20/04/2021 · Hidden Markov Model - 3

Likelihood Computation : Given an HMM Y = (A,B) and an observation sequence O, determine the likelihood P(O|Y).

E.g. given an icecream eating HMM, what is the probability of 3 1 3.

The problem with hidden states is that we dont know the underlying sequence of states.

Given $Q = q_0, q_1 \ldots q_T$ and $O = o_1, o_2 \ldots o_T$

the likelihood of observations given states

$$P(O|Q) = \prod_{i=1}^{T} P(o_i|q_i)$$

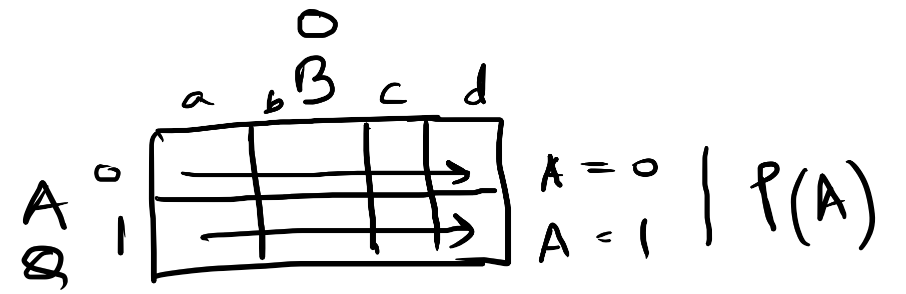
eg. $P(3\ 1\ 3 \mid \text{hot hot cold}) = P(3|\text{hot}) \cdot P(1|\text{hot}) \cdot P(3|\text{cold})$

$$P(O,Q) = P(O|Q)\cdot P(Q) = \prod_{i=1}^{T} P(O_i|q_i) \times \prod_{i=1}^{T} P(q_i|q_{i-1})$$

Eg. $P(3\,1\,3, \text{hot hot cold}) = P(\text{hot}|\text{start})\cdot P(\text{hot}|\text{hot})\cdot \ldots$

$\times\, P(3|\text{hot})\cdot P(1|\text{hot}) \ldots$

W.k.t $\quad \underline{P(A) = \sum_B P(A,B)}$



$$P(O) = \sum_Q P(O,Q)$$

$$= \sum_Q P(O|Q)\cdot P(Q)$$

$$= \sum_Q P(o_1 o_2 \ldots o_T|\underbrace{q_1 q_2 \ldots q_T}_{})\cdot P(q_1 q_2 \ldots q_T) \longrightarrow N^T$$

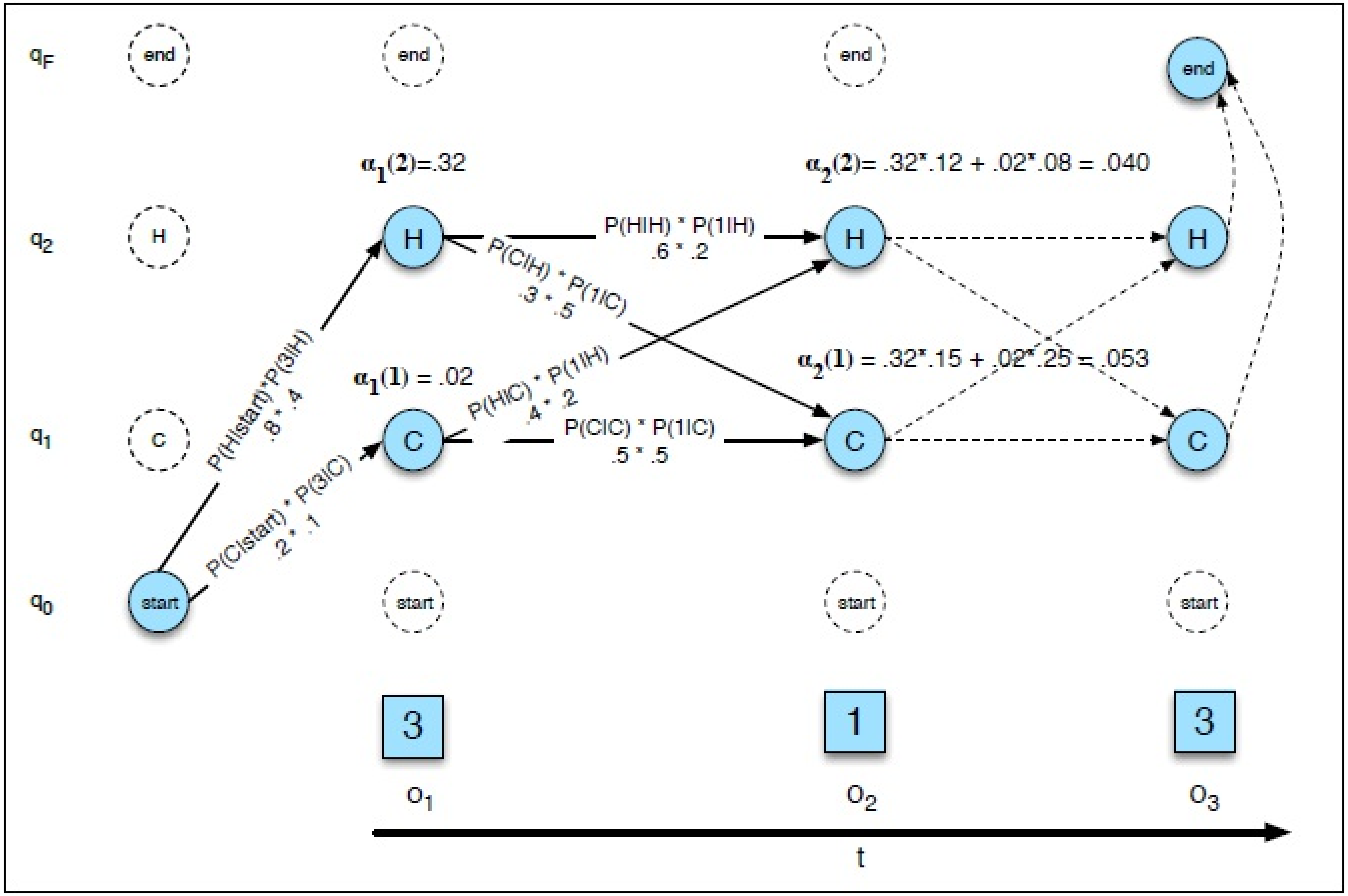
q_1	q_2	q_3	$\ldots$	q_T
hot	hot	hot		
hot	hot	cold		
hot	cold	hot		

eg. $P(3\,1\,3) = P(3\,1\,3, \text{hot hot hot}) +$

$P(3\,1\,3, \text{cold cold cold}) +$

8 combinations

$N^T = 2^3$



q_F
q_2
q_1
q_0
end
end
end
end
H
C
start
$\alpha_1(2)$=.32
$\alpha_1(1)$ = .02
H
C
P(H|start)*P(3|H)
.8 * .4
P(C|start) * P(3|C)
.2 * .1
P(H|H) * P(1|H)
.6 * .2
P(C|H) * P(1|C)
.3 * .5
P(H|C) * P(1|H)
.4 * .2
P(C|C) * P(1|C)
.5 * .5
$\alpha_2(2)$= .32*.12 + .02*.08 = .040
$\alpha_2(1)$ = .32*.15 + .02*.25 = .053
H
C
H
C
start
start
start
3
1
3
o_1
o_2
o_3
t

$\alpha_t(j)$ represents the probability of being in state j after seeing t observations, given λ.

$$\alpha_t(j) = P(o_1, o_2 \ldots o_t, q_j = j \mid \lambda)$$

the t^{th} state in the sequence of states is state j

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) \left[\underbrace{a_{ij}}_{\text{transition prob}} \; \underbrace{b_j(o_t)}_{\text{observation prob.}} \right]$$

1. Initialization

$$\alpha_1(j) = a_{0j} b_j(o_1) \qquad 1 \leq j \leq N$$

2. Recursion

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t) \qquad \begin{array}{l} 1 \leq j \leq N \\ 1 \leq t \leq T \end{array}$$

3. Termination

$$P(O/\underset{\lambda}{\gamma}) = \alpha_T(q_F) = \sum_{i=1}^{N} \alpha_T(i) a_{iF}$$

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
$\quad forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$
for each time step t **from** 2 **to** T **do** ; recursion step
$\quad$ **for each** state s **from** 1 **to** N **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,\mathrm{T}] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step

return $forward[q_F,T]$

$\alpha_t(s) \longrightarrow forward[s,t]$