Q1. Prove that for a hexagonal geometry, the co-channel reuse ratio is given by $Q = \sqrt{3N}$, where $N = i^2 + ij + j^2$. Hint : Use the cosine law and the hexagonal cell geometry.

Generally, for $N = i^2 + i \cdot j + j^2$, we can do the following to find the nearest co-channel neighbors of a particular cell:

(1) move i cells along any chain of hexagons and than

(2) turn 60 degree counter-clockwise and move j cells

From the following figure, using the cosine law, we have

$$D^2 = [i \cdot (2R')]^2 + [j \cdot (2R')]^2 - 2i \cdot (2R') \cdot j \cdot (2R') \cdot \cos 120^\circ$$

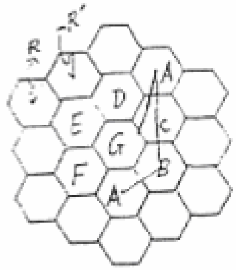
where $R' = \frac{\sqrt{3}}{2} R$, therefore

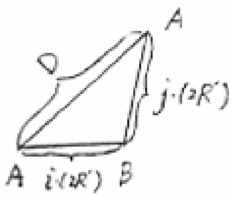
$$D = \sqrt{3i^2 \cdot R^2 + 3j^2 \cdot R^2 + i \cdot j \cdot 3R^2}$$

$$= \sqrt{3(i^2 + i \cdot j + j^2)} \cdot R$$

$$= \sqrt{3N} \cdot R$$

Hence, $Q = \frac{D}{R} = \sqrt{3N}$





Q2. A cellular service provider decides to use a digital TDMA scheme which can tolerate a signal-to-interference ratio of 15 dB in the worst case. Find the optimal value of N for (a) omnidirectional antennas, (b) 120^o sectoring, and (c) 60^o sectoring. Should sectoring be used? If so, which case (120^o or 60^o) should be used? (Assume a path loss exponent of n=4 and consider trunking efficiency).

(a) Let i_0 be the number of co-channel interfering cells, for omni-derectional antennas, $i_0=6$. Assume $n=4$,

we have $\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 15\,dB = 31.623 \Rightarrow N > 4.59$

$\Rightarrow \underline{\underline{N=7}}$

(b) For $120°$ sectoring, $i_0 = 2$.

$\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 31.623 \Rightarrow N > 2.65 \Rightarrow \underline{\underline{N=3}}$.

(c) For $60°$ sectoring, $i_0 = 1$.

$\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 31.623 \Rightarrow N > 1.87 \Rightarrow \underline{\underline{N=3}}$

From (a), (b) and (c) we can see that using $120°$ sectoring can increase the capacity by a factor of $7/3$, or 2.333. Although using $60°$ sectoring can also increase the capacity by the same factor, it will decrease the trunking efficiency therefore we choose the $120°$ sectoring.

Q3. A receiver in an urban cellular radio system detects a 1 mW signal at d=d_0=1 meter from the transmitter. In order to mitigate co-channel interference effects, it is required that the signal received at any base station receiver from another base station transmitter which operates with the same channel must be below -120 dB. A measurement team has determined that the average path loss exponent in the system is n=3. Determine the major radius of each cell if a seven-cell reuse pattern is used. What is the major radius if a four-cell reuse pattern is used?

For 7 cell reuse pattern, the interference signal power from another transmitter is

$$P_I = P_t \cdot \left(\frac{D}{d_0}\right)^{-n} = P_t \cdot \left(\frac{\sqrt{3N} \cdot r}{d_0}\right)^{-n}$$

where P_t is the transmit power in base station, D is the distance to the center of the nearest co-channel cells, r is the major radius. In this case, $P_t = 1mW$, $N=7$, $d_0 = 1m$, $n=3$, thus we have

$$10 \log_{10} \frac{P_t \left(\frac{\sqrt{3N} \cdot r}{d_0}\right)^{-n}}{1mW} < -120dBm$$

$$10 \log_{10} \frac{1mW \times \left(\frac{\sqrt{3 \times 7} \cdot r}{1m}\right)^{-3}}{1mW} < -120dBm$$

$$\therefore \quad r > 218217.9 \text{ m}$$

For 4 cell

$$r > 288675 \text{ m}$$

Q4. A cellular system using a cluster size of seven is described in Q3. It is operated with 660 channels, 30 of which are designated as setup (control) channels so that there are about 90 voice channels available per cell. If there is a potential user density of 9000 users/km^2 in the system, and each user makes an average of one call per hour and each call lasts 1 minute during peak hours, determine the probability that a user will experience a delay greater than 20 seconds if all calls are queued.

area of a cell (hexagon) $= \frac{3\sqrt{3}}{2} \cdot r^2 = \frac{3\sqrt{3}}{2} \cdot (0.4701)^2 \doteq 0.574 \text{ km}^2$

number of users in a cell $U =$ area of a cell $\times$ user density

$= 0.574 \times 9000 \doteq 5167$ users

$\Rightarrow A = U \cdot \mu \cdot H = 5167 \times \frac{1}{60} \times 1 \doteq 86.1$ Erlangs.

Given $C = 90$, from Erlang C chart, we have the probability that a call will be delayed

$Pr[\text{delay} > 0] \doteq 0.5$

$\Rightarrow Pr[\text{delay} > 20 \text{ sec}] = Pr[\text{delay} > 0] \cdot Pr[\text{delay} > 20 \mid \text{delay}]$

$= 0.5 \times \exp[-(90 - 86.1) \times 20/60]$

$\doteq 0.136$

Q5. The U.S. AMPS system is allocated 50 MHz of spectrum in the 800MHz range and provides 832 channels. 42 of those channels are control channels. The forward channel frequency is exactly 45 MHz greater than the reverse channel frequency.

(a) Is the AMPS simplex, half-duplex, of duplex? What is the bandwidth for each channel?

(b) Assume a base station transmits control information on channel 352, operating at 880.56 MHz. What is the transmission frequency of a subscriber unit transmitting on channel 352.

(c) The A-side and B-side cellular carriers evenly split the AMPS channels. Find the number if voice channels and number of control channels for each carrier.

(d) For an ideal hexagonal cellular layout which has identical cell coverage, what is the distance between the centers of two nearest co-channel cells for seven-cell reuse? For four-cell reuse?

(a) The AMPS system is duplex.

Given total bandwidth $BW_{total} = 50MHz$, total number of channel $N = 832$ channels, we have the bandwidth for each channel $B_{rv} = \frac{BW_{total}}{N} = \frac{50 \times 10^6}{832} = 60KHz$

This bandwidth of 60KHz for the duplex channel is split into two one-way channels, a forward channel (from the base-station to the subscriber) and a reverse channel (from the subscriber to the base station), each with bandwidth of 30kHz. The forward channel is exactly 45 MHz higher than the reverse channel.

(b) For $F_{fw} = 880.560\,MHz \Rightarrow F_{reverse} = F_{fw} - 45 = \underline{\underline{835.560\,MHz}}$

(c) Given $N = 832$, total number of control channel $N_{con} = 42$, we have total number of voice channel

$N_{vo} = N - N_{con} = 832 - 42 = 790$.

$\Rightarrow$ number of voice channels for each carrier

$$N_{vo,A} = N_{vo,B} = \frac{N_{vo}}{2} = \frac{790}{2} = \underline{\underline{395 \text{ channels}}}$$

number of control channels for each carrier

$$N_{con,A} = N_{con,B} = \frac{N_{con}}{2} = \frac{42}{2} = \underline{\underline{21 \text{ channels}}}$$

(d) See example 3.3

Q6. Assume a receiver is located 10km from a 50 W transmitter. The carrier frequency is 6 GHz and free space propagation is assumed, $G_t = 1$ and $G_r = 1$.

(a) Find the power at the receiver.

(b) Find the magnitude of the E-field at the receiver antenna.

(c) Find the rms voltage applied to the receiver input, assuming that the receiver antenna has a purely real impedance of $50\,\Omega$ and is matched to the receiver.

a) $P_r = \dfrac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2} = \dfrac{(50)(1)(1)(1/20)^2}{(4\pi)^2 (10^4)^2} = 7.9\,10^{-12}$ W $= -81$ dBm

b) $P_r = P_d \cdot A_e = \left[\dfrac{|E|^2}{120\pi}\right] \cdot A_e = \dfrac{|E|^2}{120\pi} \cdot \dfrac{G_r \lambda^2}{4\pi}$

$A_e = \dfrac{G \lambda^2}{4\pi}$ $\quad E = P_r \cdot (120\pi)(4\pi)/\lambda^2 G_r = 3.9\times10^{-2}$ V/m

c) $P_r = \dfrac{\left[\dfrac{V_{ant}}{2}\right]^2}{50\Omega}$ $\Rightarrow \sqrt{7.9\,10^{-12} \cdot 50 \cdot 4} = V_{ant}$ open circuit

$V_{ant} = 4 \cdot 10^{-5}$ Volts rms open circuit

$V_{rcvr} = \dfrac{V_{ant}}{2} = 2 \cdot 10^{-5}$ Volts rms

Q7. Free space propagation: Assume the transmitter power is 1W at 60 GHz fed into the transmitter antenna (Assume $G_t = G_r = 29$ dB and $P_t = 30$ dBm).

(a) Calculate the free space path loss at 1m, 100m, 1000m.

(b) Calculate the received signal power at these distances.

(c) What is the rms voltage received at the antenna if the receiver antenna has purely real impedance of $50\,\Omega$ and is matched to the receiver?

$$
\begin{aligned}
G_t &= G_r = 29 \text{ dB} \\
P_t &= 30 \text{ dBm} \\
\lambda &= \frac{c}{f} = 0.005 \text{ m} \\
d_0 &= 1 \text{ m} \\
d_1 &= 100 \text{ m} \\
d_2 &= 1000 \text{ m} \\
PL(d_0) &= 20\log_{10}\frac{4\pi d_0}{\lambda} = 20\log_{10}\frac{4\pi}{0.005} = 68 \text{ dB} \\
PL(d_1) &= PL(d_0) + 20\log_{10}\frac{d_1}{d_0} = 108 \text{ dB} \\
PL(d_2) &= PL(d_0) + 20\log_{10}\frac{d_2}{d_0} = 128 \text{ dB}
\end{aligned}
$$

$$
\begin{aligned}
P_r &= P_t + G_t + G_r - PL = 30 + 29 + 29 - PL = 88 - PL \\
P_r(d_0) &= 88 - 68 = 20 \text{ dBm} \\
P_r(d_1) &= 88 - 108 = -20 \text{ dBm} \\
P_r(d_2) &= 88 - 128 = -40 \text{ dBm}
\end{aligned}
$$

$$
\begin{aligned}
V &= \sqrt{4\, P_r\, R_{ant}} \\
V(d_1) &= 0.0447 \text{ v} \\
V(d_2) &= 0.0045 \text{ v}
\end{aligned}
$$

Q8. From the knife-edge diffraction model, show how the diffracted power depends on frequency. Assume $d_1 = d_2 = 500m$ and $h = 10m$ in Figure 1. (4.18)

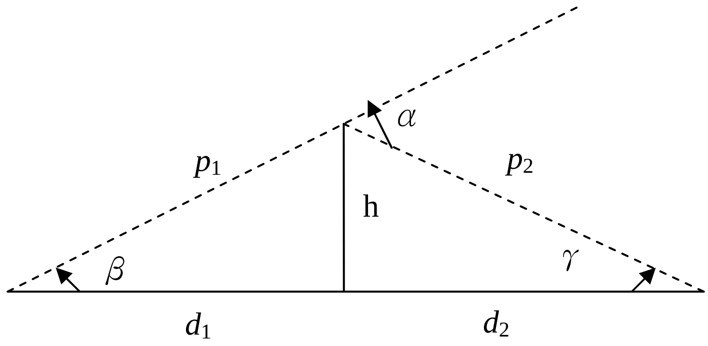


Figure 1

Diffracted power decreases with the increase of the frequency as shown in Figure 1.

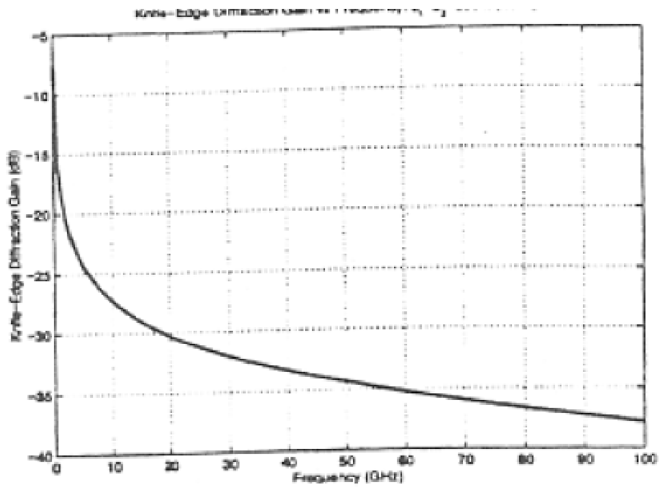


Figure 1: Diffracted power vs. frequency.

Q9. If the received power at a reference distance $d_0 = 1km$ is equal to 1 microwatt, find the received power at distance of 2*km*, 5*km*, 10*km*, and 20*km* from the same transmitter for the following path loss models: (a) Free space; (b)n=3; (c)n=4; (d)two-ray ground reflection using the exact expression. Assume f=1800MHz, h_t=40m, h_r=3m, $G_t = G_r = 0$ dB.

(a) For free space, $P_r = P_o\left(\frac{d_o}{d}\right)^2$

Given $P_o = 10^{-6}(W) = -30\,dBm$, $d_o = 1Km$.

For $d = 2Km$, $P_r = 10^{-6}\cdot\left(\frac{1}{2}\right)^2 = 2.5\times10^{-7}(W) \doteq -36\,dBm$

Similarly, For $d = 5Km$, $P_r \doteq -44\,dBm$

For $d = 10Km$, $P_r \doteq -50\,dBm$

For $d = 20Km$, $P_r = -56\,dBm$

(b) For $n=3$, $P_r = P_o\cdot\left(\frac{d_o}{d}\right)^3$

For $d = 2Km$, $P_r = 10^{-6} \cdot (\frac{1}{2})^3 = 1.27 \times 10^{-7} (W) = \underline{\underline{-39\ dBm}}$

For $d = 5Km$, $P_r = \underline{\underline{-51\ dBm}}$

For $d = 10Km$, $P_r = \underline{\underline{-60\ dBm}}$

For $d = 20Km$, $P_r = \underline{\underline{-69\ dBm}}$

(c) For $n = 4$, $P_r = P_0 (\frac{d_0}{d})^4$

For $d = 2Km$, $P_r = \underline{\underline{-42\ dBm}}$ For $d = 5Km$, $P_r = \underline{\underline{-58\ dBm}}$

For $d = 10Km$, $P_r = \underline{\underline{-70\ dBm}}$ For $d = 20km$, $P_r = \underline{\underline{-82\ dBm}}$

(d) For two ray ground reflection model using the exact expression

$$P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 d_0^2} \Rightarrow P_t = \frac{P_r(d_0) \cdot (4\pi)^2 \cdot d_0^2}{G_t \cdot G_r \cdot \lambda^2}$$

Given $P_r = 10^{-6} W$, $d_0 = 1Km$, $G_t = G_r = 0dB = 1$. $\lambda = \frac{c}{f_c} \doteq 0.1667m$,

$$\Rightarrow P_t = \frac{10^{-6} \times (4\pi)^2 \times (1000)^2}{1 \times 1 \times 0.1667^2} \doteq 5.679 \times 10^3 (W)$$

From problem 4.12, the exact expression is

$$P_r(d) = \frac{P_t \cdot G_t\, G_r\, \lambda^2}{(4\pi)^2 \cdot d^2} \cdot 4 \cdot \sin^2(\frac{\theta_\Delta}{2}), \quad \text{where } \theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2h_t \cdot h_r}{d}$$

For $d = 2km$, $\theta_\Delta = \frac{2\pi}{0.1667} \times \frac{2 \times 40 \times 3}{2000} = 4.5216$ rads

$$\Rightarrow P_r = \frac{5.679 \times 10^3 \times 1 \times 1 \times (0.1667)^2}{(4\pi)^2 \times (2000)^2} \times 4 \times \sin^2(\frac{4.5216}{2})$$

$$\doteq 5.97 \times 10^{-7} (W) \doteq \underline{\underline{-32.25\ dBm}}$$

Similarly, For $d = 5Km$, $\theta_\Delta \doteq 1.809$ rads

$\Rightarrow P_r \doteq 9.88 \times 10^{-8} (W) \doteq \underline{\underline{-40\ dBm}}$

For $d = 10km$, $\theta_\Delta = 0.904$ rads

$\Rightarrow P_r = 7.64 \times 10^{-9} (W) \doteq \underline{\underline{-51.17\ dBm}}$

Q10. Assume a SNR of 25dB is desired at the receiver. If a 900MHz cellular transmitter has an EIRP of 100W, and the AMPS receiver uses a 0dB gain antenna and has a 10dB noise figure, find the percentage of time that the desired SNR is achieved at distance of 10*km* from the transmitter. Assume *n*=4, σ=8dB, and $d_0 = 1km$. (4.28)

$$\text{noise floor} = K \cdot B_W \cdot F \cdot T_0$$
$$= 1.38 \times 10^{-23} \times 30 \times 10^3 \times 10 \times 290 \doteq 1.2 \times 10^{-15} (W)$$
$$\doteq -119.2 \, (dBm)$$

$$\Rightarrow \text{threshold } \gamma = \text{noise floor}(dBm) + SNR(dB)$$
$$= -119.2 + 25 = -94.2 \, (dBm)$$

Given $EIRP = P_t \cdot G_t = 100W$, $G_r = 0 dB = 1$, $d_0 = 1Km$, $\lambda = \frac{c}{f} = 0.333m$

$$P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2} = \frac{100 \times 1 \times 0.333^2}{(4\pi)^2 \times (1000)^2} \doteq 7.04 \times 10^{-8} (W)$$
$$\doteq -41.5 \, dBm$$

For $d = 10 Km$, $n = 4$.

$$\overline{P_r(d)} = P_r(d_0) - 10 \cdot n \cdot \log_{10}\left(\frac{d}{d_0}\right) = -41.5 - 40 = -81.5 \, dBm$$

$$\Rightarrow Pr(P_r(d) > \gamma) = Q\left[\frac{\gamma - \overline{P_r(d)}}{\sigma}\right] = Q\left[\frac{-94.2 - (-81.5)}{8}\right]$$
$$= Q(-1.5875) \doteq \underline{\underline{0.944}}$$