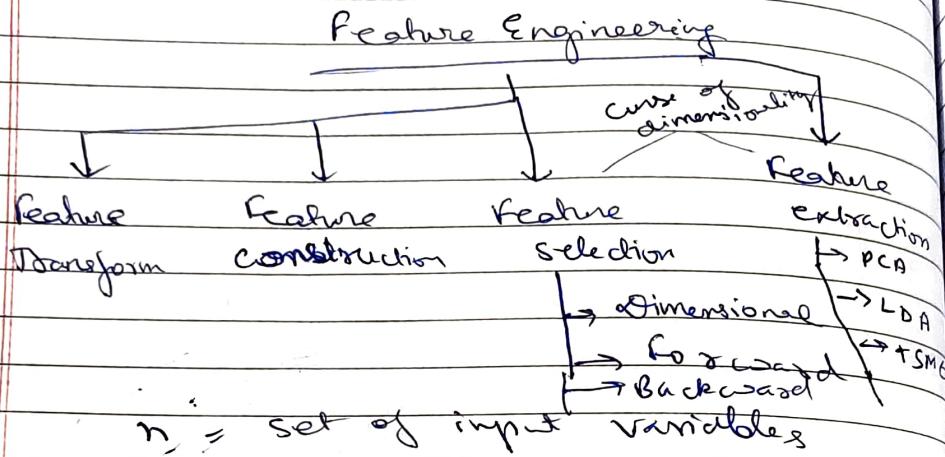


## Cause of dimensionality Reduction:-

X



1. Performance  $\rightarrow$  Select those attribute that effect  $\rightarrow$

2. Visualization  $\rightarrow$  set of input  $\downarrow$ , more the visualization  $\uparrow$

$f_1, f_2, \dots, f_n$

$F_1, F_2, F_3, \dots$

new features built on the basis of previous ones.

mean squared error =  $\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$

PCA  $\rightarrow$   $y$

g

1.  $f_i = \emptyset$   $f = \infty$
2. Select feature one by one.  
Select  $F_i$ , check whether it reduces the error or not.
3. If error  $\downarrow$ , add it to set of features.

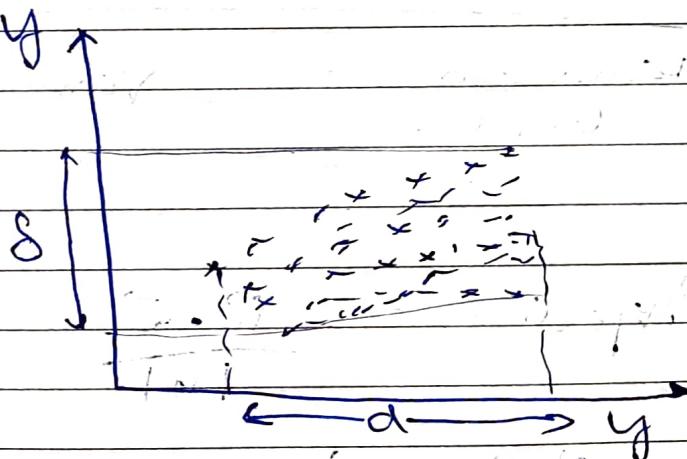
1. Set  $F_0 = \emptyset$  &  $E(F_0) = \infty$
2. for  $i = 0$ , repeat until  $E(F_{i+1}) > E(F_i)$

(a)  $F_i \cup \{x_j\}$   
Calc.  $E(F_i \cup \{x_j\})$

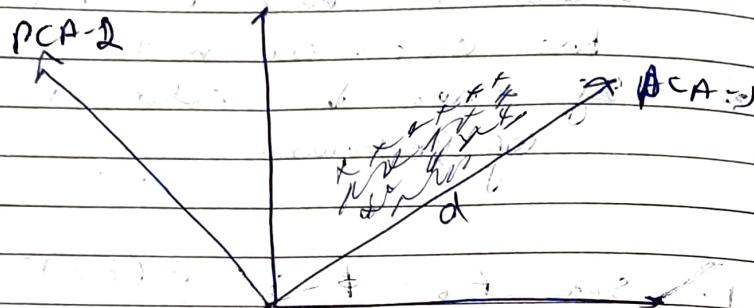
(b)  $m = \min_j E(F_i \cup \{x_j\})$

(c)  $F_{i+1} = F_i \cup \{x_j\}$   
int. val.

PCA  $\rightarrow$  Principle Component Analysis :-



linearly separable data  $\rightarrow$  both have  
the same spread,



$$\text{mean} = \bar{x} = \frac{1}{n} \sum x_i$$

$$\text{Variance} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

Variance (zayda)  $\rightarrow$  data points  
dus dus hai

Türe hoge, variance will a zayda

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

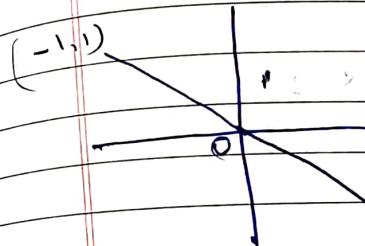
$$\text{Projector} = \frac{\vec{u} \cdot \vec{x}}{|\vec{u}|} \vec{u}$$

$$\therefore \vec{u} \cdot \vec{x} = \vec{u} \cdot \vec{x} = \vec{u}^T \vec{x}$$

$$[\vec{u}^T \vec{u}] =$$

$$\text{Variance} =$$

co-variance



co-variance

multipl  
by

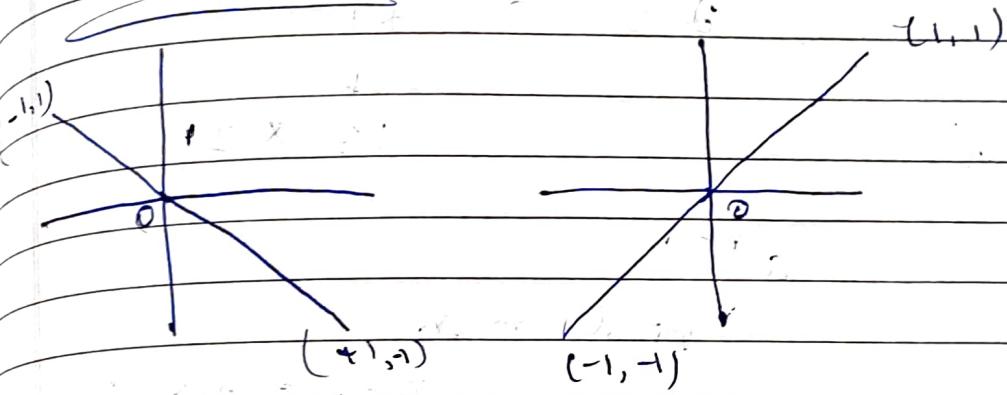
Covariance

$$[U^T u] = [U^T u_1] \dots [U^T u_n]$$

$$\text{Variance} = \frac{1}{n} \sum_{i=1}^n (U^T u_i - \bar{u}^T u_m)^2$$

projection of

co-variance is



$$\text{Covariance} = \frac{(-1*1) + 0 + (1*-1)}{3}$$

$$= -1 - 1 - 2 = -\frac{2}{3}$$

Multiply the signs and divide by 3

Covariance

	$u_1$	$u_2$	$u_3$
$u_1$	$\text{Cov}(u_1, u_1)$	$\text{Cov}(u_1, u_2)$	$\text{Cov}(u_1, u_3)$
$u_2$	$\text{Cov}(u_2, u_1)$	$\text{Cov}(u_2, u_2)$	$\text{Cov}(u_2, u_3)$
$u_3$	$\text{Cov}(u_3, u_1)$	$\text{Cov}(u_3, u_2)$	$\text{Cov}(u_3, u_3)$

for non-linear separated data, we cannot apply PCA to it.

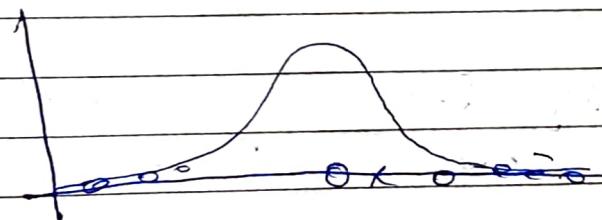
LDA  $\rightarrow$  supervised learning  
PCA  $\rightarrow$  unsupervised learning

t-SNE :- for high dimensional data to 2d, it is applied in those area where we can't use PCA

t-distributed Stochastic Neighbour Embedding

Score  
Sum of all score }  $\rightarrow$  the more bigger the ratio, the more it is closer to the point.

t-distribution, the tail is a bit longer than the normal distribution. And, some points also lie on the tail.



Check whether  $\mathbf{x}_j$  point is neighbour of  $\mathbf{x}_i$  or not. for this we use the formula:

$$P_{\mathbf{x}_j} = \exp(-\|\mathbf{x}_i - \mathbf{x}_j\|^2 / 2\sigma_i^2)$$

$$\sum_{k \neq i} \exp(-\|\mathbf{x}_i - \mathbf{x}_k\|^2 / 2\sigma_i^2)$$

$$a_{ij} = \frac{(1 + \|\mathbf{y}_i - \mathbf{y}_j\|^2)^{-1}}{\sum_k \sum_{l \neq k} (1 + \|\mathbf{y}_k - \mathbf{y}_l\|^2)^{-1}}$$

$$\sum_i P_{\mathbf{x}_i} = 1 \text{ for all } i$$

This is  $\Leftrightarrow$   
so me  
"loss" dimension  
for  $\mathbf{y}_i$

Loss given by  $\chi^2$  divergence

$\chi^2 \rightarrow$  kullback leibler

$$\chi^2(\mathbf{p} \parallel \mathbf{q}) = \sum_{i \neq j} P_{\mathbf{x}_i} \log \frac{P_{\mathbf{x}_i}}{q_{\mathbf{x}_i}}$$

If  $P_{\mathbf{x}_i}$  is more which near  $\mathbf{x}_j$  is the neighbour of  $\mathbf{x}_i$

Loss needs to be less for that  $q_{\mathbf{x}_i}$  should be ~~minimum~~ maximum

$P_{\mathbf{x}_i} \rightarrow$  minimum  
more force to minimum.

Stack Quest

M.L

Logis

If the points are not shifting

BT

the  $\mathbf{c}^m$  of line by adding or  
On adding or  
on subtraction

$A_n + B_n$

$A_n + B_n$

$w, x', + w$

$y =$

$y =$

$P_{ji} \rightarrow \text{minimum}$

more false log should be minimum

Stack Overflow

→ Checks the video links

~~M L~~

## Logistic Regression

If the points are correctly classified, no shifting is done.

Bt, if they are wrongly classified, the "c" of lines are changed either by adding or subtracting the co-ordinates. On adding, the line shifts upwards, on subtraction, it goes below.

$$A u_1 + B u_2 + C = 0$$

$$A u_1 + B u_1 + C u_3 + D u_4 + E u_5 + C_0 = 0$$

$$w_0 x_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5 = 0$$

$$y = w_0 + w_1 x_1$$

$$y = w_0 + w_1 x_1 + w_2 x_2$$

P.J.I  $\rightarrow$  minimum

more force log should be minimum.

Stack Overflow

$\rightarrow$  checks the video links

M.L

## Logistic Regression

If the points are correctly classified, no shifting is done.

Bt, if they are wrongly classified, the "eq" of lines are changed either by adding or subtracting the coordinates. On adding, the line shifts upwards, on subtraction, it goes below.

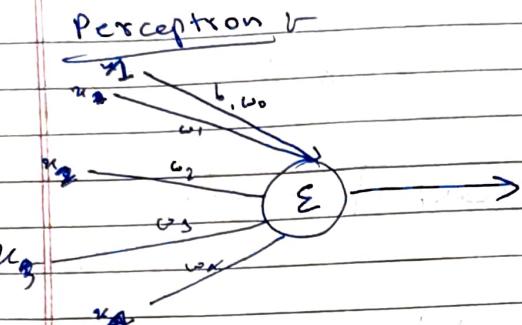
$$A u_1 + B u_2 + C = 0$$

$$A u_1 + B u_2 + C u_3 + D u_4 + E u_5 + G = 0$$

$$w_0 x_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5 = 0$$

$$y = w_0 + w_1 x_1$$

$$y = w_0 + w_1 x_1 + w_2 x_2$$



$$y = \sum_{i=1}^r w_i x_i + b$$

$$y = w_0 u_0 + w_1 u_1 + w_2 u_2 + w_3 u_3 + w_4 u_4 + b$$

$$y = \sum_{i=1}^n w_i u_i + b$$

$u_i$	$u$	$y$	$u_i$	$y$
1	..	..	0	Yes
1	-	-	1	No

$$w_{\text{new}} = w_{\text{old}} - \text{GD}$$

$$\begin{cases} 2x + 3y + 5 \\ 7x + 5y + 6 \end{cases}$$

Gradient descent

Program :-

for i in range (

randomly select

i if  $u_i \in N$

else

predict

but it  
should  
be in  
range

Actual

thus A, B, C

$w_{\text{new}} =$

$w_{new} = w_{old} - \text{gradient} \times \text{learning rate}$

$$\begin{aligned} 2x_n + 3y + 5 &= 0 \\ 4x_n + 5y + 6 &= 0 \end{aligned}$$

Gradient descent = new value - old value  $\times$  learning rate

Program :-

for i in range ( epochs )

randomly select point = 0

if  $w_i \in N$  or  $\sum w_i u_i > 0$

else 11

predict

if it

should

be in  
negative

$y > 0$

but is  
positive

in actual

thus A, B, C needs to be changed

$$w_{new} = w_{old} - \eta u_i$$

Learning rate

case 2 :-

$$y_i \leq y_i^* \text{ and } w_i < 0$$

↓  
in positive  
region acted

$y_i < y_i^*$   
but now  $y_i$  will be  
in negative  
region  
has to make  
it positive  
we add

$$w_{new} = w_{old} + \eta y_i$$

$$w_{new} = w_{old} + \eta (y_i - y_i^*) y_i$$

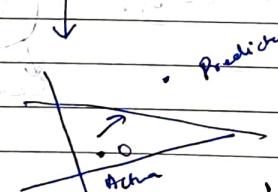
$$w_{new} = w_{old}$$

Actual	Predicted
$y_i$	$y_i^*$
1	1
0	0

for 1

out

for 0



$$w_{new} = w_{old} + \eta y_i$$

$$w_{new} = w_{old} - \eta y_i$$

$$y^* = \sum_{i=0}^n w_i x_i$$

$$y^* < 0 \rightarrow$$

$$y^* > 0 \rightarrow$$

classifications

new line  
↓

$$w_2 = -y_1 \log y_1$$

gross  
entropy  
or  
log  
function

$$y_1 = 1 \rightarrow$$

$$\therefore \text{for } y_1 = 0$$

$$\log(0.5) = -0.693$$

G → green

B → black



$$\hat{y} = \sum_{i=0}^n w_i u_i$$

$\hat{y} < 0 \rightarrow$  below the -ve region

$\hat{y} > 0 \rightarrow$  in the +ve region

[demos.com](https://demos.com)

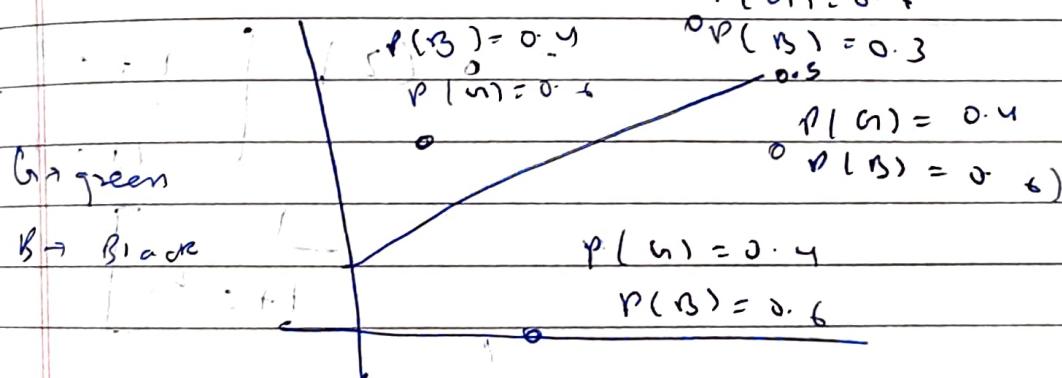
for entropy of fusion

$$H_2 = -y_1 \log(y_1) - (1-y_1) \log(1-y_1)$$

$$y_1 = 1 - \log(0.7)$$

$$\begin{aligned} \text{for } y_1 = 0 &= -\log(1-0.0) \\ &= -\log 0.4 \end{aligned}$$

$$\log(ab) = \log(a) + \log(b)$$



P = 2^24 (a)

M (Date)

- (1)
- (2)
- (3)
- (4)
- (5)
- (6)

RSA

NS

Knapsack

ECC &amp; Elliptic

Rabin Cryptography

(5) General(6) Message Digest(5)  
(MD5)ML

$$\text{Likelihood} = \prod_{i=1}^n [y_i \log(\hat{y}_i) + (1-y_i) \log(1-\hat{y}_i)]$$

$$L = \frac{1}{N} \sum_{i=1}^N [-y_i \log(y_i) - (1-y_i) \log(1-y_i)]$$

$$\sigma^{-1}(z) = \sigma(z) \left[ \frac{1+e^{-z}}{1+e^z} \right]$$

$$= \sigma(z) \left[ \frac{1+e^{-z}}{1+e^z} \right]$$

ML

Data  
Prop.

Loss  
function

$$L = -\frac{1}{n} \sum_{i=1}^n y_i \log \hat{y}_i + (1 - y_i) \log (1 - \hat{y}_i)$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma'(z) = \sigma(z) [1 - \sigma(z)]$$

Cost function  $\Rightarrow$  is a total loss

$$y_{\text{predicted}} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{bmatrix}$$

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \omega_n \end{bmatrix} \text{ coefficients}$$

$$\hat{y}_i = \sigma(x_{11}\omega_1 + x_{12}\omega_2 + x_{13}\omega_3 + \dots + x_{1n}\omega_n)$$

$$\hat{y}_i = \sigma(x_{21}\omega_1 + x_{22}\omega_2 + x_{23}\omega_3 + \dots + x_{2n}\omega_n)$$

Taking

$$Y = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \vdots \\ \hat{y}_n \end{bmatrix} =$$

$$\delta = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & & x_{2n} \\ \vdots & & & & \\ x_{m1} & x_{m2} & x_{m3} & \dots & x_{mn} \end{bmatrix} X$$

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \omega_n \end{bmatrix}$$

$$Z = \sum_{i=1}^n \delta(\omega_i, x_i)$$

$$\hat{Y} = \delta(\hat{X}, \omega)$$

$$\text{Taking } \sum_{i=1}^n y_i \log \hat{y}_i = y_1 \log \hat{y}_1 + y_2 \log \hat{y}_2$$

$$+ y_3 \log \hat{y}_3 + \dots + y_n \log \hat{y}_n$$

$$= \begin{bmatrix} y_1 & y_2 & y_3 & \dots & y_n \end{bmatrix} \begin{bmatrix} \log y_1 \\ \log y_2 \\ \vdots \\ \log y_n \end{bmatrix}$$

$$= \begin{bmatrix} y_1 & y_2 & \dots & y_n \end{bmatrix} \log \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$= Y \log X$$

$$\text{Taking } \sum_{i=1}^n (1-y_i) \log (1-\hat{y}_i)$$

$$= (1-\gamma) \log (1-\hat{y})$$

$$L = \frac{1}{m} \left[ \gamma \log \hat{y} + (1-\gamma) \log (1-\hat{y}) \right]$$

Differentiating

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial w}$$

$$w_{new} =$$

$$w_1 =$$

$$\frac{\partial L}{\partial w} = \gamma$$

$$= \frac{\gamma}{\hat{y}}$$

=

differentiating w.r.t "w"

$$\frac{\partial L}{\partial \omega} = \frac{\partial L}{\partial \omega_1} + \frac{\partial L}{\partial \omega_2} + \frac{\partial L}{\partial \omega_3} + \dots + \frac{\partial L}{\partial \omega_n}$$

$$\omega_{new} = \omega_{old} - n \frac{\partial L}{\partial \omega_{old}}$$

$$\omega'_i = \omega_i - n \frac{\partial L}{\partial \omega_i}$$

$$\frac{\partial L}{\partial \omega_i} = \frac{\partial (\gamma \log \hat{y})}{\partial \omega_i} = \gamma \frac{\partial \log \hat{y}}{\partial \omega_i}$$

$$= \frac{\gamma}{\hat{y}} \frac{\partial (\sigma(x\omega))}{\partial \omega_i}$$

$$= \frac{\gamma}{\hat{y}} \sigma(x\omega) [1 - \sigma(x\omega)] x$$

$$\frac{\partial x\omega}{\partial \omega_i}$$

$$= \frac{\gamma}{\hat{y}} \hat{y} (1 - \hat{y}) x = \gamma (1 - \hat{y}) x$$

$$\frac{\partial L}{\partial \omega} = \gamma - (1-\hat{\gamma})x$$

This is the differentiation part of  $\log \hat{\gamma}$

$$\frac{d(1-\gamma) \log(1-\sigma(x\omega))}{d\omega}$$

$$= \frac{d(1-\gamma) \log(1-\hat{\gamma})}{d\omega}$$

$$= \frac{(1-\gamma)}{(1-\hat{\gamma})} \times \frac{d(1-\gamma)}{d\omega}$$

$$= \frac{(1-\gamma)}{(1-\hat{\gamma})} \times -1 \left( \frac{d \cancel{\log(1-\sigma(x\omega))}}{d\omega} \right)$$

$$= - \frac{(1-\gamma)}{(1-\hat{\gamma})} \times \sigma(x\omega) \left[ 1 - \sigma(x\omega) \right]$$

$$= - \frac{(1-\gamma)}{(1-\hat{\gamma})} \times \hat{\gamma} \left[ 1 - \hat{\gamma} \right] \times \frac{d\cancel{\gamma}}{d\omega}$$

$$= -\hat{\gamma} (1-\gamma) x$$

$$\frac{\partial L}{\partial \omega} = \frac{-1}{m}$$

$$= -\frac{1}{m}$$

$\omega_{\text{new}}$

$\omega_{\text{new}}$



Softmax  
regression

$$\frac{\partial L}{\partial w} = \frac{-1}{m} \sum \left[ \gamma (1 - \hat{\gamma}) x - \hat{\gamma} (1 - \gamma) x \right]$$

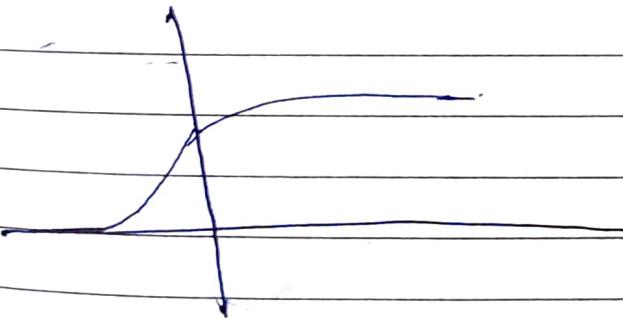
$$= -\frac{1}{m} \left[ \gamma - \gamma \hat{\gamma} - \hat{\gamma} + \hat{\gamma} \gamma \right] x$$

$$= -\frac{1}{m} [\gamma - \hat{\gamma}] x$$

$$w_{\text{new}} = w_{\text{old}} - \eta \left( \frac{\gamma - \hat{\gamma}}{m} \right) x$$

iteration number:  $n$

$$\therefore w_{\text{new}} = w_{\text{old}} - \eta \frac{\partial L}{\partial w}$$



Softmax Regression / Multinomial Logistic Regression

## Softmax Regression

$$L = -\frac{1}{n} \sum_{i=1}^m y_i \log \hat{y}_i + (1-y_i) \log (1-\hat{y}_i)$$

$$\frac{\partial L}{\partial w} = -\frac{1}{m} (y - \hat{y}) x$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \quad \hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_m \end{bmatrix}$$

$$w_{new} = w_{old} + \eta (y_i - \hat{y}_i) x_i$$

$$w_{new} = w_{old} - \eta (y - \hat{y}) x$$

$$\text{Sigmoid function: } \sigma(z) = \frac{1}{1+e^{-z}}$$

Softmax Reg.  $\rightarrow$  is when for  $K$  no. of classes

Logistic for binary class classification

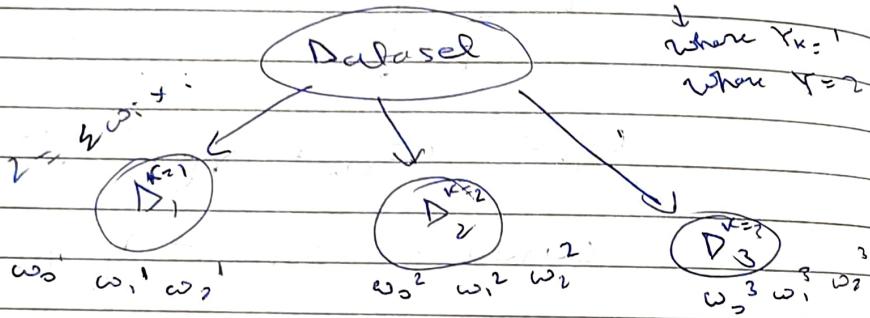
Softmax function

$$\frac{e^{z_1}}{\sum_{j=1}^K e^{z_j}}$$

$$\frac{e^{z_1}}{\sum_{j=1}^K e^{z_j}}$$

One hot encoding :-

$x_1$	$x_2$	$y$	$y_{k=1}$	$y_{k=2}$	$y_{k=3}$
6	7	1	1	0	0
5	3	1	1	0	0
3	4	2	0	1	0
1	2	1	1	0	0
6	8	3	0	0	1



$$\sigma(2)_{k=1} = \frac{e^{2_1}}{e^{2_1} + e^{2_2} + e^{2_3}} = 0.4 \Rightarrow \text{for } k=3$$

$$\therefore 2 = \sum_{i=1}^n w_i x_i$$

$$\sigma(2)_{k=2} = \frac{e^{2_2}}{e^{2_1} + e^{2_2} + e^{2_3}} = 0.30$$

$$\sigma(2_1) + \sigma(2) + \sigma(2_3) = 1$$

$$L = -\frac{1}{m} \sum_{i=1}^m$$

$x_1$

$x_{11}$

$x_{21}$

$x_{31}$

$$= y_1$$

$$0$$

+

$$y_2$$

$$0$$

$$0$$

+

$$y_3$$

$$0$$

+

$$y_2$$

$$0$$

+

$$y_1$$

$$0$$

+

$$y_3$$

$$0$$

$$L = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K y_i^{(i)} \log \hat{y}_i^{(i)}$$

$x_1$	$x_2$	$y$	$y_1$	$y_2$	$y_3$
$x_{11}$	$x_{12}$	1	1	0	0
$x_{21}$	$x_{22}$	2	0	1	0
$x_{31}$	$x_{32}$	3	0	0	1

$$\begin{aligned}
 &= y_1^{(1)} \log \hat{y}_1^{(1)} + y_2^{(1)} \log \hat{y}_2^{(1)} \\
 &+ (y_3^{(1)} \log \hat{y}_3^{(1)}) + \\
 &0 \left( y_1^{(2)} \log \hat{y}_1^{(2)} + y_2^{(2)} \log \hat{y}_2^{(2)} + \right. \\
 &\left. y_3^{(2)} \log \hat{y}_3^{(2)} \right) + 0 \\
 &0 \left( y_1^{(3)} \log \hat{y}_1^{(3)} + y_2^{(3)} \log \hat{y}_2^{(3)} + \right. \\
 &\left. y_3^{(3)} \log \hat{y}_3^{(3)} \right)
 \end{aligned}$$

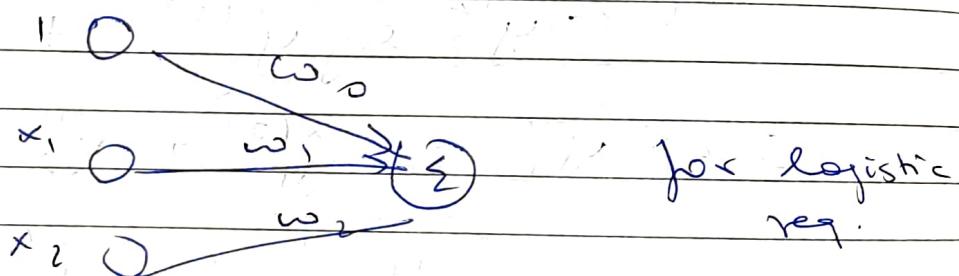
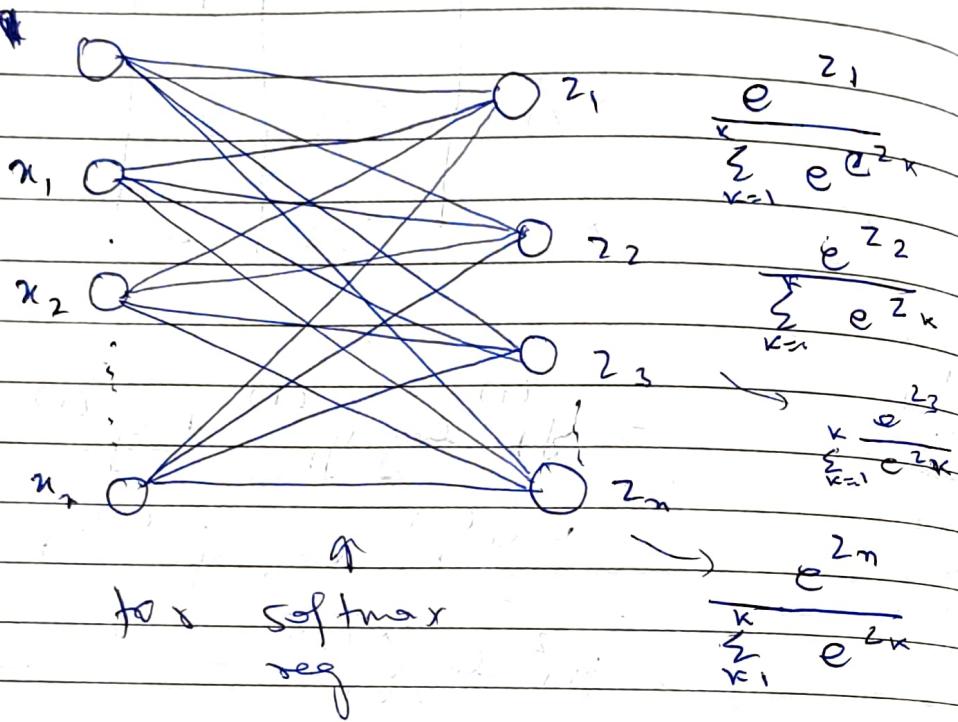
$y_2^{(1)}$  → means second row  $y_2^{(1)}$  <sup>1st</sup> col →  $= 0$

$y_3^{(1)}$  → mean 1st row of  $y_3 = 0$

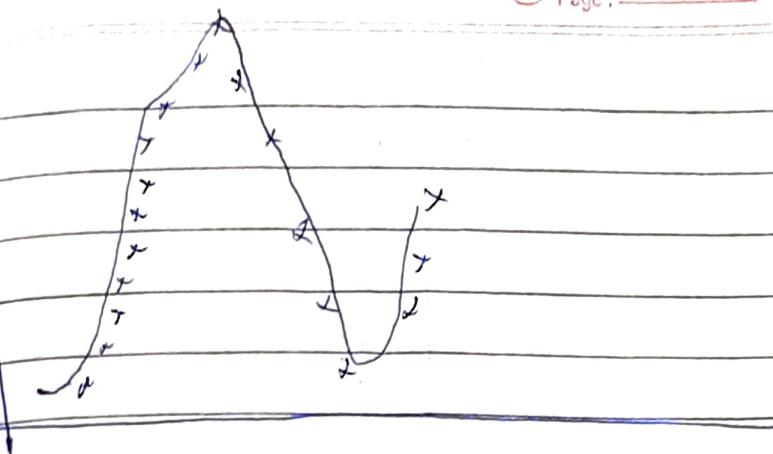
$y_1^{(2)} \rightarrow 0$

$y_3^{(2)} = 0$ ;  $y_1^{(3)} = 0$ ,  $y_2^{(3)} = 0$

$$L = y_1^{(1)} \log \hat{y}_1^{(1)} + y_2^{(2)} \log \hat{y}_2^{(2)} + y_3^{(3)} \log \hat{y}_3^{(3)}$$



for non-linear data  $\rightarrow$  polynomial  
~~logistic~~ reg. used, but it's not giving  
 best results  $\rightarrow$  efficiency is  
 less



Sessional-II  $\Rightarrow$  everything after sessional I

Accuracy =  $\frac{\text{no. of correctly classified}}{\text{Total no. of predictions}}$

$$= \frac{6}{10}$$

$$= \frac{TP + TN}{TP + TN + FN + FP} = \frac{12 + 14}{14 + 12 + 2 + 3}$$

Confusion matrix :- Predicted

		0	1
Actual	0	TP 12	FP 3
	1	FN 2	TN 14

$$\begin{aligned} \text{SST} &= 1 \\ u_1 &= 70 \\ u_2 &= 105 \\ \text{SST} &= 951 \end{aligned}$$

$y$	$\hat{y}$
0	0 ✓
0	0 ✓
1	0 ✗
0	1
1	0
0	1
1	1
0	0
1	1
0	0

Precision :-

predicted

Model A =

Model B =

Sent to SPM  
Not sent to SPM

Not SPM 0	100	170
Not SPM 1	30	700

Model A

0	0
0	190
0	10
0	700

Model B

Model A will give galti on 25%

Precision

Recall

Model

Model

Precision :- What portion of predicted true is true true. =  $\frac{TP}{TP+FP}$

$$\text{Model A} = \frac{TP}{TP+FP} = \frac{100}{100+30} = \frac{100}{130}$$

$$\text{Model B} = \frac{100}{100+10} = \frac{100}{110}$$

Detected		Not detected	
cancer		0	0
Has cancer	1000	TP	200 FN
No cancer	800	FP	800 TN

Model A

$$\begin{array}{cc} 1000 & 500 \\ 500 & 800 \end{array}$$

Model B

Precision of model B > model A

$$\text{Recall} :- \frac{TP}{TP+FN}$$

$$\text{Model A} = 1000 / 1200$$

$$\text{Model B} = 100 / 150$$

F1-score  $\rightarrow$  picks that side where the value is low

$$= 2 \times \left( \frac{\text{Precision} * \text{Recall}}{\text{Precision} + \text{Recall}} \right)$$

If a model predicts that a cat is dog or a dog is cat, for that F1 score worsens.