



## Performance Laws

- a) A uniprocessor computer can operate in either scalar or vector mode. Certain benchmark program took time  $t$  to run on the comp. Further it's found that 25% of  $t$  was attributed to vector mode. In the remaining time machine op. in scalar mode.
- a) Calc. effective speedup under above condition as compared with when vector mode not used at all.
- c) Also calculate  $\alpha$ , % of code vectorized in above program.

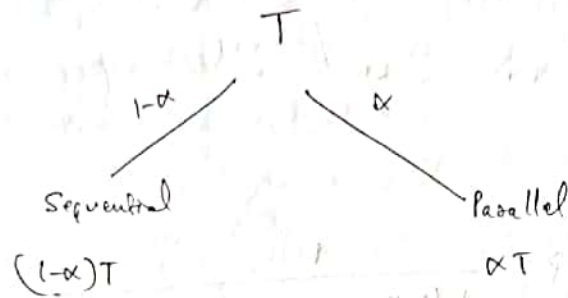
$$a) \text{ speed} = \frac{\text{time without enhancement}}{\text{time with enhancement}}$$

$$T = 0.75T(\text{scalar}) + 0.25T(\text{vector})$$

One could. - Vector mode is 9 times faster than scalar mode

$$\Rightarrow \text{speedup} = \frac{0.75t + 0.25 \times 9t}{t} = 3$$

$$b) \quad \alpha = \frac{0.25 \times 9}{3 \times 9} = 0.75 \cdot 100 = 75\%$$



Enhancement only reduces parallel time

$$T_{\text{enhancement with } n \text{ processors}} = (1-\alpha)T + \frac{\alpha T}{n}$$

$$\text{Speedup} = \frac{T}{(1-\alpha)T + \frac{\alpha T}{n}} = \frac{1}{1-\alpha + \frac{\alpha}{n}}$$

$$3 \left( 1 - \alpha + \frac{\alpha}{3} \right) = 1$$

$$3 - 3\alpha + \frac{\alpha}{1} = 1$$

$$2 = 3\alpha - \frac{\alpha}{1} \Rightarrow \alpha = \frac{3}{4}$$

$$\alpha = 75\%$$

Enhancement 1	$\alpha_1$	$s_1$
Enhancement 2	$\alpha_2$	$s_2$
...		
Enhancement n	$\alpha_n$	$s_n$

Generalised  
Amdahl's Law

$$\text{Overall speedup} = \frac{1}{1 - (\alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_n) + \frac{\alpha_1}{s_1} + \frac{\alpha_2}{s_2} + \frac{\alpha_3}{s_3} + \dots + \frac{\alpha_n}{s_n}}$$

$$\text{Fraction of reduced time no enhancement is used} = \frac{1 - (\alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_n)}{1 - (\alpha_1 + \alpha_2 + \alpha_3 + \dots + \alpha_n) + \frac{\alpha_1}{s_1} + \frac{\alpha_2}{s_2} + \frac{\alpha_3}{s_3} + \dots + \frac{\alpha_n}{s_n}}$$

c) Suppose we double speed ratio b/w vector & scalar mode by hardware improvement. Calc. effective speedup that can be achieved.

d) Suppose same speedup obtained in c) must be obtained by compiler improvement instead of hardware. What would be new vectorization ratio  $\alpha$  that should be supported by vectorization compiler for the same benchmark program.

$$c) \text{ Speedup} = \frac{1}{1 - \alpha + \frac{\alpha}{S}} = \frac{1}{1 - 0.75 + \frac{0.75}{18}} = 3.43$$

$$d) 3.43 = \frac{1}{1 - \alpha + \frac{\alpha}{9}}$$

$$3.43 \left( 1 - \alpha + \frac{\alpha}{9} \right) = 1$$

$$3.43\alpha - \frac{3.43\alpha}{9} = 2.43$$

$$\alpha \left( 3.43 - \frac{3.43}{9} \right)$$

$$3.43 - 0.38$$

$$\alpha = \frac{2.43}{3.05} = 0.79$$

$\therefore 79\%$

Let  $\alpha$  be the fraction of program code that can be executed simultaneously by  $n$  processors in a comp. sys.

Assume that remaining code must be executed sequentially by a single processor. Each processor has an execution rate of  $\alpha$  MIPS & all processors are assumed to be equally capable.

a) Derive exp for effective MIPS rate when using sys. for exclusive execution of this prog. in  $n, \alpha, \alpha$

Let no. of instructions to be executed  $k$  millions

Processors Parallel  
 $\downarrow$   
 $n \times \alpha$  MIPS  $\alpha k$

Sequential  
 $(1-\alpha)k$   $\alpha$  MIPS

$$\Rightarrow T = \frac{\alpha k}{n\alpha} + \frac{(1-\alpha)k}{\alpha} \text{ sec}$$

$$= \frac{k [\alpha + (1-\alpha)n]}{n\alpha}$$

$$\text{Effective MIPS} = \frac{k}{\frac{k [\alpha + (1-\alpha)n]}{n\alpha}} = \frac{n\alpha}{\alpha + (1-\alpha)n}$$

but can be  
comp. sys.  
fed  
so has  
assumed  
sys.

Consider comp which executed in 2 modes - regular & enhanced <sup>with</sup> probability distribution of  $\alpha$ ,  $1-\alpha$  resp.  
If  $\alpha$  varies b/w  $a$  and  $b$  & ~~assumed~~  $0 \leq a < b \leq 1$   
Derive exp. for avg speed of ~~exp.~~ <sup>factor</sup> using harmonic mean concept.