Judges' Commentary: The Ultimate Brownie Pan Papers

Kelly Black
Dept. of Mathematics
Clarkson University
P.O. Box 5815
Potsdam, NY 13699–5815
kjblack@gmail.com

Introduction

The focus of this year's problem was how to balance the compromise between quality versus quantity. The teams were asked to determine the best way to strike the balance between the two competing interests for a given compromise. The context for this decision was to design the pans used to bake brownies.

On the side of quality, the pans themselves should be designed so that the temperature distribution of the brownie mix should be as uniform as possible. The goal was to reduce the large temperature gradients that can occur near the corners of rectangular pans. For this aspect of the problem, a circular pan is the best choice to achieve the most even heating and the best brownies.

On the side of quantity, the pans should be designed to reduce the amount of empty space on one of the racks in the oven. The goal here is to put as many pans as possible into an oven that has a rectangular shape. For this aspect of the problem, the best choice, as a way to achieve the least amount of empty space, is a pan with an aspect ratio that matches the aspect ratio of a horizontal cross section of the oven.

In this commentary, we first describe the judging process, then discuss the three questions that the teams were asked to address. The next topic is the issue of sensitivity and assumptions, followed by a discussion on identifying the strengths and weaknesses of a given approach. Finally, we give a brief discussion on the difference between references and citations.

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The Process

I give an overview of the judging process, which has two primary activities: the triage rounds and the final rounds. The papers that are to be judged to be the best must make it through both sets of rounds, and the criteria used to identify good papers gradually change as the judging progresses. The papers that make it to the final stages of the judging must stand out as the best under a wide variety of criteria.

Triage

The first rounds of the judging are referred to as the *triage rounds*. The primary idea in these initial rounds is to identify the papers that should be given more detailed consideration. Every paper is read at least twice. When reading a paper, a judge's primary question is whether or not the paper contains all of the necessary ingredients that make it a candidate for the most detailed readings.

In these initial rounds, the judges' time is constrained, and we try to give a paper the benefit of doubt. If a paper addresses all of the issues and appears to have a reasonable model, then the judge will likely identify it as a paper that deserves more attention. Some papers in these early rounds receive a mixed review. In those cases, the paper will be read by other judges with the intent to give the paper every chance possible.

In these early rounds, it is most important that a paper be clear and concise. It must be clear what the approach is, and the results should be clearly spelled out. In these early rounds, the importance of the summary is accentuated. In particular, a good summary should have a brief overview of the problem, plus an overview of the paper and approach that the team will discuss, and specific results should be stated or expressed in some way. There should be no surprises as a judge reads a paper.

Small things that can make a difference in the earlier rounds include having a table of contents. It makes it much easier for a judge to look at the paper and decide what to expect while reading the paper.

It is also important that all of the questions asked be answered. If a paper does not address all of the questions, then the judge is more likely to decide that a team's efforts do not compare well with the better papers.

Finally, it is vital that the team express their general approach and results as clearly and concisely as possible. This means providing in the introduction a broad overview of the problem, the paper, and specific results. Additionally, at the start of each section, the team should provide an overview of the section itself.

These small things will make it much easier for a judge to identify what the team did, and will increase the probability that the judge will correctly identify and reward good work. It is important to remember that the competition is about the *process* of modeling and is also about *expressing* the work done. The best models and the best effort are not effective if the results are not adequately communicated.

Final Rounds

For the final rounds of judging, the judges all assemble in the same location. As the rounds progress from the earlier readings, the judging criteria slowly shift away from identifying papers that need further consideration to a process of trying to identify the very best papers. As this transition unfolds, it becomes even more important that the judges are working together and reach an understanding of what is expected.

The first round in the final set of readings begins with a meeting of the judges. In this meeting, the problem is discussed, and the judges share what they think are the key aspects of the question itself. Then each judge reads a large number of papers. These papers have been given a variety of scores ranging from low to high in the previous judging rounds with a relatively uniform distribution.

After examining these papers, the judges get together again and discuss what they think should be included in a "good" paper. The judges are well aware that the teams have limited time to complete their activities, and the purpose of this extra step is to ensure that judges adequately compensate for the limitations and restrictions imposed on the teams. This competition is about rewarding the teams' efforts and recognizing excellent work under tremendous limitations, and we make every effort to consider this situation from the students' point of view.

Once the judges agree on a set of minimal criteria, the final rounds begin in earnest. Each paper is read multiple times. As the rounds progress, the number of papers is decreased, and the entries come under increased scrutiny. Also, the time dedicated to reading each paper steadily increases.

In the very last rounds, the papers that remain are given the highest levels of attention. The amount of time that a judge spends on reading the paper increases, and multiple judges can be reading copies of the same paper at the same time. By this time, the papers that remain generally have excellent summaries and are well-written. The judges are then able to focus almost solely on the modeling process and the mathematical integrity of a paper.

The Questions

This year's problem can be boiled down to three different questions.

- The first question is to determine the heat distribution of the brownies for a given shape of the pan.

- The second question is to determine the best way to load the oven with pans of a given shape.
- The last question is to determine the best pan shape and oven configuration, given the relative trade-off between the quality of the brownies and the quantity that can be made.

We examine each of these three questions separately. We first examine the heat distribution, then address the question of how to best load the oven, and finally examine the best way to determine a compromise.

Heat Distribution

Determining the heat distribution of the pan of brownies after it has been placed in the oven is inherently a question of a relationship in space that changes in time. In particular, the temperature is assumed to change in time and vary with position in the pan.

The most common model for this was the heat equation. The primary question that resulted was whether or not to approximate the brownie mix as a two-dimensional object or use a three-dimensional approximation. Given the time and computational restrictions, either approach was OK; but a team should have provided some kind of discussion as to why they made the decision that they did.

Some teams started with the heat equation and added a body force based on Newton's Law of Cooling. A couple of the Outstanding teams did this. The judges recognized that this causes a significant problem with the model; but given the nature of the mathematics and the short time span, we decided that the balance of the team's other efforts could offset this error.

For most teams, the modeling efforts tended to be on the boundary conditions. Some sort of mixed boundary conditions are most appropriate, since the heat flux through the sides of the pan contributes to the primary exchange of energy between the oven and the brownie mix. Many teams used Dirichlet boundary conditions, and this was acceptable as long as the team provided some kind of justification for their decision.

Another aspect to the problem was how to decide what shape to use for the pan. The majority of entries took one of two approaches. The first approach was to use a regular polygon with n sides and increase n as a way to reduce the temperature gradients that occur at corners. Another common approach was to use rectangular pans with rounded corners. Some groups made use of elongated ellipses or other shapes, but they were in the minority.

The judges did not place any kind of preference on the decision about shape. The primary judging emphasis for this aspect of the problem was to determine if the shape of the pans was adequately described and whether or not a team remained consistent with the other two questions associated with the problem.

Nonrectangular and the noncircular pan shapes required the teams to approximate the heat equation. This was done in a wide variety of ways. Many teams used a finite element approximation and either MATLAB [2013] or COMSOL Multiphysics [2012]. Other teams wrote their own finite-difference approximations; and some teams made up their own technique, of which cell-averaging techniques were the most common approach.

Finally, another aspect of this problem is that the teams were required to define how to measure the temperature variation. A wide variety of techniques were used to determine this aspect of their approximation. A large number of teams simply said that they would use the "standard deviation" of the temperature. This approach was problematic, since it is not clear what this term means in a nonstochastic situation and what it means in terms of interpreting the numerical approximation.

Some teams found ways to use the gradient of the temperature distribution. Some teams just focused on what they decided were critical locations within the brownie mix. The judges did not attribute different weights to the different approaches. The primary criterion was that the technique employed was clearly stated and used in a consistent manner throughout the paper.

Loading the Oven

Another critical aspect of the problem was to decide how to arrange the brownie pans within the oven. The pans had to be distributed on the racks within the oven, and the horizontal cross-section of the oven is assumed to be rectangular with a predetermined aspect ratio. Trying to arrange oddly shaped pans in this context is an extremely difficult problem.

The majority of teams tried to employ a pre-existing technique to this part of the problem. There are many "off the shelf" approaches that are available. Many teams simply tried to use an existing pallet-loading approach. Other teams attempted to make up their own technique or simply made gross approximations.

The judges made every effort to take into consideration the extreme time constraints associated with the event. The judges did not impose any preference over the different approaches employed here. Again, the primary concern was whether or not the method was described well and implemented in a way consistent for the third question.

It was also important that the teams adequately describe the results for the packing algorithm that they employed. For this purpose, a figure was a tremendous aid in conveying to the readers what the team had implemented and what their results were. Additionally, many teams included flowcharts as an additional way to share their algorithm.

The use of figures is a great help for the judges in trying to decide what the team has decided to do, and to help decide if it makes sense and is a reasonable approach. A key aspect of this contest is to communicate complex ideas, so teams should know how to use figures and properly place them in a document.

Simple things like proper labels make a huge difference in figures. Also, every figure should include a brief description to inform the reader what it is and what to look for. Finally, every figure should be discussed and described in the narrative; otherwise the reader has no reason to look at the figure. The reader should know exactly what the team thinks is most important and revealing before searching for the figure.

The Compromise

The last question was how to decide on a shape for the pan, and an arrangement of the pans within the oven, for a given compromise between quality and quantity. This aspect of the event turned out to be one of the parts that helped differentiate the very top papers. The higher-rated papers clearly described what the teams did and clearly stated their conclusions.

The teams had to describe how they determined the balance between the two competing interests. In the original problem statement, a ratio p is briefly stated to help motivate the idea of the compromise. Many teams simply assumed that this ratio was understood and did not do a good job of explaining what it meant to them. Since it was primarily a general way to motivate the idea, the ratio was not clearly defined in the original problem statement, and most teams had a very different idea about what it meant for their approach.

For most of the papers, it was difficult to determine how the team interpreted what they thought the ratio p meant to them. A clear statement of this part of the problem was a great aid to the judges as they read through the results.

Finally, the teams were expected to state their conclusions explicitly for varying compromises between quantity versus quality. This meant for a given balance that the team should provide some way to determine what shape pan to use and a way to place the pans in the oven. Most teams had a difficult time trying to convey this complex piece of information. Those teams that were able to discuss this key piece of information in a consistent and clear manner were more likely to be noticed by the judges and recognized for their efforts.

Sensitivity and Assumptions

Every year, when the judges get together and discuss the what we think is important in a paper, we discuss assumptions and sensitivity. The basic assumptions that a team makes is the starting point for all of their efforts. We do not place many restrictions on the basic assumptions but simply ask that they make sense on some basic level.

Once those assumptions are made and clearly stated, it is expected that everything else in the paper remain consistent with the assumptions. That is a key component of the modeling process. It is important to recognize, however, that those assumptions are also a restriction and should be explicitly examined in a structured and coherent manner.

This year, the teams had to make a large number of assumptions and decisions. This is especially true in light of the computational nature of the first question, about heat distribution.

The issue of examining what happens when a small change in the assumptions takes place is a vital part of the modeling process. This year, the teams had to make decisions about a wide variety of parameters. For example, the teams had to decide the coefficient for heat conduction, how long to bake the brownies, the temperature of the oven, the coefficients associated with the boundary conditions, and a wide variety of other considerations.

The basic question that the teams should have asked is *what happens to their conclusions if they make a small change in any one assumption or make a small change in any one parameter*. If the result is a big change in one of their conclusions, then the team should indicate that as a potential weakness or at least something that deserves more consideration.

For example, the team might try to change the aspect ratio in their oven by a small percentage. The team could then ask if there are a range of values for the aspect ratio that then result in a large change with respect to the recommended shape and loading configuration. The teams could then repeat this exercise for other small changes as well.

We understand that the teams are highly constrained with respect to time and resources. We do not expect the best possible model, but we do expect the teams to explore the models that they develop in a structured way and ask critical questions associated with the models. The model itself is important, but the analysis of the model is also important.

Strengths and Weaknesses

Teams are expected to include a critical analysis of their modeling efforts. The teams should take the time to inform the judges what aspects they think are best about their effort and also what aspects they think should receive further attention. No model is perfect, and the limited time available to the teams to complete their efforts represent a considerable constraint on what they can do.

A vital part of the process of mathematical modeling is to step back and ask basic questions about the model. This ranges from examining the basic assumptions that are made to an examination of the techniques employed in the model itself. When a team explicitly provides a clear and honest examination of its own work, then it is demonstrating that they understand this basic part of the modeling process.

Citations Versus References

Finally, we give a brief word about citations and references. Over the history of this event, the teams have greatly improved on their use of references in their submissions. Most teams clearly indicate a list of resources that they have consulted in their efforts.

The use of citations, though, has not been quite as popular an activity. While we appreciate the greater use of a section of references that lists the resources used, that list is not much help if the teams do not provide citations within the narrative to indicate which resource they used for a particular idea being expressed. A team that provides proper citations that are also coupled to complete references immediately sends a message to the reader that they have paid attention to important details and are taking a responsible and professional approach to their efforts.

This issue is especially important with respect to figures and images. If a paper includes an image that is clearly copied from a source, then the paper should include a citation, together with a reference associated with the citation. The use of an appropriated image without a citation is plagiarism, and in many cases this stands out to the judges reading the papers.

Conclusions

This year, the teams were asked to determine how to balance the competing interests of quantity versus quality. They were asked to determine the best way to design a brownie pan and pack multiple pans in an oven for a given balance between the two considerations. To do this, the teams had to determine the heat distribution within a brownie mix in a pan with a complex shape. They also had to determine how to load those same pans in an oven to maximize the number of pans that could be placed in the oven. Finally, they had to determine the shape and the loading pattern to achieve a specific balance between quantity and quality.

These tasks resulted in a computationally intensive set of activities. The modeling ranged from relatively straightforward (the heat equation with attention to the boundary conditions) to extremely complex (the pallet loading problem). The teams had to bring these two approaches together and provide specific recommendations, and they had to convey complex results within their narrative.

The teams that were able to provide solutions to all three problems and provide a balanced and consistent approach were rated highest by the judges. The exercise required the teams to coordinate the competing demands of the complexity of the models developed with the time and insight required to perform an analysis of their own models. As is usually the case, the most sophisticated modeling did not necessarily result in the highest ratings; but the teams that were able to find the best balance between the model, the analysis, and clarity of their presentation received the highest ratings.

References

COMSOL Multiphysics. 2012. Version 4.3a. `http://www.comsol.com` . Stockholm, Sweden: COMSOL.

MATLAB. 2013. Version 8.1 (R2013a). Natick, MA: The Mathworks Inc.

About the Author

Kelly Black is a faculty member in the Dept. of Mathematics and Computer Science at Clarkson University. He received his undergraduate degree in Mathematics and Computer Science from Rose-Hulman Institute of Technology and his master's and Ph.D. degrees from the Applied Mathematics program at Brown University. He has wide-ranging research interests including laser simulations, ecology, and spectral methods for the approximation of partial differential equations.