

Multiple Regression using Matrices or A Matrix Formulation of the Multiple Regression Model(OLS Regression)

Q: Find a multiple regression model for the following data:

x1	x2	x3	y
4	5	4	6
7	2	3	11
2	6	4	4
1	9	6	3
3	4	5	5
7	3	4	9
8	2	5	10

Solution: See derivation in class notes

$$\beta = (X'X)^{-1} X'y$$

We know that

$$X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ 1 & x_{21} & \cdots & x_{2k} \\ \vdots & \vdots & & \vdots \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix}$$

$$\underline{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \quad y = \begin{bmatrix} 6 \\ 11 \\ 4 \\ 3 \\ 5 \\ 9 \\ 10 \end{bmatrix} \quad X = \begin{bmatrix} 1 & 4 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 1 & 2 & 6 & 4 \\ 1 & 1 & 9 & 6 \\ 1 & 3 & 4 & 5 \\ 1 & 7 & 3 & 4 \\ 1 & 8 & 2 & 5 \end{bmatrix}$$

After Solving $\beta = (X'X)^{-1} X'y$ we get

$$\underline{b} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 3.96239 \\ 1.06065 \\ 0.04396 \\ -0.48517 \end{bmatrix}$$

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

$$y = 3.96239 + 1.06065 X_1 + 0.04396 X_2 - 0.48517 X_3$$