Summary

When we are baking food, the shape of a pan is an important factor. Rectangular pans can make the most use of the oven space, while circular pans can ensure the food is evenly heated. We develop some models to find the optimal shape for baking.

In our first model, we physically characterize the heating process based on the equation of heat conduction and obtain analytic solution. In our second model, we determine a method for evaluating the evenness of the heat distribution. Combining data and analysis, we confirm that circular pan has the best uniformity, which is 18% better than a square pan with the same area, and 14% better than a equilateral triangle with the same area. We simplify the 3D problem to a 2D one, which is verified to be reasonable by our numerical comparison.

Since the rounded rectangle is the combination of rectangle and circle, it can be applied to find the optimal shape of the pan. We build an objective function F and calculate the optimal solutions for different weight p. It is worth mentioning that we find a special shape, of which the objective function remains the same for different weight p. That means, the utility value of such a pan is the same for people with different preferences.

Contents

1 Introduction

1.1 Problem Background

The oven is a kitchen appliance used for roasting and heating. We use ovens to bake food such as bread, cakes and other desserts. And there are different pans with a variety of shapes, such as round pans, rectangular pans and some other pans for special use. Empirically, the heat will concentrate on the 4 corners in a rectangular pan, while the heat will distribute evenly over the entire outer edge for a round pan. Considering the harmful effects of overheated food, some people are inclined to choose round pans to prevent food from being overcooked. However, the round pans can not make full use of the oven space. Thus it is meaningful to discuss how to design the pan shape to balance the different requirements.

1.2 Analysis of the Problem

The goal is to

1. maximize the even distribution of heat(H)
2. maximize the number of pans that can fit in the oven(N)
3. optimize the combination of the above two requirements with a weight p.

For the first requirement, we use the standard deviation σ of temperature distribution at certain time to describe the thermal uniformity for a heating pan. And the temperature distribution can be solved by applying the equation of heat conduction under certain boundary conditions, which will be discussed in the first model.

For the second requirement, we only use *the shape and the size* of the pan to determine the number of pans that can fit in a given oven. We assume that the size of the oven is large enough that we do not need to consider how to place these pans in a given oven.

We take both the requirements into considerations and construct an optimum model. We define an objective function F, which is the function of σ, N and weight p. And in our later models, we will change the σ and N to other related variables to obtain a more valuable objective function.

1.3 Assumption

To simplify the problem, we take the following assumptions:

- Ideal Oven: We do not consider the mechanism of the heater and assume that the temperature in the oven remains the same during the heating process.
- Simplified Pans: We regard the food and pan as an entirety and consider them as a flat cylinder, whose cross-section can be a circle, a square etc. All the pans in an oven have the same properties. And we prove in later models that the influence of the metal layer(the pan) has little influence on the analysis of the problem.
- Independent Pans: The heating process of each pan is not influenced by each other. The heat distribution of each pan only depends on the temperature of the environment inside the oven.
- Unchanged Food: The food is uniform. The properties of the food do not vary during the heating process. In this way, the equation of heat conduction remains to be simple.

1.4 Definitions and Notation

Let $L,\ W$ be the length and width of the oven respectively. We do not consider the influence of the ratio $\Lambda \equiv L/W$ in our models. Let

$l,\ w$ = length and width of a pan with rectangular shape, and define $\lambda \equiv l/w$,

A = the area of each pan, which is a constant in our models,

N = the maximum number of pans that can fit in the oven,

H = the distribution of heat for a pan under certain condition,

σ = the Standard Deviation with certain number of samples for a distribution, which can be used to measure the uniformity of the heat, and

p = weight for N and H.

2 Analytic Model

In our simple model, we assume that each pan is filled with food as an uniform flat cylinder(See Figure 1). Thus, we first determine the heat equations of a cylinder in the oven. Since we have assumed the temperature remains the same at each point in the oven, we can use Dirac boundary condition. We then apply separation of variables to calculate three-dimensional heat conduction equation and obtain the analytic solution.

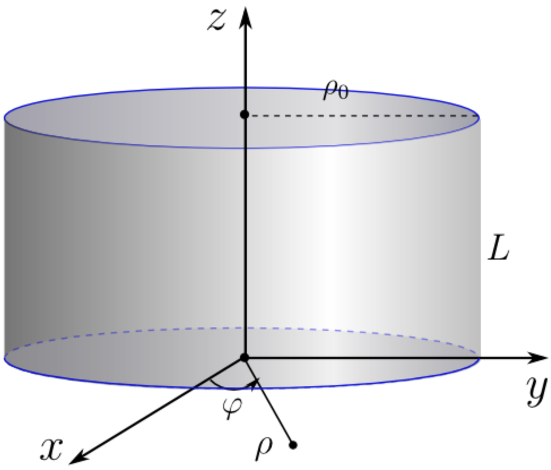

Figure 1: In this simplified model, we regard each pan as a flat cylinder. ρ_0 is the radius of the cylinder and L is the height.

The heat conduction equation:

$$u_t - a^2 \Delta_3 u = 0 \tag{1}$$

Boundary/Initial conditions:

$$\begin{cases} u(\rho = \rho_0) = u_0, & u(\rho = 0) = \text{finity} \\ u(z = 0) = u_0, & u(z = L) = u_0 \\ u(t = 0) = u_1 \end{cases}$$

where,
a^2 is the heat diffusivity of the object, which is determined by $a = \sqrt{\frac{k}{c\rho_d}}$,
ρ_d is the density of the food, which is assumed to be a constant,
u is the temperature of each point,
(ρ, φ, z) is the cylindrical coordinate,
t is the time,
ρ_0 is the radius of the cylinder,
L is the height of the cylinder.

By using method of separation of variables, let

$$u(\vec{r},t) = T(t)v(\vec{r})$$
$$v(\rho,\varphi,z) = R(\rho)\Phi(\varphi)Z(z)$$

We obtain

$$\begin{cases} T' + k^2a^2T = 0 \\ \Delta v + k^2 v = 0 \\ \Phi'' + \lambda\Phi = 0 \\ Z'' + \nu^2 Z = 0 \\ \dfrac{d^2R}{d\rho^2} + \dfrac{1}{\rho}\dfrac{dR}{d\rho} + (k^2 - \nu^2 - \dfrac{\lambda}{\rho^2})R = 0 \end{cases}$$

where, $\lambda = m^2, k^2 = \mu + \nu^2$.

Considering the boundary condition and symmetry of the object, we determine the eigenvalue of the equation and get

$$u = u_0 + \sum_{n=1}^{+\infty}\sum_{p=1}^{+\infty} A_{np} \exp\left\{-\left[\left(\frac{x_n^{(0)}}{\rho_0}\right)^2 + \frac{p^2\pi^2}{L^2}\right]a^2t\right\} J_0\left(\frac{x_n^{(0)}}{\rho_0}\right)\sin\left(\frac{p^2\pi^2}{L^2}\right)$$

where, $x_n^{(0)}$ is the nth positive zero point of zero-order Bessel function $J_0(x)$.

Determine the coefficient A_{np} by initial condition:

$$\sum_{n=1}^{+\infty}\sum_{p=1}^{+\infty} A_{np} \sin\left(\frac{p^2\pi^2}{L^2}\right) = u_1 - u_0$$

For more complex boundary conditions, it is difficult to obtain analytic solution. Thus we need to use the numerical solution to solve such kind of problem.

3 2D Simplified Model

3.1 Description of the Model

Additional Assumption:

As we know, the thermal diffusivity of metal is large enough to make the metal pan reach the oven temperature very quickly. Thus we can ignore the metal part of the flat cylinder. In this model, we simplify the 3D problem as a 2D one. By this way, we can compute the temperature distribution of a cross section more easily. Here we choose different shapes of cross section(See Figure 2):

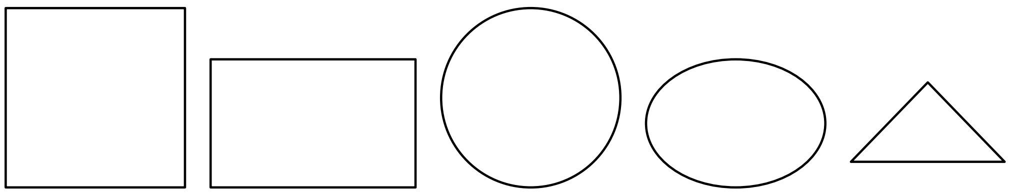

Figure 2: Different shapes of the cross section.

3.2 Analysis of the Model

The Form of Equation

Based on the two-dimensional assumption, we have the following equations:

$$u_t - a^2 \Delta_2 u = 0 \tag{2}$$

And the boundary/initial conditions:

$$\begin{cases} u(\text{boundaries}) = u_0, \quad u(\vec{r}=0) = \text{finity} \\ u(t=0) = u_1 \end{cases}$$

And we have the following parameters:

parameter	meaning	value(unit)
u_0	temperature of oven	$473(\text{ K})$
u_1	initial temperature of pan and food	$293(\text{ K})$
a^2	heat diffusivity of food [Heldman, 2007]	$1.48 \times 10^{-7}(\text{ m}^2/\text{s})$
$\tilde{a}^2$	heat diffusivity of iron	$1.172 \times 10^{-5}(\text{ m}^2/\text{s})$

The Solution of PDE

To solve the partial differential equation(PDE),we apply the *Finite Element Analysis Method* to divide the figure into fraction and then obtain the results of the equation.

We employ the *PDEtool* in *MATLAB* to solve the partial differential equation of heat diffusion. The flowchart is as following:

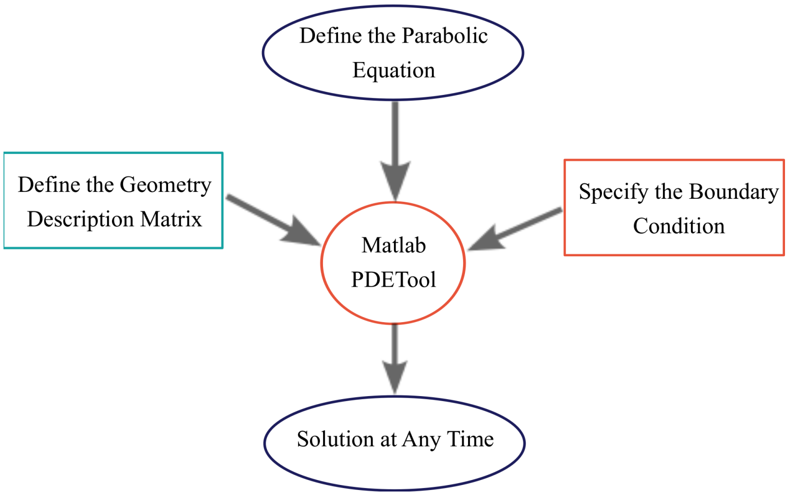

Figure 3: The flowchart of solving PDE by MATLAB PDEtool.

3.3 Calculation Parameters

We calculate the heat distribution of pans in different shapes such as circle, ellipse, square, rectangle and equilateral triangle. We let the area of each pan be the same and calculate some other parameters of shapes. The thermal diffusivity is $1.48 \times 10^{-7}(m^2/s)$.

The area of different shapes is determined by the following table.

shape	area
circle	πr^2
ellipse	πab
square	l_2^2
rectangle	lw
equilateral triangle	$\frac{\sqrt{3}}{4} l_3^2$

Some other parameters:

parameter	meaning	value/m
r	the radius of circle	0.2
a	the semi-major axis of ellipse	0.4
b	the semi-minor axis of ellipse	0.1
l_2	the side length of square	0.3545
l	the length of rectangle pan	0.2
w	the width of rectangle pan	0.6285
l_3	the side length of triangle	0.5387

3.4 The Outcome of the Problem

3.4.1 The Visualization of the Heat Distribution

By applying the *PDEtool*, we obtain the heat distribution of different kinds of pans which have the same area A.

The heat distribution of the circle/rectangle pan:

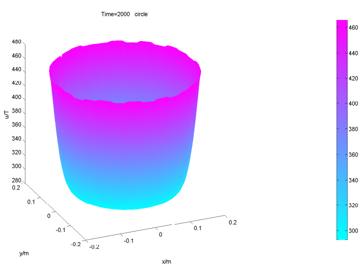

Figure 4: The temperature distribution over a circle pan.

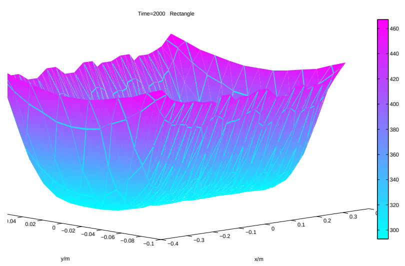

Figure 5: The temperature distribution over a rectangle pan.

The output indicates that the heat distributes evenly cross the edge of the circle pan. In contrast, the heat distribution of the rectangle pan is not uniform at all. It is obvious that the temperature at four corners are extremely high, which is consistent with our experience.

Furthermore, we also check the temperature distribution over some other shapes:

These figures indicate that the uniformity of thermal distribution varies with different shapes of pan.

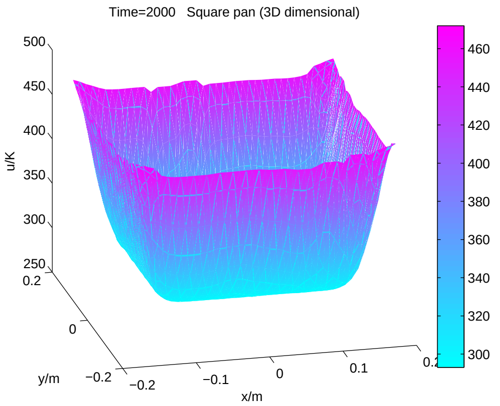

Figure 6: The temperature distribution over a square pan.

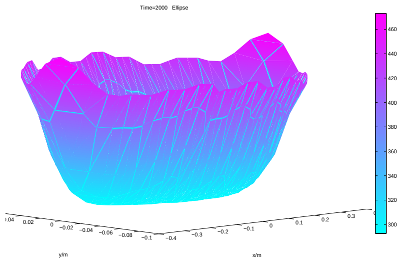

Figure 7: The temperature distribution over ellipse pan.

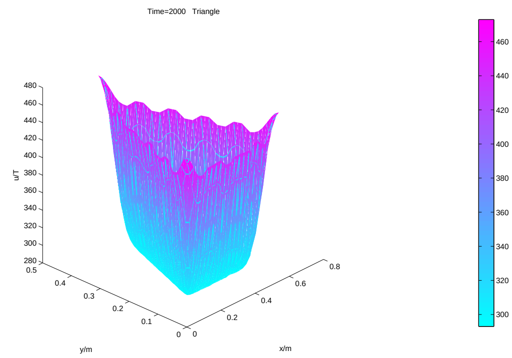

Figure 8: The temperature distribution over triangle pan.

3.4.2 Describe the Uniformity of Distribution over Pans

After solving the PDE, we choose a proper time $t = 2000$ s to study the temperature distribution. If we select enough points evenly on the temperature distribution graph, we can obtain a number of samples which can represent the temperature distribution. Then the uniformity of the distribution can be measured by using the sample's standard deviation σ.

The standard variation σ is defined as:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=0}^{N}(x_i - \mu)^2},$$

where

$$\mu = \frac{\sum_{i=0}^{N} x_i}{N}.$$

The following table shows the results of different pans:

shape	Standard Deviation(K)
circle	30.8480
ellipse	67.8390
square	43.4304
rectangle	61.9922
equilateral triangle	40.4114

As we have expected before, when we use the triangle pan, food placed on the corners of the triangle is easily overcooked because the temperature there is extremely high compared to the other place. On the contrary, when we use the circular pan, the heat is quite even cross the whole pan. As a whole, when the pan are in the shapes like square and circle, the standard deviation is comparative small.

4 Optimizing with Rounded Rectangle Model

Based on the former models, we find that the shape of pans influences the heat distribution. For circular pan, heat distributes evenly over the outer edge because of the smooth edges. While for the triangular or the rectangular pans, heat is concentrated on the corners of the pans. Comparing the results of different shapes, we conclude that shapes with less angles and more smooth edges obtain more even distributions. On the other hand, pans with smooth edge will result in low space utilization. To obtain even heat distribution and high space utilization at the same time, we carry out our model by applying a new shape: rounded rectangle(See Figure 9).

4.1 Parameters and Definitions

l is the length of the rounded rectangle,

w is the width of the rounded rectangle,

r is the radius of the rounded corner for certain rounded rectangle,

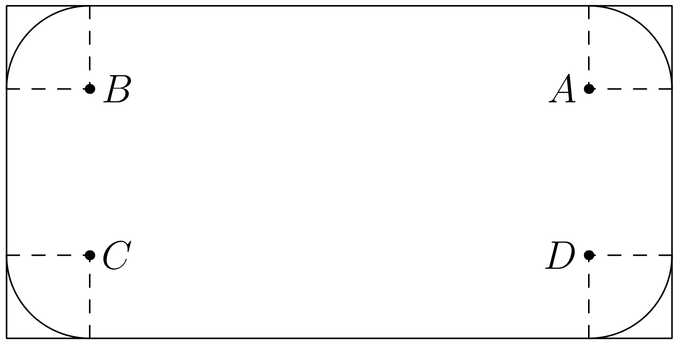

Figure 9: Rectangle with rounded corners.

A is the area of rounded rectangle(given value),

$S \equiv l \cdot w$ is the area of enclosing rectangle for certain rounded rectangle,

S_{oven} is the area of oven,

N is the number of pans that can fit in the oven,

$\alpha \equiv A/S$ is the area utilization percentage of a rounded rectangle, $\tilde{\alpha} \equiv (\alpha - \alpha_{\text{min}})/(\alpha_{\text{max}} - \alpha_{\text{min}})$ is the normalized area utilization percentage,

σ is standard deviation, $\tilde{\sigma} \equiv (\sigma_{\text{max}} - \sigma)/(\sigma_{\text{max}} - \sigma_{\text{min}})$ is the normalized σ,

p is weight to balance the N and σ,

$\lambda \equiv l/w$ is the length to width ratio for the rounded rectangle,

F is the objective function, $\frac{dF}{dr}$ is the derivative function of F.

4.2 Additional Assumptions

- Each pan with different r has the same area A. A is $0.04m^2$.
- Each pan occupies the area of its rounded rectangle. The redundant area of $(S - A)$ cannot be occupied by other pans.
- We take $\lambda = 1$ to analyze the problem at first.

4.3 Heat Distribution

For $\lambda = 1$, σ and α vary with different r. Thus we can calculate the function $\sigma(r)$ and $\alpha(r)$ for $r = 0$(turn into a square) to $r = r_{\text{max}}$(turn into a circle). We can determine the $r_{\max}$ with the following equation:

$$A = 4\pi r_{\max}^2 \Rightarrow r_{\max} = 0.1128 \text{ m}$$

The following are some of the results for function $\sigma(r)$:

r/m	σ/K	r/m	σ/K
0.00	64.9308	0.06	59.9900
0.01	65.1093	0.07	59.6811
0.02	63.3567	0.08	59.2291
0.03	63.0187	0.09	58.6396
0.04	61.2685	0.10	57.5524
0.05	60.6949	0.1128	56.9760

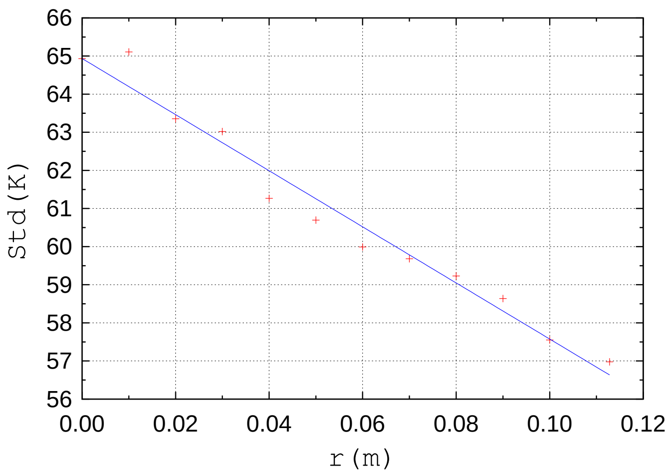

Figure 10: Linear fitting for function $\sigma(r)$.

We plot $(r, \sigma(r))$ and find that the relationship between r and σ can be approximately presented by linear function. We construct the function by linear fitting and get

$$\sigma = -73.5983r + 64.9357$$

4.4 Space Utilization

We define the number of pans N by:

$$N = \frac{S_{\text{oven}}}{S}$$

And define the area utilization percentage α by:

$$\alpha = \frac{A}{S}$$

Thus,

$$N = \frac{S_{\text{oven}}}{A}\alpha$$

Since S_{oven} and A are constant, thus $N \propto \alpha$. So we can use α to measure the degree of space utilization(Note: α is a dimensionless quantity).

And the $\alpha(r)$ can also be determined by

$$S = A + 4r^2 - \pi^2 r^2$$

thus,

$$\alpha = \frac{A}{A + 4r^2 - \pi^2 r^2} \tag{3}$$

Plot the function:

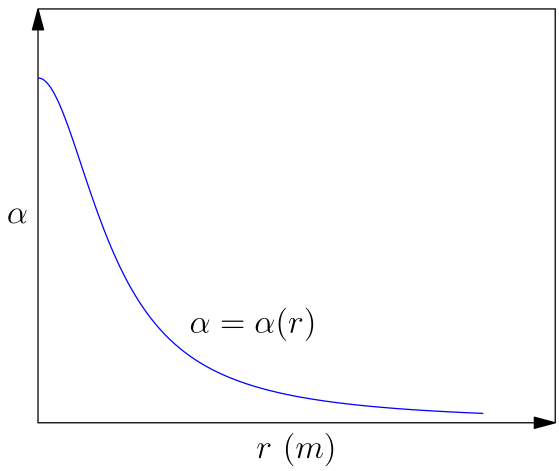

Figure 11: The function $\alpha(r)$.

From the formula, we can easily derive that α increases as r decreases. The square(when $r = 0$) has the smallest α while the circle(when $r = r_{\text{max}}$) has the largest α.

4.5 Combination

We have already chosen two quantities $\sigma(r)$ and $\alpha(r)$ to measure the uniformity of heat distribution and space efficiency. Now, we want to take both factors into consideration and optimize the combination of two conditions by change the weight p. First, we have to normalize the σ and α by the following rules.

$$\alpha(\text{square}) \longrightarrow 1, \alpha(\text{circle}) \longrightarrow 0$$
$$\sigma(\text{square}) \longrightarrow 0, \sigma(\text{circle}) \longrightarrow 1$$

So we define:

$$\tilde{\alpha} = \frac{\alpha(r) - \alpha_{\text{min}}}{\alpha_{\text{max}} - \alpha_{\text{min}}}$$
$$\tilde{\sigma} = \frac{\sigma_{\text{max}} - \sigma(r)}{\sigma_{\text{max}} - \sigma_{\text{min}}}$$

After defining the $\tilde{\alpha}$ and $\tilde{\sigma}$, we can change weight p to compare the results. Now we can define the objective function $F(r, p)$:

$$F(r, p) = p \times \tilde{\alpha} + (1 - p) \times \tilde{\sigma} \tag{4}$$

Or,

$$F(r, p) = p \times \frac{\alpha(r) - \alpha_{\text{min}}}{\alpha_{\text{max}} - \alpha_{\text{min}}} + (1 - p) \times \frac{\sigma_{\text{max}} - \sigma(r)}{\sigma_{\text{max}} - \sigma_{\text{min}}}$$

In this formula, the condition $p > 0.5$ means maximizing N is more important while the condition $p < 0.5$ means the heat distribution evenness is more important.

We write a code to find the optimal r according to p. Here is the flow chart.

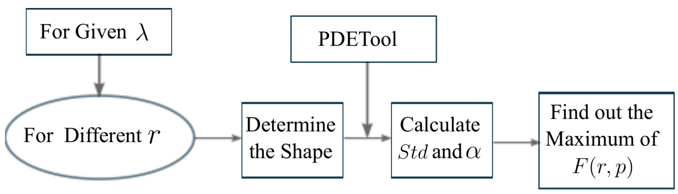

Figure 12: The flow chart of finding optimal value of r for different p.

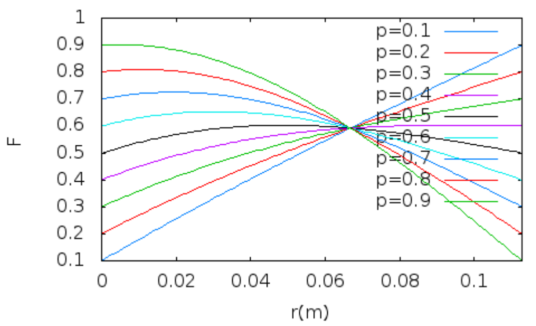

Figure 13: Function $F(r,p)$ with different p when $\lambda = 1$.

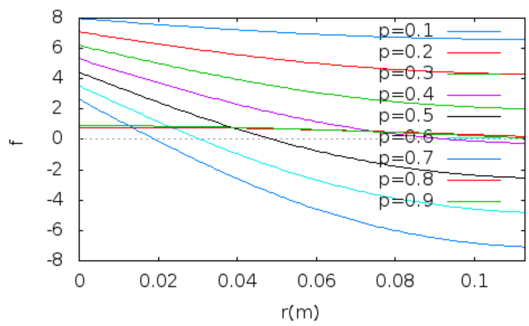

Figure 14: Function $\frac{dF}{dr}$ with different p when $\lambda = 1$.

4.6 Results and Analysis

We choose different weight p and obtain the corresponding function $F(r,p)$.

When $0 < p < 0.4$, the function $F(r,p)$ shows the trend of monotonic increasing, which implies that round pan is optimum. When $0.4 < p < 1$, the function $F(r,p)$ reaches a maximum value and achieves the optimum with a certain radius. From the table below, we can get the optimum radius under different p. When $p = 1$, the function $F(r,p)$ shows the trend of monotonic decreasing, which implies that square pan is optimum.

For given p, we get the optimum radius by solving the equation $\frac{dF}{dr} = 0$. Here, we list several results:

p	0.0	0.1	0.2	0.3	0.4	0.5
r/m	0.1128	0.1128	0.1128	0.1128	0.0942	0.0490
$F(r,p)$	1.0000	0.9000	0.8000	0.7000	0.6027	0.6029
p	0.6	0.7	0.8	0.9	1.0	
r/m	0.0308	0.0193	0.0111	0.0049	0.0000	
$F(r,p)$	0.6534	0.7254	0.8098	0.9022	1.0000	

More interestingly, we find that there is a fixed point in the Figure 13. We calculate the coordinate of this fixed point $(0.0668, 0.592)$ by setting the coefficient of p as zero. No matter how p changes, $F(r,p)$ remains the same at this radius. It indicates that the results of this certain shape will not be influenced by different demand on heat distribution and space efficiency. From this point of view, it is wise for companies to produce the rounded square pans with about $0.06 \backsim 0.07$ m radius because such kind of shape can meet most customers' demands.

5 Practical Consideration

5.1 Relaxing the Assumption

In the previous models, we make several assumptions to refine the model, which simplify the calculation a lot. However, those assumptions may also make the model less practical. We now consider the pan and food separatively as the following figure shows:

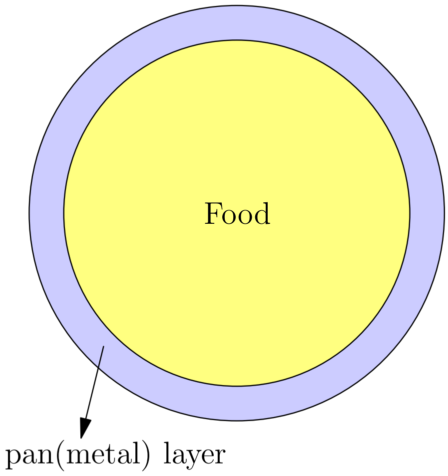

Figure 15: The Figure of Relaxing Assumption Model

5.2 The Solution

The only change in the equation of heat conduction is the difference between food and metal pan in thermal diffusivity. For pan, we assume it is made of steel, whose thermal diffusivity is $1.172 \times 10^{-5}(\mathrm{m}^2/\mathrm{s})$ [Wikipedia]. For food, the thermal diffusivity is $1.48 \times 10^{-7}(\mathrm{m}^2/\mathrm{s})$.

Then we calculate the heat distribution over the pan and food by *PDEtool* as we have done before.

5.3 Differences in the Revised Model

The temperature distribution of some shapes at time $t = 2000$ s:

Compared with the previous results, the above figures just add a metal ring. If we choose a smaller time $t = 5$ s, the results are similar with before.

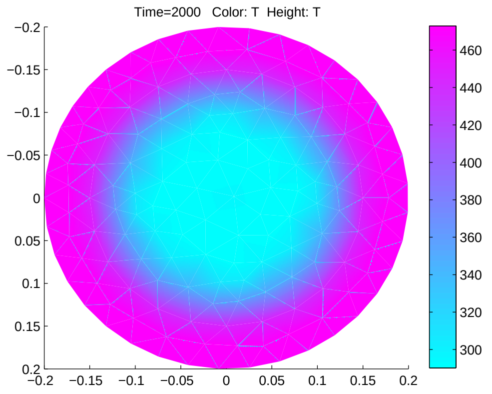

Figure 16: The top view of temperature distribution of circle pan at $t = 2000$ s.

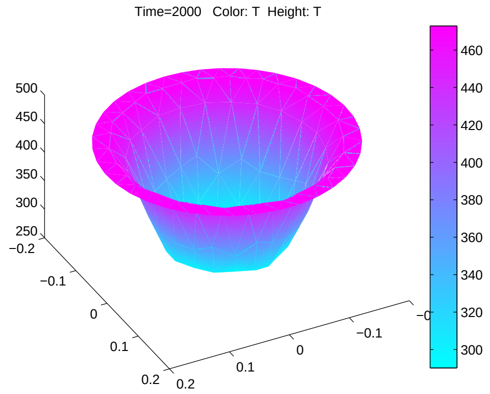

Figure 17: The 3D plot of temperature Distribution Of circle pan at $t = 2000$ s.

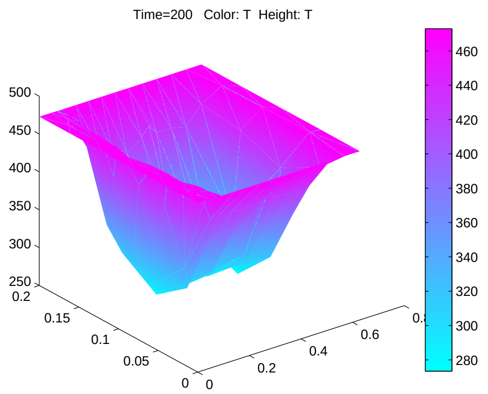

Figure 18: The 3D plot of temperature distribution of rectangle pan at $t = 200$ s.

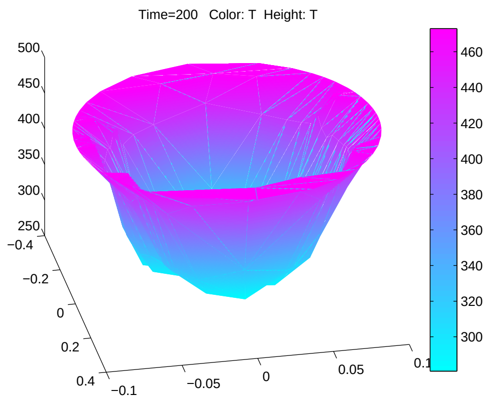

Figure 19: The 3D figure of temperature distribution of ellipse pan at $t = 200$ s.

Besides, if we only study the circle pan made of steel(without considering about the food), we will find that the temperature reaches 400 K within 500 s.

Since the temperature of the metal layer rises to the oven temperature very quickly, we do not need to consider this layer. Thus our assumption of regarding the pan and the food as an entirety is practical and meaningful.

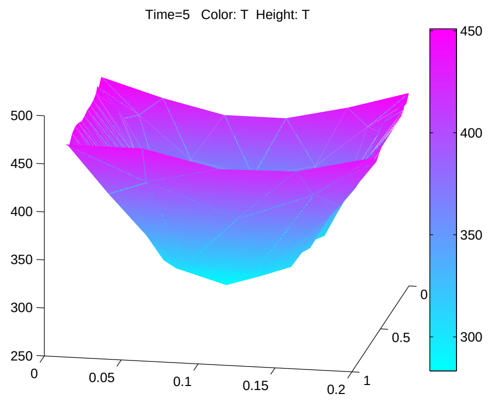

Figure 20: The 3D Figure of Temperature Distribution of rectangle Pan at $t = 5$ s.

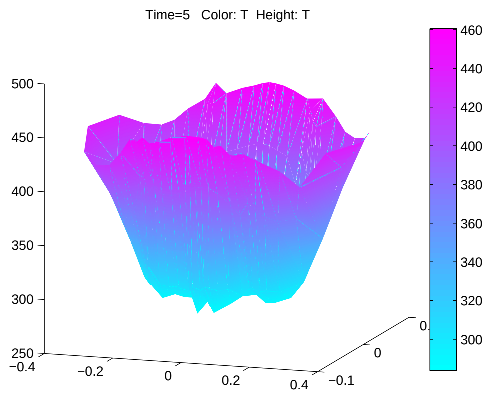

Figure 21: The 3D Figure of Temperature Distribution of ellipse Pan at $t = 5$ s.

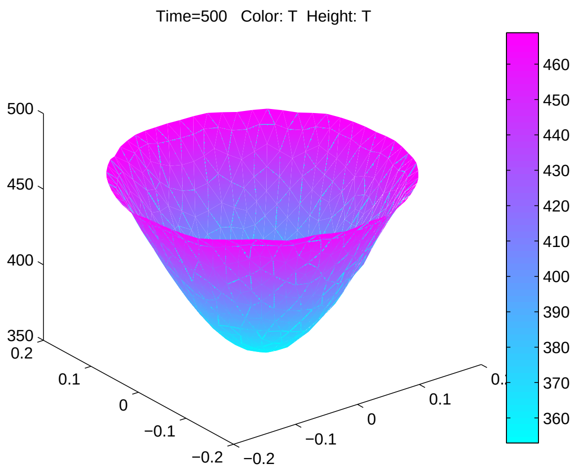

Figure 22: The temperature distribution of iron pan at $t = 500$ s.

5.4 3D Numerical Calculation

5.4.1 Relaxing the Assumption

In previous models, we simplify the problem as a 2D problem rather than a 3D one. Now we take the height into consideration to calculate the 3D shape. We use the software *ANSYS* to calculate the thermal distribution over the cube pan.

5.4.2 The Result of 3D Numerical Calculation

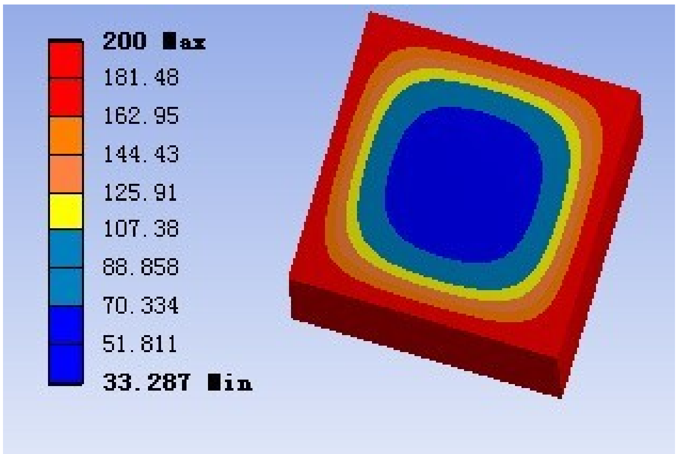

Figure 23: The temperature distribution over a cubic pan.

We can see from the figure that the temperature over the 4 corners is extremely high, which is consistent with previous results. As a consequence, the assumption that simplifies the 3D pans to 2D one is reasonable.

6 Model Validation

6.1 2D Model Solution with Different Parameters for Rectangle and Ellipse

Our 2D simplified Model solves the PDE. We validate the stability of the solution for different parameters of rectangle and ellipse.

For the rectangles:

We assume that each rectangle has the same area $0.04\ \mathrm{m}^2$. By changing the value of $\lambda = l/w$, we get:

λ	1	2	3	4
Standard Deviation(k)	58.1121	66.4303	77.7454	90.2780

As we can see from the above table, σ increases with the increase of λ. As σ increases, the feature of the corner becomes more obvious and symmetrical characteristic is destroyed. Heat is more likely to concentrate on the four corners. As a result, square is optimum since its σ is the smallest.

For the rectangles:

We also study the influence of the semi-major axis to semi-minor axis ratio a/b of the ellipse on the thermal distribution.

We assume that each ellipse has the same area $0.04\pi\ \mathrm{m}^2$. By changing the value of a/b, we get:

a/b	4	3	2	1
Standard Deviation(K)	67.8390	50.3684	41.2132	30.8480

From the table, we also can conclude that σ decreases with the decrease of a/b. As a/b increases, the circle is being extended and the curvature becomes larger. The ellipse shows the feature of the corner. Heat is more likely to concentrate on these 'corners'. As a result, circle is optimum since its σ is the smallest.

6.2 Rounded Rectangular Model Solution with Different λ

In the section 'Optimizing with Rounded Rectangle', we assume that the enclosing rectangle of the rounded rectangle is a square that $\lambda = 1$. Now we relax this assumption and let $\lambda = 1.2$. The formula of calculating α remains the same: Formula 3. We also calculate σ of shapes with different r and use linear fitting to construct the function. After normalizing both parameters, we use the equation (4) to study the function $F(r, p)$.

The results:

$r/\ \mathrm{m}$	0.0000	0.0102	0.0203	0.0304	0.0406	0.0507	0.0608	0.0710
$\sigma/\ \mathrm{k}$	68.2287	66.0022	62.3205	59.7442	58.9874	57.1369	52.6655	51.8697

And the function $F(r, p)$ and $\frac{dF}{dr}$ for certain values of p:

For given p, we get the optimum radius by solving the equation $\frac{dF}{dr} = 0$. Here, we list several results:

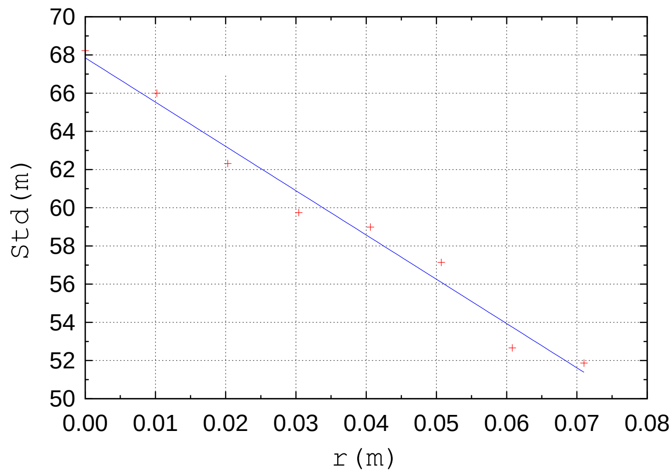

Figure 24: Linear fitting for function $\sigma(r)$.

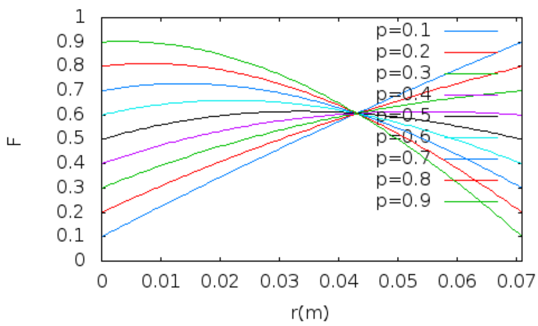

Figure 25: Function F when λ is 1.2.

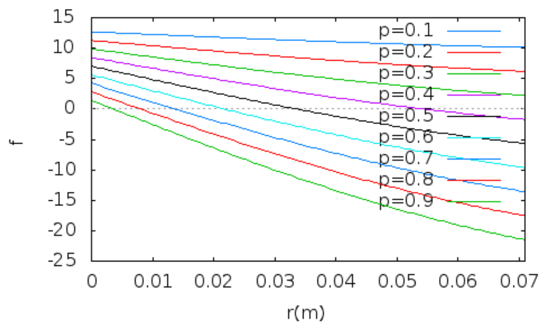

Figure 26: Function $\frac{dF}{dr}$ when λ is 1.2.

p	0.0	0.1	0.2	0.3	0.4	0.5
$r/$ m	0.0710	0.0710	0.0710	0.0710	0.0543	0.0336
$F(r,p)$	1.0000	0.9000	0.8000	0.7000	0.6150	0.6155
p	0.6	0.7	0.8	0.9	1.0	
$r/$ m	0.0218	0.0138	0.0080	0.0036	0.0000	
$F(r,p)$	0.6608	0.7291	0.8113	0.9025	1.0000	

The trend under $\lambda = 1.2$ is approximately similar to the trend under $\lambda = 1$. $F(r,p)$ reaches peak at a certain radius when $0.4 < p < 1$. $F(r,p)$ increases monotonically and the shape with the biggest r is optimum when $p < 0.4$. $F(r,p)$ decreases monotonically and rectangular shape is the best when $p = 1$. From the picture above, we also find that there is a fixed point. The coordinate of the fixed point is $(0.0431, 0.6072)$.

7 Conclusions

From the results of our models, we find that the thermal uniformity of a circular pan is 18% better than a same-area square pan, and 14% better than a same-area equilateral triangle, in the term of standard deviation. Overall, a pan with sharper corners gets worse thermal uniformity. On the other hand, compared with the square pan, a circular pan will lose 21% of the effective heated area.

7.1 Strengths

- Apply a appropriate method to combine the number of pans N and even distribution H for the pan, and determine the objective function $F(r, p)$.
- Use rounded rectangles to discuss the optimization problem. The problem is greatly simplified by this way. And more importantly, our approach is still meaningful and flexible, since the rounded rectangle is a transition from rectangle to round.
- Simplify the 3D problem to a 2D problem, and verify it is reasonable.

7.2 Weaknesses

- We do not consider the influence of different oven size $\Lambda = L/W$, which is an important parameter to determine the shape of the pans.
- It has been verified by our computation that the circle shaped pan obtains the evenest heat distribution. But we do not prove it with analytic algorithm.
- We discuss the optimization using our 2D model, which may lose some useful and practical features for the real 3D objects.
- We assume the food unchanged during the process. The food must changed its property when heating, so we would better study how the food changes with time and study the heating process more precisely.

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Appendices

Appendix A Advertising Sheet for the New Brownie Gourmet Magazine

Many people like to use oven to bake their food. We may use different shaped pans, such as circle, rectangle, triangle, ellipse or other ornamental shapes. Maybe you have not realized the importance of the shape. In this article, we will tell you how to choose the optimum shape of your pan based on your demand.

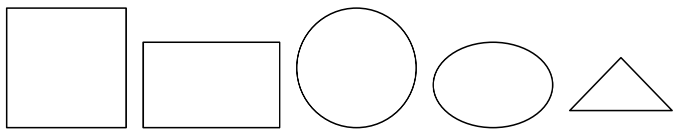

Figure 27: Different shapes.

Demand for Even Heat Distribution

Nowadays, many people are worried about harmfulness the overcooked food brings to us. A number of scientific researches have already pointed out that some specific chemical reactions would happen in the baked food under certain temperature. For example, amino acid will react with carbohydrate which is chemical composition of most food and produce acrylate at 250 Fahrenheit. Such kind of product may probably contribute to cancer. As a result, it is important to regulate the temperature of food.

In our research, we find that shape of the pans has a great influence on the temperature of food. If we set the oven's temperature about 400 degree Celsius, the corner of the pan reaches nearly 400 Fahrenheit, but the temperature of other edges remains merely 340 Fahrenheit. In order to ensure the food sufficiently cooked, people will need more time(30 mins) to heat the food. As a result, more heat will concentrate on the four corners of the edge and there is great possibility of more carcinogenic substance being produced. If we choose round pans, the situation becomes ameliorated. Heat distributes evenly over the edge of the pan. The difference between the highest temperature and the lowest temperature over the edge of the round pan is about 20 Fahrenheit. Round pan is mostly recommended for even heating demand. You had better choose the shape with arc edge.

Demand for High Space Efficiency

However, the weakness of round pan is the waste of space. If you want to cook as much food as possible at one time, round pan is not a rational choice. Based on our calculation, rectangular shape can make good use of space. Several pans can be put side by side without any interspace in the oven. We find that heat distributes more evenly as the width to length ratio becomes larger. As a result, we recommend square pans for high space efficiency demand.

Optimum Combination of both factors

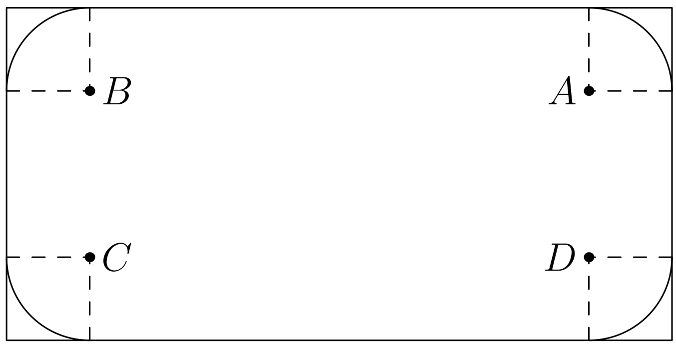

Figure 28: Rounded Rectangle.

If someone wants to take both factors into considerations, what is the optimum shape for baking? We consider rounded rectangle shape in which four corners of the rectangle are replaces by circular arc. Due to the circular arc, the difference of the highest temperature and the lowest temperature becomes smaller. Heat concentrated on the corners is gradually distributed over the whole edge. You can choose the radius of the circular arc to meet your own need. If you want to cook more food, you can choose smaller radius. If you regard healthy food as a more important factor, you can choose larger radius.

Special Design for Company

Now, we would like to recommend a special design for pan corporation.

We use p to represent the customers' demands. If the customer pay more attention to space utilization, p is larger. If the customer thinks healthy food is more important, p is smaller. According to their favorites, different customers will choose different p. However, from the standpoint of companies, they must meet most people's demands. Based on rounded rectangular shape, we find a certain radius to solve this problem.

We define F as our objective function. We achieve the optimum radius when F reaches its peak. In the following illustration, we give the functional image under different p. You can easily find that there is a fixed point. This fixed point does not change with p, which means that the quantity of F remains the same according to different demands. It is wise for companies to design such kind of shapes with this certain radius. For the pan which area is $0.04m^2$, radius is 0.044m. The company can also calculate radius for different rectangle.

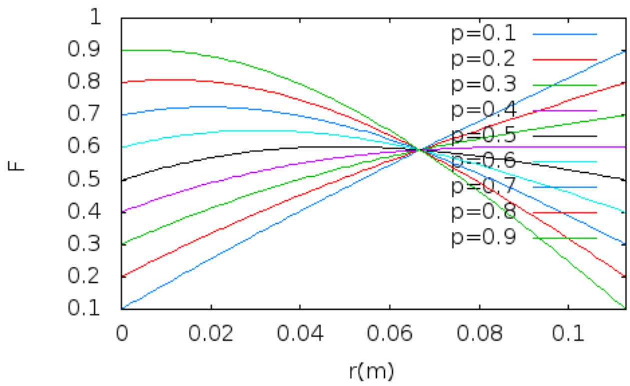

Figure 29: Objective function $F(r, p)$.