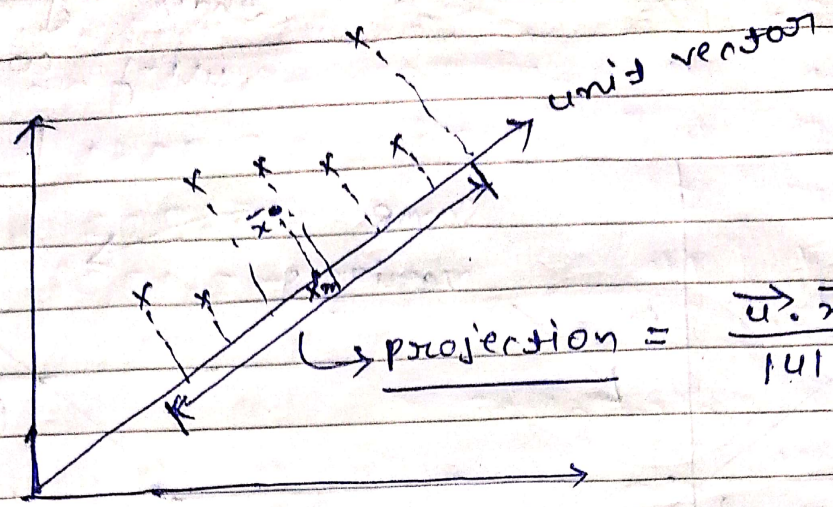
☆ PCA (principal Component Analysis) :-



$\hookrightarrow$ projection $= \dfrac{\vec{u}\cdot\vec{x}}{|u|} = \vec{u}\cdot\vec{x}$

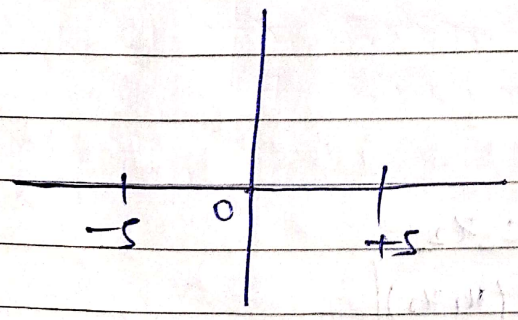
$= \vec{u}\cdot x$

$= u^T\cdot x$

$[u^T\cdot x_1]$, $[u^T\cdot x_2]$, $[u^T\cdot x_3]$ ..... $[u^T\cdot x_m]$

$$\text{Variance} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

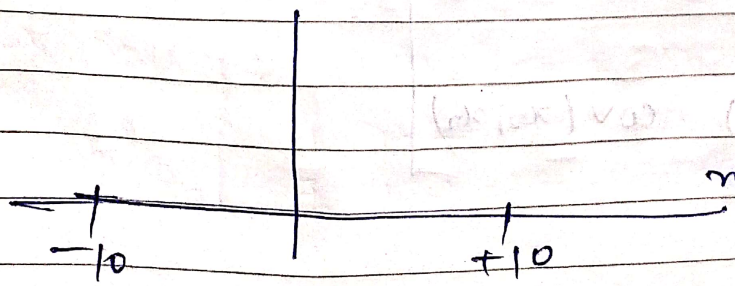
$$= \frac{\sum(u^T\cdot x_i - u^T x_m)^2}{n}$$



mean = 0

Variance $= \dfrac{50}{3}$

We have to maximize the variance



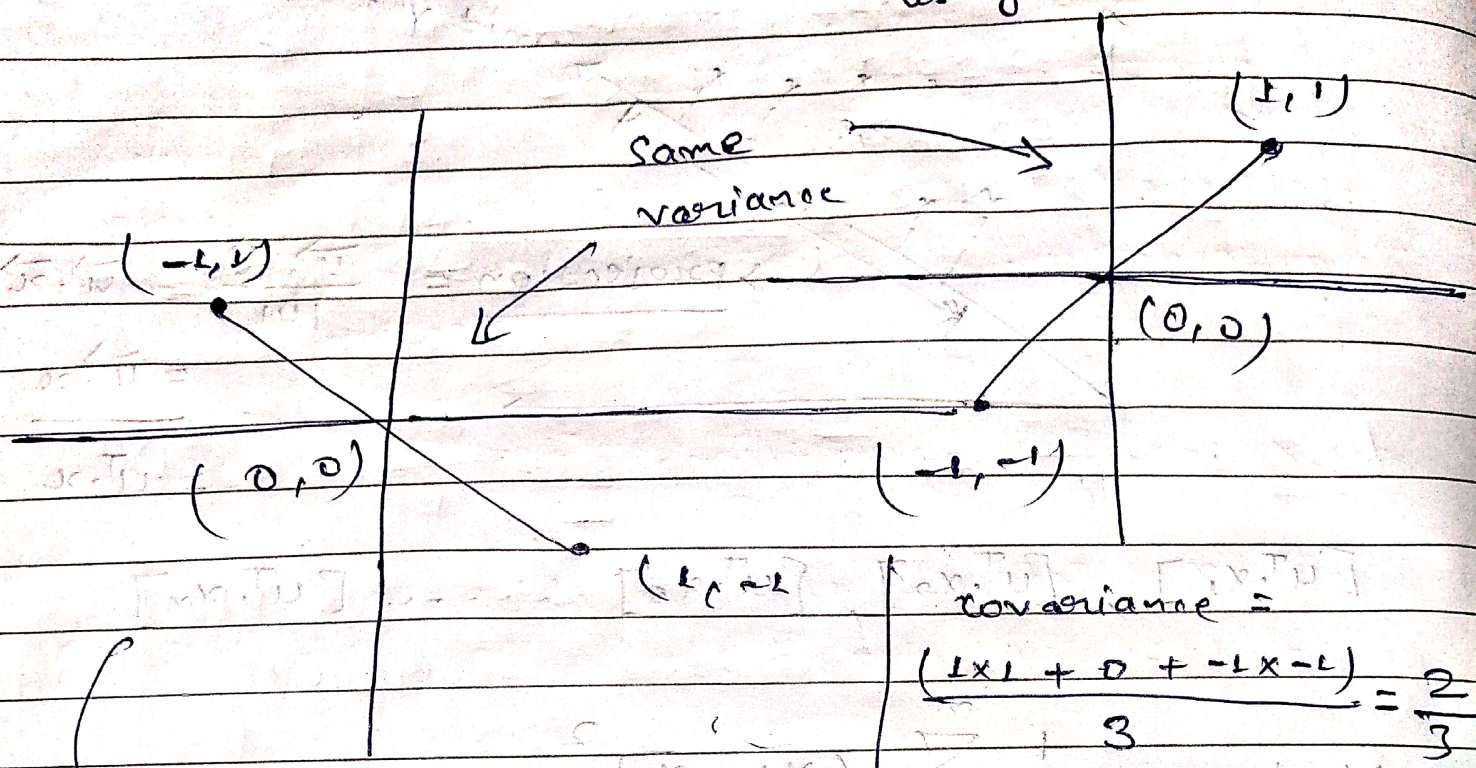
mean = 0

Variance $= \dfrac{200}{3}$

Spiral

★ Co-Variance :

variance doesn't about direction. that's why we are using co-variance



covariance = $\frac{(1 \times 1 + 0 + -1 \times -1)}{3} = \frac{2}{3}$

covariance $= \frac{(-1 \times 1 + 0 + 1 \times -1)}{3} = \frac{-2}{3}$

Covariance Matrix :

$$\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & cov(x_1, x_1) & & \\ x_2 & & cov(x_2 x_2) & \\ x_3 & & & cov(x_3, x_3) \end{array}$$

$cov(x_2, x_1) = cov(x_1, x_2)$

$$\begin{array}{c|cc} & x_1 & x_2 \\ \hline x_1 & cov(x_1, x_1) & cov(x_1, x_2) \\ x_2 & cov(x_2, x_1) & cov(x_2, x_2) \end{array}$$

☆ PCA :-

Transformation

Date

$$\text{Variance} = \frac{1}{n} \sum_{i=1}^{n} \left(u^T x_i - u^T \bar{x} \right)^2$$

$cov(x_2, x_1) = cov(x_1, x_2)$

$cov(x_1, x_1) = cov(x_1)$

$$\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & cov(x_1, x_1) & & \\ & \cancel{cov(x_2, x_1)} & \cancel{cov(x_2, x_2)} & \\ x_2 & cov(x_2, x_1) & cov(x_2, x_2) & \\ & & & \\ x_3 & & & cov(x_3, x_3) \end{array} \quad 3 \times 3$$

→ covariance matrix

find &

~~Select~~

☆ Eigen value & Eigen vector

⟹ find Eigen value & Eigen vector of co-variance matrix.

→

2D → we have ... into 1D

Date

ex

	x_1	x_2	Target	New feature
	4	11		-4.30535
	8	4.		3.7361
	13	5.		5.6922
	7.	14		-5.1238
sum =	32	34		

Step 1: mean calculation

$$\bar{x}_1 = \frac{32}{4} = 8 \qquad \bar{x}_2 = \frac{34}{4} = 8.5$$

Step 2 Covariance matrix

$$\begin{array}{c|cc} & x_1 & x_2 \\ \hline x_1 & \text{cov}(x_1, x_1) & \text{cov}(x_1, x_2) \\ x_2 & \text{cov}(x_2, x_1) & \text{cov}(x_2, x_2) \end{array}$$

$$\text{cov}(x_1, x_1) = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x}_i)^2$$

$$= \frac{(4-8)^2 + (8-8)^2 + (13-8)^2 + (7-8)^2}{4-1}$$

$$= \frac{16 + 0 + 25 + 1}{3}$$

$$= 14$$

Date

$\text{cov}(x_2, x_1) = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_1 - \bar{x}_1)(x_2 - \bar{x}_2)$

$$= \frac{(4-8)(11-8.5) + (8-8)(4-8.5) + (13-8)(5-8.5) + (7-8)(14-8.5)}{3}$$

$= -11$

Now, covariance matrix (x_1, x_2)

$$\text{cov}(x_1, x_2) = \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix}_{2\times 2}$$

$\text{cov}(x_2, x_2) = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$$= \frac{(11-8.5)^2 + (4-8.5)^2 + (5-8.5)^2 + (14-8.5)^2}{3}$$

$= \underline{23}$

Date

Step 3: Eigen vector & Eigen value

$$|S - \lambda I| = 0$$

$$\left| \begin{bmatrix} 14 & -11 \\ -11 & 23 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (14-\lambda)(23-\lambda) - 121 = 0$$

$$\lambda_1 = \underline{30.38}, \quad \lambda_2 = \underline{6.61}$$

$$\downarrow \qquad\qquad \downarrow$$

$$PC1 \qquad\qquad PC2$$

$$(S - \lambda I)X = \begin{bmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$$

$$e_1 = \begin{bmatrix} 0.5574 \\ -0.8303 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0.8303 \\ 0.5574 \end{bmatrix}$$

Date

$$\underline{e_1^T} = \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix}$$

1st

$$\begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} x_{11} - \bar{x}_1 \\ x_{21} - \bar{x}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 4-8 \\ 11-8.5 \end{bmatrix}$$

$$= \underset{1\times 2}{\begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix}} \times \underset{2\times 1}{\begin{bmatrix} -4 \\ 2.5 \end{bmatrix}} = \underline{-4.30535}$$

1x1 → single value

2nd

$$\begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} x_{12} - \bar{x}_1 \\ x_{22} - \bar{x}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 8-8 \\ 4-8.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5574 & -0.8303 \end{bmatrix} \begin{bmatrix} 0 \\ -4.5 \end{bmatrix} = 3.7361$$

★ LDA (Linear Discriminant Analysis)

Date

Supervised Learning

diff. b/w PCA & LDA ?

★ Fisher Discriminant Ratio :-

$$J(m) = \frac{(\tilde{\mu}_1 - \tilde{\mu}_2)^2}{\tilde{S}_1^2 + \tilde{S}_2^2}$$

S_1 & S_2 are class discriminant scatter matrix

minimize It should be maximum

$S_w \rightarrow$ Scatter matrix within the class

$S_B \rightarrow$ between Class scatter matrix

$$S_w = \sum_{i=1}^{C} S_i$$

for class (C) = 2

$$S_w = S_1 + S_2$$

S_1 & S_2 are scatter matrix

Date

$(\mu_1 - \mu_2) = \omega^T S_B \omega$

#

$$\boxed{J(m) = \frac{\omega^T S_B \omega}{\omega^T S_\omega \omega}}$$

$$\frac{d(J(m))}{d\omega} = 0$$

$$S_\omega^{-1} S_B \omega - J\omega = 0$$

$A v = \lambda v$

λ → eigen value

A → matrix

v → Eigen vector

Q)

x_1	x_2	Class
4	1	1
9	10	2
2	4	1
2	3	1
6	8	2
3	6	1
9	5	2
8	7	2
10	8	2
4	4	1

Date

Class $(C_1) = (x_1, x_2) \{ (4,1), (2,4), (2,3), (3,6), (4,4) \}$

Class $(C_2) = (x_1, x_2) \{ (9,10), (6,8), (9,5), (8,7), (10,8) \}$

<u>Step 1:-</u> Compute the d-dimensional mean vector.

$$\mu_1 = \left[\frac{4+2+2+3+4}{5}, \frac{1+4+3+6+4}{5} \right]$$

$$= [3 \quad 3.6]$$

$$\mu_2 = \left[\frac{9+6+9+8+10}{5}, \frac{10+8+5+7+8}{5} \right]$$

$$= [8.4 \quad 7.6]$$

<u>Step 2:-</u> Compute the Scatter matrix.

$$S_1 = \sum_{x \in \omega_1} (x - \mu_1)(x - \mu_1)^T$$

$$= \begin{bmatrix} 1 & -1 & -1 & 0 & 1 \\ -2.6 & 0.4 & -0.6 & 2.4 & 0.4 \end{bmatrix}$$

points of class 1

Date

(i) $\begin{bmatrix} 1 \\ -2.6 \end{bmatrix}_{2\times1} \begin{bmatrix} 1 & -2.6 \end{bmatrix}_{1\times2} = \begin{bmatrix} 1 & -2.6 \\ -2.6 & 6.76 \end{bmatrix}_{2\times2}$

(ii) $\begin{bmatrix} -1 \\ 0.4 \end{bmatrix} \begin{bmatrix} -1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & -0.4 \\ -0.4 & 0.16 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 \\ -0.6 \end{bmatrix} \begin{bmatrix} -1 & -0.6 \end{bmatrix} = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 0.36 \end{bmatrix}$

(iv) $\begin{bmatrix} 0 \\ 2.4 \end{bmatrix} \begin{bmatrix} 0 & 2.4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5.76 \end{bmatrix}$

(v) $\begin{bmatrix} 1 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0.4 \end{bmatrix} = \begin{bmatrix} 1 & 0.4 \\ 0.4 & 0.16 \end{bmatrix}$

$$S_1 = \begin{bmatrix} 1+1+1+1 & -2.6 \cancel{-0.4} \cancel{+0.6} \cancel{+0.4} \\ -2.6 \cancel{-0.4} \cancel{+0.6} \cancel{+0.4} & 6.76+0.16+0.36+5.76+0.16 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 4 & -2 \\ -2 & 13.2 \end{bmatrix}_{2\times2} = \begin{bmatrix} 0.8 & -0.4 \\ -0.4 & 2.64 \end{bmatrix}$$

no. of points (normalize)

Date

for Class 2

$$\begin{bmatrix} 0.6 & -2.4 & 0.6 & -0.4 & 1.6 \\ 2.4 & 0.4 & -2.6 & -0.6 & 0.4 \end{bmatrix}$$

(i) $$\begin{bmatrix} 0.6 \\ 2.4 \end{bmatrix} \begin{bmatrix} 0.6 & 2.4 \end{bmatrix} = \begin{bmatrix} 0.36 & 1.44 \\ 1.44 & 5.76 \end{bmatrix}$$

(ii) $$\begin{bmatrix} -2.4 \\ 0.4 \end{bmatrix} \begin{bmatrix} -2.4 & 0.4 \end{bmatrix} = \begin{bmatrix} 5.76 & -0.96 \\ -0.96 & 0.16 \end{bmatrix}$$

(iii) $$\begin{bmatrix} 0.6 \\ -2.6 \end{bmatrix} \begin{bmatrix} 0.6 & -2.6 \end{bmatrix} = \begin{bmatrix} 0.36 & -1.56 \\ -1.56 & 6.76 \end{bmatrix}$$

(iv) $$\begin{bmatrix} -0.4 \\ -0.6 \end{bmatrix} \begin{bmatrix} -0.4 & -0.6 \end{bmatrix} = \begin{bmatrix} 0.16 & 0.24 \\ 0.24 & 0.96 \end{bmatrix}$$

(v) $$\begin{bmatrix} 1.6 \\ 0.4 \end{bmatrix} \begin{bmatrix} 1.6 & 0.4 \end{bmatrix} = \begin{bmatrix} 2.56 & \cancel{0.01}\ 0.64 \\ \cancel{0.04}\ 0.64 & 0.16 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1.84 & -0.04 \\ -0.04 & 2.64 \end{bmatrix}$$

$$S_2 = \frac{(i) + (ii) + (iii) + (iv) + (v)}{5}$$

Spiral

within class scatter matrix

Date

$$S_w = \sum_{i=1}^{C} S_i = \sum_{i=1}^{2} S_i = S_1 + S_2$$

$$= \begin{bmatrix} 2.64 & -0.44 \\ -0.44 & 5.28 \end{bmatrix}_{2\times 2}$$

<u>Step 3:</u> Compute S_B (b/w class scatter matrix)

$$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

$$= \begin{bmatrix} (3 - 8.4) \\ (3.6 - 7.6) \end{bmatrix} \begin{bmatrix} (3-8.4) & (3.6 - 7.6) \end{bmatrix}^T$$

$$= \begin{bmatrix} -5.4 \\ -4 \end{bmatrix} \begin{bmatrix} -5.4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix}_{2\times 2}$$

Date

Step 4: find the best LDA projection vector.

$$S_\omega^{-1} S_B \, \omega = \lambda \, \omega \qquad [\because Av = \lambda v]$$

$$\left| S_\omega^{-1} S_B - \lambda I \right| = 0$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$S_\omega^{-1} = \frac{1}{(2.64 \times 5.28 - 0.44 \times 0.44)} \begin{bmatrix} 5.28 & 0.44 \\ 0.44 & 2.64 \end{bmatrix}$$

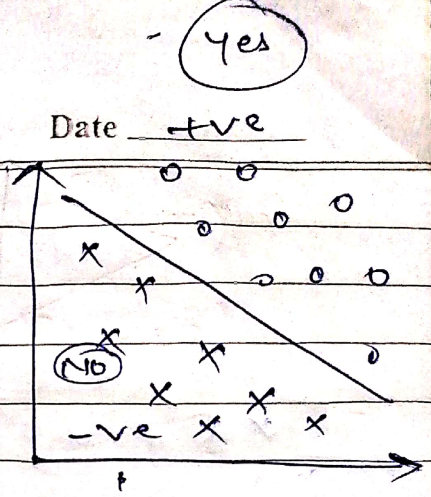
$$= \begin{bmatrix} 0.384 & 0.032 \\ 0.082 & 0.192 \end{bmatrix}$$

$$\left| \begin{bmatrix} 0.384 & 0.032 \\ 0.082 & 0.192 \end{bmatrix} \begin{bmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\lambda = \underline{15.65}$$

☆ Logistic Regression :-

Date



$$Ax + By + C = 0$$

$$\Rightarrow By = -Ax - C$$

$$\Rightarrow y = \underbrace{\left(\frac{-A}{B}\right)}_{m} x + \underbrace{\left(\frac{-C}{B}\right)}_{b}$$

for multiple inputs

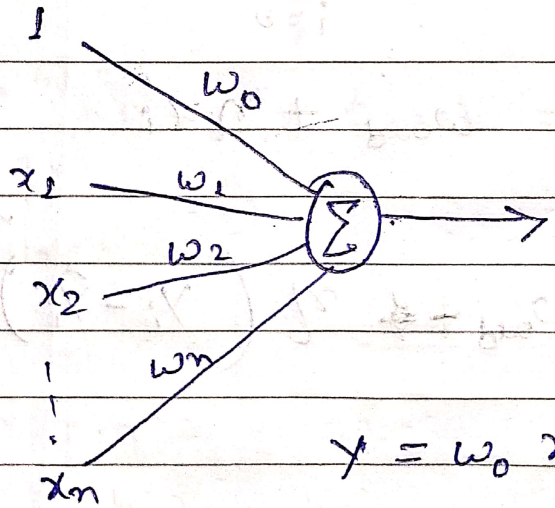
$$Ax_1 + Bx_2 + Cx_3 + D = 0$$

five points:

$$Ax_1 + Bx_2 + Cx_3 + Dx_4 + Ex_5 + F = 0$$

$$w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_0 = 0$$

$$y = w_0 + w_1x_1 + w_2x_2 \quad \Big\} \text{ for two points.}$$

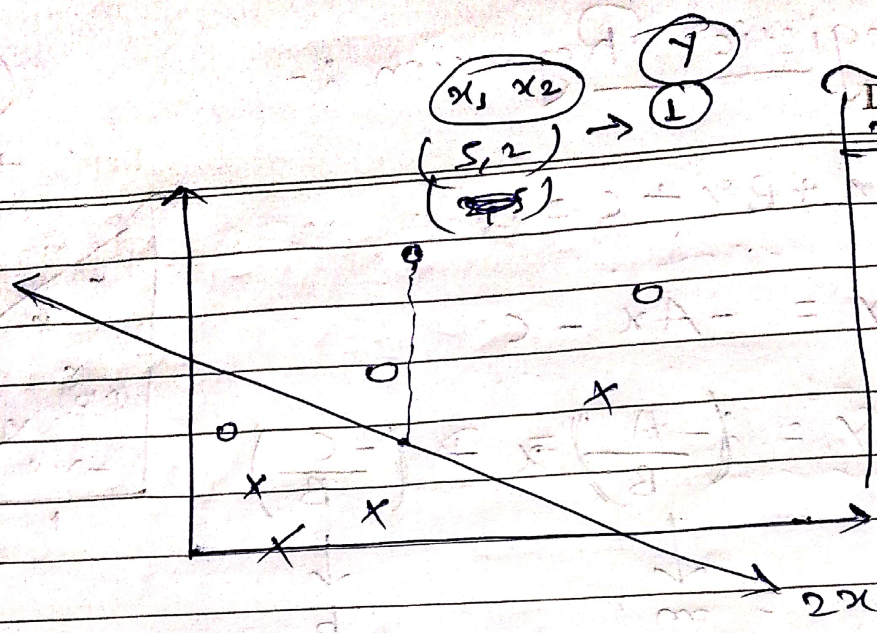


$$y = \sum_{i=0}^{n} w_i x_i$$

$$y = w_0 x_0 + w_1 x_1 + w_2 x_2$$

$$y = b + w_1 x_1 + w_2 x_2$$

$$\left[\because w_0 = b \;\middle|\; x_0 = 1\right]$$



i/p Date x_0	x_1	x_2	o/p y
1			
1			
1			
1			

$7x + 5y + 6 = 0$

for i in range(epoch):

randomly select points

if $x_i \in N$ and $\sum_{i=0}^{n} w_i x_i \geq 0$

(w_0, w_1, w_2) $w_{new} = w_{old} - \eta x_i$

if $x_i \in P$ and $\sum_{i=0}^{n} w_i x_i < 0$

$w_{new} = w_{old} + \eta x_i$

replaced by

$w_{new} = w_{old} + \eta (y_i - \hat{y}_i) x_i$

desmos.com

actual	predicted	$(y_i - \hat{y}_i)$
y	$\hat{y}$	
1	1	$\rightarrow (1-1) = 0$
0	0	$\rightarrow (0-0) = 0$
1	0	$\rightarrow (1-0) = 1$
0	1	$\rightarrow (0-1) = -1$

$$\hat{y}_i = \sum_{i=0}^{n} w_i x_i$$

~~Cost~~ ~~Loss~~ Loss function :

$$L = -\frac{1}{n} \sum_{i=1}^{n} y_i \log y_i + (1-y_i) \log (1-\hat{y}_i)$$

Sigmoid function :-

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

~~Loss~~ cost function :

Date

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & x_{24} & \cdots & x_{2n} \\ \vdots & & & & & \\ x_{m1} & x_{m2} & x_{m3} & x_{m4} & \cdots & x_{mn} \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ \vdots \\ w_n \end{bmatrix}$$

$$\cancel{\begin{bmatrix} w_1 & w_2 & w_3 & w_4 & \cdots & w_n \end{bmatrix}}$$

$$\hat{y} = \sigma(z) \quad \Big| \quad z = \sum_{i=1}^{n} w_i x_i$$

$$\hat{y}_1 = \sigma\left(x_{11} w_1 + x_{12} w_2 + x_{13} w_3 + \cdots + x_{1n} w_n\right)$$

$$\hat{y}_2 = \sigma\left(x_{21} w_1 + x_{22} w_2 + x_{23} w_3 + \cdots + x_{2n} w_n\right)$$

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \sigma\left(\underbrace{\begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ \vdots & & & & \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix}}_{X} \times \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix}}_{w} \right)$$

$$\hat{y} = \sigma(Xw)$$

Date

Taking $\sum_{i=1}^{m} y_i \log \hat{y}_i$

$$= y_1 \log \hat{y}_1 + y_2 \log \hat{y}_2 + \cdots + y_m \log \hat{y}_m$$

$$= \begin{bmatrix} y_1 & y_2 & \cdots & y_m \end{bmatrix} \times \begin{bmatrix} \log \hat{y}_1 \\ \log \hat{y}_2 \\ \vdots \\ \log \hat{y}_m \end{bmatrix}$$

$$= \begin{bmatrix} y_1 & y_2 & \cdots & y_m \end{bmatrix} \times \log \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_m \end{bmatrix}$$

$$= y \log \hat{y}$$

Date

Taking $\sum_{i=1}^{n} (1-x_i) \log(1-\hat{y}_i)$

$$= (1-Y) \log\left[\sigma(1-\underbrace{Xw}_{\hat{y}_i})\right]$$

$$L = -\frac{1}{m}\left[Y \log \hat{Y} + (1-Y) \log\left[\sigma(1-Xw)\right]\right]$$

$$\frac{\partial L}{\partial w_0}, \frac{\partial L}{\partial w_1}, \frac{\partial L}{\partial w_2} \text{ ---- } \frac{\partial L}{\partial w_n}$$

$$w_{new} = w_{old} - \eta \times \frac{\partial L}{\partial w_{old}}$$

$$w_1' = w_1 - \eta \frac{\partial L}{\partial w_1}$$

$$w_0' = w_0 - \eta \frac{\partial L}{\partial w_0}$$

$$w_2' = w_2 - \eta \frac{\partial L}{\partial w_2}$$

x_1	x_2

$w_0 x_0 + w_1 x_1 + w_2 x_2$

★ Taking first term

Date

$$\frac{\partial L}{\partial w} = \frac{\partial (y \log \hat{y})}{\partial w} \quad \Big| \quad \hat{y} = \sigma(xw)$$

$$= y \frac{\partial (\log \hat{y})}{\partial w} = y \frac{\partial (\log [\sigma(xw)])}{\partial w}$$

$$= \frac{y}{\hat{y}} \cdot \frac{\partial (\sigma(xw))}{\partial w}$$

$$\because \left[\sigma'(z) = \sigma(z)[1 - \sigma(z)] \right]$$

$$= \frac{y}{\hat{y}} \sigma(xw)[1 - \sigma(xw)] \times \frac{\partial (xw)}{\partial w}$$

$$= \frac{y}{\cancel{\hat{y}}} \times \cancel{\hat{y}} (1 - \hat{y}) \times x$$

$$= y(1 - \hat{y}) x$$

Date

Taking $(1-y)\log(1-\sigma(xw))$

$$\frac{\partial (1-y)\log(1-\hat{y})}{\partial w} = \frac{(1-y)}{(1-\hat{y})} \times \frac{\partial (1-\hat{y})}{\partial w}$$

$$= \frac{(1-y)}{(1-\hat{y})} \; -1 \times \frac{\partial (\sigma(xw))}{\partial w}$$

$$= -\frac{(1-y)}{(1-\hat{y})} \times \sigma(xw)\left[1-\sigma(xw)\right]\frac{\partial (xw)}{\partial w}$$

$$= -\frac{(1-y)}{\cancel{(1-\hat{y})}} \times \hat{y}\,\cancel{(1-\hat{y})}\,x$$

$$= -(1-y)\,\hat{y}\,x$$

$$L = -\frac{1}{m} \times \left[y(1-\hat{y})x - \hat{y}(1-y)x\right]$$

$$= -\frac{1}{m}\left[y(1-\hat{y}) - \hat{y}(1-y)\right]x$$

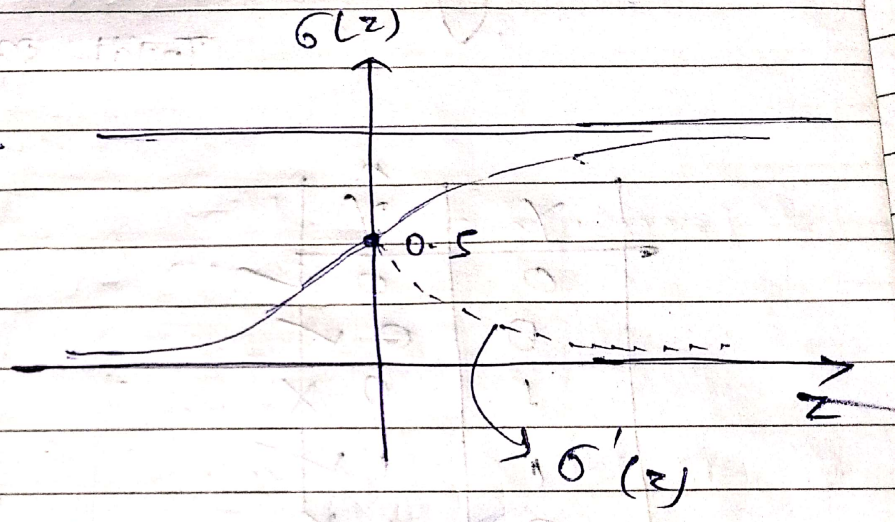
Date

$$= -\frac{1}{m}\left[y - \cancel{y\hat{y}} - \hat{y} + \cancel{\hat{y}y} \right] x$$

$$= -\frac{1}{m} \times (y - \hat{y})\, x$$

$$w_{new} = w_{old} - \eta \frac{(y - \hat{y})}{m} x$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



☆ Classification Matrix :-

Date _____

→ Accuracy F_1 Score

→ precision macro F_1 score

→ Recall weighted F_1 Score

☆ Accuracy = $\dfrac{\text{No. of correctly classified (prediction)}}{\text{Total no. of prediction}}$

y	$\hat{y}$	
0	0	✓
0	0	✓
1	0	✗
1	1	✓
0	1	✗
1	0	✗

$= \dfrac{3}{6}$

Confusion Matrix

		predicted 0	predicted 1
Actual	0	TN 12	FP 3
	1	FN 2	TP 14

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FN + FP}$$

$$= \frac{12+14}{12+3+2+14}$$

		Sent to Spam 0	Not Sent to Spam 1
Spam	0	100	170
Not Spam	1	30	700

Model A

$$\text{Accuracy} = \frac{100+700}{1000}$$

$$= 80\%$$

	0	1
0	100	190
1	10	700

Model B

$$\text{Accuracy} = \frac{100+700}{100}$$

$$= 80\%$$

Same Accuracy.

→ ~~Accura~~ Accuracy can't decide which Model is better (A or B).

Date

☆ Precision :

what proportion of predicted positive is true positive.

$$= \frac{TP}{TP + FP}$$

	0	1
0	12	3
1	2	14

$$\text{Precision} = \frac{14}{3+14} = \frac{14}{17}$$

Model A (precision) $= \frac{700}{870}$

Model B precision (P_B) $= \frac{700}{890}$

Date

	1	0
1		
0		

☆ Recall :-

	Detected cancer 1	Not Detected 0
1 Has cancer	1000	200
0 No cancer	800	8000

Has cancer → 1

No cancer → 0

1000 cases has cancer but ~~det~~ not detected 200 ← Model A

	1	0
1	1000	500
0	500	8000

→ 1000 cases has cancer but not detected is 500

Model B

$$P_A = \frac{1000}{1800} = \frac{10}{18}$$

$$P_B = \frac{1000}{1500} = \frac{10}{15}$$

$$\boxed{\text{Recall} = \frac{TP}{TP + FN}}$$ ☆

$$R_A = \frac{1000}{1200}$$

$$R_B = \frac{1000}{1500}$$

Date

☆ F_1 Score :- (Harmonic Mean of P & R)

High precision ↓ Low precision

Low Recall — High Recall

$$F_1 \text{ Score} = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

$P = R = 80$

$$F_1 = \frac{2 \times 80 \times 80}{80 + 80} = \frac{\cancel{2} \times \cancel{80} \times 80}{\cancel{160}_{2}}$$

$$= \underline{80}$$

$P = 60, R = 100$

$$F_1 \text{ Score} = \frac{2 \times 60 \times 100}{60 + 100} = \frac{\cancel{2} \times \cancel{60}^{3} \times \cancel{100}^{25}}{\cancel{160}_{4}}$$

$$= \underline{75}$$

☆ Multi class :-

Date

Actual \ Predicted	Dog	Cat	Rabbit	
Dog	25	5	10	$\Sigma = 40$
Cat	10	30	4	$\Sigma = 34$
Rabbit	4	10	20	$\Sigma = 34$
	$\Sigma = 29$	$\Sigma = 45$	$\Sigma = 34$	3X3

$$\begin{array}{r} 45 \\ +\ 34 \\ 29 \\ \hline 108 \end{array}$$

$$P_{Dog} = \frac{25}{29} = 0.86$$

$$P_{cat} = \frac{30}{45} = \frac{2}{3} = 0.66$$

$$P_{Rabbit} = \frac{20}{34} = 0.588$$

Macro precision $= \dfrac{0.86 + 0.66 + 0.58}{3} = \dfrac{2.1}{7}$

$= \underline{0.7}$

weighted precision $= \dfrac{40}{108} \times 0.86 + \dfrac{34}{108} \times 0.66 + \dfrac{34}{108} \times 0.58$

$= \cancel{0.71}\ \underline{0.71}$

Date

$$R_D = \frac{25}{40} = 0.62$$

$$R_C = \frac{30}{34} = 0.88$$

$$R_{Rb} = \frac{20}{34} = 0.58$$

Macro Recall $= \dfrac{0.62 + 0.88 + 0.58}{3}$

$$= \frac{2.08}{3} = \underline{0.693}$$

weighted Recall $= \dfrac{29}{108} \times 0.62 + \dfrac{45}{108} \times 0.88 + \dfrac{34}{108} \times 0.58$

$$= 0.166 + 0.366 + 0.182$$

$$= \underline{0.714}$$