A killer is at large, and the police need our help! Lovers of mystery fiction might jump at the chance to engage in a hot pursuit of forensic clues and shady characters, but our reality is not so glamorous. Our task: given a set of crime site locations and times, narrow down a killer's location and try to predict where the next murder might be.

We present two mathematical models to assist in the investigation. The models take the locations of previous crime sites and the times that they occurred as variables. These data are few compared to the seemingly infinite possibilities of the criminal's actions, but we know at least one thing: the criminal will do his best to avoid being caught. We infer then, in accordance with past research, that he will kill where he knows the surroundings. This motivates the assumption that the killer is more likely to kill closer to his home or to places where he has killed before. We further assume that the future locations of attacks depend *only* on the location of the killer's home and the location and relative time of past crimes.

The first of the two models, the Centrographic model, builds upon earlier work by LeBeau. It uses an analogy to kinematics to derive the location and movement of the "centroid" of a series of crime sites. We consider the change in the location of the centroid over time to predict future crime site locations. In other words, we model the series to have a "velocity" and "momentum" and use this information to predict the future location of the centroid.

Our second model, the Rational Serial Criminal model, uses methods from microeconomic theory to model the criminal as a rational agent. This model formalizes the process of looking at a series of crime locations and assessing where the criminal prefers to find victims or dump bodies. Using data on a series of crimes, the model outputs contour surfaces representing areas where the killer is more or less likely to live. From here, we extend the model to predict where a killer is likely to strike next.

We run simulations on crime site data for different serial killers and compare our location predictions to the actual next kill in the series. The centrographic model performs well as measured by the error distance. Unfortunately, the model is sensitive to a parameter that must be determined for each individual series. The rational choice model also performs well predicting home location as measured by the Hit Score %, a metric which compares the efficiency of a search informed by prediction to a random search.

Our model could be another useful tool for police chasing a serial killer. While we do not incorporate data such as population density or geographic features, we believe that police officers will have no trouble filling in these gaps. The model provides a good starting point when data are too few to utilize more complex methods.

From Kills to Kilometers
Using Centrographic Techniques and Rational Choice Theory for Geographical Profiling of Serial Killers

February 22, 2010

Abstract

We present two mathematical models to assist in the investigation of a serial killer. The models take the locations of previous crime sites and the order that they occurred as variables. These data are few compared to the seemingly infinite possibilities of the criminal's actions, but at least one thing is known: the criminal will do his best to avoid being caught. We infer then, in accordance with past research, that he will kill where he knows the surroundings. This motivates the assumption that the killer is more likely to kill closer to his home or places he has killed before. We further assume that the future locations of attacks depend *only* on the location of the killer's home and the location and relative time of past crimes.

The first of the two models, the "centrographic" model, builds upon earlier work by LeBeau. It uses an analogy to kinematics to derive the location and movement of the "centroid" of a series of crime sites. We consider the change in the location of the centroid over time to predict future crime site locations. In other words, we model the series to have a "velocity" and "momentum" and use this information to predict the future location of the centroid.

Our second model, the Rational Serial Criminal model, uses methods from microeconomic theory to model the criminal as a rational agent. This model formalizes the process of looking at a series of crime locations and assessing where the criminal prefers to find victims or dump bodies. Using data on a series of crimes, the model outputs contoured surfaces representing areas where the killer is more or less likely to live. From here, we extend the model to predict where a killer is likely to strike next.

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1 Introduction

When a serial killer is on the prowl [1], the police deploy every available resource to stop the killings and bring the murderer to justice. Serial killers provide an interesting challenge to geographic profilers, due to the relative paucity of data on these criminals and their unique geographic behavior. Statistical prediction methods in widespread use for other types of serial crimes such as burglaries [8] or auto thefts [39] are less useful for finding serial killers, as these methods rely on a large initial data set to generate predictions.

We attempt to predict a serial killer's future behavior and location data on their crime sites and initial assumptions about their behavior. In Section 3, we adapt two complementary models to predict future crime sites and point to the location of a serial killer's *anchor point* [32].

We tested our models by implementing each one in a computer simulation, and running them on partial data sets from two historical serial killers. We describe our data collection process and provide examples in Section 5. We compared our models' predictions for the locations of later crimes to the actual locations where later crimes in each series occurred. We describe the results of these simulations in Section 6. Finally, in Section 7, we note some shortcomings of our approach and propose directions for further research.

2 Background Information

2.1 Serial Killing

The FBI defines serial murder as three or more killings with "cooling-off" periods in between [32, 27]. Experience shows that the behavior of such serial criminals follows some predictable patterns. Specifically, serial killers tend to operate near their homes, in areas with which they are familiar [32]. For example, one study of 126 U.S. serial killers found that 89% of them lived within a circle drawn around the locations where they disposed of bodies [23]. The inclination of a serial killer to work close to a home base or *anchor point* [32] often leads to a spatial clustering of their crime sites. Therefore, by looking at the geographic locations of crimes, we can say something about where the criminal who committed them is likely to live or might strike next.

In the past two decades, "geographical profiling" has exploded, and many law-enforcement agencies now use spatial modeling techniques in the pursuit of serial criminals [33]. The most popular approach includes the assumption that the criminal lives near his crime scenes, and has not moved place of residence since the crime spree began [32, 33].

[1] During such an investigation, even if the same *modus operandi* is followed, the police might not acknowledge a series of crimes as being the work of one person. Thus, one might be inclined to refer to this suspect as an alleged serial killer for precision. For the duration of this paper, we will assume that all such series are the work of one person, and will drop the "alleged".

2.2 Previous Work

At first blush, criminals seem difficult, if not impossible, to predict. The criminal, however, must follow *some* sort of thought process in order to commit his acts and, as such, “crime pattern theory combines rational choice, routine activity theory, and environmental principles to explain the distribution of crimes” [32]. From these ideas, geographical profiling was developed. When analyzing serial crimes, few data are available other than that at the crime scene and disposal site of the body. This motivates Lundrigan and Canter’s research into a serial killer’s disposal site location choice, which statistically analyzed data on apprehended serial killers to determine how a killer’s home location related to his body disposal sites [23]. Many factors can influence the serial killer’s disposal site selection. In a study conducted in Germany, the median distance between the disposal site and the killer’s home decreased as age of killer increased, a killer’s higher IQ corresponds to a further distance of disposal site from the home base, though the number of murders and distance between a killer’s home and disposal site was not statistically significant [38].

Another method of determining the location of a serial killer is to consider the daily routines of the victims. Geographic profilers can use mapping software to determine where victims’ routines overlap, and these locations can be interpreted as places where the killer might have seen the victims. These data, in turn, can be used to hypothesize the killer’s daily whereabouts [37]. In an age where most people are connected to the rest of the world through cellular telephones and internet, it is possible now more than ever to gather objective data regarding where a person was at a certain time.

In modeling serial crime, a primary concern of geographical profiling is the prediction of areas at high risk of future crime, known colloquially as crime “hotspots”. Past models for geographical profiling of serial offenders have shown varying results. Due to the nature of serial crimes as discrete events, the majority of methods for displaying and analyzing data on serial criminals utilize a technique known as point mapping [10]. A problem with simple point mapping, however, as described by Chainey et al., is that “it is difficult to clearly identify the location, relative scale, size and shape of hotspots when crime data are presented as points”. As such, it becomes necessary to find alternate methods of display and analysis.

3 Models

3.1 A Centrographic Model

3.1.1 Background

A standard analytical measure in the geographical profiling toolbox is the spatial mean of point data, obtained by summing the geographic coordinates over all observations independently in each spatial dimension and dividing by the total number of observations [32]. This effectively generates a ‘center’ to the data set that can be used in conjunction with other methods to describe the concentration of a distribution. A long-utilized and relatively simple tool, the spatial mean remains in widespread use today in geographical profiling due to its ease of use. Despite the simplicity of calculation, previous studies have shown the spatial mean to perform comparably to alternative, more complex algorithms and even human judgment [39].

One drawback to the spatial mean is that it doesn't intrinsically use time to describe and forecast an evolving distribution such as is seen with serial killings. To utilize the temporal aspect of such a data set, we must extend the model. One extension that arises naturally is to consider the change in the spatial mean over time [21]. Previous work has used analogy to simple physical phenomenon to come up with parallels to velocity (change in position over time), acceleration (change in velocity over time), and momentum (the product of velocity and mass) [32, 21]. In this model we will use several of these concepts to come up with a method of predicting the locations of future kills or crimes in a spree.

3.1.2 Assumptions

To justify a centrographic approach to geographical profiling, a few assumptions must be made regarding the behavior of serial killers:

- A serial killer is more likely to kill in an area he's familiar with, (i. e., a place he lives near or has killed near before.)
- The distribution of probable locations of next kill changes with time, even if the distribution of known kills does not.
- The order in which and time at which kill events in a distribution occur are a factor in determining probable locations of next kill, not just the spatial location.

3.1.3 Model

Define $\mathbf{X}_{K_j}$ to be the position vector corresponding to the site of the j-th observed crime, K_j. If we consider the reach or area of activity of a serial killer to be represented by the distribution of the observed events, we can define the 'center of mass' or centroid of the area of activity at the time of the of the j-th event, $\mathbf{C}_j$, to be the geographic mean of $\mathbf{X}_{K_1} \dots \mathbf{X}_{K_j}$,

$$\mathbf{C}_j = \sum_{i=1}^{j} \frac{\mathbf{X}_{K_i}}{j}$$

Utilizing this, a reasonable estimate for a probable area of future encounter with the criminal, A, can be obtained by finding the mean distance from $\mathbf{C}_j$ to each event $\mathbf{K}_i$ in each spatial direction and defining A to be the area within the polygon formed by extending from the mean this distance in each corresponding direction.

Since $\mathbf{C}_j$ is a spatial location that changes with time, it is natural to calculate the change in position per unit time—the geographic analogue of the velocity of $\mathbf{C}$ at the time of the j-th event, t_j [32],

$$\mathbf{V}_{C_j} = \frac{d\mathbf{C}_j}{dt},$$

as well as an analogue of the 'linear momentum' of $\mathbf{C}$ at time t_j [21],

$$\mathbf{P}_{C_j} = j\mathbf{V}_{C_j},$$

where the 'mass' of the distribution is equal to the current number of serial events, j.

The concept of linear momentum $\mathbf{P}_{C_j}$ is important to our model as it allows for a mathematical analysis of the spatial direction in which the kill spree seems to be moving, relating $\mathbf{P}_{C_j}$ to $\mathbf{P}_{C_{j-1}}$ by considering the law of conservation of momentum. We define the velocity of serial event K_j as the distance between event K_j and the previous centroid divided by the time between event K_j and K_{j-1}

$$\mathbf{V}_{K_j} = \frac{\mathbf{X}_{K_j} - \mathbf{C}_{j-1}}{t_j - t_{j-1}}.$$

It follows from the definition of mass above that the mass of a single event is 1, and thus the momentum of a single event K_j is equal to its velocity,

$$\mathbf{P}_{K_j} = \mathbf{V}_{K_j}.$$

Modeling the state of C_j at time t_j as the result of a completely inelastic collision between C_{j-1} and K_j (i.e., K_j is absorbed and joins C_{j-1} to form C_j), conservation of momentum tells us that

$$\mathbf{P}_{C_j} = j\mathbf{V}_{C_j} = \mathbf{P}_{C_{j-1}} + \mathbf{P}_{K_j} = (j-1)\mathbf{V}_{C_{j-1}} + \mathbf{V}_{K_j},$$

or,

$$\mathbf{V}_{C_j} = \frac{(j-1)\mathbf{V}_{C_{j-1}} + \mathbf{V}_{K_j}}{j},$$

where it follows trivially that

$$\mathbf{V}_{C_1} = 0,$$

which follows expectations for a spree consisting of only one kill (non-serial).

To make a prediction for the next kill site in a series K_{j+1}, we can incorporate both the current centroid $\mathbf{C}_n$ and the current velocity of the centroid $\mathbf{V}_{C_j}$ by defining $A(t)$ as the polygon A as defined above with a center dependent on time according to $\mathbf{V}_{C_n}$,

$$Center_{A(t)} = \mathbf{C}_j + f(t)\mathbf{V}_{C_j} + g(t)\mathbf{\hat{a}_j},$$

where $\hat{\mathbf{a}}_{\mathbf{j}}$ is an estimate of the acceleration of C_j based on a linear regression of velocity data obtained from $K_1 \ldots K_j$ and $f(t)$ and $g(t)$ are parameter coefficients chosen to match the constraints:

$$\lim_{t\to\infty} \frac{d(Center_{A(t)})}{dt} = 0 \tag{1}$$

$$f(t) \approx t\text{for } t < \bar{t} \tag{2}$$

$$g(t) \approx t^2/2\text{for } t < \bar{t} \tag{3}$$

where $\bar{t}$ is the mean time between K_i's.

Constraint 1 is necessary to keep our prediction reasonably centered on the observed events over long spans of time, and conditions 2 and 3 are necessary for the kinematic analogue. One possible choice of $f(t)$ and $g(t)$ explored in this paper is

$$f(t) = \frac{2t}{1+e^{\beta t}}, \qquad g(t) = \frac{2t^2}{(1+e^{\beta t})^2}$$

for some real constant $\beta > 0$.

Note that the final model yields a prediction $A(t)$ that is a function of time, consistent with our assumption of an evolving killer. To get a general prediction, it is possible to evaluate $A(\bar{t})$, yielding the prediction for where the next kill will occur should it happen after the mean time $\bar{t}$.

3.2 A Rational Choice Model

3.2.1 Background

A number of criminologists have suggested that much criminal behavior can be modeled as rational choices made by agents who derive some utility from their crimes [6, 12, 29]. On one hand, criminal activity has historically been seen as defying rationality [12]. If we grant, however, that a given rapist, burglar, or other criminal will rape, burglarize, or otherwise break the law, then their choices of whom to rape or which houses to burglarize may appear rational [16], and may in fact be the result of much planning and deliberation. This makes rational choice theory as implemented in economics particularly well-suited for our purpose here of looking at a collection of crimes to draw inferences about the criminal's future behavior.

Previous studies have applied economic theory to criminology by using Expected Utility Theory [40] and Prospect Theory [20] to model a criminal who weighs the benefits to himself of committing a crime against the odds of getting caught and its associated harm. While our model here is based on the idea from microeconomics of using observed *revealed preference* data to make inferences about unobserved *preferences* [34], it does not resemble a traditional economic model so nearly as do these earlier works.

The microeconomic model seeks to make predictions about the responses of dependent variables (such as quantities consumed of various goods) to possible future values of independent variables (such as prices) based on past observations of both [34]. In this model, the agent selects a bundle of goods to purchase by assigning quantities to each available good; these quantities form a consumption vector. In our model, the agent assigns probabilities to each possible location for his next crime, forming a probability vector. The economic agent faces a scalar budget and a vector of prices. The dot-product of her chosen consumption vector and the environmental price vector must not exceed her budget. She is assumed to maintain a consistent ranking of all possible consumption bundles. For our criminal agent, the analogue of price is perceived risk of getting caught if he commits his next crime in a given location. While we cannot observe this the same way that we observe prices, it is analogous in that it differs across observations, since we expect the risks of getting caught in various locations to change each time the criminal acts (due to increased public awareness or police presence, e.g.). His (unrelated) constraint is that the entries in his probability vector must sum to 1. Rather than ranking consumption bundles, he ranks probability distributions.

Choosing a probability distribution as a means of agency has precedent in the notion of a mixed strategy in game theory [15]. We believe this reflects the nature of the actual choices the criminal makes, particularly about location. Geographical profiling expert D. Kim Rossmo notes that "for a direct-contact predatory crime to occur, the paths of the offender and victim must intersect in time and space, within an environment appropriate for criminal activity." [32]. Since the criminal cannot significantly control the flow of potential victims or the suitability of different locations, he does not have complete control over where and when he breaks the law. Rather, when he walks down a particular street, he is choosing to increase the probability that he will commit his next crime there.

Since a rational agent must have consistent preferences, if our serial criminal is to evolve, then each crime must change his incentive structure. We model this by including a perceived extra risk of capture at any given potential target as a function of the locations of previous acts: you can never go back to the scene of a perfect crime [11]. Since the overall risk of getting caught should already be a significant factor in the agent's preference for one crime scene over another, it is only this additional risk which we call analogous to price.[2]

When we talk about a "rational serial criminal", we do not mean to suggest that there are good reasons for committing series of crimes.[3] We merely hypothesize that by assuming this series of crimes to be determined by a consistent preference field [30], we may make useful predictions about the criminal. Additionally, when we talk about "utility" as a function of the probability vector described above, we do not mean to imply that committing crimes fulfills some desire of the criminal's to a variable extent depending on where he does it. The

[2] One could argue that a fully rational criminal adding up the costs of committing a crime at a particular place would consider the loss in welfare he would experience when it became riskier for him to commit other crimes there in the future. While technically true, we do not consider this point relevant to our analysis, or at all.

[3] But nor do we deny it.

utility function is just a convenient mathematical tool, which is equivalent to making direct inferences from revealed preference [17].

To summarize, we construct a model of a rational serial criminal who controls his behavior by determining a probability distribution defined over a bounded geographical range of activity. This is the distribution of a random variable representing the location of his next crime. The model does not include the decision to commit a crime, as we are interested in modeling the behavior of a serial offender in the course of a spree. We assume the distribution to be static over the period between any two crimes, and to change instantaneously at the instant that a crime is committed.

3.2.2 Rational Choice: Theory

Divide the criminal's assumed range into n cells, numbered 1 through n. Included in our data, then, is a vector $\mathbf{K}$, where each $K_j \in \{1, ..., n\}$ denotes the cell in which the criminal committed his j^{th} crime. Assume the criminal is a rational agent with a utility function

$$U(\mathbf{p}, \mathbf{c}),$$

where $\mathbf{p}$ is a vector defined over all cells of the grid such that p_i = the probability that next crime will happen in cell i, and $\mathbf{c}$ is a vector of the same dimension such that c_i = the criminal's perception of the increased risk that he will be caught if he commits his next crime in cell i.

He chooses $\mathbf{p}$ to maximize U subject to the constraint

$$\sum_{i=1}^{n} p_i = 1.$$

This yields the first-order conditions

$$\begin{array}{lcc} \frac{\partial U}{\partial p_1} \quad (p_1, \ldots, p_n, c_1, \ldots, c_n) & = & 0 \\ \vdots & & \vdots \\ \frac{\partial U}{\partial p_n} \quad (p_1, \ldots, p_n, c_1, \ldots, c_n) & = & 0 \end{array}$$

Assume that U is well-behaved, so that we may invoke the Implicit Function Theorem to define $\mathbf{k}(\mathbf{c})$ such that

$$\begin{array}{lcll} p_1 & = & k_1 & (c_1, ..., c_n) \\ \vdots & & \vdots & \\ p_n & = & k_n & (c_1, ..., c_n) \end{array}.$$

Assume that each time the criminal commits a crime, $\mathbf{c}$ changes, and therefore $\mathbf{k}$ changes. As the function $\mathbf{k}(\mathbf{c})$ is arbitrary, it may contain any number of parameters α: these may reflect any way in which we think the criminal prefers one location for his crime over another. It is these parameters which we hope to find.

We begin by assuming a form for $\mathbf{c}(x)$. It may be as simple as

$$\mathbf{c}(X_j) = \begin{cases} 1 & \text{if } i \in X_j \\ 0 & \text{otherwise} \end{cases}, \tag{4}$$

where $X_j = \{i|$ the criminal committed one of his first j crimes in cell $i\}$. This form captures the belief, "If I commit a crime twice in the same spot, I will be arrested for sure, but otherwise my past crimes have no impact on my chances of being caught." Next, assume some form for the k_i functions. For a criminal assumed to prefer working near a home base (allowing for a buffer zone), a possible one-parameter function would be

$$k_i(\mathbf{c}, \alpha) = \begin{cases} 0 & \text{if } \alpha = i \\ \frac{1-c_i}{d(\alpha,i)\bar{k}} & \text{otherwise} \end{cases} \tag{5}$$

where α represents the location of a home base, d is a metric, and $\bar{k}$ is a normalization constant. Then for any value(s) of α, we can calculate the probability $P(K_j|X_{j-1}, \alpha)$ that the rational criminal characterized by the functions $\mathbf{k}$ and $\mathbf{c}$, having committed $j-1$ crimes in the set of locations X_{j-1} where the first $j-1$ crimes occurred, would have committed his j^{th} crime in the location K_j where the j^{th} crime actually took place. Finally, define

$$P(\mathbf{K}|\alpha) = \bigcap_j k_{K_j}(\mathbf{c}(X_{j-1}), \alpha). \tag{6}$$

Then $P(\mathbf{K}|\alpha)$ is the probability that the observed set of crimes $\mathbf{K}$ would have been committed by our rational criminal, given α. By evaluating this function over a range of possible values for α, we can get an idea of which parameter values are more probable than others.

3.2.3 Rational Serial Criminal: Practice

The theory above could be used to model any serial criminal who commits crimes at different locations. In the current paper, we are interested in helping a police department catch a serial killer, and in this section we describe our specific implementation of a Rational Serial Criminal (RSC) model for this task.

When a serial killer is at large, law enforcement has two goals: to prevent future killings and to find the killer. The rational choice model can help with both of these goals, by making predictions about where the killer lives and about where he may kill again, both based on previous killings. Implementing the model requires making explicit our assumptions about the killer's behavior. For both purposes, we make the following assumptions:

- The killer prefers to kill near, but not at, an anchor point α. This anchor point does not change over the course of the spree.
- The killer's taste for killing in a given location, k_i, is a function only of that location's distance from his anchor point and its distance from previous kill sites X.

- k_i is given by Equations 4 and 5.

First, we estimate the likelihood that the killer lives in each grid-cell of the city. We make the following additional assumptions:

- The prior odds $P(\alpha)$ of the killer living in place α are equal for all α.
- The prior odds $P(K_j = i)$ of the killer committing his j^{th} murder in place i are equal for all i.

Given these assumptions, Bayes' Formula

$$P(\alpha = i|\mathbf{K}) = \frac{P(\mathbf{K}|\alpha = i)P(\alpha = i)}{P(\mathbf{K}|\alpha = 1)P(\alpha = 1) + \cdots + P(\mathbf{K}|\alpha = n)P(\alpha = n)}$$

implies

$$P(\alpha = i|\mathbf{K}) \propto P(\mathbf{K}|\alpha = i). \tag{7}$$

Our goal, and perhaps the limit of our abilities, is to rank potential anchor points in order of likelihood. As such, Equation 7 is sufficient to say that the isopleth produced by plotting Equation 6 may be taken as a jeopardy surface, representing the relative likelihood of the killer residing within the region associated with each grid cell [32]. Such a plot could help the police find the killer faster by telling them where to look first [32].

Next, we estimate the likelihood that the killer will strike next in each grid-cell. For each grid cell i, this is equivalent to p_i. Again, we assume $P(\alpha|K_1, \ldots, K_{j-1}) \propto P(K_1, \ldots, K_{j-1}|\alpha)$ for all α. Using data from $J-1$ attacks, we estimate the likelihood that the J^{th} attack will occur in any given cell i by

$$\hat{p_i} \propto \sum_{j=1}^{J-1} \sum_{\alpha=1}^{n} k_i(\mathbf{c}(X_{j-1}), \alpha) P(K_1, \ldots, K_{j-1}|\alpha),$$

where $P(K_1, \ldots, K_{j-1}|\alpha)$ is computed by Equation 6. Again, our goal is to rank possible locations for the next attack by likelihood, so we do not bother with normalization constants.

4 Data

5 Data

To calibrate and test our models, we constructed data sets from data on three real serial killers: David Berkowitz, the “Son of Sam”, Angelo Buono and Kenneth Bianchi, the “Hillside Strangler” (police originally thought there was a single killer), and John Allen Muhammad, the “D.C. Sniper”. For each one, we began with a list of the killer's victims taken from a Wikipedia [3, 4, 2], and then filled in details for each murder or attempted murder with details found in newspaper articles from the time. The Berkowitz and Sniper data sets included address where people were shot, while the Hillside dataset included locations where strangled bodies were found. For calculations, we converted the street addresses to latitude-longitude coordinate pairs (Table 1).

Kill#	Longitude	Latitude	RelativeDay	Victim(s)	Address
1	-73.833846°	40.847067°	0	Donna Lauria and Jody Valenti	2860 Burhe Ave, Bronx, NY
2	-73.804809°	40.768099°	25	Carl Denaro and Rosemary Keenan	160th St and 33rd Ave, Queens, NY
3	-73.706561°	40.738631°	121	Joanne Lomino and Donna Demasi	83-31 262nd St, Queens, NY
4	-73.845048°	40.718511°	185	Christine Freund and John Diel	1 Station Square, Queens, NY
5	-73.847051°	40.718598°	222	Virginia Voskerichian	4 Dartmouth St, Queens, NY
6	-73.835418°	40.847445°	262	Alexander Esau and Valentina Suriani	1878 Hutchinson River Parkway, Bronx, NY
7	-73.771343°	40.758704°	332	Sal Lubo and Judy Placido	45-39 211th St, Queens, NY
8	-74.011712°	40.612735°	367	Stacy Moskowitz and Robert Violante	86th St and 14th Ave, Brooklyn, NY

Table 1: Example data set: Berkowitz murders [36, 3]

6 Results

6.1 Centrographic Model

To test the centrographic model's predictive capabilities, we implemented it via a Python script and ran it on the data from the Hillside kills sequentially, using the first $j-1$ kills to predict the j^{th} starting with $j=4$ (Figure 1). As the output from the model changes with time we used the actual time of the subsequent kill K_j in evaluating the prediction function. In our simulations we adopted an β value of $\beta = \frac{\bar{t_j}}{\ln 15}$ where $\bar{t_j}$ is the mean time between kills up to kill j. In practice a value of β must be determined according to the previous crime data for the specific series.

To measure accuracy of the model, we calculated the distance between the center of the area of probable next kill and the actual location of the next kill (Table 2). The error distance has been criticized in past works [32], but recent comparisons between the error distance and alternative accuracy measures have shown it to perform comparably [39]. The metric we use in calculating error distance is the L1 or Manhattan metric,

$$d_1(p,q) = ||p-q||_1 = |p_x - q_x| + |p_y - q_y|$$

Our choice of the Manhattan metric arises from previous work indicating that it most accurately models actual travel distance in urban settings, a more important factor than strict Euclidean distance in geographical profiling [32].

To estimate the acceleration of the spatial centroid we perform a linear regression on the calculated centroidal velocity data for each estimate, in both the longitudinal (Table 3) and latitudinal (Table 4) directions independently.

Kill#	**Distance from center of predicted area (km)**
4	19.9032477532
5	13.1624956104
6	26.1455165479
7	12.1278772065
8	4.41086910109

Table 2: Distance between center of predicted area and actual location of subsequent kill, measured with Manhattan metric. Hillside data set [19].

Kill#	$\hat{\mathbf{a}}_{\mathbf{x}}$	$\mathbf{R^2}$	p
4	0.00119566020314	1.0	N/A
5	0.00110767788734	0.99360376805	0.0720425343571
6	0.000787815342892	0.829782525827	0.170217474173
7	0.000494702112585	0.592338457701	0.292588804185
8	0.000422872375405	0.571437451732	0.236142632648

Table 3: Linear regression data for the centrographic model, x-direction (longitudinal). Hillside data set [19].

Victim#	$\hat{\mathbf{a}}_{\mathbf{y}}$	$\mathbf{R^2}$	p
4	0.000665961218837	1.0	N/A
5	0.000983419577946	0.90783947974	0.275461438702
6	0.00105866206185	0.93384684203	0.0661531579701
7	0.000983719311558	0.935632003161	0.0194133564982
8	0.000950274912901	0.943853948827	0.00464007177548

Table 4: Linear regression data for the centrographic model, y-direction (latitudinal). Hillside data set [19].

6.2 Rational Serial Criminal Model

Like Rossmo's criminal geographic targeting (CGT) model [32], the RSC model yields a jeopardy surface representing the estimated relative probabilities that the killer lives within each grid cell on a map. For this to be useful, it should allow law enforcement to search for the killer more efficiently. We tested our results using Microsoft Excel macros in the same way that Rossmo tested the

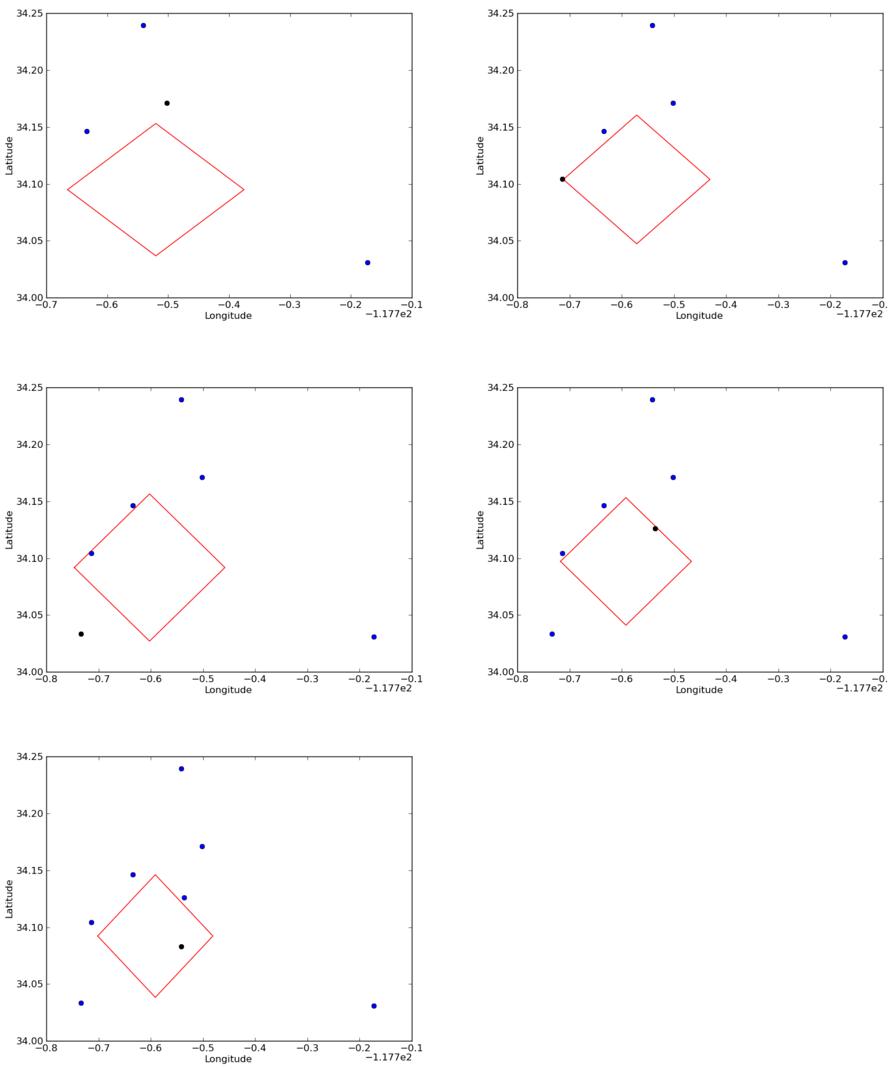

Figure 1: Simulation results for the centrographic model with the Hillside Strangler data set [4]. The blue points represent the kills used to calculate the centroid, centroidal velocity and centroidal acceleration. The red diamond represents the estimated area of next probable event (within one mean distance of the centroid along each axis in each direction). The black point represents the actual location of the next event.

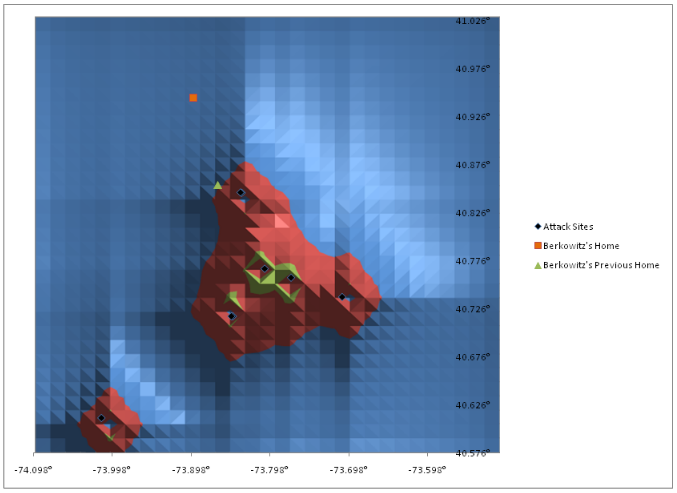

Figure 2: Attack sites, jeopardy surface, and homes of David Berkowitz, the "Son of Sam," using the Berkowitz data set [24]. The ranking of quadrant coloring in order from least probable location to most probable location is: blue, red, green, purple.

CGT model: by drawing a box called a "hunting area" around the attack sites, plotting the jeopardy surface based on the attack locations, and then asking the question: if the police were to search every cell in the hunting area in order of highest to lowest on the jeopardy surface, what percentage of the grid would they have to search before they got to the killer? This number is called the Hit Score % [32]. Multiplying this by the size of the hunting area yields the effective search area: the actual area the police would have to search, following the jeopardy surface, before finding the killer. This method of testing allows the modeler some degree of flexibility, in deciding which locations would have been counted as crime sites and in guessing where the killer would have been found.

For David Berkowitz, we ran the RSC model with the locations of the eight shooting incidents attributed to him at the time of his capture [24]. The resulting jeopardy surface is shown in Figure 2, with locations of shootings and Berkowitz's last two residences overlaid. While on most occasions he shot or shot at two people, we counted each shooting only once. Berkowitz lived in the Bronx for several years, near his first and sixth crime scenes, but moved to Yonkers before shooting his first victims [25]. He later admitted to having stabbed two people near his Bronx home while living there [25], but newspaper accounts from the time [36, 24] show no indication that the stabbings had been connected to his shootings before his arrest. Therefore, we did not include them in our analysis, as their location would not have been available for consideration by law enforcement at the time. We computed two scores, one using a grid defined by the crime sites to look for Berkowitz former house in the Bronx, and one using a grid contrived to include his Yonkers house to find him there. The results are tabulated in Table 5.

Home	Hunting Area	Hit Score %	Search Area
Bronx	1,414 km^2	18.1%	255 km^2
Yonkers	2,514 km^2	63.2%	1,588 km^2

Table 5: Performance of RSC model on data for David Berkowitz [24]

In October and November of 1977, the bodies of 11 strangled women were discovered in Los Angeles, and were all believed to be the victims of a single "Hillside Strangler" [19]. Using the exact same parameters and assumptions, we ran the RSC model on the 9 of these locations (two of the bodies were found in the same location, which we only counted once; for one body we could not find an exact location of discovery). The resulting jeopardy surface is plotted in Figure 3, and the Hit Score % and accompanying data for the house of one of the two offenders, Angelo Buono, [7] can be found in Table 6.

There were two main differences between this data and the data for the Berkowitz shootings: first, these were locations where bodies were found dumped, not locations where killings actually took place. Second, the hunting zone around these bodies, determined by their location, stretched over twice as far North to South as it did East to West, while the Berkowitz hunting zone was essentially square. Nonetheless, the model still performed reasonably well on the Hillside Strangler data without modification.

To test the predictive powers of the RSC model, we used the jeopardy surface with the first six kills from the Berkowitz data set and the modified utility and cost functions (see Section 3) to try to predict the location of the 7th kill (Figure 4). In this case it doesn't make sense to use a Hit Score % to analyze accuracy as the killer will not necessarily be stationed at the predicted site during the course of an extended search, but it is evident from the jeopardy surface that the model performed favorably, as the kill occurred in one of the regions of highest probability. Of course, this is just one data point, and a month later Berkowitz shot his final victims in Brooklyn: this point would not even have been on our map.

Hunting Area	Hit Score %	Search Area
2,721 km^2	13.9%	379 km^2

Table 6: Performance of RSC model on data for Buono & Bianchi [19]

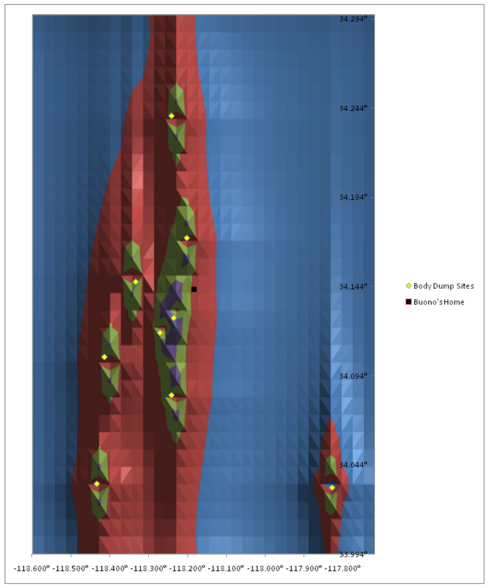

Figure 3: Attack sites, jeopardy surface, and home of Angelo Buono [7], one half of the “Hillside Strangler” duo. The ranking of quadrant coloring in order from least probable location to most probable location is: blue, red, green, purple.

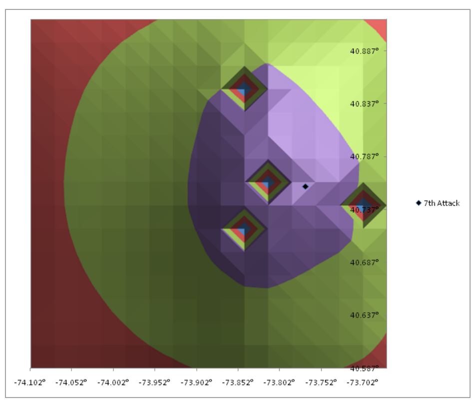

Figure 4: Prediction results with the RSC model for the 7th Berkowitz kill using the first six kills from the Berkowitz data [24] to generate the jeopardy surface. The ranking of quadrant coloring in order from least probable location to most probable location is: blue, red, green, purple. The black point is the actual location.

6.3 Combined Models

As a brief example of using the RSC model and the Centrographic model in conjunction with each other, we used each model on the DC Sniper data set [2] and graphed the outputs (Figures 5 and 6).

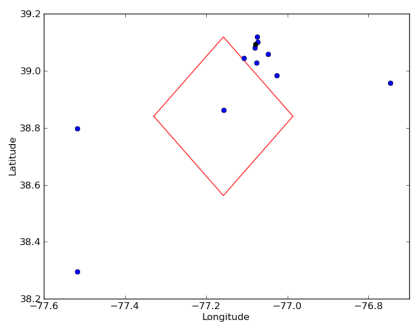

Figure 5: Simulation results for the centrographic model with the DC Sniper data [2]. The blue points represent the kills used to calculate the centroid, centroidal velocity and centroidal acceleration. The red diamond represents the estimated area of next probable event (within one mean distance of the centroid along each axis in each direction). The black point represents the actual location of the next event.

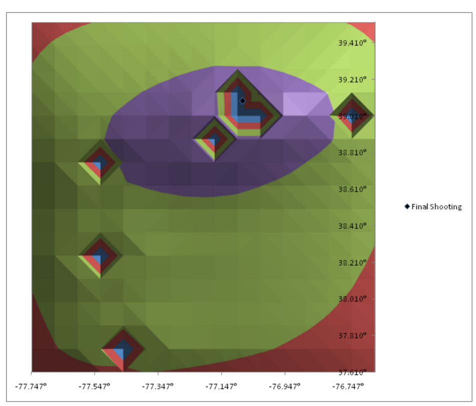

Figure 6: Prediction results with the RSC model for the last DC Sniper kill using the previous kills from the DC Sniper data [2]. The ranking of quadrant coloring in order from least probable location to most probable location is: blue, red, green, purple. The black point is the actual location.

7 Discussion

Any model of serial killer behavior must be somewhat *ad hoc*. While Rossmo's macrolevel serial killer dataset lists 225 cases of serial killers in 9 countries going back to the late nineteenth century, of these, he found only 21 to be suitable for testing his model [31].

Looking at the data for the Centrographic model, we see in Figure 1 that the predictions seem to get better over time, meaning that as more data become available the model is becoming more accurate. Indeed, the error distances (Table 2) are decreasing with each additional kill, and the mean error distance is only roughly 15.12 km, a small distance considering the search area was the city of Los Angeles.

A key part of the Centrographic model is its use of a linear regression to estimate the acceleration, $\hat{\mathbf{a}}$, which brings up a question of how valid a linear regression on the velocity data actually is. Evident in Table 4, our test demonstrated a high correlation coefficient ($\approx 90\%$ correlation) for the Y-acceleration, $\hat{\mathbf{a}}_{\mathbf{y}}$, as well as small p-values for the later crimes, implying that the linear regression is a statistically significant model that accounts for a large part of the variation in the data. This indicates the linear regression model to be a mathematically valid approach in this case. However, in the case of the X-acceleration, $\hat{\mathbf{a}}_{\mathbf{x}}$ (Table 3), the correlation coefficient was smaller and the p-values larger, yielding a statistically insignificant model that doesn't account for a large degree of the variation in the data.

The problems with the model's acceleration approximation illustrate once again the problems with performing statistical analysis on a small data set. Even though the $\hat{\mathbf{a}}_{\mathbf{x}}$ obtained from regression would seem to account for at least 60% of the data variation, it could just be random chance. If we disregard the question of statistical significance for a second and just look at the correlation, however, it's obvious that the linear model does not seem to be enough in the case of $\hat{\mathbf{a}}_{\mathbf{x}}$. This indicates that perhaps it is necessary to reconsider the idea of a linear regression to calculate $\hat{\mathbf{a}}$—perhaps the idea of a centroidal acceleration of a data set is not entirely conducive to geographical profiling, or perhaps even a higher-order regression is necessary, introducing analogues to physical concepts such as jerk (the change in acceleration over time) and jounce (the change in jerk over time)[4].

The Centrographic model performs well on the data available to us, but more extensive testing is needed to determine its validity. To improve the model, it would be advisable to conduct future research on many more data sets in order to determine appropriate choices of the $f(t)$ and $g(t)$ functions used. A weakness of the model as it stands is its sensitivity to the choice of β, so perhaps our choices of the functions in this simulation were ill-advised. Additionally, the model does not incorporate as much temporal modeling as it could, which could strengthen its predictive powers.

The Rational Serial Criminal model performed reasonably well on the data

[4] But maybe this is just silly.

for which we tested it. With the Hillside Strangler data, the Hit Score % of our jeopardy surface for the location of the killers home was only 13.9% (Table 6), meaning only 13.9% of the hunting area would need to be searched before finding the home of Angelo Buono, a vast reduction. With our choice of probability density function k, the RSC model made predictions qualitatively similar to those made by the CGT model [31]. In the case of the Berkowitz data, however, the Hit Score % for our prediction for finding Berkowitz's home at the time of the murders was only 63.2% (Table 5)—worse than random chance. This is due to the fact that shortly before the murders took place Berkowitz moved his place of residence. Using Berkowitz's old home, the RSC model yields a Hit Score % of 18.1%, a much more reasonable figure.

Conducting a χ^2 goodness of fit test of the Hit Score % from the three tests against the uniform probability density function implied that the data are not uniformly distributed at the 95% level of significance. This implies that the RSC model developed is statistically superior to a random search method for the location of the murderer's home.

The RSC model's performance in our tests was hampered by a few simplifying assumptions we made:

- We assumed the initial probability distribution over the hunting area for the location of a serial killer's home was uniform.
- We assumed the initial probability distribution over the hunting area for the location of a murder was uniform.

Obviously, as in the case of the Berkowitz data, this is not always the case—half of the area included in our simulation was underwater, indicating zero probability of the killer's home being in that area, and zero probability of the next kill occurring in much of that area[5]. To improve the RSC model, it is possible to modify the initial guess of the probability distribution for both of these cases based on simple factors such as population density or geographic landmarks (ie, lakes and harbors).

The real strength of our Rational Serial Criminal model is its theoretical indifference to the choice of utility function. This means that it can be adapted to the pursuit of almost any serial criminal. Besides the well-documented tendency for criminals to act locally, police investigating a particular fugitive may have any number of other ideas about the crook's preferences. If they can write these down as functions, the RSC model can help them as they develop a search plan based on their evidence and intuition. Since one might reasonably consider a group engaged in organized crime to be a single agent, one could also use the RSC model to investigate gang behavior, for example.

Independent of any specific investigation, we could improve the RSC model significantly by putting other common themes and population trends [28] about crime sites into functional form. For instance, criminals tend to prefer to act in areas which are like the areas they are familiar with [23]. If we included zoning

[5] Disregarding serial criminals with fishing boats or scuba certification.

and demographic data in our grid, we could include a term in the **k** function which operationalizes this prediction. We could also change the meaning of p to include more sophisticated information about the victims, so that we could model a criminal who tended to find his victims in a particular region but then assaulted them or disposed of their bodies in more diverse locations [13]. This could augment the current method of using GIS software to determine how several victims' daily lives might have led them to interact with a killer, and from this information determining areas where the killer might spend a lot of time [37].

In Figures 5 and 6 we see the results of running both the RSC and Centrographic model on the DC Sniper data set. Using both methods generates two zones each with theoretically high probability of encounter with the serial killer in question. Looking at the graphs, we see that the models did not predict the actual crime site. This can be explained by the nature of the data. In the RSC model case, there is a crime site in the same location of a previous crime site. This violates the assumptions of the model. In the centrographic model, the killer took an unexpected turn south, resulting in a crime site far from any of the others. Both of the problems above stem from the size of the hunting area for the DC Sniper, which contains large areas of Maryland and parts of Virginia. This, however, is simply a characteristic of the DC Sniper data that makes it difficult to make predictions regarding his actions in general. Additionally, the DC Sniper data differs from the other sets in that many of the kills took place on the same day at different times, making some of the calculations involved in the centrographic model difficult or nonsensical (ie, predicting infinite velocity). With a more accurate data set (to the time of day, for example) it is possible the model would perform better.

To improve the way the models work together, a number of possible adaptations could be made in future models. As the RSC is a resource-intense method, it is possible that using it for a large area with a fine enough quadrant resolution could be simply too slow. Since the Centrographic model theoretically narrows down the search window, it could be useful to consider using a variable quadrant resolution when analyzing large areas, where the resolution outside the area predicted by the Centrographic model is larger than that inside, yielding a faster runtime while still providing a high level of accuracy in high probability areas. Using the models in the other direction, probability data from the RSC model could be used to create a sort of resistance to the motion of the centroid in the Centrographic model to cause it to stay near regions of higher probability.

Overall, this model provides a starting point for a police search. It is meant to be another tool to the police, to give a general guideline of where to allocate police sources when very few data are available. In reality, police experience is impossible to replace with any kind of mathematical method, and when dealing with a subject as unpredictable as a serial murderer, experience is necessary to determine the most efficient way to catch the killer.

8 Executive Summary

This method is a computer program that takes crime site (either body disposal site or scene of the crime) data in absolute coordinates and returns a prediction of the offender's residence and a prediction of the next crime site. This should be used for two things: to narrow the search for the offender in his daily routine and to narrow the search for the next crime site. It is possible that, after discovering a crime site, that there is another crime site in a location that has not yet been discovered. Furthermore, a search that starts in the most probable area and radiates from this center is more likely to find the offender's residence, or next crime site, than a random search would [32]. The method will provide a starting point and an area for a police force to concentrate the efforts of determining both the location of an offender and where the next crime site might be.

In employing the method, the police force makes the same assumptions that are involved with developing the program. The assumptions made are:

- The series will occur in an area that the offender is familiar with, meaning a place where he lives, has lived, or has committed crimes before.
- The most probable location of the next crime site can change with time, even without new data being acquired regarding former sites in the series.
- The order and time at which former crime sites occur affects the location of the predicted next crime site in the series.
- Given no crime site data, the offender is equally likely to live or commit a crime at any location.

In using the program to predict the offender's home, it is assumed that the offender has a home base within or close to the crime site, as opposed to commuting to the area of the crime site. This assumption can be relaxed when predicting where the next crime site might be, as this prediction is made solely on earlier crime site data and thus does not offer additional information. An example of when the method would not apply is when an offender picks up prostitutes in a red light district in order to kill them. If, as in the John Wayne Gacy Jr. case, the crime sites all share a location the model will not apply as no usable predictions could be gleaned from that data.

The program utilizes two different types of models to make predictions, a centrographic model and a rational choice model. The centrographic model has an analogy in physics, where the crime sites are treated as point masses in a system, and the "centroid" is determined by finding the center of mass of the system. From this model is output a graph that includes the points of the crime sites and a diamond. Within the diamond lies the prediction for the next crime site. The rational choice model has an analogy in economics. This model uses the idea that the offender is deciding the location of the crime site in order to satisfy his personal goals (mostly of committing another crime) and to minimize his risk of being caught. Thus, it is assumed that the series will continue and that there will not be two crime sites in the same location. There are two types of this model included in the program, one that predicts where the offender lives and the other predicts the location of the next crime site. In both cases, the model outputs a contour plot. In the two cases, the peaks correspond to either high probability of the offender's home base and to the location of the next crime site, respectively. There will always be a valley at the location of a previous crime site. Then, when determining where to search for the next crime site, the contour plot from the rational choice model and the plot from the centrographic model can be utilized in conjunction to narrow the initial search area. A key item to note is that the centrographic model makes use of a parameter, β, that is determined by the data of known crime times and must be derived individually for each serial killer. An example value might be $\beta = \frac{\bar{t}}{\ln 15}$, where $\bar{t}$ is the mean time between crimes.

The method will always output graphs that can be used to begin a police search, but as it is a computerized process, it only responds to the data set provided. Thus, there is an element of subjectivity in determining which crime sites in a given case are useful for predictions [32]. When predicting where the offender lives, the following should be used as a guideline for inputting crime site data:

- A minimum of five crime sites should be used in order to provide a basis for the prediction. Certainly, the program will work with as few as three crime sites, but will not serve to improve the search area greatly. Furthermore, when invoking the program, one assumes that the offender has not changed the home base since the series began. Else, more crime sites are necessary to determine a valid prediction.

- Only crime sites that are accurately known should be input to the program. For example, using data determined from victims acquaintances as to where the victim was last seen to determine where the offender might have met the victim should not be counted as valid input.

- If it is clear that two crime sites relate to the same crime (i.e. a site is found where the offender killed the victim, but the body disposal site in a different location) one or the other location should be input in to the model, in accordance with the other crime sites [32].

When predicting the location of the next crime site the previous also apply, though several crime sites in the same area at different points in time might be considered as several different sites, as this might be an indication of where the offender's next act in the series might occur.

One of the most powerful tools to be incorporated alongside this program is a GIS mapping software. Using such software will retain perspective of the crime site data when considering the predictions set forth from the program. Furthermore, the program does not account for any sort of victim data that might be gathered during the investigation. Considering these two additional elements will lead to more efficient searches.

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