Quadratic Polynomial Regression Model Solved Example in Machine Learning

Regression modeling is a process of determining a relationship between one or more independent variables and one dependent or output variable.

Example:
1. Predicting the price of the car given the car model, year of manufacturing, mileage, engine capacity.
2. Predicting the height of a person given the age of the person.

Polynomial Regression

Let there be only one independent variable x and the relationship between x, and dependent variable y, be modeled as,

See also Dimensionality reduction in Machine Learning

$$y=a+a_1*x+a_2*x^2 +.........+ +a_n*x^n$$

for some positive integer n >1, then we have a polynomial regression.

Problem Deninition:

Find a quadratic regression model for the following data:

X	Y
3	2.5
4	3.2
5	3.8
6	6.5
7	11.5

Solution:

Let the quadratic polynomial regression model be

$$y=a+a_1*x+a_2*x^2$$

The values of **a, a$_1$, and a$_2$** are calculated using the following system of equations:

$$\sum y_i = na_0 + a_1\left(\sum x_i\right) + a_2\left(\sum x_i^2\right)$$

$$\sum y_i x_i = a_0\left(\sum x_i\right) + a_1\left(\sum x_i^2\right) + a_2\left(\sum x_i^3\right)$$

$$\sum y_i x_i^2 = a_0\left(\sum x_i^2\right) + a_1\left(\sum x_i^3\right) + a_2\left(\sum x_i^4\right)$$

First, we calculate the required variables and note them in the following table.

	x	y	x²	x³	x⁴	y*x	y*x²
	3	2.5	9	27	81	7.5	22.5
	4	3.2	16	64	256	12.8	51.2
	5	3.8	25	125	625	19	95
	6	6.5	36	216	1296	39	234
	7	12	49	343	2401	80.5	563.5
Σ	25	27.5	135	775	4659	158.8	966.2

Using the given data we,

$$27.5 = 5a_0 + 25a_1 + 135a_2$$

$$158.8 = 25a_0 + 135a_1 + 775a_2$$

$$966.2 = 135a_0 + 775a_1 + 4659a_2$$

Solving this system of equations, we get

a=12.4285714
a_1=-5.5128571
a_2=0.7642857
The required quadratic polynomial model is

y=12.4285714 -5.5128571 * x +0.7642857 * x²

Now, given the value of x (independent variable), we can calculate the value of y (dependent or output variable).