Judges' Commentary: The Outstanding Snowboard Course Papers

Kelly Black
Dept. of Mathematics
Clarkson University
P.O. Box 5815
Potsdam, NY 13699–5815
kjblack@gmail.com

Overview

The focus of the problem was design of a snowboard course that allows snowboarders to achieve the largest possible vertical jump. The problem also required that teams identify issues associated with athletes performing other tricks and identify potential tradeoffs for other considerations.

We provide an overview of a few select observations of some of the judges. Students are required to put together a well-formed report on a complex topic in only a few days. Every year there are inconsistencies and errors in even the best reports, and the judges always struggle to find ways to balance the positive and negative aspects of each team's submission.

The problem examined this year is no exception; this problem is even more complex than usual. The Outstanding papers represent remarkable work by talented teams. Careful reading of the reports can reveal specific errors, but it is important to recognize the limitations of the event and examine the report as a whole.

This overview is divided into four parts:

- a broad overview of the judging process,
- an overview of the models and their derivation that were submitted by many teams,
- issues of examining the sensitivity of the resulting mathematical models, and
- an overview of how some student teams presented their overall results.

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The Judging Process

The judging process proceeds in three sets of rounds.

- **The triage process.** Every paper is read several times by different people. The goal is to determine which papers should be given more careful attention and could possibly achieve a higher rating.

  The amount of time available per paper during the triage round is limited. The main concern is whether or not a team has answered the question. The importance of the summary is amplified for these initial readings. A paper that provides a good overview of the entire paper, is written well, and provides a good overview within each section has a stronger probability of being passed on to the later rounds.

- **Screening rounds.** The judges are given more time to read each paper. In the triage round, papers perceived to be good tend to be given the benefit of doubt and be passed on; in the screening round, this is still true, but the goal begins to shift from removing papers that are not likely to achieve a higher ranking to trying to identify good papers that require more careful reading.

  During the screening rounds, the judges spend more time examining the mathematical model. Papers that provide a clear description of the model and offer substantial analysis of it tend to receive higher marks. The judges can begin to spend more time and focus on the whole submission. There is a higher expectation that the analysis, results, and writing be more consistent.

- **Final rounds.** The judges are given an increased amount of time to focus on the teams' submissions. During this set of rounds a judge may spend between half an hour to a full hour reading a single paper. During these rounds, the complete focus is on identifying the best papers. The judges focus on particular details and are able to make detailed comparisons between papers.

At the end of the final rounds, there are typically 12 to 16, and each remaining paper is given a rating of Finalist. Time is allotted so that each paper is read by every judge. At the end of the reading time, the judges assemble, and together they discuss each paper in order. The judges then make the final decision about which papers receive a rating of Outstanding. After deciding which teams receive Outstanding, the members of each of the sponsoring societies assemble in smaller groups to decide which paper should receive their award.

Modeling

This competition requires students to examine a nontrivial problem and identify a potential solution in a short amount of time. The problem this year required that the teams put together a nontrivial physical model and then apply mathematical tools in the analysis of it.

This overview of the modeling issues is broken up into two parts. First the physics of the various approaches is examined. Then the mathematics associated with the various approaches is examined. The majority of the teams used one of two approaches to the physics. The types of analysis cannot be easily divided with respect to the approach that the teams took to derive the physical model.

Physics

One of the difficulties in this problem is that it required the teams to model nontrivial dynamics. The first task required the teams to describe the physical situation and describe the terms found within the complex equations describing the physical situation.

Overall, the teams tended to take one of two approaches to develop a model, centering on use of either:

- the work-energy relationship, or
- Newton's Second Law.

Each team then had to translate the approach into a system of equations. For this second issue, the teams made use of a wide variety of techniques.

Deriving the Physical Situation

The first task for the teams was to describe the physical setup of a snowboarding "half-pipe." The International Olympic Committee has specific restrictions on the design of a half-pipe, and the majority of teams tried to stay consistent with the Olympic specifications.

The majority of teams broke down the construction of the half-pipe into a small number of parts. For example, a common construction included a middle flat part down the center of the half-pipe, round corners at the ends, and a flat lip along the top of the sides. Describing those restrictions can be difficult, and in this case most teams made use of a diagram that greatly simplified the task of translating those restrictions for the reader.

Once the parameters associated with the half-pipe were defined a coordinate system also had to be defined. Different teams used different coordinate systems, and there is no one obvious coordinate system to use. Teams that clearly indicated the coordinate system and showed it in a diagram had an immediate advantage when it came to describing the derivation of their model.

The parameters for the snowboarder had to be defined. An immediate discriminator for a paper in this regard was whether or not a free body diagram of the snowboarder was included. Teams that included one made it much easier for the judges to understand the resulting model.

Physical Principles

The teams whose primary approach made use of the relationship between work and energy faced a number of difficulties. The first is that the relationship between work and energy is a scalar relationship, and it does not easily lend itself to determining the height in a multivariate setting.

The teams also had to determine the total work done on a person while traveling on a snowboard, which then required that they model the system using Newton's Second Law. Upon successfully modeling the motion of a rider, the team then had to decide which forces were relevant to calculating the work integral for the friction forces and then approximate the integral. By itself, calculating the work integral was a difficult task to accomplish.

Teams that focused solely on the differential equations derived from Newton's Second Law had fewer complications. Even using a the free body diagram, the team still needed to do a correct derivation of the differential equations. It also required determining how to represent the forces in the different parts of the half-pipe including the straight section, the corners, the upper lip, and moving through the air.

Some papers used different parametrizations for the different sections, which caused a number of difficulties. Also, a common mistake found in even the best papers was to use mv^2/r to represent the magnitude of the radial force in the round corners. This is only true for constant radial velocity, which is not the case in this situation.

Mathematical Models of the Physical Principles

Once a team decided which physical principle to use and which terms were most important, the team had to formulate a system of equations. The entries that tended to receive the most positive attention used systems of differential equations. Given the complex paths different teams made use of different ways to express these equations and divided them into the various situations in different ways.

For example, some teams broke up the equations in terms of the location of the snowboarder in the half-pipe. Also, teams parametrized in terms of time, position, or other quantities. Because of the structure of the course and the multivariate nature of the problem, it was important for a team to describe carefully the parametrization and what equation was used for different portions of the half-pipe.

Bringing all of the physical principles together, keeping them consistent for the whole of the path within the half-pipe, translating the motion correctly into a system of equations, and then implementing the model in a consistent way was an extremely difficult task. Every team's entry included errors, and some of those errors were basic problems dealing with details such as the multivariate chain rule, numerical approximation, or assumptions about the values of physical terms.

The judges made every effort to try to balance the difficulty of the problem and the short time allotted to the teams with the desire to have a clear, correct solution. This not possible in the best of situations, and the judges had a difficult task in comparing entries to decide which team put together a better solution. In the end, it was a matter of judgment, and the work of the teams that more clearly discussed how they were able to arrive at a conclusion and justify their work made a more positive impression.

Sensitivity

The exploration of the sensitivity of the models tends to mark a significant difference between the top tier of the submissions and the rest of the entries. The judges expect that the best papers will include some indication of which parameters are most important and are the most sensitive in terms of what happens to the predictions in the presence of small changes in their values or what happens under slightly different assumptions. This year, the physical situation offered a rich set of options to explore the sensitivity of the resulting models.

The goal is to determine what happens to the snowboarder's performance for small changes in one or more parameters. The impact in terms of both the height of the jumps and safety for the snowboarders are important questions to address through the sensitivity analysis.

The exploration can take many forms. The most straightforward approach is to examine small changes in the results when different individual parameters are changed. For example, a team might examine what happens when the width of the half-pipe is changed by some small amount.

The sensitivity of different parameters is *always* an important aspect to the development of a mathematical model. *Every year*, the judges look closely at this aspect of the problem; *every year*, very few teams explore this aspect of the problem. A simple way for a team to have their submission stand out from the other submissions is to include a coherent exploration of the sensitivity of the mathematical model.

Discussion of Results

The majority of the teams used one of a few standard approaches. The differences between the entries were the combination of techniques used and how extensively the model was analyzed. The three things that make an entry stand out and receive positive attention from the judges are the following:

- the combination of techniques to assemble a mathematical model,
- the analysis of the model, and
- the writing and presentation of the model and results.

Advice

The first impression that a team can make on a judge comes from how the material is presented. To make a positive impression a team must provide a coherent structure to their document. The summary must be coherent and include an overview of the problem, an overview of the paper, and the team's specific results. The document itself should follow some basic rules and maintain a consistent presentation throughout the paper.

There are some simple rules for any entry:

- The nomenclature adopted by a team should be clearly described. (Keep in mind that different teams use different terms and variable names, which can make it difficult for a judge to compare different papers.)
- Every graph, table, or plot should be clearly described in the text, and the teams should explicitly explain what to look for in it and why it is relevant to the paper.
- Every equation should be numbered and proper punctuation employed to integrate the equations within the text.
- A picture can make complicated ideas much easier to understand.
- A free body diagram and a clear picture that shows the coordinate system can make it much easier for a judge to determine what a team was able to accomplish.
- When a plot is used, the axes should be clearly labeled and the units stated.
- Just having a table of contents at the beginning of the document can make it much easier for a judge reading a paper in the early rounds.
- Finally, team members should know the difference between a citation and a reference. The references are the sources listed at the end of the document and are a vital part of a paper. Citations are the indications within the text that help the reader decide which references are associated with specific ideas. A vast number of entries include a list of references but do not include citations within the text. Simply including consistent citations is an easy way to make a team's entry stand apart from the other entries.

Conclusions

The problem this year was difficult. Determining the important parameters and designing a half-pipe for snowboarders is a challenge that required the teams to bring together complex physics principles and use a wide array of mathematical topics. Every team was unable to avoid some basic pitfalls, but most of the submissions reflected the teams' overall desire to complete an excellent submission for this event.

The majority of teams made use of similar physical principles, but the different ways that those principles were applied and translated into a mathematical model made the difference between submissions. The judges were aware that this is a difficult problem, and the teams had a limited time to explore the topic. Despite these difficulties, the teams were able to bring together a high level of talent and desire that resulted in an impressive collection of entries.

In the end, the difference between the papers judged to be the top entries came down to the analysis of the subsequent models and the way in which the teams conveyed their results.

About the Author

Kelly Black is a faculty member in the Dept. of Mathematics and Computer Science at Clarkson University. He received his undergraduate degree in Mathematics and Computer Science from Rose-Hulman Institute of Technology and his Master's and Ph.D. from the Applied Mathematics program at Brown University. He has wide-ranging research interests including laser simulations, ecology, and spectral methods for the approximation of partial differential equations.