Judge's Commentary: Game of Ecology

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Introduction

The questions for Problem A of the 2019 MCM®required student teams to examine what would happen if the fictional dragons from the television series *Game of Thrones* [Benioff and Weiss 2019], which is based on the epic fantasy novels in the series *A Song of Ice and Fire* [Martin 2012], were living today.

Contestants were asked to analyze the dragons' characteristics, behavior, habits, diet, and interactions with their environment. Specifically, they considered the energy expenditures of the dragons, the ecological impacts of the dragon on the environment, and how climate or community assistance might change the system. The ultimate goal was to provide guidance to the author, George R.R. Martin, about how to maintain ecological realism in the fictional story.

This problem was based on a fictional creature. This means that it was even more important for assumptions to be clearly communicated and rationalized. Many of the student teams developed dragon growth equations based on existing models. In some cases, they considered dragon length; but the majority considered dragon mass. To understand energy needs, many teams used proportionality laws to relate mass to basal energy requirements and then used a wide variety of methods to address the energy requirements for the dragon's long-distance flight, fire-breathing, hunting, and other daily activities. Finally, teams considered how community assistance could be used to support the dragons and how changes in climate affect the dragons' energy requirements or intakes. There was a range of successful approaches to the problem, including predator-prey

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models, agent-based models or simulations, and models built from physical or chemical laws.

We consider an overview of the approaches. We first discuss the problem as stated and give an overview of approaches used. Next, we comment on issues associated with how results for the models were presented. Finally, we address a variety of important topics highlighted by this year's judges.

This article is not intended to provide an overview of the judging process. For more information about this important aspect of the contest, please see commentaries from previous years [Black 2015, Shannon 2016, Wilhelm 2017, Bieri 2018]. We highly recommend that advisors and students read some of the commentaries from previous years. These reports include information about the judging process and it can be helpful to understand the importance of the various parts of the report.

The Modeling Problem

Judges are aware of the time and resource constraints that student teams face when participating in the contest and we make sure that judging is based on student efforts rather than on judges' expectations.

Judges read the problem statement carefully, read a random sample of student papers, and work from a broad common rubric for judging that is based on what student teams were able to achieve.

The Questions

The first part of the problem asked students to analyze the dragons' characteristics and behavior. Teams were asked to consider the ecological impacts and requirements, along with the energy expenditure in terms of caloric intake requirements. The goal was to develop a model for how much area would be required to support the dragons and how large of a community would be necessary at varying levels of community assistance. The analysis of ecological impacts and energy expenditure was one of the most important aspects of the modeling process.

The second part of the problem asked teams to consider the impact that climate might have, given that dragons might migrate to different regions of the world.

Next, students were asked to prepare a two-page letter to the author, George R.R. Martin, that provides guidance about how to maintain realistic ecological underpinnings in the story. Teams were asked to focus on how movement between regions might affect the dragon ecologically.

Finally, the problem asked teams to describe and discuss a real-world situation in which their model, based on the fictional dragon, might be applied.

Models

Part I: Dragon Energy and Ecological Impacts

To address the questions in the first part of the problem, teams had to consider four broad categories:

- Dragon growth,
- Energy requirements,
- Ecological interactions and impacts, and
- Community support.

Dragon Growth

The majority of teams used existing models for dragon growth. Models using growth functions such as the logistic equation, the von Bertalanffy equation [von Bertalanffy 1957], or the Gompertz equation [Gompertz 1833] were widely used. One of the more complicated tasks was finding reasonable assumptions in parametrizing these growth equations. Two data points were given for dragon growth—when hatched, the dragons are roughly 10 kg, and after a year they grow to roughly 30–40 kg—but teams fitting nonlinear models had to build strong arguments for long-term growth assumptions. Highly-successful teams fell into two broad categories:

- They used data generated by fans [Balerion n.d.] of the fictional series to determine the size of the largest fictional dragon, or
- they considered the maximum size of existing species as a comparison for limitations on dragon growth.

Energy Requirements

Teams used a wide range of comparisons for size and energy requirements. Some examples include: birds to model flight energy, pterosaurs or Komodo dragon to model heat loss or basic energy needs, and wolves to model hunting efficiency. These approaches led to results that ranged from dragons approximately the size of an African elephant to dragons more than three times as large as a blue whale. Judges were particularly impressed when teams related their mass or length findings to real-world examples, which supported the plausibility of their assumptions and result.

Most teams approached the energy requirements by breaking them into the sum of basal metabolic needs and additional needs due to activity. A majority of teams use proportionality rules for converting mass into energy requirements, for example, Kleiber's law [Kleiber 1932]:

$$E \propto M^{\gamma},$$

where E is energy requirements and M is mass. For the vast majority of animals, we have $\gamma = \frac{3}{4}$. Here E is energy requirements and M is mass. Highly successful teams took care in describing the assumptions that went into both Kleiber's law and the rate constants used.

From this point, teams needed to describe the range of activities that might increase the energy needs of the dragon. In attempting this part of the problem, teams had to rely on creativity and judges were looking for good logical assumptions to support this creative process. Highly-successful teams clearly indicated multiple activities that might impact energy needs. Examples include fire-breathing, walking, flying, hunting, and cooking food. Activities listed directly in the problem statement—flying great distances and breathing fire—were particularly important to include in the energy equations. Successful approaches included relating fire-breathing to the energy required for operating a flame thrower or for cooking meat, relating dragon flying to energy used by existing flying animals, and using physical laws to determine the energy required to fly. In some cases, teams considered energy losses due to temperature differences between the dragon and its environment.

Ecological Impacts

Next, teams investigated the problems of ecological impacts. Again, there was a wide range of approaches; but the end result should have reported a reasonable estimate for the area required to support the dragons. Some teams approached this problem by considering food web and trophic-level models [Pimm 1979]; but because of time constraints, this analysis was often simplified to consider just an herbivore feeding on grassland followed by the dragon feeding on the herbivore. Other teams built predator-prey models, such as Lotka-Volterra [Lotka 1920], to demonstrate the impact that the dragon may have on a prey species. Some teams even considered the water needs of the dragon. In all cases, it was important to tie the ecological impacts back to the energy equations developed previously.

Community Support

Finally, teams were asked to consider community support. Approaches ranged widely. Examples include agent-based models of supporting dragons in battle situations, and communities raising prey to supplement the dragon diet. It was important that results for the amount of area required and the community support needed were at least plausible.

Part II: Climate Change

In addressing the effect that climate has on the dragons, most teams modified their results from the first part to consider how climate in different regions (arid, tropical, temperate, or arctic) might change the results. Approaches included changing the type of prey available, investigating how temperature and regional conditions affect growth rates, and full migration models allowing seasonal or temporal migration of the dragons through various ranges. Teams that were highly successful in this aspect modified their model to address climate for the full range of results: mass, energy, area needed, and community support required—for example, reporting how the entire growth and feeding requirements for the dragon would change if it were moved to, or through, different climates.

Part III: Letter to the Author

Most years, contestants are asked to write a second synopsis of their results. An important reason for this requirement is to show that you can interpret and tailor your results to the audience for whom you are writing. So the third part of this year's problem asked students to write a synopsis of the results as a note to George R.R. Martin. The teams that scored highest in this effort shared some important characteristics:

- The synopsis was not a simple restatement of the one-page summary, although it included some of the same introductory information.
- The synopsis included detailed information about the results but presented for a nonmathematical audience. It was important that the letter described the assumptions and modeling equations in simple language and related the results in a way that would be useful for an author creating a fictional series.

The most-successful teams found a good balance between mathematical precision and clear communication without too much mathematical jargon.

Part IV: Other Applications

This aspect of the problem was the most forgotten part. This year, models were based on a fictional creature but applied in our real world, thus the teams were asked to discuss how the model might be applied to a different situation. This process can show the ability to take the ideas from a basic model and extend them to new situations. Teams successful in this part of the problem were able to describe clearly how the assumptions and equations in their model might be extended to another application. Some examples include extending the dragon model to a model of invasive species in an ecosystem or using the model equations to model the energy needs of large farm animals across different climates.

Model Development and Presenting Results

One of the biggest challenges for teams this year was coming up with logical assumptions for a mythical creature without relying too heavily on magic. Not only is the dragon a fictional creature, but depending on your cultural viewpoint, it may or may not breath fire or it may or may not be gentle. Other assumptions included whether or not the dragon was cold-blooded, the maximum size of the dragon, and how humans and dragons interact. As long as teams were clear about their assumptions and addressed all of the points in the problem description, their model was judged favorably. It was particularly important this year to justify assumptions and address why the results were at least plausible in the real world. Another aspect that really set top papers apart from the others was a clear description of how the team parametrized the model. Whenever a constant is used in an equation, the team should report the value of the constant, justify why it is a reasonable number, and ideally discuss error involved in the value used.

Another issue that teams always face in this competition is the limited amount of time available to create a model, test the model, and write up results. Judges understand that time is limited, and this year no single paper had a perfect result for every part of the problem. However, judges ranked papers that addressed all of the topics in the prompt higher than papers that completely skipped or glossed over parts of the problem, even if the modeling methods of the former were simpler.

One piece of advice for contestants is to start very simple, addressing all aspects of the prompt ,and then with remaining time, revise the portions of the model that the team deems too simple or that produce unrealistic results. This means that teams should make sure to "sanity check" their results at every step. Always ask if the results are making sense. It helps to compare to other physical systems; for example, "Our results indicate that a dragon will grow to be about the size of a blue whale under ideal conditions."

In presenting the results, it is important to write only about parts of the model that you use. For example, if an assumption is stated, it should be used later to help you develop your model; or if models are developed in one part of the paper, they should be used later to generate results. Papers that built and revised their model throughout the paper, and clearly explained this progression, scored higher than papers that addressed each piece of the problem separately and never tied the pieces together. Papers that gave detailed explanations for parts of the model that were never used also received less favorable scores than papers that clearly used all equations developed.

Other Comments

We discuss a number of topics that were more subtly important during the judging process. These themes came up during final judging and should serve to add to the important themes from past judges commentaries [Black 2015, Shannon 2016, Wilhelm 2017, Bieri 2018].

General Modeling Comments

Every year, when selecting Finalists, the judges seem to bring up the idea of "value added." Modeling approaches that add something unique, or layer in multiple approaches, are adding value to the equations that they take directly from their references. Judges often ask the question, "What did the team add?" to their model. If a team assumed the Gompertz law for dragon growth, they could have added more detail to the parameters in that equation. For example, modelers might allow the growth rate to be a function of energy needs or hunting efficiency, or they might let the asymptotic growth term be a function of available area or climate.

Explaining the various aspects of the modeling process is important for each piece of the model. Every time an equation is introduced, teams should explain why that equation makes sense as a model for the system. Also, make sure to define the parameters and variables used when the equation is first shown. It is helpful to include a table of variables used; but remember that your reader will not memorize this list, so you should still define the variables within the text. Clarity in describing your mathematical process is just as important as your assumptions or the executive summary. Important parts of clarity include:

- citing your sources,
- defining variables,
- explaining in words all equations used,
- describing what the figures or tables mean in terms of the model,
- and discussing your results.

Teams were better this year about not including too many results in an appendix, but teams should remember that any result that is required for your paper should not be relegated to an appendix. Appropriate uses for the appendix include code written, extensive data tables, or parts of your model that were not used after revising. Judges should not need to look at the appendix.

The Summary

The executive summary is the contestants' chance to make a great first impression on the judges. The goal is to communicate to another mathematician the most important results. Give a very brief interpretation of the problem, give a summary of the most important modeling methods used, and present the most important results.

Sensitivity

One of the most important parts of mathematical modeling is assessing how well your model actually works. This is why including sensitivity analysis is so important. Teams are getting better every year at doing sensitivity analysis. Teams that were highly successful at this analyzed a wide range of parameters and interpreted their results.

Sensitivity analysis does not have to be done as a separate section at the very end. In some cases, it is more appropriate to do sensitivity analysis throughout the paper, as each part of the model is introduced. An analysis of how changing a parameter changes your final results is one of the best ways to show your model in use. Not only can you test your choice of parameters, but you can also show that your model applies to a wide range of values, environments, or applications.

Writing and Formatting

Formatting, writing, and communicating mathematics is an extremely important part of mathematical modeling. It is always important to have proper grammar and spelling along with clear overall structure. Here is a quick list of some things to avoid when writing a mathematical paper:

- Misspellings. (But remember that sometimes a spellchecker will autocorrect to the wrong word!)
- Incomplete sentences or poor structure. Often this can be avoided by reading your paper out loud before submitting it, to make sure it sounds correct.
- Equations without a clear written description of the meaning of the equation or of the variables, parameters, and constants in it.
- Inconsistent fonts throughout the paper.
- Putting too many of your words in bold font.
- Making your figures so small that the font of the labels is illegible.

Conclusion

This year, Problem A in the 2019 MCM asked students to develop a model to investigate what might happen if fictional dragons were living today. Most teams had successful models for at least one part of the problem (usually dragon growth), but the best approaches considered all parts of the problem, rationalized their choices, revised and expanded their model in each section, were very clear in explaining their modeling process, and presented their results including details like sensitivity testing and "sanity checking" their final values.

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