20/04/2021

Hidden Markov Model - 3

Likelihood Computation : Given an HMM Y = (A,B) and an observation sequence O, determine the likelihood P(O|Y).

E.g. given an icecream eating HMM, what is the probability of 3 1 3.

The problem with hidden states is that we dont know the underlying sequence of states.

Given $Q = q_0, q_1 \ldots q_T$ and $O = o_1, o_2 \ldots o_T$

the likelihood of observations given states

$$P(O|Q) = \prod_{i=1}^{T} P(o_i|q_i)$$

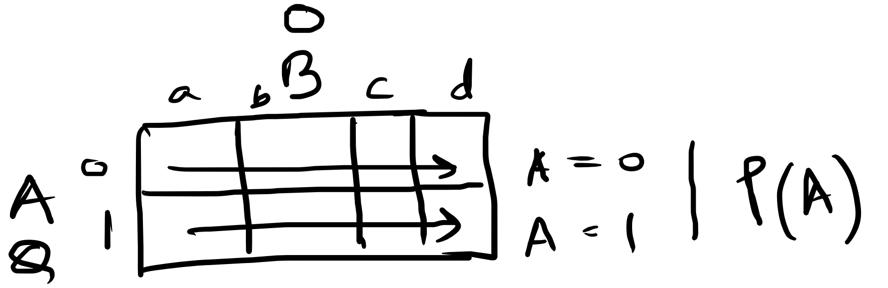
eg. $P(3\ 1\ 3 \mid \text{hot hot cold}) = P(3|\text{hot}) \cdot P(1|\text{hot}) \cdot P(3|\text{cold})$

$$P(O,Q) = P(O|Q)\cdot P(Q) = \prod_{i=1}^{T} P(O_i|q_i) \times \prod_{i=1}^{T} P(q_i|q_{i-1})$$

Eg. $P(3\ 1\ 3, \text{hot hot cold}) = P(\text{hot}|\text{start})\cdot P(\text{hot}|\text{hot})\cdot \ldots$

$\times P(3|\text{hot})\cdot P(1|\text{hot}) \ldots$

w.r.t $\quad \underline{P(A) = \sum_{B} P(A,B)}$



$$P(O) = \sum_{Q} P(O,Q)$$

$$= \sum_{Q} P(O|Q)\cdot P(Q)$$

$$= \sum_{Q} P(o_1 o_2 \ldots o_T | \underbrace{q_1 q_2 \ldots q_T})\cdot P(q_1 q_2 \ldots q_T) \longrightarrow N^T$$

q_1	q_2	q_3 ·	q_T	
hot	hot	hot		
hot	hot	cold		
hot	cold	hot		

eg. $P(3\ 1\ 3) = P(3\ 1\ 3, \text{hot hot hot}) +$

$P(3\ 1\ 3, \text{cold cold cold}) +$

$\downarrow$ 8 combinations

$N^T = 2^3$