $\cos^{-1}x$ takes values $[0,\pi]$, which of the following options is/are correct?
    a) $[\lim_{n\to\infty} f(n) = \frac{1}{2}]$
    b) $f(4) = \frac{\sqrt{3}}{2}$
    c) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$
    d) $\sin(7\cos^{-1} f(5)) = 0$
14. Find the value of:
$$\cos(2\cos^{-1}x + \sin^{-1}x) at x = \frac{1}{5}$$
    where $0 \le \cos^{-1}x \le \pi$ and $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$.
15. Find all the solutions of
$$4\cos^2 x\sin x - 2\sin^2 x = 3\sin x$$
16. Prove that $\cos\tan^{-1}\sin\cot^{-1}x = \sqrt{\frac{x^2+1}{x^2+2}}$

Integer Value Correct Type:

17. The number of real solutions of the equation
$$\sin^{-1}(\sum_{i=1}^{\infty}x^{i+1} - x\sum_{i=1}^{\infty}(\frac{x}{2})^i)$$
$$= \frac{\pi}{2} - \cos^{-1}(\sum_{i=1}^{\infty}(\frac{-x}{2})^i - \sum_{i=1}^{\infty}(-x)^i)$$
    lying in the interval $(\frac{-1}{2},\frac{1}{2})$ is.....
18. The value of
$$\sec^{-1}\frac{1}{4}\sum_{k=0}^{10}\sec(\frac{7\pi}{12}+\frac{k\pi}{2})\sec(\frac{7\pi}{12}+\frac{(k+1)\pi}{2})$$
    in the interval $[\frac{-\pi}{4},\frac{3\pi}{4}]$ equals.....

Section-B

19. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x$ =
    a) $\tan^2(\frac{\alpha}{2})$
    b) $\cot^2(\frac{\alpha}{2})$
    c) $\tan\alpha$
    d) $\cot(\frac{\alpha}{2})$
20. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for
    a) $|a| \ge \frac{1}{\sqrt{2}}$
    b) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$
    c) all real values of a
    d) $|a| < \frac{1}{2}$
21. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to
    a) $2\sin 2\alpha$
    b) 4
    c) $4\sin^2\alpha$
    d) $-4\sin^2\alpha$
22. If $\sin^{-1}(\frac{x}{5}) + \text{cosec}^{-1}(\frac{5}{4}) = \frac{\pi}{2}$, then the value of x is
    a) 4
    b) 5
    c) 1
    d) 3
23. The value of $\cot(\text{cosec}^{-1}(\frac{5}{3}) + \tan^{-1}(\frac{2}{3}))$ is
    a) $\frac{6}{17}$
    b) $\frac{3}{17}$
    c) $\frac{4}{17}$
    d) $\frac{5}{17}$
24. If x, y, z are in A.P and $\tan^{-1}y$, $\tan^{-1}z$ are also in A.P., then
    a) x = y = z
    b) 2x = 3y = 6z
    c) 6x = 3y = 2z
    d) 6x = 4y = 3z
25. Let
$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}(\frac{2x}{1-x^2})$$
    where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is
    a) $\frac{3x-x^3}{1+3x^2}$
    b) $\frac{3x+x^3}{1+3x^2}$
    c) $\frac{3x-x^3}{1-3x^2}$
    d) $\frac{3x+x^3}{1-3x^2}$
26. If $\cos^{-1}(\frac{2}{3x}) + \cos^{-1}(\frac{3}{4x}) = \frac{\pi}{2}(x > \frac{3}{4})$, then x is equal to
    a) $\frac{\sqrt{145}}{12}$
    b) $\frac{\sqrt{145}}{10}$
    c) $\frac{\sqrt{146}}{12}$
    d) $\frac{\sqrt{145}}{11}$

27. Match the following:

Column I	Column II
A. $\sum_{n=1}^{23} \tan^{-1}(\frac{1}{2i^2}) = t$, then $\tan t =$	(p). 1
B. Sides a, b, c of a triangle ABC are in A.P. $\cos\theta_1 = \frac{a}{b+c}$, $\cos\theta_2 = \frac{b}{a+c}$, $\cos\theta_3 = \frac{c}{a+b}$, then $\tan^2(\frac{\theta_1}{2}) + \tan^2(\frac{\theta_3}{2}) =$	(q). $\frac{\sqrt{5}}{3}$
C. A line is perpendicular to x + 2y + 2z = 0 and passes through (0, 1, 0) Then the perpendicular distance of this line from the origin is	(r). $\frac{2}{3}$

28. Match the following:

Column I	Column II
A. If a = 1 and b = 0, then (x, y)	(p). lies on the circle $x^2 + y^2 = 1$
B. If a = 1 and b = 1, then (x, y)	(q). lies on $(x^2 - 1)(y^2 - 1) = 0$
C. If a = 1 and b = 2, then (x, y)	(r). lies on $y = x$
D. If a = 2 and b = 2, then (x, y)	(s). lies on $(4x^2 - 1)(y^2 - 1) = 0$

29. Match the following:

Column I	Column II
P. $(\frac{1}{y^2}(\frac{\cos(\tan^{-1} y)+y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y)+\tan(\sin^{-1} y)})^2 + y^4)^{\frac{1}{2}}$ takes value is	(i) $\frac{1}{2}\sqrt{\frac{5}{3}}$
Q. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\frac{x-y}{2}$ is	(ii) $\sqrt{2}$
R. If $\cos(\frac{\pi}{4} - x)\cos 2x + \sin x \sin 2 \sec x = \cos x \sin 2x \sec x + \cos(\frac{\pi}{4} + x)$ then possible value of $\sec x$ is	(iii) $\frac{1}{2}$
S. If $\cot(\sin^{-1}\sqrt{1-x^2}) = \sin(\tan^{-1} x\sqrt{6})$, $x \neq 0$ then possible value of $\sec x$ is	(iv) 1

	P	Q	R	S
	(a) 4	3	1	2
codes:	(b) 4	3	2	1
	(c) 3	4	2	1
	(d) 3	4	1	2