Comparison of the Analytic N-Burst Model with Other Approximations to Self-similar Telecommunications Traffic

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ABSTRACT

The N-Burst model describes traffic in telecommunication systems as the superposition of N packet (or cell) streams of ON/OFF type, i.e., during its ON-time each source generates packets according to a Poisson Process with intra-burst packet rate λ_p. As such, the N-Burst is an analytic Point-Process modeling network traffic on the packet level. When using Power-Tail Distributions for the duration of the ON periods, self-similar properties are observed. A variety of widely used approximate traffic models are shown to be limiting cases of N-Burst/G/1 queues. For very low intra-burst packet rates, the N-Burst/G/1 model reduces to an M/G/1 queue. For $\lambda_p \to \infty$ all packets in a burst arrive simultaneously and the model reduces to a Bulk arrival, or $M^{(X)}/G/1$, queue. In the same limit, the packet-based model can be compared to a model on the burst level, an M/G/1 queue where the individual customers represent complete bursts rather than individual packets. Thus the mean system time describes the mean delay for the *last* packet in a burst rather than the average over all packets. By letting the number of packets in a burst, n_p, and the router's packet service rate, ν, go to infinity while holding their ratio constant, the *continuous flow* model is obtained. Numerical results are presented comparing the steady-state results for Mean Packet Delay (mPD) and for Buffer Overflow Probabilities (BOP) of the different analytic models. They collectively show the critical importance of the *Burstiness Parameter* (the fraction of time that a burst is OFF). The self-similar N-Burst/M/1 model shows drastically changing steady-state performance for specific values of the Burstiness Parameter. The limiting models are incapable of describing the detailed structure of the performance in this transition region.

Abbreviations: S-S := Self-Similar; PT := Power-Tail; TPT := Truncated Power-Tail;
r.v. := random variable; SM := Semi-Markov; iid := Independent, Identically Distributed;
CLT := Central Limit Theorem

Key Words: Self-Similar Processes, Power-Tail distributions, Telecommunications Networks, Bursts, Packet Delays, Buffer Overflows

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1 Introduction

It has become increasingly clear in the last few years that the standard models of queueing theory are not adequate for modeling many telecommunications networks/services [LELA94], [CROV96]. One major factor is the way data is transmitted, namely in *bursts* of *cells* or *packets* (used synonymously here), rather than continuously. But this by itself is not sufficient to explain the *erratic* performance of the various routers through which the data flows. The erratic traffic must not only be *bursty* but also *self-similar*. This latter property can in turn be explained and modeled by recognizing that the size of the bursts varies over many orders of magnitude, as is the case if the distribution of burst sizes is *power-tailed*. That is, if $R(x)$ is the fraction of transmitted bursts whose size exceeds x, then $R(\cdot)$ is *power-tailed* (PT) if

$$R(x) \longrightarrow \frac{c}{x^{\alpha}}.$$

The distribution is said to have a *Truncated* Power-Tail (TPT) if $x^{\alpha}R(x)$ is approximately constant over several orders of magnitude before it finally drops to zero.

Accommodating both burstiness and self-similarity in an analytic point-process model is not easy, and thus many approximations have been used by various researchers to understand buffer overflow problems and packet delay. Some examples include the $M/G/1$ queue where the service time has infinite variance [HEYM98], *continuous flow models* during bursts [JELE97, DUMAS00], and *batch arrivals*. The burst models are also known as ON-OFF models. The diversity of these models, all purporting to model the same traffic, can leave the reader with a confused picture of just what is happening.

In a series of papers, we have developed an analytic N*-Burst* model within the Markovian framework where independent sources each emit bursts of packets to a traffic line feeding into a server (here a *router* or *switch*). The arrival processes turn out to be semi-Markov, falling into the general category of MMPP's. Thus we have a class of analytical (multiple) ON-OFF point-process models with PT (or TPT) ON times. In previous papers, we discussed such models within the context of ATM. However, the models are general enough to be used for any kind of packet traffic. In particular, the investigations about the impact of packet sizes as discussed in Sect. 4.2 are of value with respect to IP.

In the succeeding sections, we first describe the N-Burst model, and then give a simple example of the 1-Burst/M/1 queue and show how it connects up with the simple M/M/1 and $M^{(X)}/M/1$ bulk-arrival queues. We then show how the distribution of burst sizes can affect the performance greatly. Special consideration is given to the transition area around the so-called *blow-up point*, where system performance changes from acceptable to unacceptable with a very small change in what is called here, the *Burstiness Parameter*. We then give examples showing that continuous flow models are limiting cases of the N-Burst, as the number of packets in a burst and the packet service rate approach infinity in a constant ratio. It is seen that the models are extremely close above the blow-up point, but the transition from acceptable to unacceptable is not properly described at all by continuous flow models.

We finally consider the M/G/1 model as an alternate view of telecommunications traffic. It may describe some performance characteristics better than models at the packet level. For in this case, each burst is one customer, so the *Mean Burst Delay* (mBD) describes the mean delay of the *last* packet in each burst, rather than the average delay of all packets. From the router's point of view, packet delay may appear to be the more relevant, but from the users' point of view, transmission of the entire burst is what counts. However, for buffer overflow probabilities, the number of *packets* waiting to be served is the important concern.

2 Description of the N-Burst Model

The N-Burst model, introduced in [LIPS99, SCHW99A] is a variant of the many ON-OFF models described in the literature. The N-*Burst* arrival process is a superposition of traffic

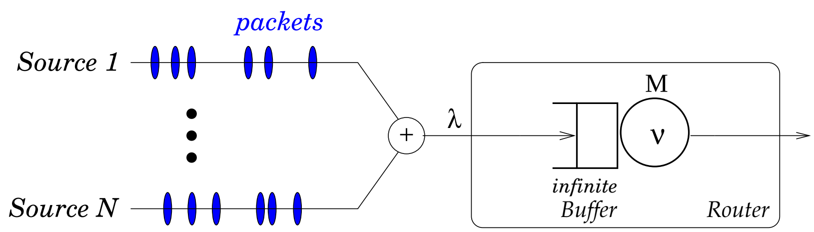

Figure 1: **Diagram of N-Burst/M/1 Queue**. Several independent sources send packets in bursts to a router where they merge and are temporarily stored in a buffer until they can be sent on to their ultimate destination.

streams from N independent, identical sources of ON/OFF type, as shown in Figure 1. Each source emits packets at a rate of λ_p (peak-rate) during its ON-time (a *burst*), and then transmits nothing during its OFF-time. This arrival process, with arbitrary ON and OFF time distributions (having *Matrix-Exponential* (ME) representations, see [LIPS92]) is analytically modeled as a Semi-Markov process of the *Markov Modulated Poisson Process* (MMPP) type. The details of this model can be found in [LIPS99] and [SCHW99A]. Somewhat more general processes (where at least one packet is guaranteed in each burst) will be presented in future work.

Let κ be the mean rate for each source (the average for ON- and OFF-times together), then the N sources collectively generate packets at the mean rate of

$$\lambda = N\,\kappa\,.$$

In our model, the packets generated by the N-Burst process arrive at a single server (called a *router* or *switch*), where they queue up until they can be dispatched elsewhere. The system can then be classified as an SM/G/1 queue. The analytic details are available in [LIPS99].

For the first part of this paper we assume that $N = 1$, then the following definitions are straight forward:

$\lambda :=$ Overall arrival rate (packets / time unit);
$\bar{n}_p :=$ Mean number of packets during a burst;
$\lambda_p :=$ peak transmission rate during a burst (packets / time unit);
$\overline{ON} := \bar{n}_p/\lambda_p =$ Mean ON time for a burst (time units);
$\overline{OFF} :=$ Mean OFF time between bursts (time units);
$\nu :=$ Mean packet service rate of router (packets / time unit);
$\lambda_b := \lambda/\bar{n}_p =$ Mean burst arrival rate (bursts / time unit);

$\nu_b := \nu/\bar{n}_p =$ Mean burst service rate (bursts / time unit);
$\bar{x}_b := \bar{n}_p/\nu = 1/\nu_b =$ Mean time to service a burst (time units);
$\rho := \lambda/\nu = \lambda_b/\nu_b =$ Router utilization.

In addition, we introduce the *Burstiness Parameter*, b, defined as:

$$b := \frac{\overline{OFF}}{\overline{ON} + \overline{OFF}} = 1 - \frac{\kappa}{\lambda_p}\cdot \tag{1}$$

This parameter can be thought of as a *shape* parameter, since λ, or the amount of data sent per unit time, and ρ can be held constant as b is varied over its range, [0, 1]. As will be seen below, if $b = 0$ then the bursts abut each other, and the ON/OFF process reduces to a simpler (in our model, a renewal) process. On the other hand, if $b = 1$ then all packets in a burst arrive simultaneously, i.e., they arrive as *bulk* process.

2.1 Distributions of Sub-Processes

From the above description, the 1-Burst model depends on four separate distributions, with random variables denoted by X_{SV}, X_{OF}, X_{ON} and X_{IN}, respectively. They are:

SV: **Packet Service Time Distribution** with mean $1/\nu$ (distribution depends on packet-size distribution, service rate ν depends upon router speed and size of packets);

OF: **OFF-Time Distribution** with mean $\overline{OFF}$ (depends on how bursts are generated, and how often);

ON: **ON-Time distribution** with mean $\overline{ON}$ causing a mean number of $\bar{n}_p = \lambda_p \cdot \overline{ON}$ packets in a burst (e.g. ON-Time distribution depends on file-size distribution, $\bar{n}_p$ depends on mean size of files, and on packet size);

IN: **Inter-packet Time Distribution** during a burst, with mean $1/\lambda_p$.

Clearly, for a comparison study of models to be useful, the parameters to be varied must be selected judiciously. It is intuitively clear that two distributions can have the same "shape", but have different means, with higher moments scaled proportionately. Formally this can be described in the following way. Let

$$R_X(x) := Pr(X > x)$$

be the *Reliability Function* (or *Complementary Cumulative Distributive Function*) for some nonnegative random variable, X, with mean, $\mathrm{E}(X)$. Let Y be a nonnegative random variable whose distribution has the same shape as $R_X(\cdot)$, but with mean $\mathrm{E}(Y)$. Then

$$R_Y(x) = R_X(r\,x), \quad \text{where} \quad r = \mathrm{E}(X)/\mathrm{E}(Y),$$

for

$$\mathrm{E}(Y) = \int_0^\infty R_Y(x)\,dx = \int_0^\infty R_X(r\,x)\,dx = \int_0^\infty R_X(u)\,du/r = \mathrm{E}(X)/r = \mathrm{E}(Y).$$

In general, functions with the same shape have moments that scale according to $\mathrm{E}(Y^n) = r^n \mathrm{E}(X^n)$, or

$$\frac{\mathrm{E}(Y^n)}{[\mathrm{E}(Y)]^n} = \frac{\mathrm{E}(X^n)}{[\mathrm{E}(X)]^n}.$$

In this way, the means for each of the four distributions above can be varied, while keeping the shapes unchanged. For Markovian-type models (as ours is), the number of phases needed to represent each distribution stays the same.

2.2 Limiting Cases for $b \to 0$ and $b \to 1$

From (1) it is easy to see that the slowest rate at which packets can be transmitted during $\overline{ON}$ times ($\lambda_p = \kappa$) corresponds to $\overline{OFF} = 0$ and $b = 0$. Clearly, in this case the OFF-time distribution is irrelevant. It should become clear that the ON-time distribution also has no impact on the system, since there is no pause between bursts. Therefore, for $b \to 0$ the 1-Burst process reduces from a $SM/G_{(SV)}/1$ queue (where SM depends on X_{OF}, X_{ON}, and X_{IN}), to a $G_{(IN)}/G_{(SV)}/1$ queue. (The notation, $G_{(IN)}$ means *general distribution with inter-arrival times according to* X_{IN}. Interpretation of similar symbols should be straight forward.)

In the other extreme, if λ_p is allowed to become unboundedly large, then $\overline{ON} \to 0$, $b \to 1$, and all packets in a given burst arrive at the router at the same time (or at least before the router can finish serving any packet). This now corresponds to a *Bulk Arrival* process, with bulk size distributed proportional to the ON-time distribution, but with mean, $\bar{n}_p$. The time between bulk arrivals is distributed according to the OFF-time distribution, with mean $1/\lambda_b$. Now the 1-Burst process reduces to a $G_{(OF)}^{[(ON)]}/G_{(SV)}/1$ queue. (The notation, $G_{(OF)}^{[(ON)]}$ stands for: *the bulk arrivals have inter-arrival times* X_{OF}*, and their sizes are taken proportional to* X_{ON}.)

To keep our *base* model as simple (and as small in dimension as possible), in this paper we assume that X_{SV}, X_{OF}, and X_{IN} are all exponentially distributed. Only the ON-time distribution, X_{ON} will be varied. Nevertheless, the mean times for each of these distributions will be parameters in our models. In this case, for

$$b \to 0 \quad \text{the system becomes an} \quad M_\lambda/M_\nu/1 \quad \text{queue,} \tag{2}$$

and for

$$b \to 1 \quad \text{the system becomes an} \quad M_{\lambda_b}^{[(ON)]}/M_\nu/1 \quad \text{queue.} \tag{3}$$

(M_λ stands for *exponential inter-arrival times with rate,* λ.)

3 Delay Time for a Packet

Although the analytic N-Burst model can be evaluated for all possible distributions (within the ME class), the calculations are not trivial. But it is possible to get much insight by looking at the limiting cases for $b = 0$ and $b = 1$. In our parametric studies to be described below, we will hold ρ constant, while varying b, for various ON-time distributions. That is, the *average* load is held constant, while the packets in a burst are bunched up or spread out as much as possible according to the value of b. It is not hard to see then, that for mean packet delay (mPD) (as well as for buffer overflow probabilities) $b = 0$ corresponds to the best performance (smallest mPD), while $b = 1$ corresponds to worst case (largest mPD). Furthermore, the mPD is a monotonically increasing function of b. Therefore, it is useful to discuss some well-known formulas in queueing theory.

As mentioned in the previous section, for $b = 0$ the SM/M/1 queue reduces to the $M_\lambda/M_\nu/1$ queue, in which case, the mean packet delay is given by the elementary formula:

$$\mathrm{mPD}(b=0) = \frac{1}{\nu - \lambda} = \frac{1/\nu}{1-\rho}, \tag{4}$$

where $\rho = \lambda/\nu$. At the other extreme ($b = 1$) is the bulk arrival $M_{\lambda_b}^{[(ON)]}/M_\nu/1$ queue. This behavior is also well known (see, e.g., [COOP81]), and can be written as:

$$\mathrm{mPD}(b=1) = \frac{1/\nu}{1-\rho}\, C, \tag{5}$$

where

$$C := \left[\frac{\mathrm{E}\left(\frac{L(L+1)}{2}\right)}{\mathrm{E}(L)} \right],$$

and r.v. L counts the number of packets in one burst. Note that $\mathrm{mPD}(b = 1) = C \cdot \mathrm{mPD}(b = 0)$, a property we will take advantage of in discussing results. C/ν is the mean packet delay if all packets in a burst arrive together, and processing begins immediately. Put another way, it is the mean time to service one packet in a burst, beginning at the time the first one begins service. Observe that if all bursts are the same size, then

$$C = \frac{\bar{n}_p + 1}{2},$$

i.e., the mean packet delay after service begins for the burst is just over 1/2 the time until the last packet is finished. If the number of packets in a burst is geometrically distributed (i.e., $Pr(L = n) = (1 - p)p^{n-1}$, where $p = 1 - 1/\bar{n}_p$, and $\mathrm{E}(L) = \bar{n}_p$), then

$$C = \bar{n}_p.$$

For distributions with higher variance, C can be unboundedly large. The mPD, in turn, can be unboundedly large, even for small ρ! Therefore, the distribution of the number of packets, or equivalently the ON-Time distribution, X_{ON}, is critical for understanding router behavior when b is close to 1. As in previous papers [GARG92], [LIPS97], [GREI99], we examine the system performance using a family of *Truncated Power-Tail Distributions* (TPT), which we describe briefly below. For complete details see [GREI99].

3.1 ON-Time (Burst Size) Distribution, $G_{(ON)}$

Power-Tail (PT) distributions with exponent, α, have the following properties:

$$\mathrm{E}(X^\ell) = \int_0^\infty x^\ell f(x)\, dx \quad \begin{cases} = \infty & \text{for } \ell \geq \alpha \\ < \infty & \text{for } \ell < \alpha \end{cases}$$

Equivalently,

$$\lim_{x\to\infty} x^{\ell}\, R(x) = \begin{cases} \infty & \text{for} \quad \ell > \alpha \\ 0 & \text{for} \quad \ell < \alpha \end{cases}$$

Reliability functions with *tail behavior* (behavior for large x) of the form:

$$R(x) \longrightarrow \frac{c}{x^{\alpha}} \qquad (c > 0),$$

are the simplest functions having this property, and are called *Power-Tail* distributions. They are also known as *heavy-tailed*, *long-tailed Pareto functions*, or *Levy functions*. There is good reason to believe that such functions are involved in network traffic. First of all, self-similar arrival patterns in ON-OFF systems can be caused by processes with PT ON times [KLIN97], [SCHW97], [WILL95]. Secondly, it has been shown in numerous places that the file size distribution in many facilities, including those transmitted over the Internet, tend to have PT (or at least TPT) behavior [GARG92], [HATEM97], [CROV96]. We now briefly describe a hyperexponential-type function (which is matrix exponential) that in the limit becomes a PT distribution, and that we have used very successfully in standard queueing models [LIPS97], [LIPS99], [SCHW99A], [SCHW99B]. For full details, see [GREI99].

Consider the following family of functions,

$$R_T(x) := \frac{1-\theta}{1-\theta^T} \sum_{j=0}^{T-1} \theta^j \exp(-\mu x/\gamma^j), \tag{6}$$

where $0 < \theta < 1$ and $1 < \gamma$. It is easy to show that

$$\lim_{T\to\infty} \mathrm{E}(X_T^{\ell}) = \infty \qquad \text{for} \qquad \ell \geq \alpha$$

and α, θ and γ are related by the equation, $\theta\gamma^{\alpha} = 1$. $R_T(\cdot)$ is called a *Truncated Power-Tail* (TPT) distribution, with moments given by:

$$\mathrm{E}(X_T^{\ell}) = \frac{1-\theta}{1-\theta^T} \frac{1-(\theta\gamma^{\ell})^T}{1-(\theta\gamma^{\ell})} \frac{\ell!}{\mu^{\ell}}.$$

$T = 1$ corresponds to the exponential distribution. Figure 2 shows $R_T(x)$ as a function of x for various values of T, but plotted on log-log scale. The straight-line behavior over many orders of magnitude is a characteristic of TPT's and PT's generally. Clearly, the larger T is, the longer the straight line persists before it finally drops off exponentially. Furthermore, we believe that infinite tails are not practically relevant: both the limited time-scales for real networks (busy period of e.g. 6 hours) and physical limitations on the sizes of files lead to a truncation of the relevant tails.

4 Results for 1-Burst Processes

For something as complex as the N-Burst model, one must be judicious in selecting parameters that can reveal system behavior in a clear manner. We have chosen to look first at the 1-Burst

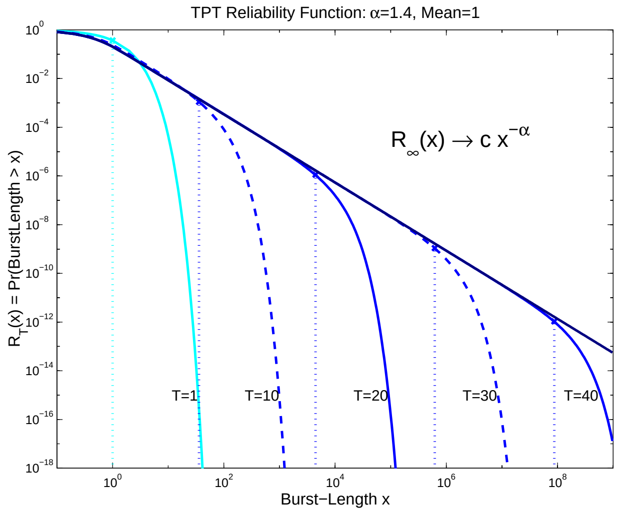

Figure 2: **Truncated Power-Tail Distributions**. The TPT function in Equation (6) is plotted on log-log scale, for various values of T, where $\theta = 1/2$, and $\alpha = 1.4$. The straight-line behavior with negative slope α is characteristic of functions that behave according to $R(x) \to c/x^{\alpha}$ for large x. The larger T is, the further out this straight line extends before dropping off exponentially. Note that for $T = 40$, the straight line extends to 10^8 times the mean. One would have to take more than 10^{12} samples in order to find one sample that falls beyond that range.

process, with fixed utilization parameter, ρ, and exponential distributions for all but the ON-times. The ON-times are distributed according to (6). This class of functions is robust enough to demonstrate the qualitative differences among distributions with different variances. In all cases presented here, we set $\theta = 0.5$, and $\alpha = 1.4$. As mentioned previously, $T = 1$ corresponds to exponential ON-times (or equivalently, geometric distribution of number of packets in a burst). Figure 3 shows the mean packet delay as a function of b, the Burstiness Parameter, as given in (1), for various values of T. Keep in mind that *all* points on this graph have the same value for ρ, which in this case is 0.5. That is, in all cases the router is busy only 1/2 the time.

4.1 Mean Packet Delay and the Burstiness Parameter

As can be seen in Figure 3 when b is small enough, delay is negligible and is insensitive to the ON-time distribution. In this case λ_p is small (close to λ), so the packets of a burst are spread over the whole time between burst starts. This is not necessarily desirable, since there may be considerable delay between the first and last packets of a burst. As the packet rate is increased, thereby decreasing the time for the source to transmit a burst, then the mPD increases gradually to the bulk-arrival limit at $b = 1$, but only if the ON-time distribution is well behaved (e.g., when $T \leq 10$).

The behavior of mPD changes when truncated Power-Tail distributions with larger truncation parameter T are considered. Now, when λ_p exceeds the service-rate ν, the mPD literally blows-up. That is, the mPD jumps by two or more orders of magnitude. In fact, as $T \to \infty$,

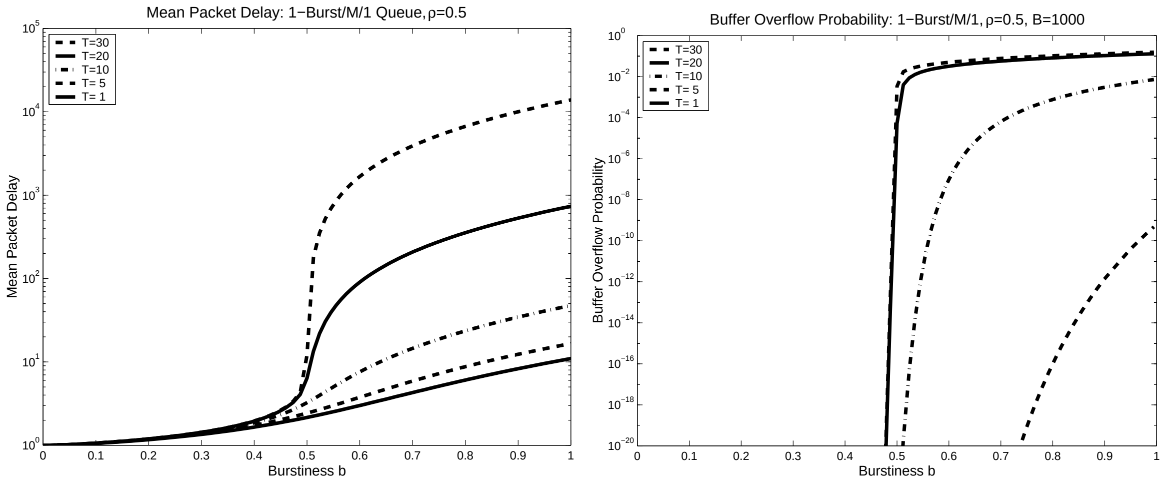

Figure 3: **Comparison of Mean Packet Delay and Buffer Overflow Probability for $1 - \mathbf{Burst}/M_\nu/1$ Queues with different ON-Time Distributions**. Because of the wide variation of performance, the mPD is plotted on log scale. In all instances, the router utilization, $\rho = 0.5$. For $b \to 0$ the packet source is never off, and the 1-Burst behaves like an $M_\lambda/M_\nu/1$ queue for all ON-time distributions. For $b \to 1$ the system behaves like a $M_{\lambda_b}^{[(ON)]}/M_\nu/1$, and mPD depends critically on the first and second moments of the burst size distribution.

the jump becomes unboundedly large. By the definition of b, the condition $\lambda_p > \nu$ corresponds to

$$b > 1 - \rho\,.$$

The point, $b_1 = 1 - \rho$, is called the *blow-up point*. The region $b > 1 - \rho$ is called *blow-up region 1*. For $N > 1$ (more than one source), several blow-up regions exist, see Sect. 5 and [SCHW99A, SCHW99B].

It is clear then, that if the distribution of number of packets in a burst corresponds to a distribution with large T, then router performance will be unacceptable in the blow-up region. This is true for all ρ. Figure 4 illustrates, how the location of the blow-up point depends on the utilization ρ, namely that blow-up occurs near $b_1 = 1 - \rho$. It also makes use of the discussion following (5) where it was shown that the ratio of $\mathrm{mPD}(b = 1)$ to $\mathrm{mPD}(b = 0)$ is independent of ρ. With this scaling, all the curves have the same value for $b = 0$ and $b = 1$. Keep in mind, though, that $\mathrm{mPD}(\rho = 0.9)$ is actually nine times larger than $\mathrm{mPD}(\rho = 0.1)$ for those extreme values of b.

4.2 Packet Size and Fluid Flow Models

Fluid flow ON-OFF models have become popular in the last few years, see e.g. [JELE97] and [DUMAS00]. However, it is not clear how accurate they are in describing phenomena such as packet transfers that are actually point processes. From our point of view, a fluid is the flow of an infinite number of infinitesimally small packets whose sum is the same size as one burst. A closely related issue is the question as to optimal packet size. Of course, in reality, each packet in a burst must have a fixed amount of *overhead* information irrespective of its

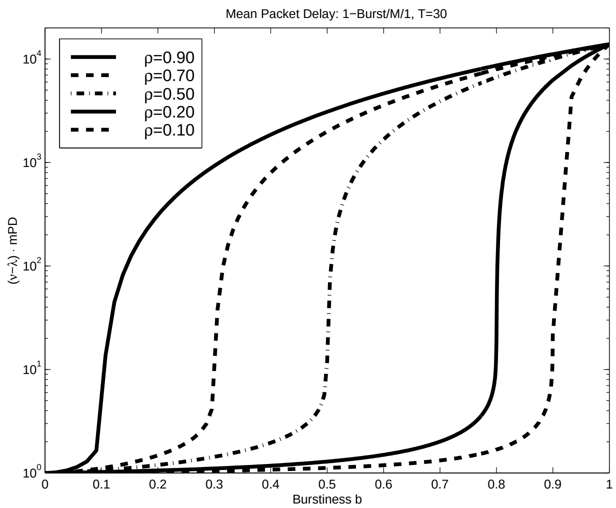

Figure 4: mPD **as a function of b for various values of ρ.** Note that the delay is scaled by the factor $\nu - \lambda$ so that all curves have the same limit at both ends ($b = 0$ and $b = 1$). Actually, the mPD for $\rho = 0.9$ is nine times larger than the mPD for $\rho = 0.1$ at those ends. The blow-up clearly occurs near $b_1 = 1 - \rho$ for each curve.

size. Therefore the smaller each packet is, the more total information must be transmitted. Since we are primarily interested in the fluid approximation, in what follows we assume that the overhead information is negligible, and reserve the question for a forthcoming paper.

Let us take the examples already given as the *norm*. Now we ask what performance will be if each packet is broken into k parts, called packets again. Then the system parameters become:

$k :=$ Packet-size divisor (each packet is divided into k parts);
$\bar{n}_p(k) := k \cdot \bar{n}_p =$ New number of packets during a burst;
$\lambda_p(k) := k \cdot \lambda_p =$ New peak transmission rate during a burst (packets / time unit);
$\lambda(k) := k \cdot \lambda =$ New overall arrival rate (packets / time unit);
$\nu(k) := k \cdot \nu =$ New mean packet service rate of router (packets / time unit);
$\rho(k) := \lambda(k)/\nu(k) = \lambda/\nu$ (same as before).

In all cases, the new parameter reduces to the old one for $k = 1$. For instance, $\lambda_p(k = 1) = \lambda_p$. The various *burst* parameters ($\overline{ON}$, $\overline{OFF}$, λ_b, ν_b, and $\bar{x}_b$) remain unchanged. With these definitions, for $k \to \infty$, the fluid-flow model is obtained.

It is not hard to show that the mean packet delay when $b = 0$ for any k becomes, from (4)

$$\mathrm{mPD}(b = 0,\, k) = \frac{1/(k\nu)}{1 - \rho} = \frac{1}{k} \cdot \mathrm{mPD}(b = 0).$$

(By definition, $\mathrm{mPD}(b,\, k = 1) = \mathrm{mPD}(b)$). The corresponding relation for $b = 1$ is not so easy to get. However, by replacing the random variable, L (number of packets in a burst) with $k\,L$

everywhere in (5), the following can be shown to be true.

$$\mathrm{mPD}(b=1,\ k) = \mathrm{mPD}(b=1) - \frac{1/\nu}{1-\rho}\left[\frac{1}{2} - \frac{1}{2k}\right].$$

It also follows that the quantity, $\nu(1-\rho)\cdot \mathrm{mPD}(b=1,\ k)$ is independent of ν and ρ for all k. This formula shows quite clearly that if one ignores the overhead factor, the mPD becomes insensitive to packet size for bulk arrivals and large truncation parameter, T. The difference between the fluid-flow bulk arrival limit ($k \to \infty$) and the point-process bulk arrival model ($k = 1$) is given by $1/[2(\nu - \lambda)]$.

Figure 5 plots the mean delay as a function of burstiness b for different values of k. It is clear that k has great impact below the blow-up point $b_1 = (1-\rho)$, but in the blow-up region of the 1-Burst/M/1 model, its value does not matter with respect to the absolute values of the mPD's, particularly for large T. The fluid flow model, then, is a good approximation to the 1-Burst model in the blow-up region, but at, and below the blow-up point, its results become meaningless. In fact, the fluid model ($k \to \infty$) yields a value of 0 for mPD there.

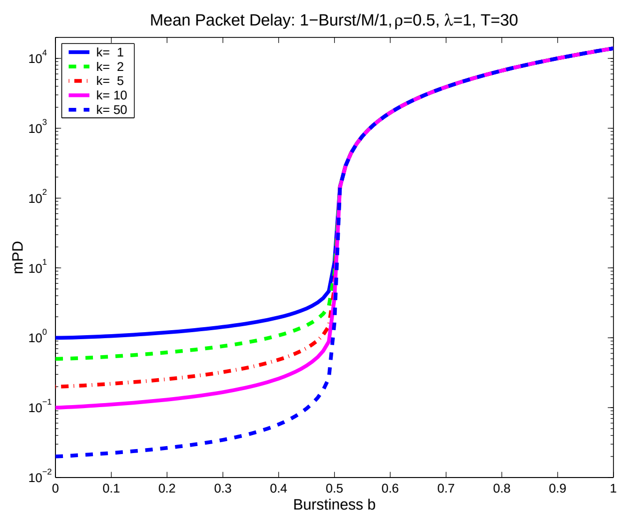

Figure 5: **Impact of Packet Size on mean packet delay**. In all cases the ON-time distribution is a TPT distributed with $T = 30$, and $\rho = 0.5$. The curve for $k = 1$ is the same as the corresponding ones in Figures 3 and 4. All the curves are very close above the blow-up point, but are not equal, even at $b = 1$. Below the blow-up point the fluid flow model ($k \to \infty$) provides no insight into system performance.

5 2-Burst Models

When two or more sources supply bursts to the same router, the mPD structure becomes more complicated, particularly if the ON-time is TPT distributed with large T. We only supply a few results here. The overall behavior and the causes of the blow-ups are described in detail in [SCHW99B].

5.1 Blow-up Regions

First consider the case $b = 0$. Then, as with the 1-Burst, each source is a Poisson process if the intra-burst distribution, X_{IN}, is exponential. As is well known, the merging of several Poisson Processes is still a Poisson Process whose rate is the sum of the individual rates. Thus, for small b, behavior is as simple as one could hope, for any number of sources result in an $M_\lambda/M_\nu/1$ model where $\lambda = N\kappa$. We mention here that if the intra-burst distributions are not exponential, then the collective process is not a renewal process. But if the distributions are well behaved, then mean packet delay should be reasonable low for small b, even if the ON-Time distribution is PT.

At the other end, where $b = 1$, if the OFF-Time distribution is exponential, then each source yields a Poisson process of bulk-arrivals. Again, the merged process is also Poisson, yielding the same performance as the 1-Burst limit, with cumulative bulk arrival rate. If the OFF-Time distribution is not exponential, but still well behaved, then we would expect performance comparable to the exponential case. But several models have considered the case where OFF-Times are PT. Performance can then be very poor, even if ON-Times are not PT. The system behaves in a manner similar to a PT/M/1 queue. See [GREI99] for details.

The above discussion asserts that router behavior in the regions near $b = 0$ and $b = 1$ is the same whether there is one, or more than one independent sources. This is shown in Figure 6 where our results for a 2-Burst calculation are presented, together with results already presented for the 1-Burst process. It is necessary to compare the 1- and 2-Burst processes with some care. To maintain the same load on the router, the 1-Burst model submits packets at twice the rate of each of the sources of the 2-Burst model ($\lambda = 2 \cdot \kappa$). Furthermore, in order to have the same Burstiness Parameter, the 1-Burst must have a peak transmission rate that is also twice that of each of the 2-Burst sources. In general, for a N-Burst model:

$$b = 1 - \frac{\kappa}{\lambda_p} = 1 - \frac{\lambda}{N\,\lambda_p} \cdot$$

Figure 6 shows clearly that for small b, router performance is largely independent of N, X_{OF} and X_{ON}. Similarly, for b close to 1, it is again independent of N and X_{IN}. But the structure of the blow-up regions is very dependent upon N. We see that at the leftmost blow-up point ($b_2 = 1 - \rho = 0.5$) the 2-Burst process does not exhibit nearly so large a jump as the 1-Burst. Part of the reason is that both sources must simultaneously be in a long burst for the blow-up to occur. The other reason is that the distribution of this double event is power-tailed with exponent $2\alpha - 1$, which is larger than α for $\alpha > 1$. For an explanation of this phenomenon, see [SCHW99A] and [SCHW99B].

For large T, Figure 6 shows a second blow-up point for the 2-Burst process, at $b_1 \approx 2/3$. One would expect a blow-up to occur when either source transmits at a rate exceeding the router's capacity. This would occur at $\hat{b}_1 = 1 - \rho/2 = 0.75$, but it actually occurs when the sum of the average of one source together with the other being always ON is sufficient to saturate the router. That is, when $\nu = \lambda_p + \kappa$, which translates to $b_1 = 1 - \rho/(2 - \rho)$, which for $\rho = 0.5$ is $b_1 = 2/3$. In general, the location of the N blow-up points is given by

$$b_i = N \cdot \frac{1 - \rho}{N - \rho\,(N - i)}, \quad i = 1, ..., N \cdot$$

The blow-up points are located by the condition

$$\nu = \Lambda_i \quad \text{where} \quad \Lambda_i := i\lambda_p + (N - i)\,\kappa\,,$$

which leads to the above formula for b_i. We call the interval $b_i < b < b_{i-1}$ the blow-up *region* i. See [SCHW99A] for more details about the location of the blow-up points for $N > 1$.

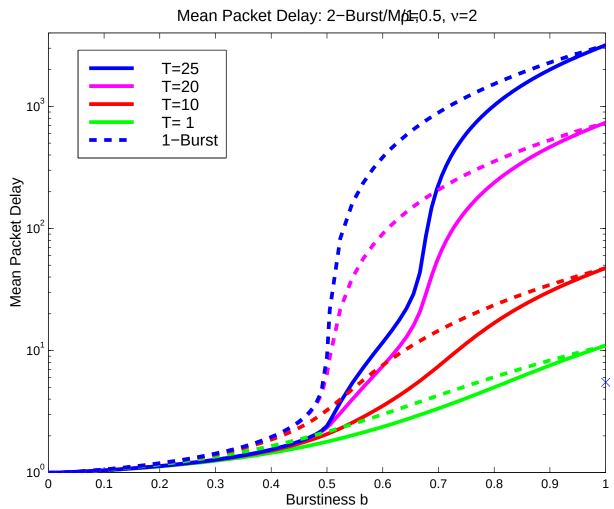

Figure 6: **Comparison of Mean Packet Delay of 1-Burst- and 2-Burst/M/1 Queues with different ON-Time Distributions**, $b = 1 - \kappa/\lambda_p$. Note that κ of the 2-Burst model is only half the value of the 1-Burst model in order to have equal $\lambda = 1$. The behavior near the ends ($b = 0$ and $b = 1$) is similar to before, but now there are two blow-up points, at $b_2 = 1/2$, and $b_1 = 2/3$. See text for full discussion.

5.2 Packet Size and Fluid Flow Model

In this section we do the same thing that was done in Section 4.2, except now for the 2-Burst model. Recall that each original packet is broken into k new packets. Also, for the purposes of comparison, we have made the simplifying assumption that the total size is unchanged (no extra header information). As before, Figure 7 shows that mPD is insensitive to packet size for b close to 1. But below $b = 2/3$ (blow-up point b_1) the packet size is of critical importance. It also indicates that the fluid flow model ($k \to \infty$) provides little information in this region, other than where the blow-up points are.

6 The $M_\lambda/G_{(ON)}/1$ Queue

We finally consider the M/G/1 model as an alternate view of telecommunications traffic. It may describe some performance characteristics better than models at the packet level, particularly if λ_p is so large that all packets in a burst arrive at the router before very many of the early ones have been processed (i.e., b is close to 1). For in this case, each burst is one customer, so the *Mean Burst Delay* (mBD) describes the mean delay of the *last* packet in each

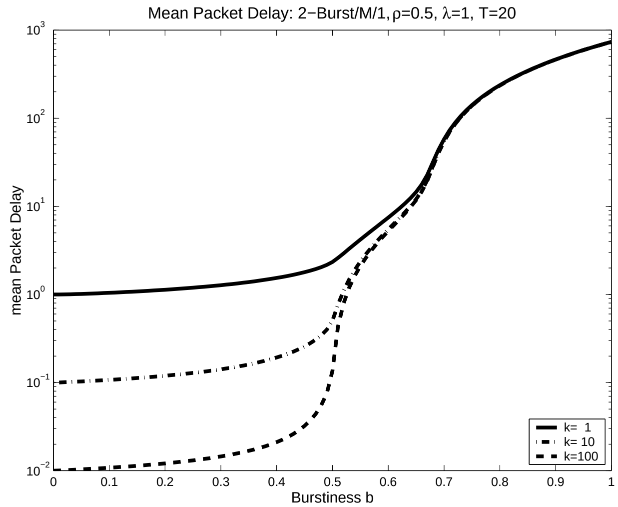

Figure 7: **Effect of Packet Size on Mean Packet Delay for 2-Burst Processes.** The ON-time distribution is TPT-20, and $\rho = 0.5$ in all cases. As the Burstiness Parameter decreases from 1 the packet size becomes increasingly important for determining mPD. The blow-up points, however, are the same for all k. $k = 1$ is the same curve as that in Figure 6 labeled $T = 20$.

burst, rather than the average delay of all packets. From the router's point of view, packet delay may appear to be the more relevant, but from the users' point of view, transmission of the entire burst is what counts. However, for buffer overflow probabilities, the number of *packets* waiting to be served is the important concern.

Denote the random variable for burst service time as X_b, with mean service time of $\bar{x}_b := 1/\nu_b = \bar{n}_p/\nu$. The service time distribution, appropriately scaled, is the same as the ON-Time distribution. Then the P-K Formula (see, e.g., [COOP81]) becomes

$$\text{mBD} = \bar{x}_b + \frac{\rho \bar{x}_b}{1-\rho}\left[\frac{\text{E}(X_b^2)}{2\bar{x}_b^2}\right]$$

The appropriate formula with which to compare is the bulk arrival formula, (5), with νX_b replacing L.

$$\text{mPD}(b=1) = \frac{\bar{x}_b}{1-\rho}\left[\frac{\text{E}(X_b(X_b+1))}{2\bar{x}_b^2}\right].$$

A comparison of the two formulas should provide some insight into last packet's delay, but it should be remembered that both are only meaningful as approximations to ON-OFF processes when b is very close to 1.0.

The $M_\lambda/G_{(ON)}/1$ queue provides some insight as to buffer overflow probabilities, and why a steady-state solution can exist, even if $\text{E}(X_b^2) = \infty$. [HATEM97] showed by calculation that

$$a(n) \to \frac{c}{n^\alpha}$$

for PT service time distributions, where $a(n)$ is the probability that an arriving burst will find n bursts already in the buffer. Clearly, if $\alpha > 1$ then $\sum a(n)$ converges (i.e., there is a steady-state solution). But if $\alpha \leq 2$ then $\sum n \cdot a(n)$ diverges (i.e., the mean system time is infinite). We have found the same properties for N-Burst queues above the first blow-up point. This has serious implications for buffer overflow performance. For well behaved systems, the overflow probability is known to drop off geometrically with increasing buffer size K:

$$P(K) = Prob(\text{"an arriving packet will see a full buffer"}) = \sum_{n=K}^{\infty} a(n) \to c\, s^K,$$

where $s < 1$ and monotonically increases with ρ. Clearly, if $P(K)$ is small, say 0.01, then doubling the buffer size, K, will reduce the probability by a factor of 100, when assuming $c = 1$. But for full PT service times,

$$P(K) \to \frac{c}{K^{\alpha-1}}.$$

Now if the buffer size is doubled, then for example, if $\alpha = 1.4$, the overflow probability will only be reduced by a factor of $1/2^{0.4} \approx 0.758$, less than 25%. Of course, for TPT's the asymptotic formula given above only holds up to some large value for K_T, and then the formula reverts to the *well-behaved* form. But, depending on T, $P(K)$ could still be extremely large. This tells us then, that overflow problems in the blow-up regions (with large T) cannot be solved by *throwing more buffer* at it, but faster routers could help (increase ν and thereby also shift the leftmost blow-up point $b_N = 1 - \rho$ to the right, such that b eventually is below all blow-up regions).

7 Summary and Conclusion

We introduced briefly the so-called N-Burst model for representing real telecommunications traffic fed into switches or routers of the Internet. The N-Burst model is an ON/OFF-model with data transmission phase during ON-times and silent phase during OFF-times. By introducing Power-Tail distributions for the distributions of ON-times (the lengths of bursts) the N-Burst model is able to represent those traffic characteristics that dominantly determine performance of telecommunication components, namely long-range dependence and self-similarity. Furthermore, we consider the N-Burst model as constructive in the sense that its input parameters correspond to real-world measurements, so it can be useful as a base model in a wide spectrum of real traffic models. One such parameter is the burstiness, b, defined as the ratio of OFF-time to total time (or equivalently, $b = 1 - \kappa/\lambda_p$, i.e., one minus the ratio of average arrival rate to peak arrival rate, for one source).

In Section 2 we showed, that setting this parameter b to its extreme value 0 results in a Poisson process of packet arrivals and at the other extreme, 1, in a Poisson process of bulk arrivals with bulk size distribution according to the burst length distribution. Within those two extremes, some values of the Burstiness Parameter define so-called blow-up points where performance, e.g. mean packet delay, Figure 6, is drastically worsened. Those blow-up points start blow-up regions where the overall stream of incoming packets oversaturate the router if a particular number, i, of sources send at their peak rate for an inordinately long ON-time. The important argument here is that this type of event (very large bursts) does not have extremely low probability if burst lengths (or bulk sizes) are PT distributed. The same is true for Truncated Power-Tail (TPT) distributions (Section 3) if the truncation does not occur until x is very large, as determined by a large value of the truncation parameter, T.

The reason for using TPT distributions in our model is twofold: (1) TPT distributions have a matrix exponential representation and thus allow matrix geometric/algebraic solutions; and (2) make it possible to do parametric evaluation not only w.r.t. the power index α (which is related to the Hurst parameter, H) but also to vary the truncation parameter T over a range of interest corresponding to the fitting to real data. An example of such a parametric evaluation is displayed in Figure 3: it shows the dramatic increase of mean packet delay in the region of high burstiness – the higher the value of the truncation parameter T the more the mPD increases.

Another parameter of interest is packet size, for two reasons: (1) as a comparison of the N-Burst model with fluid-flow models; and (2) to investigate the impact of packet size on mean packet delay. Figure 5, again, shows the blow-up effect with almost no differences in the mean delay for packets of different sizes when $b > 1 - \rho$, but noticeable differences in the mean delay of packets of different sizes in the region of low burstiness. Note, that a fluid-flow model is not able to show these differences.

Of course, a very relevant parameter of our N-Burst model is the number of sources, N, which merge together to feed the server. The question is, whether there is a difference in performance for different number of sources – even when the overall server utilization, ρ, is kept at the same level. For comparison, it is well known that the merging of two pure Poisson sources with rates λ_1 and λ_2, respectively, produces a pure Poisson source with rate $\lambda = \lambda_1 + \lambda_2$. Figure 6 shows that for ON/OFF processes with exponential ON and OFF times ($T = 1$), the 1- and 2-Burst processes are different in the intermediate region, but not significantly so. However, if the ON-times are PT or TPT distributed (large T), even though the overall server utilization, ρ, is kept at the same level, then the mean packet delay in the 1-Burst model and the 2-Burst model both show sharp increases (blow-up points), but at different burstiness values and of different sizes. For the 2-Burst model there are two such blow-up points compared to one in the 1-Burst model (c.f. Figure 6). This effect tells that it matters whether the incoming data streams from different sources to a switch is modeled (and measured) as one combined stream or as separate streams.

In summary, the N-Burst ON/OFF model covers a wide spectrum of point processes for modeling real telecommunication traffic, and has already revealed some hidden subtle dependences of switch/router performance upon different traffic characteristics. On the other hand, simpler models (M/M/1, Bulk/M/1, M/G/1, Fluid flow) are not capable of providing meaningful information in the intermediate region (at the first blow-up and below the last blow-up), where performance models are likely to be most important.

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