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Guest Editorial

An Open Letter

Project INTERMATH Staff

The letter below is being sent to engineering deans and chairs of mathematics departments, together with a brochure; however, the information is intended for all math-science-engineering faculty and therefore is published here in adapted form.

Dear Dean or Chair:

The National Science Foundation is engaged in a systemic initiative entitled Mathematics Across the Curriculum (MATC). Among the goals of the initiative is the desire to change the culture in which undergraduate mathematics, science, and engineering curricula are designed and presented. From the perspective of the mathematics departments, the desire is for the science and engineering departments to become partners rather than clients in determining what happens on a daily basis in the mathematics classroom.

Project INTERMATH is one of the projects funded under the MATC initiative. During the past 6 years, various initiatives have been created, tested and adapted for use across the country. Three of the schools in the 15-school consortium, the United States Military Academy, Carroll College (Montana), and Harvey Mudd College, have developed comprehensive 2-year 15-credit mathematics experiences in response to the ABET's Engineering Criteria 2000. Each of these curricula integrates discrete mathematics and includes dynamical systems, linear algebra, probability and statistics, calculus through vector integral calculus, and differential equations while achieving the outcomes suggested in ABET 2000. Recently one of the schools, Carroll College, underwent an ABET visit. The visitors were quite impressed with the interaction among math, science, and engineering in the curriculum at Carroll. Subsequently, ABET awarded the ABET Innovation Award to Carroll:

> For the adoption of student goals for mathematics majors that embrace the principles of ABET's Engineering Criteria 2000 and for development of an innovative, cross-disciplinary curriculum tailored to the needs of mathematics and other disciplines.

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Presently, a consortium is being formed to apply for NSF funds to adapt and implement the curricula mentioned above. Course guides are available for each of the three curricula to help schools adapt the model programs to their needs. The consortium envisions having advisors available from the three projects to assist as needed with the adaptation. The enclosed brochure describes the curricula in more detail. While the current curricula are taught from existing textual materials, a follow-on project is preparing a unified set of materials from which the curriculum can be taught. The project also seeks field testers for the unified materials. For additional information on the Consortium being formed, please contact Frank Giordano at `FrankCOMAP@aol.com`.

Additionally, Macalester College has developed a scientific programming course for freshman year and a collection of scientific programming projects (with databases available). The course introduces the student to the solution of problems too large to be solved by conventional means. Finally, all INTERMATH schools have worked with their science and engineering departments to design an impressive collection of interdisciplinary projects. Many of these projects can be introduced simultaneously in freshman mathematics and science courses and revisited later in engineering science and engineering courses. All projects are available on Project INTERMATH's website and many are available in hard copy.

If you would like more information on Project INTERMATH, please visit our website at `www.ProjectIntermath.com` or `www.ProjectIntermath.org`. For copies of the brochure concerning a specific INTERMATH Project, or a copy of the *Tools for Teaching* volume featuring Project INTERMATH and including representative interdisciplinary projects [Campbell 2001], please contact COMAP at (800) 772–6627 or visit the website `www.comap.com`.

Sincerely,

Project INTERMATH Staff

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Campbell, Paul J. (ed.). 2001. *Tools for Teaching 2000: ILAP Modules, Interdisciplinary Lively Applications Projects*. Lexington, MA: COMAP.

Editor's Note

Quick responses from the Contest Director, the authors of the Outstanding papers, and the commentators allow publication in this issue of the results of the 2002 Interdisciplinary Contest in Modeling.

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INTERMATH Forum

A Model for Academic Change

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Introduction

Project INTERMATH, through the utilization of interdisciplinary lively application projects (ILAPs), promotes the development of integrated and interdisciplinary-based mathematics courses, programs, and curricula. To do this, faculty and departments must implement academic change. This article is based on our experiences (and hopefully insights) in implementing these kinds of academic changes and on the current status of Project INTERMATH. We provide a model to describe the people and organizations involved in academic change processes. For some of the details of the dynamics, methods, and results of this type of academic change for Project INTERMATH, we refer you to West [2002] (in this issue) and to earlier articles in Campbell [2001].

Types of Curriculum Change

Types of curriculum change can be partitioned into two broad categories: evolutionary or revolutionary. Evolutionary change usually involves placing more or less emphasis on a concept. For example, decreasing the emphasis on integration techniques is an example of evolutionary change, as is increasing

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the emphasis on numerical or graphical representation of functions. Evolutionary change can be thought of as continuous change, whereas revolutionary change is discontinuous change. Replacing a standard calculus-dominated core program with a modeling-dominated program that integrates the treatment of data analysis, discrete dynamical systems, matrix algebra, calculus, and differential equations is an example of discontinuous change. Another example is to reform a traditional college algebra course by replacing the focus on symbolic manipulations and abstract drill exercises with a focus on elementary data analysis, functions, modeling, and problem solving in context.

The Model

The following change model and comments are based on experience gained from a decade and a half of conducting faculty development and dissemination workshops. The first workshops were focused on integrating the use of computer algebra systems (CAS) into instruction and were followed by calculus reform dissemination workshops, then INTERMATH workshops, and finally college algebra reform workshops. The change model that we describe applies to implementing discontinuous change. It is adapted from a business model for marketing, called the Technology Adoption Life Cycle, described in Moore and McKenna [1999]. They describe the business model pictorially by a bell-shaped curve:

> The divisions in the curve are roughly equivalent to where standard deviations would fall. That is, the early majority and the late majority fall within one standard deviation of the mean, early adopters and the laggards within two, and way out there, at the very onset of a new technology, about three standard deviations from the norm are the innovators.

Although the division points in the curriculum-change model may not correspond to standard deviations, the groups and their order are the same. The makeups of the groups span both academic ranks and years of experience. Typically, a department contains representatives from two or three different groups.

Applying the Model

Characterizing the Faculty

A first step in discussing the application of this model to curriculum change is to describe the characteristics of the faculty in each of the groups.

- *Innovators* (change agents) are visionaries who have studied a particular program and believe that they can develop a much better program. Their

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goals are usually in clear contrast to the results of the existing curriculum. They usually argue for their vision on a philosophical level, while citing the shortcomings of the present curriculums on a practical level. Visionaries are not content with evolutionary change but want breakthroughs—strategic leaps forward guided by their vision. They are revolutionaries in their field and are willing to devote tremendous efforts in order to translate their visions into curriculum materials.

- *Early adopters* are risk-takers. They are seriously concerned about the shortcomings of the present curriculum and are willing to take a risk on a new, untried curriculum. They are often recruited by one or more innovators and do not wait to be convinced by assessment studies or formal, established programs.

- *Early-majority faculty* are conservative and are driven by a strong sense of practicality. They are content to wait and see how the new curriculum works out in schools similar to theirs. They want to hear reports from well-respected references before making a decision to change but are willing to make small adaptations to make the change work in their local environment. They don't expert perfection in the course material but they do expect quality in the content of the materials.

- *Late-majority faculty* wait until the new curriculum becomes well established, particularly at several large schools. They are interested only in a "tried and true" curriculum that does not require any adaptation on their part. They expect thoroughly edited, high-quality, carefully-presented textbooks and material to be in place before they make the change.

- *Laggards* are people who do not want to change for any reason and will change only when there is no other choice available.

The Start-Up

Grant funding plays a critical role in developing and implementing curriculum change. Although there have been exceptions in which missionary zeal and idealism have sustained innovators to proceed without outside funding, the norm is for innovators to be supported by grant funding. An example is the large number of National Science Foundation (NSF) pilot grants followed by a few multi-year development grants that helped initiate and sustain the calculus reform movement for several years.

Most of the responsibility for moving from the innovators stage to the early-adopters stage rests with the innovators. They are the ones who recruit early adopters and who publicize their new programs through sessions at professional meetings, newsletters, and journal articles. Their primary recruiting device is the dissemination workshop. These workshops also serve to support existing adopters and to further publicize the new curriculum. Supporting

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recruits through additional faculty development workshops, personal visits, and ongoing communication is essential to helping a recruit become an early adopter. The early-adopters stage may last for five or more years during which time curriculum materials are class-tested, refined, expanded to provide more instructional help, and, hopefully, picked up by a commercial publisher. This stage is characterized by departments offering experimental sections based on the new program materials.

Crossing the Chasm

The largest challenge or "chasm," to use Moore and McKenna's term, is to move a curriculum from the early-adopters stage to the early-majority stage. In general, innovators are not successful in orchestrating this move. Whereas innovators and early adopters often share a similar entrepreneurial spirit, the same is not true with innovators and early-majority faculty. The conservative and pragmatic early-majority faculty are not risk-takers. They demand convincing evidence, not just the results of special sections. Early-majority faculty are often looking for reasons *not* to adopt the new curriculum, in contrast to the early adopters, who are looking for reasons to try out the new program. An example is a department chairman of a prestigious college who had discovered a mistake in a beta copy of a new computer algebra system (CAS) during a dissemination workshop. He announced that his department would not use any CAS until it had been proven to be completely accurate, which he later translated to never. This type of resistance to some academic changes is widespread.

Another source of difficulty in crossing the chasm is the change-diversity of the faculty. This is particularly true in large departments. When departments attempt to change collectively, not all faculty members are necessarily in the same change group, so the timing and implementation of collective change for large programs and courses are even more difficult. For example, several years ago we were helping a department adopt and implement a new calculus reformed course text with the utilization of technology. The faculty with whom we spoke acted like early-majority faculty in regard to their state of reform, in that they learned how the book was used at a couple of other schools and were willing to take on a few special adaptations to insure local success. However, we later discovered when the implementation didn't go well that the majority of the teaching faculty in the project were in fact late-majority faculty. Collectively, they were not willing to stray beyond the established boundaries of a textbook, nor were they willing to wait until all the difficulties and new ideas were resolved. When there are differences of levels of commitment in a group, it takes more time to resolve those difference before the change can be fully and successfully implemented.

Crossing the chasm into the early majority and then into the late majority require strong efforts by sales forces of commercial publishers and strong administrative support by department chairs, deans, and provosts. Faculty

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need to understand their institution's commitment to the change and the value assigned to the extra faculty time required. Unfortunately, the limited profit potential of a new curriculum restricts commercial sales efforts, while the pressure of budgets and student opinion often restricts the necessary administrative support.

Leadership

Leadership is essential in guiding change through the different stages. Innovators provide this leadership for moving from the innovator stage to the early-adopters stage but usually are not successful in leading departments across the chasm. Innovators and early-majority faculty have different goals and agendas, early-majority faculty do not usually participate in dissemination workshops, and early-majority faculty demand assessment studies that usually do not exist. Crossing the chasm then rests on the combined efforts of people, such as administrators and commercial sales people, who may have had no involvement with the development of the new curriculum and thus do not have the personal commitment of an innovator or early adopter. The result is that most discontinuous change programs fail to cross the chasm and thus never achieve their goal of revolutionary change.

Applying the Model to INTERMATH

Like all reform projects, Project INTERMATH began with several high-energy innovators, who built a bold new integrated and interdisciplinary mathematics program at the United States Military Academy (using ILAPs) and made it work well enough that early adopters at places like Carroll College (Montana) and Harvey Mudd College took the risks to build their own versions of integrated, interdisciplinary curricula.

INTERMATH, now with ILAP publications by COMAP and MAA and more textual and course materials being published (see the **References**), seeks early-majority departments and faculty who would like to develop their students with more integrated and interdisciplinary curricular methods. Interested faculty should contact either Donald Small (`ad5712@usma.edu`) or Gary Krahn (`ag2609@usma.edu`) at the United States Military Academy for more information.

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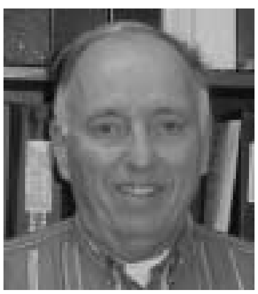
About the Authors



Don Small graduated from Middlebury Collegeand received his Ph.D. degree in mathematics from the University of Connecticut. He taught mathematics at Colby College for 23 years before joining the Mathematics Department at the U.S. Military Academy in 1991. He is active in the calculus and college algebra reform movements—developing curricula, authoring texts, and leading faculty development workshops. Active in the Mathematical Association of America (MAA), Don served a term as Chairman and two terms as Governor of the Northeast Section and has been a long-term member of the MAA's CRAFTY committee. His interests are in developing curriculum that focuses on student growth while meeting the needs of partner disciplines, society, and the workplace. He is also a member of AMATYC and AMS.

Chris Arney has an undergraduate degree from the United States Military Academy (USMA) and a Ph.D. from Rensselaer Polytechnic Institute. He taught mathematics at USMA for many years and is the author of several mathematics textbooks and laboratory manuals. Recently, he edited Arney [1997]. His areas of research interest include applied mathematics, numerical analysis, andthe history of mathematics. His teaching interests include using computers, writing, and interdisciplinary applications in the mathematics and science curricula.

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Mathematical Modeling as a Thread

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As part of the national movement toward improving mathematics education, intense curriculum reform has taken place at the college level since about 1989. One of the major purposes of this ongoing reform has been to change how students are taught mathematics, with the primary focus being to empower students. At the institutions where this type of reform has been successful, a cultural change around mathematics has taken place. One institution where the culture has changed is the United States Military Academy at West Point.

This may seem odd, as West Point is steeped in tradition and has four large (over 1,000 students) standardized courses in mathematics that all students must take. Yet the leaders of this curriculum change looked at these supposed obstacles to change as an opportunity and sought to create an integrated mathematics program out of the four distinct courses. To accomplish this metamorphosis, outcome goals and intermediate course objectives were set along five educational threads:

- mathematical modeling,
- scientific computing,
- mathematical reasoning,
- communication, and
- history of mathematics.

The key to the success of reforming into a four-semester program that empowers students and faculty was the mathematical modeling thread. In the words of many faculty and students, "Modeling made the mathematics relevant and put life into the courses." As a result, students retained more mathematics and made progress toward becoming confident, aggressive problem solvers.

The courses at West Point have always been applied, as all students must be prepared for an engineering minor. So, many problems used in mathematics

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class routinely come from other disciplines. The stated goal of the mathematical modeling thread is to get students to

- recognize opportunities for quantitative modeling,
- apply the modeling process in solving relevant problems, and
- test and analyze their models for sensitivity of variables and assumptions.

As the curriculum changed and the faculty endeavored to grow students along these threads, the need for more significant problems, or projects, became obvious. Since 1992, each of the four courses has used interdisciplinary projects developed in conjunction with other disciplines. These projects are called Interdisciplinary Lively Application Projects, or ILAPs. To keep these projects "lively" and relevant, they are changed each year. Also, one of the reform ideas was to compress the curriculum (make it leaner). The faculty found that the ILAPs enabled instructors to become more efficient in covering material and empowered students as modelers and better problem solvers.

As a participant in the reform process at West Point, I developed a teaching style that involves modeling as a thread and integrates projects throughout my courses. Teaching in a public university the past two years, I have adapted this teaching style to all of my new courses.

New interdisciplinary projects have become the norm, even in large standardized college algebra courses. The integration of mathematical modeling has the same benefits at many different levels, from college algebra and calculus courses to graduate mathematics content courses for teachers. In all cases, the success of this curricular change is evident in greater student retention and improved attitudes toward the learning of mathematics.

The purposes and results may differ from one institution to the next, but establishing mathematical modeling as a thread through the use of ILAPs appears to be a successful method to motivate students toward becoming confident, aggressive problem solvers, a goal of many reform projects.

About the Author

After spending 14 years as a mathematics professor at the United States Military Academy at West Point, Rich West retired from the Army to Florence, SC, in 1999. There he continues to teach full-time at Francis Marion University. He received his Ph.D. in Mathematics Education from New York University in 1995. He is interested in curriculum reform, program assessment, and teaching at the college level. He has served as the Managing Director of Project Intermath for the past seven years.

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Modeling Forum

Results of the 2002 Interdisciplinary Contest in Modeling

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Introduction

A total of 106 teams of undergraduates, from 71 institutions in 5 countries, spent the second weekend in February working on an applied mathematics problem in the 4th Interdisciplinary Contest in Modeling (ICM).

This year's contest began at 8:00 P.M. on Friday, Feb. 7, and ended at 8:00 P.M. on Monday, Feb. 11. During that time, the teams of up to three undergraduates or high-school students researched and submitted their optimal solutions for an open-ended interdisciplinary modeling problem involving environmental science. After a weekend of hard work, solution papers were sent to COMAP.

The two of the papers that were judged to be Outstanding appear in this issue of *The UMAP Journal*. Results and winning papers from the first three contests were published in special issues of *The UMAP Journal* in 1999 through 2001.

In addition to the ICM, COMAP also sponsors the Mathematical Contest in Modeling (MCM), which runs concurrently with the ICM. Information about the two contests can be found at

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www.comap.com/undergraduate/contests/icm
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The ICM and the MCM are the only international modeling contests in which students work in teams to find a solution.

Centering its educational philosophy on mathematical modeling, COMAP uses mathematical tools to explore real-world problems. It serves the educational community as well as the world of work by preparing students to become better informed and better-prepared citizens, consumers, and workers.

This year's problem, which involved understanding and managing the habitat of the Florida scrub lizard, proved to be particularly challenging. It contained various data sets to analyze, had several challenging requirements needing scientific and mathematical connections, and also had the ever-present requirements to use creativity, precision, and effective communication. The author of the problem, environmental scientist Grant Hokit, was one of the final judges, and his commentary appears in this issue.

All the competing teams are to be congratulated for their excellent work and dedication to scientific modeling and problem solving. This year's judges remarked that the quality of the papers was extremely high, making it difficult to choose the two Outstanding papers.

In 2002 the ICM continued to grow as an online contest, where teams registered, obtained contest instructions, and downloaded the problem through COMAP's ICM Website.

Problem: The Scrub Lizard Problem

Figure 1. Florida scrub lizard. Photo by Grant Hokit.

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If We SCRUB Our Land Too much, We May Lose the LIZARDs

The Florida scrub lizard is a small, gray, or gray-brown lizard that lives throughout upland sandy areas in the Central and Atlantic coast regions of Florida. The Florida Committee on Rare and Endangered Plants classified the scrub lizard as endangered.

You will find a fact sheet on the Florida Scrub Lizard at `http://www.comap/undergraduate/contestsicm/2002problem/scrublizard.pdf`. [EDITOR'S NOTE: We do not reproduce that document here.]

The long-term survival of the Florida scrub lizard is dependent upon preservation of the proper spatial configuration and size of scrub habitat patches.

Task 1

Discuss factors that may contribute to the loss of appropriate habitat for scrub lizards in Florida. What recommendations would you make to the state of Florida to preserve these habitats and discuss obstacles to the implementation of your recommendations?

Task 2

Utilize the data provided in **Table 1** to estimate the value for F_a (the average fecundity of adult lizards), S_j (the survivorship of juvenile lizards between birth and the first reproductive season), and S_a (the average adult survivorship).

Table 1.

Summary data for a cohort of scrub lizards captured and followed for 4 consecutive years. Hatchling lizards (age 0) do not produce eggs during the summer they are born. Average clutch size for all other females is proportional to body size according to the function $y = 0.21(\text{SVL}) - 7.5$, where y is the clutch size and SVL is the snout-to-vent length in mm.

Year	Age	Total number living	Number of living females	Avg. female size (mm)
1	0	972	495	30.3
2	1	180	92	45.8
3	2	20	11	55.8
4	3	2	2	56.0

Task 3

It has been conjectured that the parameters F_a , S_j , and S_a are related to the size and amount of open sandy area of a scrub patch. Utilize the data provided in **Table 2** to develop functions that estimate F_a, S_j , and S_a for different patches. In addition, develop a function that estimates C, the carrying capacity of scrub lizards for a given patch.

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Table 2.

Summary data for 8 scrub patches including vital rate data for scrub lizards. Annual female fecundity (F_a), juvenile survivorship (S_j), and adult survivorship (S_a) are presented for each patch along with patch size and the amount of open sandy habitat.

Patch	Patch size (ha)	Sandy habitat (ha)	F_a	S_j	S_a	Density (lizards/ha)
a	11.31	4.80	5.6	.12	.06	58
b	35.54	11.31	6.6	.16	.10	60
c	141.76	51.55	9.5	.17	.13	75
d	14.65	7.55	4.8	.15	.09	55
e	63.24	2.12	9.7	.17	.11	80
f	132.35	54.14	9.9	.18	.14	82
g	8.46	1.67	5.5	.11	.05	40
h	278.26	84.32	11.0	.19	.15	115

Task 4

Many animal studies indicate that food, space, shelter, or even reproductive partners may be limited within a habitat patch, causing individuals to migrate between patches. There is no conclusive evidence on why scrub lizards migrate. However, about 10% of juvenile lizards do migrate between patches, and this immigration can influence the size of the population within a patch. Adult lizards apparently do not migrate. Utilizing the data provided in the histogram in **Figure 2**, estimate the probability of lizards surviving the migration between any two patches i and patch j.

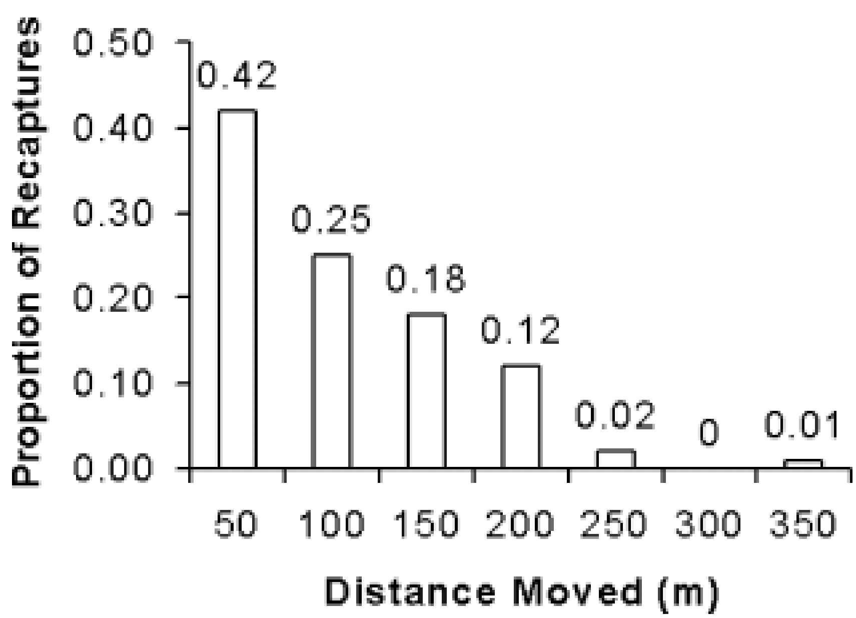

Figure 2. Migration data for juvenile lizards marked, released, and recaptured up to 6 months later. Surveys for recapture were conducted up to 750 m from release sites.

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Task 5

Develop a model to estimate the overall population size of scrub lizards for the landscape given in **Table 3**. Also, determine which patches are suitable for occupation by scrub lizards and which patches would not support a viable population.

Table 3.

Patch size and amount of open sandy habitat for a landscape of 29 patches located on the Avon Park Air Force Range. See **Figure 3** for a map of the landscape.

Patch identification	Patch size (ha)	Sandy habitat (ha)
1	13.66	5.38
2	32.74	11.91
3	1.39	0.23
4	2.28	0.76
5	7.03	3.62
6	14.47	4.38
7	2.52	1.99
8	5.87	2.49
9	22.27	8.44
10	19.25	7.58
11	11.31	4.80
12	74.35	19.15
13	21.57	7.52
14	15.50	2.82
15	35.54	11.31
16	2.93	1.15
17	47.21	10.73
18	1.67	0.13
19	9.80	2.23
20	39.31	7.15
21	2.23	0.78
22	3.73	1.02
23	8.46	1.67
24	3.89	1.89
25	1.33	1.11
26	0.85	0.79
27	8.75	5.30
28	9.77	6.22
29	13.45	4.69

Task 6

It has been determined from aerial photographs that vegetation density increases by about 6% a year within the Florida scrub areas. Please make a recommendation on a policy for controlled burning.

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Figure 3. Map of landscape of 29 patches located on the Avon Park Air Force Range.

The Results

Solution papers were coded at COMAP headquarters so that names and affiliations of authors would be unknown to the judges. Each paper was read preliminarily by two "triage" judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree on a score, a third judge evaluated the paper.

Final judging took place at the United States Military Academy, West Point, NY. The judges classified the papers as follows:

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Scrub Lizard	2	16	28	60	106

The two papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with commentaries. We list those teams and the Meritorious teams (and advisors) below; the list of all participating schools, advisors, and results is in the **Appendix**.

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Outstanding Teams

Institution and Advisor	Team Members
"Where's the Scrub? Aye, There's the Rub"	
Maggie L. Walker Governor's School	Victoria L. Chiou
Richmond, VA	Andrew Carroll
John Barnes	Jessamyn J. Liu
"Cleaning Up the Scrub: Saving the Florida Scrub Lizard"	Nicole Hori
Olin College of Engineering	Steven Krumholtz
Needham, MA	Daniel Lindquist
Burt Tilley	

Meritorious Teams (16 teams)

Beijing University of Posts & Telecommunications, Beijing, China (He Zuguo)
Carroll College, Helena, MT (Sam R. Alvey)
Central South University, Changsha, China (Zhang Hongyan and Zheng Zhoushun)
Dickinson College, Carlisle, PA (Brian S. Pedersen)
Elon University, Elon, NC (Crista Coles) (two teams)
Fudan University, Shanghai, China (Cao Yuan)
Harvey Mudd College, Claremont, CA (Michael E. Moody)
Monmouth College, Monmouth, IL (Christopher Fasano)
Northwestern Polytechnical University, Xian, China (Xiao Hua Yong)
Tsinghua University, Beijing, China (Hu Zhiming)
United States Air Force Academy, USAF Academy, CO (Jim West)
University of Missouri, Rolla, MO (Mohamed Ben Rhouma)
University of Science and Technology of China, Hefei, China (Tao Dacheng)
University of Science and Technology of China, Hefei, China (Zhang Hong)
Youngtown State University, Youngstown, OH (Scott Martin)

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Directors and by the Head Judge. Additional awards were presented to the Governors School team from Institute for Operations Research and the Management Sciences (INFORMS).

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Judging

Director
Chris Arney, Dean of the School of Mathematics and Sciences,
The College of Saint Rose, Albany, NY

Associate Directors
Michael Kelley, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY
Gary W. Krahn, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Judges
Richard Cassidy, Dept. of Industrial Engineering, University of Arkansas,
Fayetteville, AR
Grant Hokit, Dept. of Biology, Carroll College, Helena, MT
Marie Vanisko, Dept. of Mathematics, Carroll College, Helena, MT

Triage Judges
Darryl Ahner, Eric Drake, Alex Heidenberg, D. Jacobs, Alan Johnson, Gary Krahn, E. Lesinski, Joe Myers, Mike Phillips, K. Romano, Kathi Snook, B. Stewart, Ani Velo, and Brian Winkel, all of the U.S. Military Academy, West Point, NY.

Source of the Problem

The Scrub Lizard Problem was contributed by Grant Hokit, Dept. of Biology, Carroll College, Helena, MT.

Acknowledgments

Major funding for the ICM is provided by a grant from the National Science Foundation through COMAP. Additional support is provided by the Institute for Operations Research and the Management Sciences (INFORMS).

We thank:

- the ICM judges and ICM Board members for their valuable and unflagging efforts, and
- the staff of the Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY, for hosting the triage judging and the final judging.

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Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the student papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Light editing has taken place: minor errors have been corrected, wording has been altered for clarity or economy, style has been adjusted to that of *The UMAP Journal*, and the papers have been edited for length. Please peruse these student efforts in that context.

To the potential ICM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

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Appendix: Successful Participants

KEY:
P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

INSTITUTION	CITY	ADVISOR	I
ARIZONA			
McClintock	Tempe	Ivan Barkdoll	P
CALIFORNIA			
Harvey Mudd College	Claremont	Michael Moody	M, H
Sonoma State University	Rohnert Park	Elaine McDonald	P
COLORADO			
Colorado State University	Fort Collins	Michael Kirby	P
United States Air Force Academy	USAF Academy	Jim West	M
GEORGIA			
Georgia Southern University	Statesboro	Laurene Fausett	P
ILLINOIS			
Monmouth College	Monmouth	Christopher Fasano	M
INDIANA			
Earlham College	Richmond	Mic Jackson	H
KENTUCKY			
Asbury College	Wilmore	David Coulliette	H, P
Northern Kentucky University	Highland Heights	Gail Mackin	P
MASSACHUSETTS			
Babson College	Wellesley	Steven Eriksen	P
Olin College of Engineering	Needham	Burt Tilley	O
MICHIGAN			
East Grand Rapids Public Schools	Grand Rapids	Mary Elderkin	P
Lawrence Technological University	Southfield	Howard Whitston	H
		Ruth Favro	P
MINNESOTA			
St. Cloud State University	St. Cloud	Dominic Naughton	P
MISSOURI			
University of Missouri-Rolla	Rolla	Mohamed Ben Rhouma	M
MONTANA			
Carroll College	Helena	Sam Alvey	M
Montana Tech of the Univ. of Montana	Butte	Richard Rossi	H, P

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INSTITUTION	CITY	ADVISOR	I
NEW JERSEY			
Rowan University	Glassboro	Hieu Nguyen	P
		Samuel Lofland	P
NEW YORK			
U.S. Military Academy	West Point	Mike Huber	H
		Mike Johnson	H
NORTH CAROLINA			
Elon University	Elon	Crista Coles	M, M
Piedmont Community College	Roxboro	Lisa Cooley	P
OHIO			
Ohio Wesleyan University	Delaware	Richard Linder	P, P
Youngstown State University	Youngstown	Angela Spalsbury	H
		Scott Martin	M
OREGON			
Eastern Oregon University	La Grande	Jeffrey Woodford	P
Franklin High School	Portland	David Hamilton	P, P
PENNSYLVANIA			
Bloomsburg University	Bloomsburg	Kevin Ferland	P
Clarion University of Pennsylvania	Clarion	Andrew Turner	H
Dickinson College	Carlisle	Brian Pedersen	M
Lafayette College	Easton	Thomas Hill	H
TEXAS			
Texas A&M University	College Station	Jay Walton	H
VIRGINIA			
Maggie L. Walker Governor's School	Richmond	John Barnes	O, P
		Crista Hamilton	P
WASHINGTON			
Pacific Lutheran University	Tacoma	Mei Zhu	H
WISCONSIN			
Beloit College	Beloit	Paul J. Campbell	P
CANADA			
York University	Toronto, ON	Morton Abramson	P
CHINA			
Anhui University	Hefei	Cheng Junsheng	H
		Wang Dapeng	P
Beijing Union University	Beijing	Ren Kailong	P
Beijing Univ. of Chemical Technology	Beijing	Yan Cheng	H
Beijing Univ. of Posts & Telecomm.	Beijing	He Zuguo	M, P

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INSTITUTION	CITY	ADVISOR	I
Central South University	Changsha	Chen Xiaosong	H
		Zhang Hongyan and Zheng Zhoushun	M
Chongqing University Inst. of Math. & Phys.	Chongqing	Qu Gong	P
		He Renbin	P
Dalian University of Technology	Dalian	Liaoning and Yu Hongquan	P, P
East China Univ. of Science and Technnology	Shanghai	Ni Zhongxin	H, P
Experimental High School of Beijing Normal University	Beijing	Wang Jiangci	P
Fudan University	Shanghai	Cai Zhijie	H
		Cao Yuan	M
Hangzhou Univ. of Commerce	Hangzhou	Zhu Ling	H
Harbin Engineering University	Harbin	Luo Yuesheng	P
		Zhang Xiaowei	P
Harbin Institute of Technology	Harbin	Shang Shouting	P
		Zheng Tong	P
Harbin Univ. of Science andTechnology	Harbin	Chen Dongyan	H
		Li Dongmei	P
Hefei University of Technology	Hefei	Su Huaming	P
		Du Xueqiao	P
Jiamusi University College of Mathematics	Jiamusi City	HeiLong and Ji Bai Shan	P
Jilin Institute of Technology	Changchun	Lu Jin	H
		Bai Ping	P
		Li Yan	P
		Huang Qingdao	P
Jilin University	Changchun	ZhangKuiyuan	P
Jinan University	Guangzhou	Hu Daiqiang	P
		Zhang Lin	P
Nanjing University of Science and Techology	Nanjing	Qian ping	P
		Wu Min	P
Nankai Institute of Mathematics	Tianjin	Fu Lei	H
Northwestern Polytechnical University	Xi'an	Feng Nie	H
		Xiao Yong Hua	M
Peking University	Beijing	Liu Yulong	H, P
Shanxi University	Taiyuan	Wang Guang	P
		Ding Juntang	P
South China Univ. of Technology	Guangzhou	Liang Fa	H
		Hong Yi	P
Tsinghua University	Beijing	Hu Zhiming	M
		Ye Jun	P
University of Science and Technology of China	Hefei	Zhang Hong	M
		Tao Dacheng	M
Xi'an Jiaotong University	Xi'an	He Xiaoliang	H, P

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INSTITUTION	CITY	ADVISOR	I
Zhejiang University	Hangzhou	Yang Qifan	P
		Yong He	P
Zhongshan University	Guangzhou	Chen Zepeng	P
		Tang Mengxi	P
FINLAND			
Päivölä College	Tarttila	Merikki Lappi	H
IRELAND			
University College Dublin	Dublin	Michael Mackey	H, P

Editor's Note

For team advisors from China, we have endeavored to list family name first, with the help of Zheng Rong.

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Where's the Scrub? Aye, There's the Rub

Victoria L. Chiou
Andrew Carroll
Jessamyn J. Liu
Maggie L. Walker Governor's School
for Government and International Studies
Richmond, VA

Advisor: John A. Barnes

Abstract

We use data from eight patches inhabited by scrub lizards in logistic regressions to predict from the area of sandy habitat the average fecundity, juvenile and adult survivorship, and total population of a patch.

From the viewpoint of evolutionary biology, we analyze the marginal benefit and risk for an individual lizard migrating. The probability of dying during migration is 30%, with a 0.3% marginal risk per meter migrated.

We determine which patches at Avon Park Air Force Base are self-sustaining and which are sustained by migration; our model is 76% accurate in predicting whether a patch is occupied.

We recommend removing encroaching vegetation through roller-cutting as opposed to controlled burning, due to the high intensity of a fire required to burn scrub and due to the public discomfort with controlled burning.

Introduction

Because of the immense diversity within the Florida sand pine scrub ecosystem, the World Wildlife Organization has granted the Florida scrub "ecoregion" status; at a mere 3900 km^2, it is among the smallest ecoregions of the contiguous United States.

This ecoregion is a "naturally fragmented archipelago of habitat islands" [Branch et al. 1999]. Isolated light-colored patches of sandy soil, obscured by litter and lichens, are surrounded by areas of dense scrub thicket.

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The Florida scrub is rapidly deteriorating due to human development and replacement of scrub by citrus groves, pasturelands, and pine plantations. Extensive human disturbance and development of scrub areas has increased the fragmentation and isolation of scrub patches, and led to fire suppression.

Florida scrub must be maintained by periodic intense fires. Scrub patches burn naturally every 15 to 100 years [Harper and MacAllister 1998]. Because of human development, fires have been suppressed for the past 80 years; this suppression has led to a decrease in the number of available scrub patches, reduction of scrub patch size, decline of habitat quality, and increased patch isolation [Branch et al. 1999].

Conservation efforts have involved proposals for prescribed burning and buying up scrub lands for consolidation. Scientists and conservationists should combine their efforts to provide the public with critical information on the needs of imperiled, threatened, and endangered species, particularly those endemic to the Florida scrub area. While much government money has been directed towards wetland conservation, the Florida scrub contains more vulnerable species than the wetlands for which the state is known [Harper and MacAllister 1998]. If appropriate measures are not taken to protect habitat, the imperiled Florida scrub lizard will become endangered or even extinct. Before further policies on prescribed burning or mechanical methods of vegetation clearance can be implemented, the public must understand the benefits of such policies.

Food for the Brood: Lizard Fecundity and Survivorship

Assumptions

- The only factors that contribute to change in population are fecundity and survivorship.
- There are numerous definitions and levels of fecundity. We use *annual female fecundity*, the total number of offspring per female in one full year.
- We do not consider age a determinant of sexual maturity, except that lizards do not reproduce in the same season in which they were born, regardless of size [Antonio 2000].

Fecundity is affected by the size and age of the lizard, available food and nutrition sources, sex ratio, environmental fluctuations, temperature, and humidity of the area. Sex ratios of lizard populations are typically about one-to-one.

Clutches range from two to eight eggs per clutch [Branch and Hokit 2000]. The major factor affecting clutch size is the size of the lizard, which is propor-

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tional to snout-to-vent length (SVL), according to the function

$$y = 0.21(\mathrm{SVL}) - 7.5. \tag{1}$$

Body size is critical because lizards require stored energy (in the form of fat reserves) to produce eggs; body size increases with age.

Lizards lay from three to five clutches in one reproductive season; this number is the *clutch frequency*. Since direct data collection is nearly impossible, clutch frequency is often estimated as the duration of the active season divided by the time to produce a clutch. This estimate may be inaccurate due to variability in the time to produce a clutch and the reproductive season being shorter than the active season.

The incubation time is 30 days; so with a reproductive season of late March through June, clutch frequency is approximately three. This agrees with researchers who determined that there are not enough data to calculate clutch size or clutch frequency and who thus assumed an average of four eggs per clutch and three clutches per season [Branch et al. 1999].

Survivorship is the ratio of lizards surviving at age x over those who were living at age $(x-1)$; it is generally measured by sequential sampling of a marked cohort of individuals. Losses due to emigration are small compared to those from mortality and tend to be balanced by gains from immigration.

Using **Table 1** of the problem statement and applying **(1)** with the assumptions of three clutches per season of four eggs each, we find:

- $F_a = 5.33$, average annual fecundity;
- $S_j = 0.185$, the juvenile survivorship rate from age 0 to 1 (between birth and the first reproductive season); and
- $S_a = 0.106$, the average survivorship rate.

Modeling Female Lizard Growth

Female reptile growth can be split into three periods:

- growth until sexual maturity (period 1),
- growth after sexual maturity until optimal size (period 2), and
- growth after optimal size (period 3).

Growth is generally rapid until reaching sexual maturity and much slower thereafter [Heatwole 1976].

The growth rate in period 1 may be estimated from the average hatchling size, lizard size at sexual maturity, and time necessary to reach maturity. The lizard is is 21 mm at hatching but reaches 45 mm by sexual maturity in 10 to 11 months [Gans and Pough 1982; Branch et al. 1999]; hence the growth rate is 2.2 to 2.4 mm / month.

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After sexual maturity, the lizard continues to grow at a lower rate to optimal size. If the lizard is still alive after this point, its rate tapers to a growth rate that continues for the rest of its life. Since most scrub lizards do not live past two years of age [Branch et al. 1999], we assume that the growth between ages 1 and 2 years in **Table 1** of the problem statement is period-2 growth (0.83 mm / month, on average), and the growth between 2 and 3 years of age is period-3 growth (0.02 mm / month, on average).

Scrub, Sand, and Survivorship: Modeling Lizard Carrying Capacity, Fecundity, and Survivorship

Much of the variation in annual female fecundity, juvenile survivorship, adult survivorship, and density is explained by patch size and amount of sandy habitat. However, 97% of the variation in sandy habitat area is explained by the size of the patch; so in our regression analyses, we use only one of those two variables (whichever one has higher correlation with the variable of interest).

The area of the sandy habitat has a large impact on average fecundity ($r^2 = .77$), with predicted fecundity varying from 5.9 to 11.7 from the smallest to the largest patches. Area of sandy habitat also greatly affects ($r^2 = .81$) adult survival rate, with predicted values ranging from .07 to .16.

However, survivorship of juvenile lizards is less related to patch size ($r^2 = .66$), varying from .14 to .20. For juveniles, the probability of successful emigration may play a significant role in survivorship.

Juvenile survivorship and adult survivorship are closely linked ($r^2 = .96$). This is expected, since juveniles are not so different in structure and metabolism from adults in predators, habitat, or food sources.

The *proportion* of the patch occupied by sandy habitat is an extremely poor predictor of average fecundity, survivorship, or density, with the highest r^2 being .04.

We use a logistic model to predict average fecundity, juvenile survivorship, and adult survivorship from the area of sandy habitat. We choose a logistic model because as the area of sandy habitat increases, fecundity and survivorship do not continue to increase without bound, as would occur with a linear model, but instead tend toward a maximum.

The regression reveals the following relationships with area covered by sandy habitat (x):

$$\begin{aligned} \text{annual fecundity} &= 10.3\big(1 + 1.42e^{-0.096x}\big) \\ \text{juvenile survivorship} &= 0.179\big(1 + 0.89e^{-0.169x}\big) \\ \text{adult survivorship} &= 0.139\big(1 + 1.93e^{-0.123x}\big). \end{aligned} \tag{2}$$

To model carrying capacity, we calculated the number of lizards in each patch by multiplying density by patch size, using the data in **Table 1** of the problem statement. We regressed population density on patch size z ($r^2 = .87$)

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and predicted number of lizards in a patch as population density predicted times size of the sandy patch, arriving at:

$$\text{number of lizards} = 0.227z^2 + 51.2z, \tag{3}$$

with $r^2 = .999$. Does this equation also give a good estimate for the carrying capacity of a patch? Lizard populations tend to be quite stable, fluctuating only mildly from carrying capacity [Gans and Tinkle 1977]. That the actual lizard populations correspond so closely to the predicted values suggests that the populations are at carrying capacity.

Lizard Migration Motivation

It is difficult to determine the probability of a lizard dying during migration based on the proportion of lizards recaptured at various distances from location of initial capture. That the proportion recaptured decreases with increasing distance could be the result of lizards dying between each of the recapture sites or of lizards ceasing to migrate after having traveled a certain distance.

We could use the average speed of dispersal (2.5 m/day) to derive the mortality rate for each day migrated [Branch et al. 1999]. However, doing so assumes that no lizard reach its destination: All are killed en route to an ideal location that is theoretically an infinite distance away. To calculate accurately the probability of dying during migration, we need to analyze the cause of migration.

There are no conclusive data why lizards migrate. Other animals migrate based on the availability of food, space, shelter, or reproductive partners, but apparently a universal 10% of juvenile scrub lizards migrate regardless of any environmental attribute so far measured.

An individual's movement to a new patch brings genetic material. Can this influx of genes be shown to benefit the lizard population and the individual lizard?

For any species to survive, it must use a reproductive strategy that allows rapid adaptation relative to changes in the environment. Less-evolved species rely heavily on genetic diversity, natural selection, and learned behavior to maintain adaptability, as well as on producing far more offspring in a short span of time than can survive. As a result, the population of lizards in a patch exhibits little genetic diversity.

The individual lizard must derive a benefit from having the only offspring in a patch with a genetic advantage, or evolution would not select for lizards that emigrate at a rate of 10%. The potential benefits of migration are moving to an area with

- greater fecundity,
- greater survivorship, or
- an advantage for progeny over other lizards.

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The potential risks are

- moving to an area with with lesser fecundity,
- moving to an area with lesser survivorship, and
- dying en route.

Due to the strong correlation (90%) between area of sandy habitat and number of lizards in a patch, most lizards live in patches with fecundity and survivorship rates above those of the average patch. Thus, the only advantage of emigrating introducing genes with a selective advantage in the new patch.

By moving to a different patch, the predicted number of offspring falls from 10.1, the average fecundity of all lizards, to 7.8, the average fecundity of all patches. (We assume that males have the same fecundity rate as females.) The decrease of 2.3 indicates a penalty of producing 23% fewer offspring by migrating.

For 10% of juveniles emigrating, the net benefit of successful emigration should theoretically be 10% as well. The average distance traveled by migrating lizards was 105.5 m, so the marginal benefit to traveling 1 m successfully should be 100% / 105.5 m = .095% / m.

Beyond 400 m, no lizards are captured. We assert this as the distance beyond which no lizards emigrate; at this distance, the net benefit of migration is -100%.

Since the migration penalty in fecundity does not vary with distance traveled, we can relate marginal benefit per meter to the average probability of dying en route:

$$b = 0.0948d - rd - 22.8,$$

where

d = distance traveled (m),

b = the net benefit of migration (%),

r = the marginal risk per meter of dying en route, and

22.8 is the percentage fecundity penalty of migrating to an average patch.

At a distance of 400 m, the net benefit is -100%. We put $d = 400$, $b = -100$, and solve for r, finding $r = 0.288\%$ deaths / meter.

We estimate the average death rate D for all emigration based on the average distance traveled (105.5 m). Thus, the average death rate for emigration is $D = 0.288\%/\text{m} \times 105.5\ \text{m} = 30.4\%$. The average mortality rate for the juvenile population due to migration equals D (30.4%) times the propensity to migrate (10%), or 3%.

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Patch Occupation and Viable Population

Previous lizard studies at Avon Park Air Force Base use a measure of isolation S_i of a patch i:

$$S_i = \sum p_j e^{-d_{ij}} A_j,$$

with the sum taken over all patches j with $j \neq i$ [Branch et al. 1999]. Thus, isolation is a function of

- d_{ij}, the distance between patches i and j; and
- A_j, the area of patch j.

The value of p_j is 1 if patch j is occupied, 0 otherwise.

The distance between patches determines the difficulty of movement between patches, while the area of the patch determines the possible number of migrants, since area has a strong correlation with patch population.

Branch et al. [1999] determine that the probability that a patch is occupied is given by

$$P_i = \frac{\exp(0.61A_i + 0.05S_i - 5.22)}{1 + \exp(0.61A_i + 0.05S_i - 5.22)},$$

where for patch i we have

P_i = probability that the patch is occupied,

A_i = area of the sandy habitat of the patch, and

S_i = the isolation parameter for the patch.

This equation predicted patch occupancy for scrub patches within the Avon Park Air Force Range with 89% accuracy [Branch et al. 1999]. However, this equation can be used only if it is known which patches are occupied; yet the goal is to predict patch occupation without knowing which patches are occupied.

To accomplish this latter goal, we use our logistic regressions **(2)**. We formulate the ability of each patch to sustain its population by comparing average fecundity to the average number of deaths through the equation:

$$\text{Sustainability} = 1 + \left(F_a - \left[\frac{1}{S_j} + \frac{1}{S_j S_a} + \frac{1}{S_j S_a S_a} + \cdots + \frac{1}{S_j S_a^n} \right] \right).$$

This equation gives the sustainability of a patch: the average number of lizards that a single lizard will yield each year from the patch. The number of lizards in the patch will grow, stay the same, or decrease, depending on whether sustainability is below 1, equal to 1, or above 1.

According to our logistic regressions, only patches 2, 9, 12, 15, and 17 have sustainability greater than 1. They are thus the only patches capable of maintaining a population, apart from migration.

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Migration

To factor in the effects of migration, we use a long-term approach. We assume that each patch is at carrying capacity, so excess lizards that are produced in patches with sustainability greater than 1 migrate to other patches. For each patch, we estimate the number of lizards each year generated above replacement level by multiplying the predicted population by the sustainability, using **(3)**.

We assume that lizards migrate patches to other populated patches uniformly; if an inhabited patch is adjacent to two uninhabited patches, half of the migrating lizards in the inhabited patch attempt to migrate to one inhabited patch and the other half attempt to migrate to the other patch. We calculate the number of lizards that die between patches using the previously calculated average death rate for emigration, $r = 0.288\%$.

We apply these formulas in a series of "rounds" that move the number of offspring above the equilibrium number from the inhabited patches to the uninhabited ones. In each "round," the effect of migration is first calculated between adjacent inhabited patches, then the effect of migration to uninhabited patches is taken into account.

After each round, patches at equilibrium are classified as inhabited. Patches that changed from a yearly deficit of lizard production to a yearly surplus, due to migration, are placed into the next round as occupied patches that generate migration into unoccupied adjacent patches. After six rounds, all the patches are either at equilibrium or have a yearly deficit of lizards even with migration.

Our model predicts that patches 2, 3, 9, 10, 11, 12, 15, 17, 21, 22, and 23 are occupied. This accurately predicts the occupancy status of 22 of the 29 patches (76%). Furthermore, the model is not systematically biased: four unoccupied patches (2, 3, 9, and 10) are predicted as occupied, and four occupied patches (5, 13, and 23) are predicted as unoccupied.

Assuming that the population in an inhabited patch is assumed at carrying capacity, the total number of lizards in the Range is 17,679.

A Policy for Controlled Burning

Florida scrub must be maintained by periodic intense fires: Flora and fauna of the scrub require fire to disperse seeds, regenerate, and clear dense brush. As vegetation becomes increasingly dense, sandy patches experience fragmentation and may disappear [Harper and MacAllister 1998]. Natural burns occur every 15 to 100 years. The U.S. Army Corps of Engineers recommend prescribed fires every 8 to 20 years [Harper and MacAllister 1998].

Prescribed fires are a heatedly debated remedy, particularly since scrub lands have a high real-estate value. Nearby homeowners fear that prescribed fires may get out of control, as happened with recent ones in Texas and California that destroyed more than 200 homes.

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Part 1: Vegetation Model

Assumptions

- The 6% increase in vegetation density per year noted in the problem statement decreases sandy habitat and applies to scrub areas in their entirety and to all Florida scrub areas.
- The rate of increase of vegetation density remains constant for subsequent years.

We use a spreadsheet to simulate overgrowth of vegetation. Using **Table 3** of the problem statement, we calculate the percentage of sandy habitat per patch; the average is 39.2%. Per our assumption, we apply this average to the whole Florida scrub ecoregion.

Initial sandy habitat area, or sandy habitat area directly prior to the establishment of the 6% vegetation density growth rate, is represented by

$$S_o = 390{,}000(0.392) = 152773.$$

Amount of remaining sandy habitat in subsequent years, given a 6% vegetation density growth rate, is calculated from

$$S_t = 1 - .06S_{t-1},$$

whose solution is

$$S(t) = 152{,}773e^{-0.0619t}$$

for time t in years.

Assessing the Model

Our model relies strongly on statistical analyses of experimental data and evolutionary theory to create equations and theories to apply to all scrub lizard populations. This is necessary because of the scarcity of documented and quantified relationships between vital attributes of scrub lizards (such as food, shelter, and space requirements, predatory and density limitations, the influence of temperature and rainfall, or why scrub lizards migrate) and scrub lizard fecundity and survivorship. As a result, our model goes a long way with few concrete data, predicting such diverse attributes as marginal risk of dying per meter migrated and the number of years that the population of a patch can survive without encroaching vegetation being cleared.

Because we use few constants in our equations and rely more upon logistic relationships between data and basic evolutionary principles, our model should be easily adaptable to most species that live in patches. Only a few data about fecundity, survivorship, relationship to habitat, population density, and tendency to migrate are required to predict which patches are be inhabited,

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which patches are necessary to sustaining a population throughout the region, the net benefit of migrating, and the relationship between size and fecundity. Although our model analyzes the population dynamics of the scrub lizard, it could just as easily apply to the scrub jay.

Another advantage of our model is the speed and ease with which it can be run and adapted. Our model requires only a spreadsheet program, a calculator that can perform logistic regressions, and minimal data-entry time.

Although it would have been possible to relate patch size, sandy habitat area, fecundity, survivorship, and density with a multiple regression, we believe that a logistic regression better represents the diminishing returns of increases in patch size and sandy habitat on survivorship and fecundity.

A weakness of this approach is that our model is not very robust. Because there are so few data, our assumptions are flawed, and so the only accurate piece of our model is the logistic equations, which are not useful for predicting which patches are inhabited. However, all our assumptions are grounded in basic principles of biology and evolution. Also, our model is at greater risk than most if the data are inaccurate, because it relies on so few data points.

Our Proposal

The risks and opposition of controlled burning outweigh support of conservationists. There is a tremendous risk to human life and property incurred by controlled burning, such as the voluminous amounts of noxious smoke that would prove detrimental to air quality and population health [Harper and MacAllister, 1998]. Inappropriate smoke management would result in severe visibility reduction for vehicle operators, and pose a health risk to those with respiratory problems.

Alternatives include numerous upland management strategies, such as scraping, chaining, cabling, railing, rollerchopping, shredding, and rotobeating. The U.S. Army Corps of Engineers have found that many scrub flora species respond nearly equally to fire and mechanical methods. Other studies indicate that mechanical methods stimulate seed germination of some scrub species [Harper and MacAllister 1998].

We recommend that mechanical methods such as rollerchopping be implemented in place of controlled burning. Rollerchopping involves a tractor or bulldozer pulling steamroller drum with chopper blades through the brush [Payne and Bryant 1994]. Rollerchopping has resulted in reduction of coarse woody debris, increased open sandy habitat, increased stand quality—and higher lizard density.

Consolidation of scrub patches would likely have a positive effect on lizard populations [Branch et al. 1999]. The U.S. Army Corps of Engineers recommends creation of larger scrub patches [Harper and MacAllister 1998], which can be achieved through restoration of surrounding degraded scrub patches. Sand roads should be used to connect patches, to facilitate migration, to im-

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prove gene flow, and to recolonize of patches [Harper and MacAllister 1998]. Disturbances such as road creation and extensive development should be avoided. Roads and construction act as barriers that increase the fragmentation of existing scrub patches.

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Cleaning Up the Scrub: Saving the Florida Scrub Lizard

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Introduction

The Florida scrub lizard is a victim of human development and detrimental involvement in the environment. This lizard lives with its "family" of 13 other animals in the Florida scrublands (**Figure 1**). Many lizards have found that their houses of open sand are being invaded by increasing human-aided dominance of flourishing scrub. This dominance has left many lizards homeless.

Our goal is to provide information that can help save the scrub lizards by modeling many different aspects of their life and their environment, and by locating abundant safe places for occupation.

Preserving Scrub Lizard Habitat

Human development of land is the largest factor in the loss of habitat for the Florida scrub lizard (*Sceloporous woodi*). In addition to converting lizard habitat to human habitat in the form of roads, homes, and citrus fields, development prevents natural lightning-sparked fires from sweeping freely across the landscape [Smith 1999]. For decades, fires have been seen by humans as destructive, rather than beneficial, and suppressed. Human prevention of such natural fires has led to overgrowth and increased shading and leaf litter, gradually shrinking the open sandy areas in which Florida scrub lizards live.

Though there are no clear data regarding extinctions and recolonizations of lizards in the scrub, the distribution of the taxa suggests that it is frequent and may be especially common in small patches [Branch et al. 1999, 3, 22]. Human

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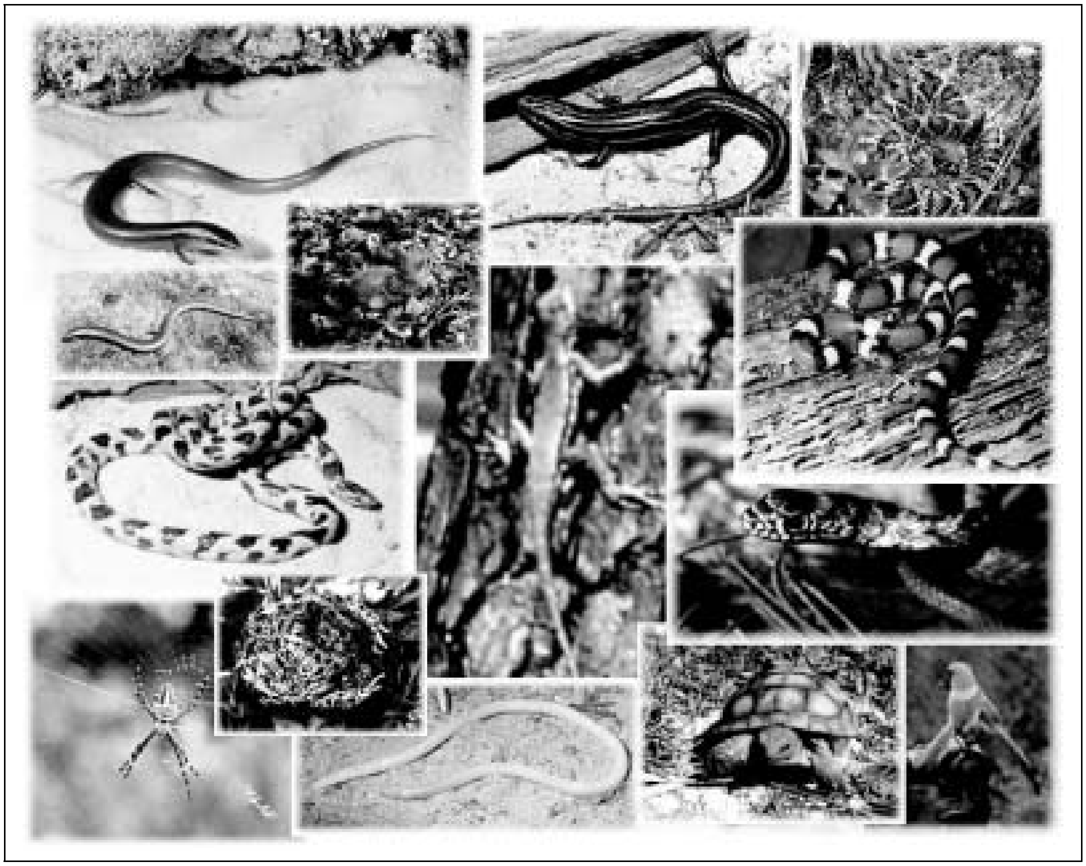

Figure 1. The Florida scrub family. From top left to bottom right: blue-tailed mole skink, southeastern five-lined skink, eastern diamondback rattlesnake, sand skink, Chuck-Will's-Widow, scarlet kingsnake, short-tailed snake, **Florida scrub lizard**, eastern coachwhip, silver-backed argiope, gopher frog, Florida worm lizard, Florida gopher tortoise, Florida scrub jay.

development has resulted in fragmentation, which creates barriers between patches of scrub that prevent lizards from migrating to repopulate areas and exchange genetic information [Branch and Hokit 2000]. Lizards in small scrub patches in urban areas in Titusville and Naples were far less genetically diverse than those in the Jonathan Dickinson State Park, which has about 1,900 ha of continuous scrub [Branch et al. 1999, 52].

Fires are an integral component of the natural scrub ecosystem and without human intervention would occur in a given area approximately once every 6–20 years. In their absence, shrubs and trees become overgrown and many species are displaced, including scrub lizards. Alterations made to the environment make the stoppage of fire-fighting insufficient for full scrubland recovery. Instead of burning thousands of acres, naturally started fires run into concrete or asphalt "firebreaks" or are extinguished to prevent damage to property, and controlled fires must take their place. Controlled burning allows the amount of fuel to be lowered to safe levels [Smith 1999] and can create areas of scrub in different stages of growth alongside one another so that there will be refuges to which small animals and insects may return [Fire in the Florida scrub 2000].

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Controlled burning must be done carefully, as accumulation of fuel may cause fires to become uncontrollable. Furthermore, scrub oaks grown to the size of trees will survive if they are not cut back first. Under natural conditions, scrub oaks would be killed before full growth by above-ground fire and sprout up again from their root systems.

The fragmentation of scrub patches has caused even more problems for scrub lizards. Lizards are much more vulnerable to local extinction in small patches; while these patches may provide good stepping stones for lizard movement between preserves, larger patches must be kept intact to sustain a stable population. The precise reasons for different survivorship, density, and recruitment rates in small and large patches are unclear [Branch et al. 1999, 71]. Some of the vulnerability experienced by small patches may be attributable to stochastic demographic processes: In the smallest patches, there are fewer than a dozen individual lizards, and they may be more susceptible to predation since there is a higher ratio of perimeter to sandy area.

In addition to controlled burning, conservation measures should include habitat preserves whose spatial distribution corresponds to the characteristics of the scrub lizards. Although an assortment of small reserves may protect as many vertebrate species as a single large reserve, the distribution of these small reserves will have a tremendous impact on individual species. The most stable populations of scrub lizards occur in patches with large amounts of bare sand that are close to other stable patches. Scrub lizards are more vulnerable than race-runners, a similar species of lizard, because they have a lower ability to disperse and are more habitat-specific, being unable to live in areas with dense grass cover, mesic flatwoods, old fields, dry depression marshes, and very barren areas [Branch et al. 1999, 24]. While race-runners have a home range of up to 13,000 m^2, home ranges for scrub lizards are 800 m^2 and 400 m^2 for males and females, respectively.

Genetic diversity correlates strongly to geographic distribution, since scrub lizards have extremely limited ranges and tend to stay in the patches in which they hatch (only 10% migrate). Lizards from the five largest scrub ridges have distinct mtDNA (mitochondrial DNA), and a representation of each should be preserved for the sake of genetic diversity. The portion of total genetic diversity observed among populations within ridges was 17.5%, and the portion that occurred within local populations is 10.4% [Branch et al. 1999, 53].

Estimating F_a, S_j, and S_a

To determine fecundity, we use the data provided, as well as additional background information on scrub lizards. Measuring fecundity—the number of hatchlings one female lizard can produce in one year—first requires knowing how many clutches of eggs a female can lay. Female lizards are capable of 3–5 clutches per year. Furthermore, mature lizards become sexually active and able earlier in the season than the younger females. Therefore, we estimate that

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young females (age 1) lay an average of 3.5 clutches per season, while mature females (age 2 and 3) lay an average of 4 clutches per season.

We determine the number of eggs per female per age group by using the equation provided in the problem statement (clutch size $= 0.21l - 7.5$) to determine clutch size, then multiplying clutch size by the estimate of number of clutches per season (**Table 1**).

Table 1.
Number of eggs laid per female, every season, divided by age group.

Age (years)	Number of Eggs
1–2	7.4
2–3	16.9
3–4	17.0

To determine how many total eggs are laid per season, we multiply the values for eggs per female per age group by the number of females in that age group and add over age groups. The sum (901.7) is divided by the total number of females (105) to get the number of eggs laid per female (8.6).

On average, 95% of eggs survive into hatchlings. Therefore, to determine fecundity, the eggs/female ratio is multiplied by 0.95, resulting in a fecundity of 8.2 eggs/female.

To determine the survival rate of juvenile lizards, the number of age-1 lizards (180) is divided by the number of age-0 lizards (972). The resulting quotient is $180 \div 972 = .185$, or 18.5% of lizards survive their first year.

Determining the survival rate of adult lizards is similar. By dividing the number of age-2 lizards (20) by the number of age-1 lizards (180), we find that the survival rate of young adult lizards is 11.1%. For the survival rate of older "senior" lizards, the number of age-3 lizards (2) is divided by the number of age 2 lizards (20), resulting in a survival rate of 10%. We assume that no age-3 lizard lives to be 4 years of age.

To determine the overall survival rate of adults for this sample, the survival rate of young adults and the survival rate of senior adults are weighted and then averaged. To weight the survival rates, the rate for each age group is multiplied by the number of members of that age group, as in **Table 2**; the resulting average adult survival rate is 11%.

Table 2.
Calculation of overall survival rate.

	Survival rate	No. of members	Weight	
young adults	0.111	20	2.22	
senior adults	0.100	2	0.20	Weighted survival rate
	Total	22	2.42	2.42/22 = 0.11

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Developing Functions for F_a, S_j,S_a, and C

Fecundity and survivorship appear to depend both on patch size A and on area h of sandy habitat. But patch size and sandy habitat are related via

$$h = .3165A + 2.31, \tag{1}$$

with correlation .986. We use area of sandy habitat as the better predictor; it makes more sense to model the lizard population by the area in which it lives instead of by the area that surrounds its living space.

Since density is measured by lizards/hectare, we must consider patch size and use **(1)** to convert to area of sandy habitat.

Since fecundity, survivorship of juveniles, and survivorship of adults all have upper bounds (levels at which physical biology presents limits), we model these quantities by logistic regressions:

$$F_a = \frac{10.33}{1 + 1.421e^{-0.0957h}}, \qquad S_j = \frac{0.179}{1 + .89e^{-0.169h}}, \qquad S_a = \frac{0.139}{1 + 1.93e^{-0.123h}}, \tag{2}$$

where F_a is the fecundity, S_j is the survival rate of juveniles (aged 0–1), S_a is the survival rate of adults (aged 1–3), and h is the sandy habitat area in hectares.

We also regress the carrying capacity of a scrub patch on the desired category and the area of sandy habitat. To do so, we make three assumptions:

- The measured of density D is in terms of lizards/hectare of scrub, not in terms of lizards/hectare of open sandy habitat.
- Since the scrub patches have existed for multiple years, each scrub patch is currently at its carrying capacity, as demonstrated by the provided density data.
- There is an upper bound to density.

Because of the third assumption, a logistic model would be the best; unfortunately, there is no way of calculating or extrapolating from the information provided the order of magnitude of such an upper bound. Unlike the vital statistics, where there are clear limits to how many eggs a female can lay and how long lizards can live, density has no clear limit. A logistic model of the given data would create an upper bound of about 80 lizards/hectare, a figure that could certainly be higher.

Therefore, for a better model we use power regression, getting for the density D of the scrub patch, in lizards/hectare,

$$D = 36.93h^{0.221}.$$

This regression has a high correlation (.937). Since carrying capacity is measured in total number of lizards, the scrub patch area of each patch must

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be multiplied by the density equation to determine the carrying capacity C for each patch:

$$C = DA = 36.93Ah^{0.221}.$$

This model can help determine if certain patches of scrub are suitable for lizard "transplantation," or if these patches are already over their capacity and should not have new lizards introduced.

Probability of Surviving During Migration

The data include a probability distribution of distances traveled by surviving lizards. That histogram gives the probability of a lizard going d meters, given that it survived, or $P(d \mid S)$. Then

$$P(d \text{ and } S) = P(S) \times P(d \mid S), \tag{3}$$

where $P(S)$ is the probability of a lizard surviving and d is distance in meters.

Using release/recapture data from the Florida Game and Fresh Water Fish Commission, we calculate the overall survival rate of the 10% of lizards who migrate:

$$P(S) = \frac{\text{lizards released}}{\text{lizards recovered}} = \frac{227}{71} = .3128 = 31.3\%.$$

Using this probability in **(3)**, we arrive at the entries in **Table 3**.

Table 3.
Probability of survival as a function of distance traveled.

Distance traveled (m)	$P(d \text{ and } S)$
50	0.1314
100	0.0782
150	0.0563
200	0.0376
250	0.0063
300	0
350	0.0031

We can now use regression to model the probability of a lizard surviving a journey of d meters. Since lizards cannot have a negative survival rate, a logistic regression seems best. We obtain

$$S = \frac{0.341}{1 + 0.873e^{0.0125d}}. \tag{4}$$

We can find the probability of a lizard surviving the migration between patch i and patch j by calculating the distance d between the two patches and substituting that value into **(4)**.

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Determining Total Landscape Population and Suitability of Patches for Inhabitation

The landscape at the Avon Park Air Force Range contains a wide range of different-sized patches, not all of which can sustain lizards. Before making the distinction, however, we first create a model to estimate the landscape's current population.

We assume that each patch is at its carrying capacity. We find the density for each patch by determining D in the equation

$$D = 36.93h^{0.221},$$

where h is the size of the sandy area (in hectares). To determine population, we multiply this density by the total patch size: $P = DA$, where P is the population and A is the area.

Using this approach on each patch, we estimate the total population to be 25,200 individuals.

We estimate the fecundity F_a, the survival rate of juveniles S_j, and the survival rate of adults S_a using the earlier regression equations **(2)**.

To determine if a scrub patch is suitable for occupation by lizards, it is important to know if the population of the patch is either increasing or decreasing. A patch that has a decaying population is most likely not a good place to which lizards should relocate, while a patch with an increasing population shows that it is flourishing and that the environment is suited to lizards.

With the fecundity (birthrate) and the survival rates of each generation of lizards, we can create a Leslie matrix for each of the 29 patches:

$$\mathcal{L} = \begin{bmatrix} 0 & F_a & F_a & F_a \\ s_j & 0 & 0 & 0 \\ 0 & s_a & 0 & 0 \\ 0 & 0 & s_a & 0 \end{bmatrix}.$$

In this matrix, the birthrates are in the top row, with each column representing one year of age. Going diagonally down to the right are the survival rates. Using MATLAB, we determined the eigenvalues for each of the individual matrices.

The eigenvalues serve as projections of change of the population. An eigenvalue greater than one indicates an increasing population, whereas an eigenvalue of less than one shows a decreasing population that without external influences would eventually die off. Most of the patches have eigenvalues of less than one and will thus eventually have no lizards. However, we must also take into account immigration.

We know that 10% of all juveniles in a given patch tend to migrate, though our results show that no lizards survive past 400 m of travel. For simplicity, we assume that the lizards emigrating from each patch distribute evenly among all patches within 400 m of the original patch. To find the number of lizards

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emigrating, we use the equation for the juvenile population j in terms of the total population P:

$$j = \frac{P - j}{2} F_a,$$

which when solved for j yields

$$j = \frac{PF_a}{2 + F_a}.$$

Since the number of lizards that emigrate is one-tenth the total juvenile population, the total number E of emigrants from a patch is

$$E = \frac{PF_a}{10(2 + F_a)}.$$

To determine whither these lizards emigrate, we need to determine what patches are within 400 m of another patch; how many survive en route depends on the distance to the other patch. The results of our measurements between patches are shown in **Figure 2**.

Figure 2 also gives a rough model of how the population distribution would play out. The green (gray) patches have an increasing population, based on the eigenvalue; they need no immigrants to sustain a population. The yellow (white) patches are less than 400 m away from one or more green patches and thus have a steady influx of immigrants from those patches. The red (dark) patches are not less than 400 m from a green patch and thus receive few immigrants. Thus, the lizard population will become concentrated almost entirely in the patches on the west side of the landscape.

Recommendation: Controlled Burning

We recommend controlled burning. Fires are an integral component of the natural scrub ecosystem and would occur in a given area approximately once every 6–20 years if allowed to spread. When an open area has been restored through controlled burning, lizards from nearby patches can migrate to the freshly burned area and repopulate it; then the highest densities of scrub lizards would be found in areas in the early stages of recovery from fire or other disturbances. As each patch of scrub matures, scrub lizards are expected to migrate to more open and sandy areas [Branch et al. 1999, 71].

Excess vegetation growth can be controlled with a combination of mechanical cutting (where the scrub oaks or other shrubs have grown too large to burn safely) and controlled burning. Some risk is involved, but millions of acres are intentionally burned each year in the United States [Cannell 1999] and the protocol is well developed.

Not only would prescribed burning increase the amount of habitat suitable for native species, it would reduce the possibility of a wild fire like the one that swept through 500,000 acres in Florida in the summer of 1998.

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Figure 2. Landscape at Avon Park Air Force Range, with distances between patches (in hundreds of meters).

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Judges' Commentary: The Outstanding Scrub Lizard Papers

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Introduction

The papers were assessed on

- the breadth and depth of the analysis on each portion of the posed problems,
- the validity and creativity in the proposed models, and
- the clarity and presentation of solutions.

Virtually all papers demonstrated a significant amount of work and thoughtful analysis by the team members. The judges were impressed with the quality of attentive research undertaken by the students on the science involving survival of scrub lizards and were pleased to see a variety of innovative attempts to solve the problems. Making sense of ecological factors affecting the scrub lizard population was essential for successful papers, but the heart of the contest problem was developing a mathematical model that might accurately determine the factors that could contribute to or detract from survival of the scrub lizards.

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The Problem

The Florida scrub lizard is a small, gray, or gray-brown lizard that lives throughout upland sandy areas in the Central and Atlantic coast regions of Florida. The Florida Committee on Rare and Endangered Plants classified the scrub lizard as endangered. The long-term survival of the Florida scrub lizard is dependent on preservation of the proper spatial configuration and size of scrub habitat patches.

The problem was written by Prof. Grant Hokit of the Dept. of Natural Sciences at Carroll College in Helena, Montana. He and his colleagues from the University of Florida have conducted extensive research on scrub lizards and their habitat, some of the results of which appear in this problem.

The Data

It is conjectured that fecundity and survival rates of the scrub lizard are related to the size and amount of open sandy area of a scrub patch where they live. Students were asked to deduce from small samples of data a pattern for fecundity and survival rates. The students were also provided the positions of scrub patches relative to one another and were asked to describe the impact of lizard migration on the survival rate of lizards in these patches. [Note: It is still not known why the scrub lizard migrates.]

The Science

Students with a background in ecology recognized that the plight of the scrub lizard is very similar to the plight of other endangered species. Research into the various factors that affect the habitat of the lizard was essential, because the maintenance of a livable habitat is just as important as the understanding of the impact that the habitat has on the survivability of the lizards. One of the problems posed related to the maintenance of the sandy areas by occasional controlled burning. After researching the habitat dynamics of the scrub lizards, students were asked to make recommendations to preserve the habitats and to discuss obstacles they might encounter to their recommendations.

Then came the students' understanding of reproductive rates within different age groups and the survivability of offspring, youths, and adults under different conditions. The term "fecundity" lent itself to more than one interpretation. The migration of juvenile lizards introduced a factor that complicated the population model.

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The Model

The teams used a variety of modeling techniques. To estimate parameters such as fecundity and survival rates, some students extrapolated from the given data and some accessed additional data. To predict long-term survivability, some teams conducted simulations and others used Leslie matrices to determine which patches could sustain the lizards.

Interesting and viable probability models, as well as informative simulations, were used to analyze the migration of lizards from one patch to another. The geometry of the migration required complex modeling, taking into account the positions and sizes of patches relative to one another.

An essential part of the modeling process is clearly stating the underlying assumptions. It was enjoyable and informative when teams interpreted the results of the model with respect to the simplifying assumptions. Often, students believe that the judges know the correct answer and have absolute knowledge about the model development, so that there is no need to fill in the details of the modeling process. This belief is wrong—good papers must carefully provide these details.

Students wrestled with their responsibility to transform an ill-defined problem into a well-defined problem. The migration component of this problem provided only ideas, and the students had nearly a clean slate to begin the analysis. The motivation and dynamics for migration of scrub lizards is almost completely unknown. Therefore, the modeling process was limited to an empirical and not explicative model.

Other interesting perspectives on modeling were seen on papers that suggested burning schemes or other ways to keep the scrub patches from being overrun and uninhabitable for the lizards.

The Analysis

Analysis distinguished the Meritorious and Outstanding papers from the others, and the thoroughness with which that analysis was done distinguished the Outstanding papers from the Meritorious. Some teams used modeling that was less sophisticated but verified their model with simulations. This is acceptable as long as they describe their modeling process and show reasonable results.

Other teams used classic models, such as developing Leslie matrices for the patches and then basing their conclusions on the eigenvalues of the matrices. Many used the exponential distribution to describe the survival pattern of the lizards.

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Presentation

There were great variations in the quality of the write-ups. Thoroughness is essential, and conciseness is necessary for the one-page summary. Some papers revealed great potential from the modeling perspective but were difficult to follow and therefore problematic to assess. Others developed good models but failed to interpret the models in the context of the issues raised.

On the other hand, some papers had page after page of well-written perspectives on the issues but failed to do adequate mathematical modeling. A qualitative approach must be accompanied with a quantitative analysis.

Failure to document sources properly kept papers from rising to the top. Papers that revealed a comprehensive review of available resources and documented where those resources were referenced showed intellectual maturity that was appreciated and valued by the judges.

Conclusion

Reading and judging the ICM papers was an enjoyable experience. It was clear that many students worked very hard on the project during the four-day period, and the judges were impressed. The interdisciplinary nature of this problem opens the door for creative solutions from many perspectives, and problems of this type enlighten students to the broader challenges associated with biodiversity and survival of endangered species.

About the Authors

Gary Krahn is the Head of the Department of Mathematical Sciences at the U.S. Military Academy at West Point. His interests include the study of generalized de Bruijn sequences for communication and coding applications. He enjoys his role as a judge and Associate Director of the ICM.

Marie Vanisko is in her 31st year of teaching undergraduate mathematics at Carroll College in Helena, Montana, and has been active in Project INTERMATH. She is interested in seeking out useful applications of mathematics to share with her students and in developing technology modules to enrich the mathematics classroom. Having served as a judge for the MCM for many years, she found it very interesting to judge the ICM for the first time this year.

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Author's Commentary: The Outstanding Scrub Lizard Papers

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Introduction

I recall watching the astronauts take the first steps on the Moon and the contagious euphoria that swept the country after such a remarkable technological achievement. Technology has not been idle in the last three decades, as we have witnessed many innovations that have truly changed the world.

Despite such achievements, human civilization is still completely dependent on natural systems. The anthropogenic systems that provide the life support for our people in space are no substitute for the natural systems that sustain the billions on earth. The natural ecosystems that provide clean air and water, food, and shelter, are unlikely to be replaced by technological systems in the foreseeable future. These natural systems must be maintained to preserve our existence.

Anthropogenic systems have one major advantage over natural systems: Their status is easily monitored with calibration instruments that we know and understand because we built them. Thermostats, carbon monoxide detectors, and computer-controlled fuel injectors are commonplace on modern automobiles. Such calibration instruments are difficult to recognize for natural systems. We must rely on our incomplete understanding of natural systems to identify when these systems are endangered. Consequently, we often find ourselves creating costly environmental problems.

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The Fragmented Landscape of Southern Florida

As one example, human activities have endangered the fresh water resources in Southern Florida. Upland habitats provide a vital function in water regulation and purification. The summer rains supply fresh water to Southern Florida and percolate through the upland soils on their way to the wetlands and swamps that act as natural reservoirs.

However, these uplands are also ideal locations for urban development and citrus agriculture. Consequently, urbanization and citrus agriculture have converted the major portion of upland habitats. Now the summer rains are augmented by agricultural runoff and especially by runoff from heavily fertilized lawns and golf courses. The downstream reservoirs, both natural and human-made, are suffering from eutrophication effects such as oxygen depletion and changes in microbial activities.

Florida and federal taxpayers are now paying hundreds of millions of dollars to design and implement a water-management system that saves the fresh water needed by Floridians and saves national treasures such as the Florida Everglades. Although the urban and agricultural development is undoubtedly an economic boom for Florida, we are now paying for the unaccounted cost of disturbing a vital natural system.

Had we an instrument to assess the impact, we might avoid further damage to this natural water system and the consequent cost of rectifying the damage. Also, such an instrument might have prevented the problem by indicating how much development was possible before experiencing significant damage.

Biodiversity may provide a barometer for assessing the impact to natural systems. Biodiversity can be considered the sum total of all the genes, species, and ecosystems that exist on our planet. Because everyone can relate to what a species is, biodiversity is often thought of in terms of species diversity. It logically follows that if all species are persisting in a natural system, then the system must be stable. Unhealthy systems experience a decrease in biodiversity.

The Florida Scrub System

The Florida scrub system is part of the mosaic of ecosystems that cover the peninsula of Florida. Scrub is a xeric (dry) upland system surrounded by the more hydric (wet) systems that dominate the South Florida landscape. Scrub is patchily distributed across the Florida landscape, occurring only on well-elevated, well-drained areas such as ancient sand dunes. Florida scrub contains the highest number of endemic species (species found in no other type of habitat) for any terrestrial ecosystem in the Southeastern United States and thus has high biodiversity. Unfortunately, because of the destruction of scrub habitat, many of these species are listed as endangered or potentially

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endangered. The Florida scrub lizard (*Sceloporus woodi*) is one such species.

Because of our poor understanding of ecological processes, we do not know how much scrub habitat is required to maintain scrub biodiversity. In fact, it is difficult for us to predict how much habitat is necessary to maintain even a single species. We are only beginning to understand the processes that influence species distribution patterns. We hope that by focusing on one or a few species, we can ultimately piece together an understanding of processes that influence overall biodiversity. If we can successfully model and predict trends in scrub lizard populations, we can better understand the processes important to the persistence of scrub lizards. More important, by understanding the habitat distribution needs of the Florida scrub lizard, we can better appreciate the needs for all scrub organisms.

Such a problem requires an integrative approach. The scrub lizard thrives in habitat that exists in patches across the Florida landscape. This spatial component requires researchers to understand not only ecological processes inside scrub patches but also processes that influence the dispersal of lizards between scrub patches. To achieve success, population biology, landscape ecology, computer modeling, and mathematics have to be integrated. This was the problem presented to students in this year's Interdisciplinary Contest in Modeling.

Formulation of the Contest Question

In 1994, the Department of Defense provided funds to conduct amphibian and reptile surveys on the Avon Park Air Force Range (APAFR). This area is set aside for bombing practice for military aircraft and ironically contains some of the best-preserved scrub habitat in Florida.

Scrub organisms are fire-adapted. In fact, many scrub organisms cannot persist without periodic fires that thin dense, senescent vegetation and open up areas of open sandy habitat. Not only are fires allowed to burn on APAFR, the natural resource staff initiates periodic controlled burns to help manage scrub habitat. Because of private land issues elsewhere in the state, prescribed burning is seldom employed and the scrub habitat suffers.

My colleagues (Lyn Branch and Brad Stith of the University of Florida) and I were awarded funds to survey for rare and endangered species on APAFR. We immediately recognized the potential to collect valuable demographic, dispersal, and habitat data concerning the Florida scrub lizard.

We mapped all 95 scrub patches on APAFR: We used a geographic information system (GIS) and infrared aerial photos to delineate the boundaries of the patches, calculate patch areas, measure vegetation density, and construct digitized maps of the landscape. We conducted surveys in each patch to determine the presence of scrub lizards and to establish a baseline to compare occupancy patterns with patterns predicted by mathematical models. We also established eight trapping grids, each one hectare in size, in eight different patches that ranged in size from 11 to 278 hectares. These trapping grids were visited ev-

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ery month for two years; mark/recapture techniques allowed us to estimate density, survivorship, and fecundity (birth rate) for lizards in all eight patches.

We also conducted dispersal studies. Although radio telemetry can provide direct measurements of dispersal behavior, scrub lizards are too small to burden with typical radio transmitters. Smaller transmitters cost too much to afford the hundreds necessary to get large sample sizes, and the batteries last for only a few weeks. We assessed dispersal indirectly. We simply marked hundreds of lizards, released them, and walked transects to recapture lizards up to months after their release date. The distance to the release site was recorded for each recaptured lizard. In this manner, we could estimate how distance from the release site was associated with recapture rates. We also tested lizards in enclosures to assess how effectively they moved through different types of vegetation and across water barriers.

This initial research was the source for all of the data provided in the Contest, and the students were assigned the task of modeling a *metapopulation* (group of populations connected by dispersal) of scrub lizards on the north end of the APAFR.

Although my colleagues and I had published a logistic regression model using the same data [Hokit et al. 1999], the model was static and did not include dynamic demographic and dispersal processes. We recently published the results of two dynamic models [Hokit et al. 2001], but both models include very general assumptions about population demographics. For example, one model assumes that vital rates (survivorship and fecundity) are equivalent for different patches. This simplifying assumption makes the modeling easier but does not incorporate what we know from other analyses: Patch size is positively correlated with survivorship, fecundity, and density.

Thus, it was up to the students to design a spatially explicit (specific for a particular landscape), dynamic landscape-scale metapopulation model that incorporates patch specific vital rates and dispersal. Such a model has yet to be published for any species on any landscape, so the Contest was truly an original challenge for the students. Furthermore, students were required to address policy and management issues concerning the scrub lizard and Florida scrub habitat.

Response to Student Solutions

I was genuinely impressed with the student solutions to the problem. The creativity and range of approaches were remarkable. I was amazed at how different approaches resulted in well-thought-out and highly accurate solutions. I could gauge the accuracy of the modeling solution by testing the model predictions against known occupancy patterns for the APAFR landscape. Many models were within one or two patches of "predicting" the actual occupancy patterns on the landscape.

Many papers introduced me to new perspectives and approaches for such

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modeling problems; as a result, I'm motivated to learn new modeling strategies. Some papers utilized a traditional Leslie matrix coupled with dispersal models. Others used an incidence function approach. Still others incorporated neural-net modeling and polygonal representations of the actual landscape. The polygons were then used to model not only dispersal rates but also the probabilities associated with the direction of dispersal.

Including dispersal dynamics was one of the more challenging aspects of the problem. Given the crude nature of the dispersal data (e.g., recapture rates vs. distance from release site), it was a challenge to estimate survival probabilities for lizards moving between patches. Although seemingly simplistic, many papers arrived at the assumption that survival probabilities were probably correlated with recapture probabilities. Many animal studies have demonstrated just such an association between recapture and survival probabilities. Currently, we can only assume that the same is true for scrub lizards.

The best papers integrated policy and management options with their metapopulation model, resulting in prescribed treatments for specific habitat patches. These papers not only predicted which patches could support scrub lizard populations but also created a schedule of controlled burns to enhance and maintain scrub habitat. This approach combined the best science, math, and policy to arrive at a truly integrative and interdisciplinary solution.

Conclusions

The problem faced by the Florida scrub lizard is not unique. Many species are endangered due to habitat destruction and fragmentation. Although not as newsworthy as global climate change, ozone depletion, or acid rain, habitat destruction is by far the leading threat to biodiversity (although the former factors may lead to habitat destruction). Some estimates project that without careful management of habitat destruction, 10% to 20% of extant species will go extinct within the next few decades. Extinction balanced by complementary speciation (evolution of new species) presents no great risk to species diversity. However, an extinction rate of 10% to 20% over a few decades rivals major extinction events of the past, including the one that saw the demise of the dinosaurs. Thus, it is the rate of extinction, not extinction itself, which is problematic. A high rate of extinction will jeopardize biodiversity. If biodiversity is an accurate barometer of ecosystem health, we may be jeopardizing more than the scrub lizard's future.

There is much work to be done before we can be confident that our modeling strategies are accurate, robust, and generally applicable to many species. We are only beginning to understand the subtlety of the processes that act across spatial and temporal scales to influence the distribution of species and the functioning of natural systems. With such talented and well-motivated students, we may reach sufficient understanding to allow for the continued maintenance of our life-support system.

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Acknowledgments

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About the Author

D. Grant Hokit is Associate Professor of Biology at Carroll College (Montana), where he has been since 1996. He has a B.S. (1986) from Colorado State University in Wildlife Biology and a Ph.D. (1994) in Zoology from Oregon State University, where he did amphibian research in behavioral and population ecology, including research on UV-b radiation and amphibian declines. He did a post-doc in Wildlife Ecology and Conservation at the University of Florida from 1994 to 1996, where he engaged in scrub lizard landscape ecology research.

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Classroom Scheduling Problems: A Discrete Optimization Approach

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Introduction

Every year, colleges and universities face the problem of assigning classrooms to satisfy the needs of courses, faculty, and students. Classrooms and space are limited, and certain conflicts must be avoided; more often than not, a solution cannot be found to satisfy everyone's requirements. Our main objective was to find an optimal solution to satisfy the majority of people involved at our campus, the University of Minnesota–Morris (UMM). In the long run, our mathematical model can benefit many secondary schools, vocational schools, colleges and universities; and it could be extended to other types of scheduling problems such as airline flights and manufacturing systems (see Kolen et al. [1987], Dondeti and Emmons [1986], and Mangoubi and Mathaisal [1985]).

At the University of Minnesota–Morris, not all courses can be scheduled at the times requested by professors. The university first needs to find a systematic way to allocate available rooms and time periods to courses in the "best" possible way. Second, each department or discipline needs to assign professors to the courses in the "best" possible way based on constraints provided by the professor or the course.

A typical classroom scheduling problem can be modeled as an integer linear programming problem (ILP) (Carter [1989], Carter and Tovey [1992], Ferland and Roy [1985], Garey and Johnson [1979], Glassey and Mizrach [1986], and Sierksma [1996]). An ILP is an optimization problem that consists of a linear

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objective function, linear constraints, and discrete variables. Mathematically, an ILP can be written as

$$\{\text{maximize}\, \vec{c} \cdot \vec{x} : A\vec{x} \leq \vec{b},\ \vec{x} \geq \vec{0},\ \vec{x} \text{ integer}\},$$

where $\vec{x}$ is the vector of decision variables, and $\vec{c}$, $\vec{b}$, and the constraint matrix A are given data. Thus, the objective function of the problem is a linear function whose value we want to optimize subject to the given constraints.

The constraints for any scheduling problem may vary from one institution to another and the differences will be reflected in the ILP. A few of the general constraints of a classroom scheduling problem include time availability or lack thereof, the size of the room, how well equipped the room is, the projected enrollment of the students in the courses, and availability of faculty members in terms of appropriate courses. Therefore, the University of Minnesota–Morris's Classroom Scheduling Problem (UMM-CSP) is defined as finding the best assignment of classrooms at a given time that meets the availability of the faculty members and the requirements of the courses.

There are two major parts to this paper. First, we show formulations of two ILPs that provide mathematical models to solve the (UMM-CSP). Then we solve the ILPs using data from the Mathematics Department at University of Minnesota–Morris and present the results.

Integer Linear Programming Model

We describe two ILPs that we then use to solve the (UMM-CSP). To formulate these ILPs as mathematical models, we derive a list of linear inequalities and an objective function that correspond to the *real* constraints of the scheduling problem in (UMM-CSP).

Time Periods, Mathematics Courses, and Classrooms

When UMM switched to the semester system in Fall 1999, a typical course was designated with 4 credit hours, meaning that in a week the total amount of class time should be about 200 minutes. Thus, classes usually meet for 65-minute periods on MWF or a 100-minute periods on TTh. Although most (about 90%) of the courses are 4 credit hours, a few meet TTh for 50 minutes each and carry 2 credits. For efficiency, *two* 2-credit courses are scheduled during the 100-minute time period on TTh.

We use the time period indices in **Table 1**.

A few rooms around campus are usually scheduled with mathematics courses. In addition, there is a set of courses that are offered during both the Fall and Spring semesters. **Tables 2** and **3** describe the indices for the classrooms and courses.

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Table 1.
Indices for time periods.

Index	Meeting Days	Actual Meeting Times
1	MWF	8:00–9:05
2	MWF	9:15–10:20
3	MWF	10:30–11:35
4	MWF	11:45–12:50
5	MWF	1:00–2:05
6	MWF	2:15–3:20
7	MWF	3:30–4:35
8	TTh	8:00–9:40
9	TTh	10:00–11:40
10	TTh	12:00–1:40
11	TTh	2:00–3:40
12	TTh	4:00–5:40

Table 2.
Indices for classrooms.

Index	Actual Classrooms	Comments (capacity of room)
1	MRC 10	Computer classroom for Calculus (37)
2	SCI 1020	General (70)
3	SCI 1030	General (40)
4	SCI 1040	General (20)
5	SCI 2185	General (24)
6	SCI 2200	General (46)
7	SS 136	General (50)
8	SS 245	General (70)

The Room-Time-Course ILP

We exhibit the ILP model for which a feasible solution assigns math courses to time periods and to classrooms based on certain criteria or constraints. The constraints are:

1. Certain math courses (Calculus 1 and 2) have to be in the computer classroom MRC 10.

2. For courses with multiple sections, the sections must be offered at different times.

3. No course with more than 20 students as its maximum can be assigned to rooms such as SCI 1040.

4. Every course must be assigned to exactly one room at some time period.

5. At most one course can be assigned to a room at any time.

6. Courses such as Linear Algebra or Differential Equations are not to be offered during time period 4 because that is when most large science lecture classes are held.

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Table 3.

Indices for mathematics courses for Fall and Spring.

Fall

Course	Sections	Index
(Calc 1) Math 1101	(5)	1,2,3,4,5
(Calc 2) Math 1102	(2)	6,7
(Pre-Calc) Math 1011	(2)	8,9
(Intro. Stat) Math 1601	(4)	10,11,12,13
(Basic Alg) Math 0901	(1)	14
(Survey Calc) Math 1021	(1)	15
(Calc 3) Math 2101	(1)	16
(Linear Alg.) Math 2111	(1)	17
(Pure Math 1)Math 2201	(1)	18
(Diff. Eq.) Math 2401	(1)	19
(Prob.) Math 2501	(1)	20
(Stats Mthd) Math 2601	(1)	21
(Geom.) Math 3211	(1)	22
(Discrete) Math 3411	(1)	23
(Data Analy.) Math 3601	(1)	24
(Real-Complex) Math 4201	(1)	25
(Topics in Stats) Math 4650	(1)	26

Spring

(Calc 1) Math 1101	(3)	1,2,3
(Calc 2) Math 1102	(4)	4,5,6,7
(Pre-Calc) Math 1011	(1)	8
(Intro. Stat) Math 1601	(4)	9,10,11,12
(Survey Math) Math 1001	(1)	13
(Calc 3) Math 2101	(1)	14
(Linear Alg.) Math 2111	(1)	15
(Hist. Math) Math 2211	(1)	16
(Math Stats) Math 2611	(1)	17
(Pure Math 2) Math 3201	(1)	18
(Op. Res.) Math 3401	(1)	19
(Mgmt Sci) Math 3501-3502	(1)	20
(Data Analy.) Math 3611	(1)	21
(Abst.–Topics) Math 4231	(1)	22
(Biostat) Math 4601	(1)	23

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7. A few courses may not be scheduled back-to-back.
8. Certain lower-level mathematics courses must be taught during the MWF time periods.

To formulate an ILP, we need to define decision variables. Without loss of generality, we illustrate the case for the Fall semester. Since for the first part we are deciding on which room and what time period to assign the math courses to, we define the decision variables as

$$x_{t,c,r} = \begin{cases} 1, & \text{if course } c \text{ is taught at period } t \text{ in classroom } r; \\ 0, & \text{otherwise} \end{cases}$$

for each time period $t = 1, \ldots, 12$, for each course $c = 1, \ldots, 26$, and for each room $r = 1, \ldots, 8$.

Each constraint corresponds to a linear inequality or an equality. We translate the constraints as follows:

1. *Every Calculus 1 and 2 section (course index* $1, \ldots, 7$*) has to be scheduled in the computer classroom MRC 10 (room index 1):*

$$\sum_{t=1}^{12} x_{t,c,1} \;=\; 1 \quad \text{for } c = 1, \ldots, 7.$$

2. *For courses with multiple sections, the sections must be offered at different times:*

$$\sum_{\substack{c\,:\,c \text{ is part} \\ \text{of a course with} \\ \text{multiple sections}}} x_{t,c,r} \;\leq\; 1 \quad \text{for } t = 1, \ldots, 12;\ r = 1, \ldots, 8.$$

3. *Every course has to be assigned to exactly one room at some time period:*

$$\sum_{t=1}^{12} \sum_{r=1}^{8} x_{tcr} \;=\; 1 \quad \text{for } c = 1, \ldots, 26.$$

4. *At most one course can be assigned to a room at any time:*

$$\sum_{c=1}^{26} x_{tcr} \;\leq\; 1 \quad \text{for } t = 1, \ldots, 12;\ r = 1, \ldots, 8.$$

5. *That a specific course* $\bar{c}$ *cannot be offered at some time* $\bar{t}$ *translates as:*

$$\sum_{r=1}^{8} x_{\bar{t},\bar{c},r} \;=\; 0.$$

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6. *That two courses $\bar{c}$ and $\tilde{c}$ cannot be offered back to back becomes:*

$$\sum_{r=1}^{8} x_{t,\bar{c},r} + x_{t+1,\tilde{c},r} \;\leq\; 1 \quad \text{for } t = 1, \ldots, 7, 9, 10, 11.$$

7. *That a lower-level math course $\bar{c}$ that cannot be scheduled on a TTh schedule can be represented as:*

$$\sum_{t=8}^{12} \sum_{r=1}^{8} x_{t,\bar{c},r} \;=\; 0.$$

8. *As in any general ILP model, the bounds and the integer constraints of the variables must be included:*

$$0 \;\leq x_{t,c,r} \;\leq 1, \quad \text{integer}, \quad \text{for } t = 1, \ldots, 12;\, c = 1, \ldots, 26;\, r = 1, \ldots, 8.$$

The objective function for room-course-time scheduling is not that crucial because the main purpose is to obtain a feasible solution that satisfies all the constraints. Thus, we choose to **maximize** the linear objective function

$$\sum_{t=1}^{12} \sum_{c=1}^{26} \sum_{r=1}^{8} x_{t,c,r}.$$

That *every course has to be assigned to exactly one room at some time period* implies an optimal value of 26 if a feasible solution exists. The more constraints or restrictions, the greater the possibility that the model is infeasible. For the (UMM-CSP), there is indeed a feasible solution.

The Professor Assignment ILP

At UMM, the class schedule for the academic year is determined during the fall of the previous year. However, when the schedule is being prepared, many departments do not know exactly who will be on leave, let alone who the new adjunct faculty will be the next academic year. Thus, in the Mathematics Department we usually start on the classroom-time-courses scheduling with few professors assigned, if any. Once the classroom-time-courses schedule is determined, we find the best way to allocate professors.

Another computational reason for splitting the project into two parts is to minimize the number of variables in each of the two models. Garey and Johnson [1979] showed that an ILPs is an NP-hard problem, meaning that its computational complexity grows exponentially with the number of input (decision) variables. The first model, on time-room-course assignment, has $13 \times 9 \times 26$ variables; the second, on professor assignment, has 11×26 variables. Partitioning into two problems makes the computations more manageable.

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Based on the outcome of the classroom-time-courses ILP model, we define

$$A = \big\{\{t, c, r\} : x_{t,c,r} = 1, \text{from the time-course-room ILP model}\big\}.$$

We have about 11 faculty. We request that each faculty member submit four preferred courses and three preferred times. For this ILP, to maximize satisfaction among all faculty, the objective function is crucial.

Many faculty put a higher weight on course preferences than on time preferences. We incorporate objective-function coefficients that reflect those preferences, with the objective function maximizing the overall *happiness* factor of the entire department. For assignment $a = \{t, c, r\}$ of time t, course c, and room r to professor p, the coefficient is:

$$k_{p,a} = \begin{cases} 10, & \text{if course } c \text{ and time } t \text{ are requested by } p; \\ 5, & \text{if course } c \text{ but not time } t \text{ are requested by } p; \\ 1, & \text{if course } c \text{ is not requested, but time } t \text{ is, by } p; \\ 0, & \text{if neither course } c \text{ nor time } t \text{ is requested by } p. \end{cases}$$

We maximize

$$\sum_{a \in A} \sum_{p=1}^{10} k_{p,a}\, x_{p,a},$$

where the decision variables are

$$x_{p,a} = \begin{cases} 1, & \text{if } p \text{ is assigned course } c \text{ at time } t; \\ 0, & \text{otherwise} \end{cases}$$

for each professor $p = 1, \ldots, 11$ and for each $a \in A$. For the constraints, we have

1. *Each professor can teach at most one course at each time period:*

$$\sum_{a \in A : a = \{t,c,r\}} x_{p,a} \;\le\; 1 \quad \text{for } p = 1, \ldots, 11.$$

2. *Each professor is assigned to at least two but at most three courses:*

$$\sum_{a \in A : a = \{t,c,r\}} x_{p,a} \;\le\; 3 \quad \text{for } p = 1, \ldots, 11;$$

$$\sum_{a \in A : a = \{t,c,r\}} x_{p,a} \;\ge\; 2 \quad \text{for } p = 1, \ldots, 11.$$

3. *Each course is taught by one and only one professor:*

$$\sum_{p=1}^{11} x_{p,a} \;=\; 1 \quad \text{for } a \in A.$$

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4. *If a particular professor* $\bar{p}$ *is prohibited from teaching course* $\bar{c}$*, then we add the constraint:*

$$x_{\bar{p},a} \;=\; 0 \;\;\text{for } a \in A \quad \text{AND} \quad a = \{t, \bar{c}, r\}.$$

5. *If one of three professors (the statisticians,* $\bar{p}$*,* $\tilde{p}$*,* $\hat{p}$*) must teach a certain course* $\bar{c}$*, then we add:*

$$x_{\bar{p},a} + x_{\tilde{p},a} + x_{\hat{p},a} \;=\; 1 \;\;\text{for } a \in A, \;\&\;\; a = \{t, \bar{c}, r\}.$$

6. *The bounds and integer constraints of the decision variables are:*

$$x_{p,a} \text{ integer}, \qquad 0 \le x_{p,a} \le 1, \;\; \text{for } a \in A;\, p = 1, \ldots, 11.$$

Results and Analysis

We solved the ILP models using optimization software called CPLEX, which stands for *simplex algorithm* written in the C *language*. The simplex algorithm is well known for solving linear programming problems [Sierksma 1996]. The simplex algorithm searches from one basic feasible solution, a solution found at a corner of a polyhedron, to another until it finds a solution that cannot be improved by any of its neighboring corner points. The algorithm also concludes if there is no feasible solution. CPLEX can also be used to solve an integer linear programming problem, although at a much slower speed, by incorporating the *branch-and-bound* algorithm for solving ILPs [Sierksma 1996]. Basically, the branch-and-bound algorithm goes through the phases of either partitioning the problem into two smaller ones and solving their individual linear relaxations (which are basically linear programming problems); or by proving that any partition problem will not yield optimal values that will be better.

Results

The results for the times-courses-rooms assignment problems (Fall and Spring semesters) are given in **Tables 5** and **6** in the Appendix, and results for the professors-courses assignments are in **Tables 7** and **8**. [EDITOR'S NOTE: We omit these tables, since their results pertain to the specific constraints at University of Minnesota–Morris.]

Analysis of the Results

There are not many rooms left for the other 5 departments in just the Division of Science and Math, let alone other departments (about 20) across the campus (and each department offers 10 to 26 sections of courses each semester)!

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Conclusions

We found an optimal and feasible solution to the classroom scheduling problem that satisfied the majority of the people involved.

It is conceivable that not all professors got their preferred times. Indeed, since we solved the room-time-course model first, the times of the courses were already fixed when we attacked to solve the professor assignment problem. In addition, our main concern in the latter problem was assigning the professors to the preferred courses.

An advantage of solving classroom scheduling problems by this mathematical model or other ILP formulations is that it allows the users the freedom to control or add constraints and adjust objective-function coefficients. For example, if a department wanted to require that a professor cannot teach two classes in consecutive time periods, then additional mathematical constraints (inequalities) could be added. Also, the hierarchy on the level of satisfaction between the "course" preference and the "time" preference can be changed by incorporating it into the objective function coefficients. Further customization is possible.

We hope that this project will benefit the University of Minnesota–Morris and other institutions by giving a better applicable solution to the scheduling of classrooms according to the requests of professors and students. In addition, the results provide a foundation based on which other types of scheduling problems can be solved or understood better.

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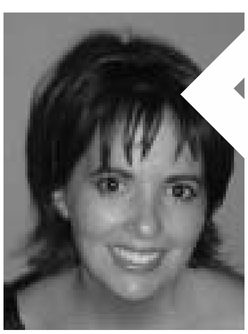

Lora Martin graduated with a B.A. in Computer Science and a minor in Mathematics from the University of Minnesota–Morris in May 1998. She currently enjoys working on a Java Virtual Machine Development Team as a software engineer for the Unisys Corporation in Roseville, MN. Her presentation and research experience proved valuable during team meetings and company functions. Future career plans include graduate coursework in a management-related field.

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The Optimal Positioning of Infielders in Baseball

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Introduction

When asked in the late 19th century to explain his success at the plate, baseball Hall-of-Famer "Wee Willie" Keeler responded quite simply, " . . . I hit 'em where they ain't." While we will always associate this famous quote with Keeler, it is the objective of all batters in the game of baseball to "hit 'em where they ain't." But who exactly are "they"? Evidently, "they" are the nine fielders on the opposing team. In each trip to the plate, a batter attempts to put the ball into play so that it will not be caught or otherwise intercepted by one of the opposing fielders. Conversely, it is the goal of the nine fielders to do just that—to catch or at least get their gloves on a batted ball.

The fielders seek to position themselves so that their collective likelihood of reaching a batted ball is maximized. When facing a right-handed batter known to pull the ball down the third-base line, for instance, the fielders tend to shift to the left.[1] Similarly, when a left-handed pull hitter steps to the plate, the fielders most likely shift to the right.

We develop a mathematical model that uses elementary probability and calculus to determine the optimal positioning of each of the four infielders—the third baseman, the shortstop, the second baseman, and the first baseman—as a function of the distribution of the batter's hits.

[1] We refer to "left" and "right" from the perspective of a spectator sitting behind home plate, not that of the fielders themselves. Hence, the left side of the infield is the area between second base and third base.

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The Model

The first step in developing the model is to establish a method for quantifying the "location" of a fielder or a batted ball in the infield. Let H represent the location of home plate. We assume that any batted ball travels along a straight line through H. Let θ be the angle between this line and the third-base line (measured in degrees) and set $x = \theta/90$. Thus, $x = 0$ represents the third-base line, $x = 1$ represents the first-base line, and $x = 0.5$ represents any point on the line from home plate through the center of second base. Since we are concerned only with fair balls, the range of values of x is limited to $[0, 1]$.

Let X be a continuous random variable representing the location of a batted ball and let $f(x)$ be the probability density function of X. While a number of different density functions might be appropriate models, we adopt one that is particularly simple—the piecewise-linear distribution, defined by:

$$f(x) = \begin{cases} \dfrac{2x}{k}, & 0 \le x \le k; \\ \\ \dfrac{2(1-x)}{1-k}, & k \le x \le 1, \end{cases} \tag{1}$$

where k is a constant.[2] Note that this density function is unimodal with mode k (see **Figure 1**). This means that the batter is most likely to hit the ball to position $x = k$ and that the likelihood of hitting the ball to position $x = j$ decreases as $|j - k|$ increases. Furthermore, since the area under any density function must equal 1, $f(k) = 2$ for every k.

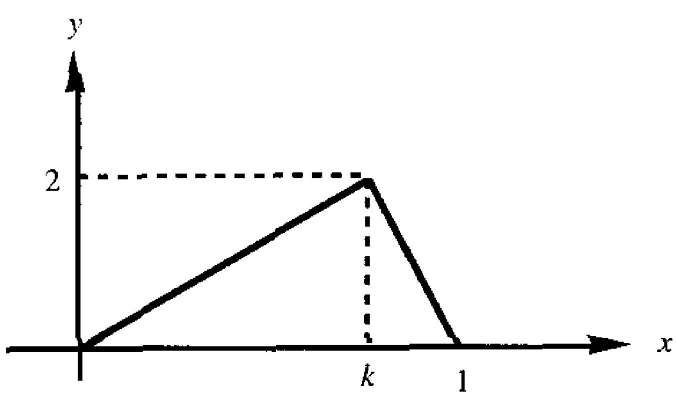

Figure 1. Piecewise-linear density function.

From the definition of density functions, the probability that a ball is hit in the sector between $x = a$ and $x = b$ is the area under the graph of over the interval $[a, b]$; in other words, $P(a \le X \le b) = \int_a^b f(x)\,dx$.

We assume that for each infielder there exists an interval of x-values such that the infielder can, with probability 1, reach any ball hit in that interval. (This

[2]Later we develop a method for estimating k based on data collected for a given batter.

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doesn't mean that the infielder will make the play and retire the batter.) We define the *range* of the infielder as the length of that interval. We accept the conventional infield positioning in the sense that the third baseman is always the leftmost infielder, the shortstop is always to the right of the third baseman, the second baseman is always to the right of the shortstop, and the first baseman is always the rightmost infielder.[3]

To distinguish between infielders, we use subscripts corresponding to the standard position numbers used in baseball scoring. Specifically, the first baseman is denoted by "3", the second baseman by "4", the third baseman by "5", and the shortstop by "6". Let $\vec{x} = (x_5, x_6, x_4, x_3)$ be a vector representing the location of the four infielders, where x_j is the right end of the interval that infielder j can cover. Let $\vec{r} = (r_5, r_6, r_4, r_3)$ be a vector representing the ranges of the infielders. Thus, infielder j can cover the interval from $x_j - r_j$ to x_j (**Figure 2**).

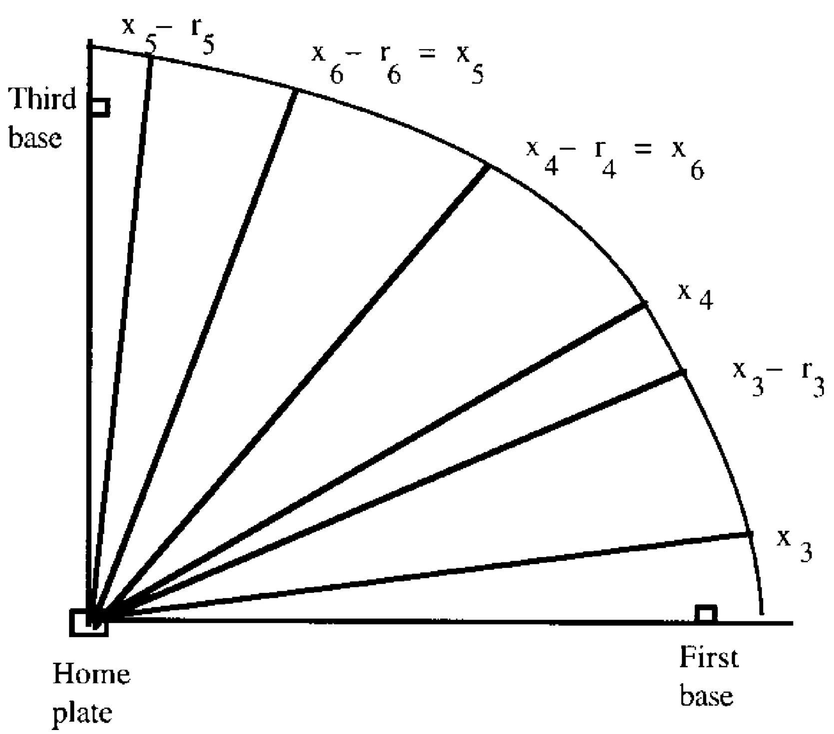

Figure 2. Positioning of infielders and their ranges.

We say that $\vec{x}$ is *feasible* if:

1. $0 \leq x_5 - r_5$,

2. $x_5 \leq x_6 - r_6$,

3. $x_6 \leq x_4 - r_4$,

[3]We are considering only the players' lateral range. Depending on the speed at which the ball is hit, the infielder may have to charge inward or retreat away from home plate to make the play. Some players are more adept at this than others.

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4. $x_4 \leq x_3 - r_3$, and

5. $x_3 \leq 1$.

These five inequalities ensure that all four infielders and their entire ranges remain in fair territory and that the ranges of no two infielders overlap, although their ends may coincide at a point.

We say that $\vec{x}$ is *gapless* if the inequalities in (2), (3), and (4) are replaced by equal signs. This means that the right end of the third baseman's range coincides with the left end of the shortstop's range, the right end of the shortstop's range coincides with the left end of the second baseman's range, and the right end of the second baseman's range coincides with the left end of the first baseman's range. The positioning in **Figure 2** is feasible but not gapless, as there is a gap between the first and second basemen.

Optimal Positioning

Let $L(\vec{x}, \vec{r})$ represent the probability that a set of infielders whose locations are given by $\vec{x}$ and whose ranges are given by $\vec{r}$ will reach a batted ball. Equivalently, $L(\vec{x}, \vec{r})$ is the probability that the batted ball will be in one of the four intervals of the form $[x_j - r_j, x_j]$, for $j = 3, 4, 5, 6$. Hence, for any feasible $\vec{x}$, we have

$$L(\vec{x}, \vec{r}) = \sum_{j=3}^{6} \int_{x_j - r_j}^{x_j} f(x)\, dx.$$

If $\vec{x}$ is gapless, then

$$L(\vec{x}, \vec{r}) = \int_{x_5 - r_5}^{x_3} f(x)\, dx = \int_{x_b}^{x_b + R} f(x)\, dx,$$

where $x_b = x_5 - r_5$ is the left end of the third baseman's interval and $R = \sum_{j=3}^{6} r_j$ is the total range of all of the infielders.

Let

$$L^* = \max\{L(\vec{x}, \vec{r}),\ \vec{x} \text{ is gapless}\}$$

be the maximum probability that can be attained by a gapless location of infielders. We claim that L^* is also the maximum probability that can be attained by any feasible location of infielders. In other words, the optimal location of infielders is always a continuous block—that is, with no gaps between any of them.

To see why this is true, observe that if $\vec{x}$ is gapless, then $L(\vec{x}, \vec{r})$ is the area of "double trapezoid" in **Figure 3**. (We are assuming that the optimal solution should have the mode of the distribution somewhere in the range that can be covered by the infielders.) This can be most easily computed by adding the areas of the two triangles outside the range and subtracting from 1.

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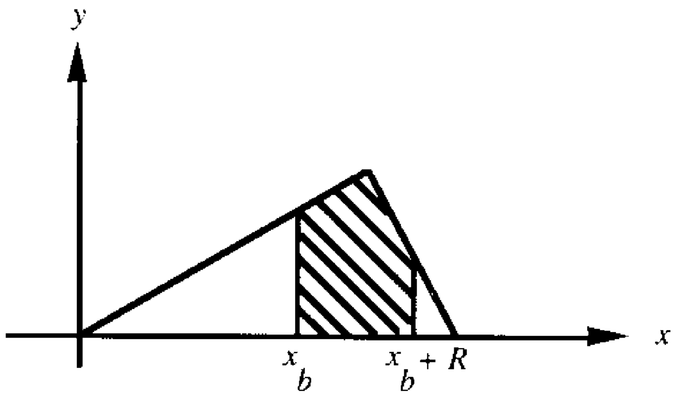

Figure 3. Area covered by gapless location of infielders.

Thus,

$$L(\vec{x},\vec{r}) = 1 - \tfrac{1}{2}x_b f(x_b) - \tfrac{1}{2}(1-x_b-R)f(x_b+R) = 1 - \frac{x_b^2}{k} - \frac{(1-x_b-R)^2}{1-k}.$$

Differentiating gives

$$L'(x_b) = \frac{-2x_b}{k} + \frac{2(1-x_b-R)}{1-k}.$$

Upon setting this equal to 0, we find that L is maximized when $x_b = k(1-R)$. Hence, the optimal gapless location of infielders covers the interval

$$[k(1-R), k(1-R)+R].$$

Furthermore, using **(1)**, we find that the optimal probability is

$$L^* = 1 - k(1-R)^2 - \left[(1-R)^2 - k(1-R)^2\right] = 2R - R^2,$$

which, not surprisingly, is independent of k. Most important, however, is the fact that $f(x_b) = f(x_b+R) = 2(1-R)$ and that $f(x) \geq 2(1-R)$ for all $x \in [x_b, x_b+R]$. Thus, the function values at the two ends of the optimal interval are the same, and the function values in the interior of the optimal interval are bigger than those at the ends.

Now imagine that we place a gap of length $d > 0$ between two of the infielders, say the first and second basemen. This will cause the first baseman to move a distance d_r to the right and the other infielders to move a distance d_l to the left, where $d_l + d_r = d$. The probability that the ball is hit in the interval covered by the infielders is

$$L = L^* - \tfrac{1}{2}d\left[f(a)+f(b)\right] + \tfrac{1}{2}d_r\left[f(x_b+R)+f(x_b+R+d_r)\right] + \tfrac{1}{2}d_l\left[f(x_b)+f(x_b-d_l)\right],$$

where a and b are the endpoints of the gap. We know that $f(x_b) > f(x_b - d_l)$, $f(x_b+R) > f(x_b+R+d_r)$, and both $f(a)$ and $f(b)$ are greater than $2(1-R)$. Therefore,

$$L < L^* - \tfrac{1}{2}d\left[f(a)+f(b)\right] + 2d(1-R) < L^*.$$

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So inserting the gap causes a decrease in the probability that one of the infielders can reach the batted ball.[4]

Estimating k

The piecewise-linear density function f in **(1)** that we've chosen to model the distribution of hits depends on one parameter, the mode k. For our results to be useful, we have to estimate k based on some data collected about a batter's past performance. In particular, suppose that we have a sample $t_1, t_2, \ldots, t_n$ of n locations at which balls were hit in the infield. We use these data to determine the maximum likelihood estimate of k. (We use the letter t to distinguish from the x's used previously to denote the location of the infielders.)

Without loss of generality, we assume that the data are in increasing numerical order, so that t_1 is the smallest observation and t_n is the largest. Then there exists some j such that $t_j \le k < t_{j+1}$. It follows that the likelihood function for k is

$$H(k) = \left[\prod_{i=1}^{j} \left(\frac{2}{k}\right) t_i\right] \left[\prod_{i=j+1}^{n} \left(\frac{2}{1-k}\right)(1-t_i)\right] = \frac{C}{k^j (1-k)^{n-j}},$$

for $t_j \le k < t_{j+1}$, where $C = 2^n \left[\prod_{i=1}^{j} t_i\right] \left[\prod_{i=j+1}^{n} (1-t_i)\right]$ is a constant independent of k. Note that to make this definition complete, we define $t_0 = 0$ and $t_{n+1} = 1$, and all vacuous products are also equal to 1.

For example, if $n = 2$, we have:

$$H(k) = \begin{cases} \dfrac{4(1-t_1)(1-t_2)}{(1-k)^2}, & 0 \le k \le t_1; \\[2ex] \dfrac{4t_1(1-t_2)}{k(1-k)}, & t_1 \le k \le t_2; \\[2ex] \dfrac{4t_1 t_2}{k^2}, & t_2 \le k \le 1. \end{cases}$$

Observe that H is a continuous function of k.

Our goal is to determine the value of k that maximizes $H(k)$. This is equivalent to minimizing the denominator $G(k) = k^j(1-k)^{n-j}$. Then $g'(k) = k^{j-1}(1-k)^{n-j-1}(j-kn)$. Setting $g'(k) = 0$ and solving for k, we see that $k = j/n$ is a critical point of g. There is no guarantee, however, that $t_j \le j/n < t_{j+1}$. If

[4] Our results do not take into account the potential damage that a hit might do. In other words, leaving a gap between the third baseman and the third-base line might allow a hitter to pull the ball down the line for an extra-base hit. This could be worse than leaving a gap in the middle of the infield through which the batter might only be able to hit a single. Often teams place their first and third basemen near the lines, especially in later innings, to prevent these extra-base hits.

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indeed that is true, then $k = j/n$ is a local *maximum* of g, not a local minimum. Consequently, $H(k)$ may (but does not necessarily) achieve a local minimum on $[t_j, t_{j+1}]$. Never does it achieve a local maximum there. So, for example, in the case $n = 2$, if $t_1 \leq t_2 < 0.5$ or $0.5 < t_1 \leq t_2$, then H is monotonic on each of the intervals $[0, t_1]$, $[t_1, t_2]$, and $[t_2, 1]$. On the other hand, if $t_1 \leq 0.5 \leq t_2$, then H is monotonic on $[0, t_1]$ and $[t_2, 1]$ and achieves a local minimum on $[t_1, t_2]$ at $k = 0.5$.

In all cases, we are left to conclude that the global maximum of $H(k)$ must occur at an endpoint of some interval $[t_j, t_{j+1}]$. In other words, the maximum likelihood estimate of k is one of the data points. Thus, all we need do is evaluate $H(t_j)$ for all j and select the one that is greatest.

Example: Suppose that a particular batter has hit 7 balls through the infield during the last two games. Examination of videotape reveals the location of these hits, in the context of our model, as: .21, .27, .30, .33, .36, .40, and .64. Furthermore, assume that the infielders' ranges are given by $\vec{r} = (.16, .20, .18, .12)$. This means that the shortstop has the widest range, followed by the second baseman and the third baseman. The first baseman has the smallest range, as is often the case, especially if there is a runner on first base.

Table 1 gives the values of $H(t_j)$ for $j = 1, 2, \ldots, 7$.

Table 1.
Values of $H(t_j)$ for $j = 1, \ldots, 7$ for the example.

t_j	$H(t_j)$
.21	24.922
.27	31.136
.30	31.108
.33	27.847
.36	22.559
.40	15.156
.64	1.506

Since $H(t_j)$ assumes its greatest value when $j = 2$, the maximum-likelihood estimate of k is $x_2 = .27$. We now can define our density function $f(x)$:

$$f(x) = \begin{cases} 7.41x, & 0 \leq x \leq .27; \\ 2.74(1-x), & .27 \leq x \leq 1. \end{cases}$$

Our goal, however, is to determine the location of our optimal "block." We know that $k = .27$ and that $R = \sum_{j=3}^{6} r_j = .66$. Thus, the left end of the block is at $t_b = k(1 - R) = .09$ and the right end of the block is at $t_b + R = .75$. The third baseman covers the interval $[.09, .25]$, the shortstop covers $[.25, .45]$, the second baseman covers $[.45, .63]$, and the first baseman covers $[.63, .75]$. Presumably, each infielder is able to range

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some given distance to the left and some given distance to the right. The optimal positioning of the infielder within the designated interval must be consistent with this ability to move to either side.

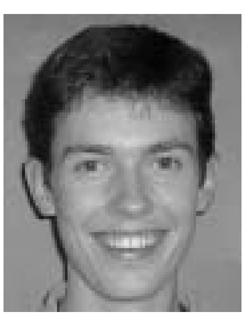
Further Analyses

There are a number of variations to the model described here to explore. First, we might try to adopt a different density function for the distribution of hits. One possibility is a beta distribution $f(x) = cx^{\alpha-1}(1-x)^{\beta-1}$, for some constant c. This density function depends on two parameters, α and β, and thus we have more control over the shape of the distribution. That the optimal positioning of infielders is gapless is still true, but the calculation of the optimal position and the estimation of the parameters are more difficult problems than in the piecewise-linear case. We could also try density functions that are not unimodal, although there are probably not many batters whose hit distribution would be described by such a function.

Our model maximizes the probability that one of the infielders will get to a batted ball. It does not take into account the possibility that even though the infielder gets to the ball, doing so may not actually retire the batter (i.e., get the batter out). So we could create a function $p(x, y)$, which is the probability that an infielder located at position x can retire a batter who hits the ball to location y. Presumably, $p(x, y)$ would be very close to 1 if $x = y$ (meaning that the ball is hit directly at the infielder) and decrease as $|x - y|$ increases. The objective would then be to maximize the collective probability that some infielder retires the batter.

About the Authors

Alan Levine received his Ph.D. in Applied Mathematics from the State University of New York at Stony Brook in 1983. Since then, he has taught in the Dept. of Mathematics at Franklin and Marshall College. Although he has never fulfilled his lifelong ambition to be the statistician for the New York Mets, he did teach a first-year seminar entitled "Math and Sports" in the fall of 2000.



Jordan Ludwick is a December 2000 graduate of Franklin and Marshall College, where he majored in mathematics. He was the preceptor for the Math and Sports course taught by Prof. Levine, and this paper is the result of an independent study project that he did in conjunction with the course. He currently works in the banking industry.

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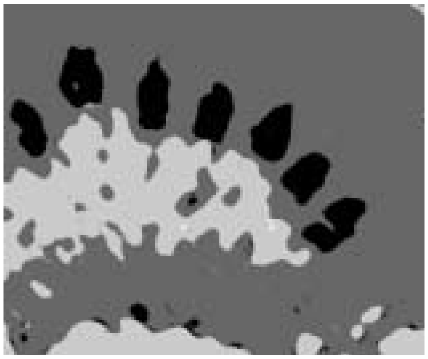

INTERDISCIPLINARY LIVELY APPLICATIONS PROJECT

AUTHOR:
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Who Falls Through the Healthcare Safety Net?

MATHEMATICS CLASSIFICATIONS:
This project is appropriate for a mathematics course intended to serve students majoring in fine arts andthe humanities, frequently referred to as a "liberal arts" mathematics course.

DISCIPLINARY CLASSIFICATIONS:
Health Information Management, Business, Ethics, Sociology, Political Science

PREREQUISITE SKILLS:
An understanding of basic statistical terms and charts

PHYSICAL CONCEPTS EXAMINED:
Actual data sets from the Census Bureau are analyzed concerning those without health insurance, first in general, then relative to those in poverty, and finally broken down by state. The emphasis is on interpreting the statistics and explaining them to others.

COMPUTING REQUIREMENT:
Either a graphing calculator or a computer with spreadsheet, computer algebra system, and/or statistical package.

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Contents

1. Introduction

Many of us take our health insurance for granted; but according to the U.S. Census Bureau report for 1999, more than 15% of the people in the United States have no health insurance, public or private. That is more than 42 million people, many of whom are children. With increasing medical costs, those without insurance frequently have to forego medical treatment or must give up their life savings to pay for it.

This Interdisciplinary Lively Application Project (ILAP) explores groups of people without health insurance in an effort to analyze the problem. All data are from the U.S. Census Bureau.

2. Instructions

group must provide detailed written reports, as instructed in the requirements, and be prepared to discuss the results in class.

This ILAP, with periodically updated data, is at `http://web.carroll.edu/mvanisko/default.htm`. Select `Carroll ILAPs` on the menu page and go to the ILAPs under `Distribution of Wealth`. The tables can be copied and pasted into an Excel spreadsheet. If direct pasting does not put the data into columns:

- Select (highlight) the first column (already highlighted after pasting) and deselect the other columns.
- Under the `Datamenu` item, select `Text to Columns` (the default is `Fixed` and that frequently works, but you can check and see).
- Click on `Finish`—and all the data are in columns. There may be a couple of details that you have to clean up, but they are minor.

3. Requirements

Requirement 1

Table 1 gives a breakdown of individuals not covered by health insurance in 1999. Develop a profile for those without insurance. Show the results graph-

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ically if appropriate. Provide a rationale for why the results turned out as they did.

Requirement 2

Table 2 gives a breakdown of 1999 health insurance coverage status for persons living below the poverty level by selected characteristics. This group represents 11.8% of the United States population, and some people in this group do not qualify for Medicaid (government health insurance for the poor) or Medicare (government health insurance for the elderly). Develop a profile for those without insurance in this group. Discuss why the results turned out as they did. Be sure to look at the percentage who are under 18. Compare and contrast these results to those in **Requirement 1**. How is the profile different for those in poverty than for the general population?

Requirement 3

Go to the Internet site `http://www.census.gov` and examine other Census Bureau data concerning poverty. Determine how "poor" is defined by the Census Bureau. Write a two- to three-page paper on what you have found and be prepared to present your results to the class.

Requirement 4

After completing the first three requirements, summarize your results in the form of a newspaper article that is approximately two pages in length. The goal is to see how effective you can be in translating the numbers into meaningful rhetoric. After all the reports are in, you will select one group's report to submit to the school and/or the city newspaper.

Requirement 5

Table 3 provides a breakdown regarding those without health insurance in each state. Categorize each state in terms of its location. Use **Table 3** to construct boxplots and scatterplots to explore differences in statistics among the states relative to their location and to their population. Compute the mean and median for the group as a whole and for each region of the country and compare the measures of central tendency within and across regions. Provide plausible explanations for variations you see.

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Requirement 6

How could the information given about health insurance coverage be used to convince someone that a change in national health policy is needed? How could the same information be used to convince someone that a change is not needed? What does this tell you about this type of information?

Table 1.

All Persons Not Covered by Health Insurance, by Selected Characteristics: 1999.

Source: U.S. Census Bureau, Current Population Surveys, March 1999 and 2000.

Numbers in thousands

Characteristic	Total Number	Not covered by health insurance: Number Not Covered	Percent Not Covered
All	274,087	42,554	15.5
Sex			
Male	133,933	22,073	16.5
Female	140,154	20,481	14.6
Age			
Under 18 years	72,325	10,023	13.9
18 to 24 years	26,532	7,688	29.0
25 to 34 years	37,786	8,755	23.2
35 to 44 years	44,805	7,377	16.5
45 to 64 years	60,018	8,288	13.8
65 years and over	32,621	422	1.3
Race and Hispanic Origin			
White	224,806	31,863	14.2
White, not of Hisp. Origin	193,633	21,363	11.0
Black	35,509	7,536	21.2
Asian / Pacific Islander	10,925	2,272	20.8
Hispanic origin[1]	32,804	10,951	33.4
Education (persons aged 18 and over)			
No high school diploma	34,087	9,111	26.7
High school graduate only	66,141	11,619	17.6
Some college, no degree	39,940	6,051	15.2
Associate degree	14,715	1,902	12.9
Bachelor's degree or higher	46,880	3,848	8.2
Work Experience (persons aged 18 to 64)			
Worked during year	139,218	24,187	17.4
Worked full-time	115,973	18,984	16.4
Worked part-time	23,245	5,204	22.4
Did not work	29,923	7,921	26.5
Nativity			
Native	245,708	33,089	13.5
Foreign-born	28,379	9,465	33.4
Naturalized citizen	10,622	1,900	17.9
Not a citizen	17,758	7,565	42.6
Household Income			
Less than $25,000	64,628	15,577	24.1
$25,000–$49,999	77,119	13,996	18.2
$50,000–$74,999	56,873	6,706	11.8
$75,000 or more	75,467	6,275	8.3

[1] Persons of Hispanic origin may be of any race.

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Table 2.

Poor Persons Not Covered by Health Insurance, by Selected Characteristics: 1999.

Source: U.S. Census Bureau, Current Population Surveys, March 1999 and 2000.

Numbers in thousands

Characteristic	Total Number	Not covered by health insurance	
		Number Not Covered	Percent Not Covered
All	32,258	10,436	32.4
Sex			
Male	13,813	4,830	35.0
Female	18,445	5,606	30.4
Age			
Under 18 years	12,109	2,825	23.3
18 to 24 years	4,603	2,088	45.4
25 to 34 years	3,968	2,059	51.9
35 to 44 years	3,733	1,672	44.8
45 to 64 years	4,678	1,686	36.0
65 years and over	3,167	107	3.4
Race and Hispanic Origin			
White	21,922	7,271	33.2
White, not of Hisp. Origin	14,875	4,158	28.0
Black	8,360	2,347	28.1
Asian / Pacific Islander	1,163	485	41.7
Hispanic origin[1]	7,439	3,254	43.7
Education (persons aged 18 and over)			
No high school diploma	7,888	2,876	36.5
High school graduate only	6,810	2,611	38.3
Some college, no degree	3,162	1,278	40.4
Associate degree	836	324	38.8
Bachelor's degree or higher	1,452	521	35.9
Work Experience (persons aged 18 to 64)			
Worked during year	8,649	4,104	47.5
Worked full-time	5,582	2,654	47.5
Worked part-time	3,066	1,450	47.3
Did not work	8,333	3,400	40.8
Nativity			
Native	27,507	7,817	28.4
Foreign-born	4,751	2,619	55.1
Naturalized citizen	968	347	35.9
Not a citizen	3,783	2,271	6.0

[1]Persons of Hispanic origin may be of any race.

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Table 3.

Number of Persons Covered and Not Covered by Health Insurance by State in 1999

Source: U.S. Census Bureau, Current Population Surveys, March 1999 and 2000.

State	Total thousands	Covered thousands	Not Covered thousands	Not Covered percent
United States	272,691	230,424	42,267	15.5
Alabama	4,370	3,745	625	14.3
Alaska	620	501	118	19.1
Arizona	4,778	3,765	1,013	21.2
Arkansas	2,551	2,176	375	14.7
California	33,145	26,417	6,728	20.3
Colorado	4,056	3,375	681	16.8
Connecticut	3,282	2,960	322	9.8
Delaware	754	668	86	11.4
District of Columbia	519	439	80	15.4
Florida	15,111	12,210	2,901	19.2
Georgia	7,788	6,534	1,254	16.1
Hawaii	1,185	1,054	132	11.1
Idaho	1,252	1,013	239	19.1
Illinois	12,128	10,418	1,710	14.1
Indiana	5,943	5,301	642	10.8
Iowa	2,869	2,631	238	8.3
Kansas	2,654	2,333	321	12.1
Kentucky	3,961	3,387	574	14.5
Louisiana	4,372	3,388	984	22.5
Maine	1,253	1,104	149	11.9
Maryland	5,172	4,561	610	11.8
Massachusetts	6,175	5,527	648	10.5
Michigan	9,864	8,759	1,105	11.2
Minnesota	4,776	4,393	382	8.0
Mississippi	2,769	2,309	460	16.6
Missouri	5,468	4,998	470	8.6
Montana	883	719	164	18.6
Nebraska	1,666	1,486	180	10.8
Nevada	1,809	1,435	375	20.7
New Hampshire	1,201	1,079	123	10.2
New Jersey	8,143	7,052	1,091	13.4
New Mexico	1,740	1,291	449	25.8
New York	18,197	15,212	2,984	16.4
North Carolina	7,651	6,473	1,178	15.4
North Dakota	634	559	75	11.8
Ohio	11,257	10,018	1,238	11.0
Oklahoma	3,358	2,770	588	17.5
Oregon	3,316	2,832	484	14.6
Pennsylvania	11,994	10,867	1,127	9.4
Rhode Island	991	922	68	6.9
South Carolina	3,886	3,202	684	17.6
South Dakota	733	647	87	733
Tennessee	5,484	4,853	631	11.8
Texas	20,044	15,374	4,670	11.5
Utah	2,130	1,827	302	14.2
Vermont	594	521	73	12.3
Virginia	6,873	5,904	969	14.1
Washington	5,756	4,847	910	15.8
West Virginia	1,807	1,498	309	17.1
Wisconsin	5,250	4,673	578	11.0
Wyoming	480	402	77	16.1

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Title: Who Falls Through the Healthcare Safety Net?

Instructors' Comments and Solutions

This ILAP requires minimal computations. The focus here is to raise the mathematical literacy of the intended audience, so that they might become a more "informed citizenry" (in the words of Thomas Jefferson). A "liberal arts" mathematics course is generally intended for students in English, fine arts, history, etc.; such individuals generally avoid mathematics and having to sift through numbers to see what the story is behind the numbers.

This ILAP provides current information about the state of the nation regarding those without health insurance. The focus of the first four Requirements is summed up in **Requirement 4**. Students must study the data and write intelligently about where the problem might be. In a similar way, **Requirement 6** asks students to interpret the data pertaining to individual states.

Requirement 5 has formal mathematical content and suggested solutions follow in **Table S1** and **Figure S1**.

Table S1.

State summary statistics.

Statistic	% Uninsured
Mean	14.4
Median	14.2
Std. Deviation	4.3
Minimum	6.9
Maximum	25.8
Percentiles:	
25th	11.1
50th	14.2
75th	17.1

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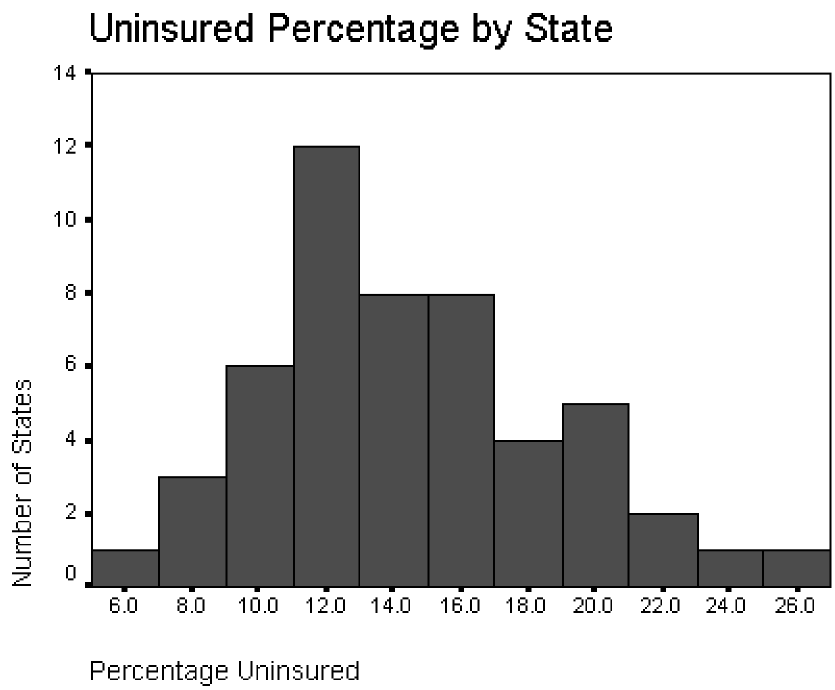

Figure S1. Histogram of state percentages.

About the Author

Marie Vanisko is a professor of mathematics at Carroll College in Helena, Montana, where she has taught for more than 30 years. As a co-director of the NSF Project INTERMATH at Carroll, she has been a primary mover in initiating the writing of interdisciplinary projects (ILAPs), and she has taken a lead role in instituting curricular reform in the undergraduate mathematics program. Marie is co-author of the technology supplement that accompanies the 10th edition of Thomas's *Calculus* (2001) and has served as a judge in COMAP's Mathematical Contests in Modeling at both the undergraduate and the high school levels. In Spring 2002, she was a visiting professor at the Department of Mathematical Sciences at the U.S. Military Academy at West Point.

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Reviews

Krantz, S. 1999. *Handbook of Complex Variables.* New York: Birkhäuser. xxiv + 290 pp, $69. ISBN 0–8176–40118.

The author is a well-known expositor; he has written about ten books in addition to the one under review, as well as being Editor of the *Notices of the American Mathematical Society*. In addition, he is a spirited participant in public issues affecting mathematics education, including the impact of technology and software. This book is modeled in many ways on his text on complex analysis, written with Robert E. Greene [1997]. The author is a well-known worker in several complex variables, and his text is a standard reference in that subject; however, the focus of this book is a single complex dimension.

This "handbook" is written in a different spirit than a traditional textbook. There are almost no proofs, and the material is presented in short, snappy paragraphs which are designed to be read somewhat independently. Thus, while not suitable as a textbook, it is a useful reference for students, teachers, or scientists who need a convenient source to check matters such as contour integrals, numerical methods in conformal mapping, basic transforms (Laplace, Fourier, Z), approximation theory (Mergelyan, Runge), potential theory (harmonic and subharmonic functions), and an overview of five computer packages (e.g., Mathematica, Maple, Matlab) that are used for computation or sketching. There are also drawings of elementary conformal mappings. Some of these topics are seldom in the textbook literature.

In addition, the book has a 37-page "Glossary" of mathematical terms at the end, with references to sections in the text in which they are used. There are also tables listing the power series expansions of the most familiar functions, the computation of Laurent coefficients, and standard definite integrals in terms of residues. Other useful features include material on numerical techniques of conformal mapping and an intuitive introduction to homotopy (in the context of analytic continuation) and Riemann surfaces (with lots of pictures). Most proofs are omitted, and although there is a list of textbooks at the end of the book—divided into the categories "classical," "modern," and "applied"—there are few specific references for proofs.

However, there are some slip-ups and inconsistencies that should be mentioned. This is not a complete list; Mitrovic [2001] has others, and Prof. R. Burckel's list appears below, with his permission. The need to compress material of interest to a wide audience into about 200 pages forces many arbitrary choices, and any reviewer will have other preferences.

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Some situations indicate a disconnect between the author's sophistication and what he reveals to his audience. One occurs at the end of the chapter on complex line integrals and the Cauchy theory: If f is analytic inside and on a simple closed curve γ and if z_0 is inside the curve, then Cauchy's formula asserts that

$$2\pi i f(z_0) = \int_\gamma \frac{f(z)}{z - z_0}\, dz.$$

The expression on the right is the Cauchy integral of f. In fact, the Cauchy integral is an analytic function of z_0 when z_0 does not lie on γ, even when $f(z)$ is replaced in the integral by any continuous function $g(z)$. We then call the left side $G(z_0)$. In his "coda" to Chapter 2, the author observes that the relation between g and G is "rather subtle." That said, we are referred to one of his other texts (no page citation), with the barest hint of the relation.

Another example: The chapter on residue integration is very extensive, and even includes the useful case of rational functions on the positive axis $0 \le x < \infty$ (thus exploiting the multivaluedness of the logarithmic function). The section devoted to

$$\int_{-\infty}^{\infty} \frac{x^{1/3}}{1 + x^2}\, dx$$

(which also uses multivaluedness to advantage) has in its heading a reference to making the "spurious part" of the integral disappear. It would seem that this refers to the contribution to the integrand near $z = 0$, but the author does not reveal what the term "spurious" means; indeed, the behavior at $z = \infty$ is just as "spurious." In contrast, the preceding section considers

$$\int \tfrac{\sin x}{x}\, dx,$$

where the singularity at zero is not "spurious." (The answer depends on the nature of the singularity at $z = 0$; the function $z^{-1}e^{iz}$ has a nonzero residue at the origin. This matter reappears on p. 124 when the author refers to an "integrable singularity" although this term is not defined.)

The careful reader will be confused about whether a sequence of constants $z_n = a$, $n = 1, 2, \ldots$, has a as a limit point—according to p. 37, the answer is "no," but p. 110 suggests that the answer is "yes." The definition of accumulation point in the glossary is simply wrong; according to any other text, the sequence $z_n = (-1)^n$ has ± 1 as accumulation points.

Perhaps the topic that causes the greatest confusion to students is the matter of multiple-valued "functions," such as the complex logarithm. The (multivalued) function $\arg z$ is defined at the outset, but the logarithm has to wait until the chapter on analytic continuation. This leads to a discussion of homotopy, and afterward (p. 135) we are told that the multivaluedness of "functions" such as $\sqrt{z}$ (or $\log z$) can be "explained" by the monodromy theorem, since the region $D = \{0 < z < 1\}$ is not simply-connected. That is not the full story, since the function $g(z) = z^{-n}$ for any positive integer $n \ge 2$ is holomorphic in D but is single-valued (and does not extend to be holomorphic at 0).

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The book encompasses a large amount of material very efficiently. Prof. Burckel suggests Herz [1996] (in German) as having a similar orientation.

References

Greene, Robert E., and Krantz, Steven G. 1997. *Function Theory of One Complex Variable*. New York: Wiley. *Mathematical Reviews* 98d: 30001.

Herz, Andreas. 1996. *Repetitorium der Funktionentheorie (mit Staatsexamensaufgaben)*. Wiesbaden, Germany: Vieweg. *Zentralblatt* 885.30002.

Mitrovic, D. 2001. Review of *Handbook of Complex Variables* by Steven G. Krantz. *Mathematical Reviews* 2001a: 30001.

Timmann, Steffen. 1998. *Repetitorium der Funktionentheorie*. Springe, Germany: Binomi Verlag. ISBN 3–923923–56–2.

David Drasin, Math Dept., Purdue University, West Lafayette, IN 47907; `drasin@math.purdue.edu` .

Mini-List of Errata

p. 7, l. 7: Polynomials have *zeros*, equations and numbers have roots.
p. 22, last sentence in 2.1.6: Makes no sense: The derivative of f appears in formula (2.1.6.1).
p. 29, l. -4: $F \longrightarrow f$
p. 36: Nonemptiness of U, V is not enough; they must each have nonempty intersection with W.
p. 58, 4.5.5: "spurious"? Strange use of the word.
p. 64, l. 5: The subtle linguistic distinction that you were at pains to point out in def. 4.6.2 is lurking here, too: S is not a "singular set" (sic), it is rather a "singularity set" (i.e., comprised of singularities).
p. 87, l. 6: "deformed to a point": meaning that the deformation never leaves U?
p. 87, l. 8: "the complement of U has only one component"? The simply-connected region $\mathbb{C}\,\{x \in \mathbb{R} \,:\, |x| \geq 1\}$ fails this criterion.
p. 88, l. -1: The slit-plane result is valid without finite-connectivity.
p. 89: Why $\equiv$ in (7.1.2.1) but simple $=$ in (7.1.2.2)? I should think either both $\equiv$ or only the second (being a definition).
p. 96, l. 7: $\zeta \longrightarrow x$
p. 104, 8.1.4: Not the customary definition. This is called *local uniform convergence*. A series $\sum x_n$ in a normed linear spaces is *normally convergent* if $\sum \|x_n\| < \infty$. Your definition does not have the desirable feature that a normally convergent series is absolutely convergent.
p. 110, 8.2.4: For a constant sequence, this would seem to be false; that is, if $\{a_n \,:\, n \in \mathbb{N}\}$ is a single number, there is then no accumulation point per definition on p. 37.
p. 113, (8.3.6.1): Is a_l^j a power, like $(z - \alpha_j)^l$ adjacent to it?
p. 117, 9.1.1: Spelling.

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p. 122, 9.3.6: “finite order” seems irrelevant: f can be any nonpolynomial entire function.
p. 124, (10.1.2.2.2): “ . . . the singularity will be integrable . . . ”? Was this defined?
p. 135, l. 5: “ . . . we can now understand . . . ” How does the Monodromy Theorem, which provides (only) a sufficient condition for analytic continuability, explain non-continuability?
p. 145, Ex. 11.1.4.2: The “uniform” clause violates the Maximum Principle.
p. 150: Paul Koebe never wrote his name as “Köbe.”
p. 162, 13.3.3: Why not mention the beautiful proof of D.J. Newman (*Mathematical Reviews* 82h: 10056, 98j: 11069, 2000b: 11110)?
p. 168, (14.2.2.10): The right-hand side is not a function of (x, y).
p. 170, Figure 14.6: The wrong triangle is shaded.
p. 187: The top figure can’t be correct, because (1) the image set is unbounded and (2) the image set is conjugate-closed.
p. 195, l. 5: Among “delightful” (and very accessible) texts you should mention T.W. Körner’s *Fourier Analysis*, Cambridge University Press, Cambridge, 1988; (*Mathematical Reviews* 89f: 42001).
p. 231: *Accumulation point* seems to describe rather *convergence*.
p. 247, l. -3: “alternate” should be “alternative”—cf. p. 251, l. -2.
p. 247, ll. 7–8: What if $x = 0$ and $y = i$? Better define *imaginary* part z as $(z + \bar{z})/2$. Similar problem on p. 252, l. 10, and in the definition of complex conjugate itself.

R.B. Burckel, Math Dept.,Kansas State University, Cardwell Hall, Manhattan, KS 66506–2602; burckel@math.ksu.edu .

Colley, Susan Jane. *Vector Calculus*. 2nd ed. Upper Saddle River, NJ: Prentice-Hall, 2002; xiv + 558 pp, $93.33. ISBN 0–13–041531–6.

According to the author, this second edition differs from the first edition in addition of exercises, a new final Chapter 8 on differential forms, and expanded discussion of several topics. The first seven chapters are devoted to vectors, differentiation, vector-valued functions, maxima and minima, multiple integrals, line integrals, and surface integrals.

The book has a number of striking good qualities. The author has made a strenuous effort to treat each topic thoroughly, to anticipate points that can cause difficulty for the student, to provide abundant illustrations (including those from applied fields), to point out computer techniques where relevant, and to supply a large body of exercises. There are also some novel ideas drawn from the literature, such as a test for constrained local extrema due to D. Spring and a discussion of Bézier curves.

For the difficult task of presenting the underlying theory, the author provides carefully stated propositions (which are highlighted), discusses their significance, and gives proofs or proof sketches for many but not all results. For example, an intuitive proof is given for the anticommutativity of the vector product, and the basic theorems on limits and continuity are stated without proof; but the differentiability of a function having continuous partial derivatives is proved, as is the theorem on interchange of order of partial derivatives.

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For theorems for which no proof is given, the reviewer wishes that references to other sources had been provided, so that a curious student could have a complete understanding of the theorems. Very few such references are given. However, at the end of the book, there is a short list of works for further reading.

The first four chapters follow traditional approaches to the subjects. The later chapters concern integration, and for them the author has made some novel choices in treating the theory. For the double integral, the author gives a fairly standard definition of the Riemann integral for a function over a rectangular region in an xy-plane. The reviewer is not satisfied with the definition, since it refers to a limit as "all Δx_i, all Δy_j approach 0," and this has no clear meaning. The standard use of a mesh would clarify this. The theorem that the integral exists if the function is continuous is then stated but not proved. This is followed by the theorem that the integral exists if the function is bounded and its set of discontinuities has zero area; again there is no proof, and "zero area" is not defined. Next comes a theorem called Fubini's theorem, which is in fact a very weak form of that theorem, but a form very adequate for most applications; a proof is given. Four basic properties of the double integral are listed and one of them is proved. Next, an "elementary region" in the xy-plane is defined (suitable for forming an iterated integral), and the integral over such a region is defined to be the integral over an enclosing rectangle of the function extended to equal zero outside the region. It is then proved that the integral just defined equals the appropriate iterated integral. The proof, occupying 4 pages, is difficult. Triple integrals are treated similarly. The theorem on change of variables for double integrals is stated only for the case of a mapping of an elementary region onto an elementary region, and a sketch of a proof is given.

It would be difficult to present this exposition of the theory of multiple integrals to students. Typically, at this stage of their mathematical education, their greatest need is for practice with many examples and for guidance in gaining an intuitive feeling for the subject. The fine points of theory, even very well presented, usually escape them.

The next chapter, on line integrals, starts by defining the integral over a smooth path in the xy-plane with respect to arclength. The proof that the integral defined is the limit of appropriate sums is not complete. The vector line integral is then defined and shown to have a value independent of parametrization of the path. The author then introduces the concept of a "curve" as the image set of a path and defines line integrals over such curves. The reviewer finds this development unnecessary and confusing; it would be far better to define a (directed) curve as an equivalence class of paths, and then the theorems already proved provide meaning for line integrals over such curves.

The word "region" appears at various points in the later chapters but apparently is not defined, and the term "connected" appears for the first time on p. 403. Noting these facts, the reviewer was led to examine the domains of functions throughout the book and was surprised to find that for the most part the domain is an arbitrary open set in n-space. Thus, even for functions of one variable, the domain is allowed to be an arbitrary open set on the real

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line, hence a union of disjoint open intervals. It seems strange to expect a student just beginning to learn about partial derivatives and multiple integrals to be thinking in terms of such sets. The tradition for one-variable calculus is to consider functions defined on intervals, in many cases closed intervals; for many-variable calculus, one considers functions defined on regions, which may be open or closed or occasionally formed of an open region plus part of its boundary. The examples in this book are all of this sort, so nothing is gained by allowing disconnected domains. The unnecessary generality can even lead to errors; for example, on p. 241 equation (2) gives a remainder theorem that is false if the domain is disconnected.

The final chapter, on differential forms, is well written, with many examples. The smoothness of the mapping functions needs clarification; on p. 489 it is suggested that they are of class C^k, but k is never specified and k is used in another context later in the chapter. One wonders how often an instructor will include this topic in a course. However, it is good to have it in the book, and it may well encourage a student to learn about Stokes's Theorem in all its generality and to pursue more advanced mathematics.

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Beatrous, Frank and Casper Curjel. *Multivariable Calculus: A Geometric Approach*. Upper Saddle River, NJ: Prentice-Hall, 2002; xii + 456 pp, \$88. ISBN 0–111–22222–3.

This work is directed at students with a very weak background who want to gain some feeling for calculus beyond the first course without having to master difficult mathematical theory. Very little is proved, and no linear algebra is assumed or even mentioned. Concepts are introduced by examples, with no attempt to provide a clear and precise formulation of the theory. The term "geometric approach" in the title refers to frequent appeals to geometric intuition, with the aid of a large number of drawings presenting 2-dimensional and 3-dimensional figures. Euclidean geometry is hardly referred to; even the Pythagorean theorem is applied without being mentioned. Analytic geometry is treated in a superficial manner, with no discussion of conic sections or quadric surfaces. The text does not refer to computer aids, except for a discussion in the preface of a computer lab, in which problems are treated graphically with the aid of computer programs.

The first chapter treats vectors and curves in the plane and in space, taking 65 pages to cover vector operations through the cross product. Chapter 2 covers partial derivatives, directional derivatives, local, global and constrained extrema. Chapter 3 treats double and triple integrals, with an intuitive approach to the limit processes. The remaining four chapters are concerned with vector fields, line and surface integrals, and the familiar theorems.

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A good instructor can certainly convey a substantial amount of mathematics with the aid of good figures and intuitive reasoning based on them. Thus, the authors' goal makes sense and the book can well meet the needs of an appropriate body of students. For those wanting to go further in mathematics, the book is inadequate, since its absence of rigor prevents it from laying the foundations for advanced courses. This reviewer found some minor flaws, which are noted in the following paragraphs.

Very explicit instructions on graphing are provided (pp. 40–41): The unit of distance is always to be 1 centimeter. The angle between two vectors is defined (p. 31) without mention of the case where one or both vectors are the zero vector; hence, there may be zeros in denominators on subsequent pages. There is similar difficulty for the cross product (p. 60).

The theorem on interchange of order of partial derivatives is stated (p. 122) with a footnote indicating that it is valid only for "certain functions" (unspecified). On p. 125, a problem on interest payments involves differentiating a function defined only for integer values. Generally, there is little attention paid to where functions are defined. In particular, intervals, open regions, and closed regions are not discussed except for the statement (p. 187) that "filled in figures are called regions or domains"; later, the term "of finite extent" is used for bounded sets, with no explanation. In evaluating double integrals, the authors use what they call "x-slices" and "y-slices," with no discussion of the sets for which the method is applicable.

The authors present a "curl test" for independence of path of a line integral; but, in the absence of a discussion of open regions, connectedness, and simple-connectedness, the reader is left with a very incomplete statement of validity of the test (pp. 318–320). Surface integrals are developed before a discussion of surface area.

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Snieder, Roel. *A Guided Tour of Mathematical Methods for the Physical Sciences.* Cambridge, U.K.: Cambridge University Press, 2001; xi + 429 pp, $30 (P). ISBN 0–521–78751–3.

This book is fun. It introduces the topics of vector analysis in an easy, discursive style with engaging examples. It is ideal for self-study or seminar-study or for the instructor who is looking for great applications of the mathematical ideas. Where else can one find out:

- How fast is the Earth growing by the accumulation of cosmic dust?
- Why is life not possible in a five-dimensional world?
- Where does lightning start?

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- What connects quantum mechanics and hydrodynamics?
- What causes the explosion of a nuclear bomb?
- Is the Earth's mantle convecting?
- How to design a frequency filter?
- How to predict the motion of a particle in syrup?
- Why pressure in a fluid is isotropic?

Most of the book is devoted to vector analysis, but there also is one chapter each on linear algebra (which gets as far as singular value decomposition), the Dirac delta function, Green's function, Fourier analysis, perturbation theory, and two chapters on complex analysis. The bulk of the examples draw on gravitational fields, electricity, and magnetism; but as the list given in the first paragraph illustrates, there is also a playfulness about the chosen illustrations. The book is a trove of interesting and richly-referenced applications to spark up mathematics classes. The first example of the first chapter takes an old chestnut—Given the rate of change of the volume of a sphere, how fast is its radius changing?—and gives it new life by asking: Given that cosmic dust is estimated to add 4×10^7 kg per annum to the earth's mass and that the density of a meteor is 2.5×10^3 kg/m^3, how fast is the radius of the Earth increasing?

What makes this book special is that it is designed to be read. The introduction of the mathematical technique and the description of the applications are done in a conversational tone. Whole paragraphs—occasionally, whole pages—go by without a displayed equation. But the author makes sure that the reader has pen and paper nearby, for he constantly interrupts his narrative to ask the reader to check something, to perform a calculation, to graph a function, to try out the theory in a particular case.

Something has to be sacrificed: This book would be difficult to use as a text in a traditional course. Nothing is labeled as a theorem or as a definition; there are no formal proofs. As a reference book, it would be awkward and frustrating, because most definitions and principal results are embedded within the text. There are no practice problems at the ends of the chapters. But the author is up-front about this drawback and suggests suitable supplementary texts with a more conventional approach.

The level is described as advanced undergraduate to beginning graduate. That may be a little high for mathematics or physics majors. I could see using the book in the junior or senior year. The author assumes that the reader is familiar with partial derivatives, multiple integrals, basic linear algebra, and vector algebra. With this preparation and a willingness to read with pen and paper at hand, any student is equipped to embark on a delightful journey.

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