

Human Capital Dynamic Network Model of Team Science

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Abstract

As an indispensable part of team science, managing human capital in organizations directly determines the company's productivity. This paper tries to combine network model with team science.

Firstly, build the human capital static network: we use ICM's 370 positions as nodes, the affiliations and cooperative relationships as edges. For different nodes of position, we define some quantified values that can reflect basic attributes of the position based on proper assumptions. By building the Post Archives Matrix(PAM), we can record the internal attributes of each position. This matrix can be really helpful to simulate the real conditions. For every edge (i, j) , the value is 1 if and only the node i and node j have the working relationship. Then we put forward the relationship compactness calculation method, which combines affiliation and cooperative relationship, and this is applied to quantify team performance.

Secondly, build the human capital dynamic network model: we regard employees' demission, promotion and recruitment as the main causes of network evolution and internal churn in the company. Based on the real conditions and some reasonable assumptions, we designed three evolution driving algorithms. For demission algorithm, there are two conditions: if the employee who wants to leave is in low level, he can dismiss directly; if he is in middle level, promotion and recruitment algorithm must be called to fill the position. What's more, we consider carefully about the declination of working enthusiasm and the increasing in churn rate for surrounding people when there is one employee churn. For promotion algorithm, we determine the promotion conditions for different level to realize the layer-by-layer promotion of internal employees. Besides, promotion is always in priority to recruitment. For recruitment algorithm, we regard the 2/3 of the vacant position as recruitment plan and we build the on-position time vector to represent delaying effects.

Thirdly, focusing on the issues that the supervisor needs us to consider, we draw the conclusions: in the next two years, the recruitment and training cost are 26.22σ and 140.15σ ; When the churn rate reaches 25% and 35%, working

人力资源管理在组织中直接决定公司的生产力, 是团队科学必不可少的一部分. 本文试图将网络模型与团队科学结合起来.

首先, 建立人力资源静态网络: 我们利用 ICM 组织的 370 个职位作为节点, 他们之间的附属和合作关系作为边. 根据适当的假设, 对不同的节点, 我们定义了一些可以反应其基本属性的量化值. 通过建立档案矩阵 (PAM), 我们可以记录每个职位的内部属性. 这对模拟真实的情况很有帮助. 对于每一个边 (i, j) , 当且仅节点 i 和节点 j 有工作关系时记为 1. 据此我们提出了关系紧密度计算方法, 该方法结合了职位之间的附属和合作关系而且可以应用到量化团队绩效上.

接着, 建立人力资源动态网络模型: 我们把员工的离职, 晋升和招聘作为公司网络的演进和内部流失的主要影响因素. 基于真实的情况和合理的假设, 我们设计了三大演进算法. 离职算法, 这里有两个条件: 如果是底层的员工想要离职, 那么 he 可以直接离职; 如果他在公司的中层, 则必须调用晋升算法和招聘算法来填补空缺职位. 更重要的是, 我们考虑到当一个员工流失, 他周围的人的工作热情会降低, 流失率会增加. 晋升算法, 对于不同的员工层级我们确定了不同的晋升条件以实现内部员工的层层晋升. 此外, 公司内员工的晋升通常优于招聘. 招聘算法, 我们将空缺位置的 2/3 作为招聘计划, 同时使用在职时间向量来表示延迟效应.

其次, 针对主管要求考虑的问题, 我们得出了结论: 在今后的两年, 公司的招聘和培训成本分别是 26.22σ 和 140.15σ ; 当流失率达到 25% 和 35% 时, 工作效率稳定在 80% 以下,

rate is steadily below 80% and bring some negative effects such as the declining of company performance and employees' enthusiasm; The no-external recruitment policy is beneficial to maintain the company performance and working rate, but we still have to admit that it will cause the declining of enthusiasm of middle-level employees.

Finally, we regard friendships, competition and Human Capital as three network layers to describe the connection compactness in the whole team. By using the main statistical indicators (such as the average length of path), we draw the conclusion that multilayer network can reflect internal connections more precisely and comprehensively than human capital network.

并且会带来诸如公司业绩下降, 员工工作热情下降的负面影响; 虽然无外部招聘的政策, 有利于保持公司业绩和工作效率, 但我们不得不承认, 这将导致的中层员工工作的积极性下降.

最后, 我们把友谊, 竞争以及人力资源作为三大网络层来描述整个团队内的紧密联系. 通过使用主要的统计指标描述 (比如路径平均长度), 我们发现多层级网络比人力资源网络更能准确, 全面地反映公司的内部联系.



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1 Introduction | 引言

Human capital is an intangible asset, it is relatively abstract and it's meaning develops with the development of our society. Pierre Bourdieu offers a nuanced conceptual alternative to human capital that includes cultural capital, social capital, economic capital, and symbolic capital [1]. Human capital, when viewed from a time perspective, consumes time in one of key activities:

- Knowledge (activities involving one employee);
- Collaboration (activities involving more than 1 employee);
- Processes (activities specifically focused on the knowledge and collaborative activities generated by organizational structure-such as silo impacts, internal politics, etc.);
- Absence (annual leave, sick leave, holidays, etc.).

As the complexity of the workplace continues to grow, organizations increasingly depend on teams. Building an organization filled with good, talented, well-trained people is one of the keys to success. More and more people gradually begin to realize its charming power. So it is extremely important for the HR office to assign employee to positions appropriate to their talents and experience. These positions are just the efficient communication systems to facilitate development of innovative ideas and quality products (commodities or services). But how to manage the human capital has always been a great challenge.

1.1 Problem analysis | 问题分析

We focus on a more practical problem on the Information Cooperative Manufacturing(ICM) organization of 370 people, which is in a highly competitive market place.

Now, the actual situation of the organization: only 85% of its 370 positions are filled at any time. Considering different position layers in which people are required to have certain years of experience, the low quality employees often stay with the company for a full career. However, mid-level positions suffer much higher turnover, and it's often the case that churn seems to diffuse from former employees to others, so they are critical ones when consider the position changes. As for the CEO, the ratio of their salary to worker is approximately 10 times. Besides, in

人力资本是一种相对抽象的无形资产，其内涵随着我们社会的发展而丰富。皮埃尔·布尔迪厄提供了一个细致的概念代替人力资本，它包括文化资本，社会资本，经济资本和象征资本 [1]。从时间角度看，人力资本在以下主要活动之一中耗费时间：

- 学习（活动涉及一名员工）；
- 协作（活动包含一名以上的员工）；
- 流程（主要集中于由组织结构产生的学习和协作活动，如仓储的影响，内部政策等）；
- 缺席（年假，病假，节日假等等）；

随着工作环境复杂性的不断增长，企业越来越依赖于团队。组建一个充满优秀的，有才能的，训练有素的员工团队是成功的关键因素之一。越来越多的人逐渐地开始意识到优秀团队的魅力。因此对于人事部门来说，根据员工的才能和经验将其分配到合适的位置是非常重要的。这些职位仅仅是促进形成创新理念和生产优质产品（商品或服务）的高效的通信系统。但是如何管理人力资本一直是个巨大的挑战。

我们着重研究一个更实际的问题，这个问题是关于拥有 370 个员工的信息协同制造 (ICM) 组织，信息协同制造是一个竞争激烈的市场。

当前，该组织的一些实际情况：370 个职位中只有 85% 的在任何时候都不空缺。考虑到不同职位层次的员工都需要一定年限的工作经验，通常低素质的员工在他的整个职业生涯都会留在公司。但是，中层级的员工更容易改变，而且这种情况通常似乎由之前离职的员工扩散传翻给其它员工，所以在考虑职位改变时，他们（之前离职的员工）是重点考查对象。而 CEO，他们的薪水大约是普通员工的 10 倍。另外，在时间



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terms of time, the earlier an employee gains the loyalty, the more productive is the organization. What's more, the annual evaluation based on performance is judged by the supervisor rather than the HR office.

Our tasks:

- Build a human capital network model of ICM organization's personnel situation that can identify dynamic processes to get certain influence such as the cost, the direct and indirect effects on organization's productively with a certain churn rate as well as simulate or predict what will happen with a changed churn rate.
- Build a multilayer network that can connect our Human Capital network to other organizational network layers such as information flow, trust, influence, and friendship.

1.2 Literature review | 文献回顾

“Human capital” has been and continues to be criticized in numerous ways. Many researchers have engaged in defining and developing this concept [1, 2]. In the early days, it is an aggregate economic view of the human being acting within economies, which is an attempt to capture the social, biological, cultural and psychological complexity as they interact in explicit and/or economic transactions. Today, most theories attempt to break down human capital into one or more components for analysis [3] – usually called “intangibles”. Accordingly much more attention is paid to factors that led to success versus failure where human management is concerned. The role of leadership, talent, even celebrity is explored. How to make the best use of human capital in the organization, more specifically, to retain good people, keep them properly trained and placed in proper positions, and eventually target new hires to replace those leaving the organization has always been a challenge work.

Network has been widely used in various aspects, especially in natural science. For example, biological networks provide a mathematical analysis of connections found in ecological, evolutionary, and physiological studies, such as neural networks [4]. In computer networks, networked computing devices pass data to each other along data connections (network links). As more and more people realized the

方面, 组织越早赢得员工的忠诚度, 组织就会更加高效. 此外, 业绩的年度考核是由上级判定而不是人力资源部门.

我们的任务:

- 以 ICM 组织的人员情况建立一个人力资本网络模型, 它可以识别动态过程来确定诸如成本造成的影响, 一定的人员流失率对公司效益的直接或间接影响, 以及模拟或者预测不同的人才流失率会导致什么样的后果.
- 建立一个多层级网络, 将人力资本网络与组织的其他网络 (如信息流网络, 信任度网络, 影响力网络和友谊网络) 层连接起来.

“人力资本”在许多方面仍饱受争议. 众多的研究者参与了这一概念的定义和发展 [1, 2]. 早期, 人力资本是一个人作用于经济的整体经济观念, 这个观念是一个试图捕捉社会, 生物, 文化和心理的复合体, 这是由于社会, 生物, 文化和心理与或多或少的与经济运作存在明确的相互作用. 当今, 大多数的理论试图将人力资本分解成一个或多个元素来分析 [3], 通常叫做“无形资产”. 于是, 有关人力资本导致的成功和失败的因素受到了更多的关注. 领导, 天才甚至是名人的作用被研究. 在组织中如何最好地利用人力资本一直是一项富有挑战性的工作, 更具体地说是, 留住优秀的人才, 适当对他们进行培训并安排在合适的职位, 甚至是招聘新员工来填补空缺职位.

网络已被广泛应用在各个方面, 尤其是自然科学. 比如, 生物网络为生态学, 进化学和生理学中的关系提供了一种数学分析方法, 如神经网络 [4]. 在计算机网络中, 网络计算设备通过数据连接将数据传递给对方 (网络链接). 随着越来越多的人意识到网络的强大, 一些复杂的网络, 如社会网络和



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power of the network, other complex networks, such as social network and business network forms the nascent field of network science [5]. For instance, a social network is a social structure made up of a set of social actors (such as individuals or organizations) and a set of the dyadic ties between these actors. Since using network in social areas is newly developed, the theory is relatively scarce and many theories are more focused on how to establish the structure of the network rather than the function-the practical use. So it is a novel perspective to build a human capital network to study how the churn and recruit employee will affect the efficiency of the organization. And as teams have increasingly become a way of life in many organizations, using the new concept of team science to build a human network has little been discussed.

What's more, our life is filled with all kinds of networks, if we do more research on these networks, we may find connection between them, especially a set of entities interact with each other in complicated patterns that can encompass multiple types of relationships, and it is important to take such “multilayer” features into account to try to improve our understanding of complex systems. So connect our Human Capital network to other organizational network layers such as information flow, trust, influence, and friendship is just our creative and challenging work.

1.3 Terminology and definitions | 术语和定义

- **Churn**: the resulting turbulence when people leaving for other jobs or retiring are replaced.
- **Churn rate(turnover rate)**: a measure of the number of individuals or items moving out of a collective group over a specific period of time, and it can be defined as follows:

$$\sigma_{CR}(\%) = \frac{2N_R}{N_S + N_E} \times 100\%$$

where, N_R is the number of employees resigned during the month, N_S is the number of employees at the start of the month and N_E is the total number of employees at the end of the month.

商业网络形成了网络科学的新兴领域 [5]. 例如, 一个社会网络是由一系列社会角色 (如集体或个人) 和这些角色之间的二元关系构成的一个社会结构. 由于在社会领域使用的网络是新近开发的, 理论相对稀缺, 而且许多理论都专注于如何建立网络结构, 而不是实际使用. 因此建立人力资本网络模型来研究员工的流失和招聘会对公司的绩效产生怎样的影响是一个新颖的角度. 随着团队已经越来越成为组织的一种生存方式, 利用团队科学的新概念去建立人力网络才刚开始被讨论.

更重要的是, 我们的生活充满了各种各样的网络, 如果我们更多的去研究这些网络, 就可以发现它们之间的联系, 特别是一组实体在复杂模式下彼此的相互作用, 这些实体间包括多种类型的关系, 并且将“多层级”特性考虑在内以试图改善我们对复杂系统的理解是重要的. 因此将我们的人力资本网络与组织的其他网络层诸如信息流, 信任度, 影响力及友谊联系起来是创造性的且富有挑战的工作.

- 流失: 人员跳槽或退休时职位被代替所产生的动荡.
- 流失率 (流动率): 在一段特定时间内离开集体的个人或物品的数量, 计算公式如下:

$$\sigma_{CR}(\%) = \frac{2N_R}{N_S + N_E} \times 100\%$$

其中, N_R 是一个月内员工辞职的数量, N_S 是月初的员工数量, N_E 是月末的员工总数.



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1.4 Our work | 我们的工作

This paper tries to combine network model with team science to deal with management issues. In section 2, we build human capital network. Firstly, in part 2.2 we use ICM's 370 positions as nodes, the affiliations and cooperative relationships as edges to build a human capital static network model. Then in part 2.3 we design three evolution driving algorithms respectively which make the network change with certain initial conditions as well as the development of algorithm in time dimension. Thirdly, focused on the issues that the supervisor need us to deal with, we revise our model's initial parameters and algorithms. Finally, in part 2.4, based on our simulation results, we solve the issues supervisor requires and analyze effects under different conditions. In section 3, after studying the theory of multilayer network, we regard friendships, competition and affiliation as three network layers to describe the connection compactness in the whole team. We also find that multilayer network can reflect internal connections more precisely and comprehensively than human capital network. In section 4, we do the sensitivity analysis for simulation methods and some parameters. In section 5, we draw the conclusion of our model and get our strength and weakness.

本文试着结合网络模型和团队科学以解决人力管理问题。在第2节，我们建立人力资本网络。首先，在第2.2部分我们用 ICM 组织的 370 个职位作节点，由它们之间的附属和合作关系形成边建立人力资本网络静态模型。然后在第2.3部分，我们设计了三大演进算法，使得网络在确定的初始条件下随着算法在时间上的推进而变化。接着，我们重点关注管理者需要解决的问题，我们修改了模型的初始参数和算法。最后，在第2.4部分，基于模拟结果，我们解决了管理者需要解决的问题并分析了不同情况下的影响。在第3节，在学习多层网络理论之后，我们将友谊，竞争和附属关系作为三大网络层级来描述整个团队的紧密关系。我们也发现多层网络比人力资本网络更能精确地，全面地反映公司内部的关系。在第4部分，我们对模拟方法和一些参数做了灵敏度分析。在第5部分，我们得出了模型的结论，并给出了我们模型的优缺点。

2 Human Capital Evolution Network Model | 人力资源演进网络模型

2.1 Model overview and concepts definition | 模型概述和概念定义

From the perspective of the human resources department, we build the human capital network model to reflect the human capital as well as its changes of each level of employees. According to the requirements, our network should have the following functions:

从人力资源部门的角度，我们建立了人力资本网络模型来反映人力资本状况以及不同层级员工的变化。根据要求，我们的网络应有以下功能：

- The network can clearly show the distribution of different levels of employees in each divisions or offices as well as the internal leading and being led relationships, which is the basic structure of the promotion mechanism in the organization.
- Since it is a human capital network, we must pay more attention to the main factors that can affect the human capital, which includes: the ability an
- 网络可以清楚的展示每一个部门或者办公室不同层级的员工分布，同时明确领导与被领导关系，这是组织中晋升机制的基本结构。
- 因为是人力资本网络，我们就必须更加重视影响人力资本的主要因素，这些因素包括：员工获得职位时已经具



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employee has had when he gets the position, the productive enthusiasm of the employee, the time an employee stays in certain position and the training he obtained.

- Our human capital network is an evolutive network model. It has three fundamental dynamic phenomena: employees' internal promotion, employees' demission and external recruitment.

The churn of the employee is especially worrying in ICM organization that the current churn rate is 18% per year. So we should emphatically consider the how the churn rate will directly and indirectly affect the total benefit and working rate (the effects on employees' relationship and enthusiasm will also be considered). To better illustrate our human capital evolution network, the important elements and concepts are defined as follows:

- **Node:** The nodes in the network model refer to 370 positions set up in the company. This is quite different from the nodes in a normal social network, which represents the actual person. In order to show the characteristic of these nodes more clearly, we build a matrix with 370 rows, and we call it "Post Archives Matrix"(denoted as PAM for the following illustration). In the matrix, each line represents a quantitative attribute of certain position. In order to satisfied the function, the quantitative attributes contains the level of position, the on-the-job condition of the position, working time and which division or office it belongs to.
- **Edge:** Edge is the ditch that two points interact with each other. Focused on the present network that regard position as points, the two most direct connections are the affiliation in the division or the office and the leading and being led relationship between divisions. In order to reflect influence of individual jobs on team work, we establish the matrix with the size of 370×370 to reflect the affiliation of different positions, which will be analyzed in the following part of this paper about the discussion of the teamwork.
- **Factor:** Based on the description of the problem and the reference given [6], the input factors of the evolution network model are the on-job-rate at the initial time and the churn rate, the output factors are the on-job-rate in the future, corporate performance, team work, the costs of recruitment and

备的能力, 员工的生产积极性, 在特定职位员工的工作时间及其所获得的培训.

- 我们的人力资本网络是不断演化的网络模型, 它有三个基本动力学行为: 员工内部的晋升, 员工的离职, 和外部招聘.

员工的流失是 ICM 组织非常担忧的问题, 当前每年的流失率为 18%. 所以我们着重考虑流失是如何直接或者间接的影响公司的总效益和在任率的 (同时也考虑对员工人际关系和工作积极性的影响). 为了更好地说明我们的人力资本演进网络, 以下给出重要因素和概念的定义:

- **节点:** 网络模型中的节点代表公司中设立的 370 个职位. 它完全不同于常规的社会网络中的节点, 它代表的是实际的人. 为了更清楚地显示这些节点的特点, 我们建立了一个 370 行的矩阵, 我们称其为“职位档案矩阵”(下图简记为 PAM) 矩阵中的每一行代表特定职位的某个定量属性. 为了满足模型的功能需求, 特定属性包括职位的层级, 在职条件, 工作时间和它属于哪个部门或办公室.
- **边:** 两个节点之间的相互作用关系称为边. 对于当前的网络我们将职位看作点, 两点之间最直接的联系是同一部门或办公室员工之间的联系和不同部门之间的领导与被领导的关系. 为了反映个人对团队工作的影响, 我们建立了一个 370×370 的矩阵来来反映不同职位间的附属关系, 并在后文分析团队合作.
- **因素:** 基于问题描述和所给参考资料 [6], 演进网络模型的输入因素为初始时间的员工在职率和流失率, 输出因素为未来时间员工的在职率, 团队绩效, 团队合作和员工的招聘培训费用等等.



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training, etc.

- **Corporate performance:** Our company's corporate performance is the summation of productivity in each position. In order to calculate changing performance, we consider that the maximum productivity is the summation of basic productivity in certain position and the individual experience. Moreover, the actual productivity is on the base of the maximum productivity and effected by the individual working enthusiasm as well as team work, which will be illustrated in detail in our model.
- **Minimum time unit:** In our model, the minimum time unit is month, which is determined by the data given. So in each month, we will consider the number of people who submit the resignation, join the company and the people should be recruited by the company respectively as well as their enthusiasm in certain month to calculate the corporate performance in that month.

2.2 Human capital static network model | 人力资源静态网络模型

In order to obtain a network model with evolution function to assess the condition to human capital in the company, we first build a complete Human capital static network model. The reason we call it static is that our network needs multiple dimensions to reflect the condition in all aspects in a certain time for the company. This static network model is the basic of evolution network. In the meanwhile, it can be used to describe the state indicator in all aspects in each time frame.

2.2.1 The main assumptions of static human capital network model | 人力资源静态网络模型的一些主要假设

- As a network that reflects human capital, each node in the network represent a position and the number of position and the affiliation in the network are fixed.
- Based on the relative data of ICM in the question, we need to divide the employee in seven levels into different divisions and office. Here, we present a reasonable way of division, as is shown in Figure 1. The problem discussed

- **公司业绩:** 公司的业绩是每个职位的生产力总和. 为了计算不断变化的业绩, 我们认为最大生产力是特定职位的基本生产力和个人经验的总和. 此外, 实际生产力以最大生产力为基础, 并且受到个人工作热情和团队合作的影响, 这将在我们的模型中详细说明.
- **最小时间单位:** 根据所给数据信息, 模型中我们以月为最小时间单位. 每个月, 我们会分别考虑公司提交辞职申请, 加入公司和公司需要招聘的人数, 同时以员工们在特定月份的工作积极性来计算该月公司的业绩.

为得到具有演进功能的网络模型来评估公司的人力资本现状, 我们首先建立完全静态的人力资本网络模型. 称为静态的原因是因为我们的网络需要从多个维度来反映公司在某个特定时刻内各个方面的条件状况. 这个静态网络模型是演进网络模型的基础. 同时它可以用来描述每一个时间段内公司各个方面的状态.

- 作为一个反映人力资本的网络, 网络中的每个节点代表一个职位, 并且网络中职位的数目及其附属关系是固定的.
- 根据问题中的 ICM 组织的相关数据, 我们需要将 7 个层级员工划分进不同的部门和办公室. 这里我们给出一种合理地划分方式, 如图1所示. 后文中关于此问题



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in the following paper is based on this division structure.

的讨论都是基于这种划分结构.

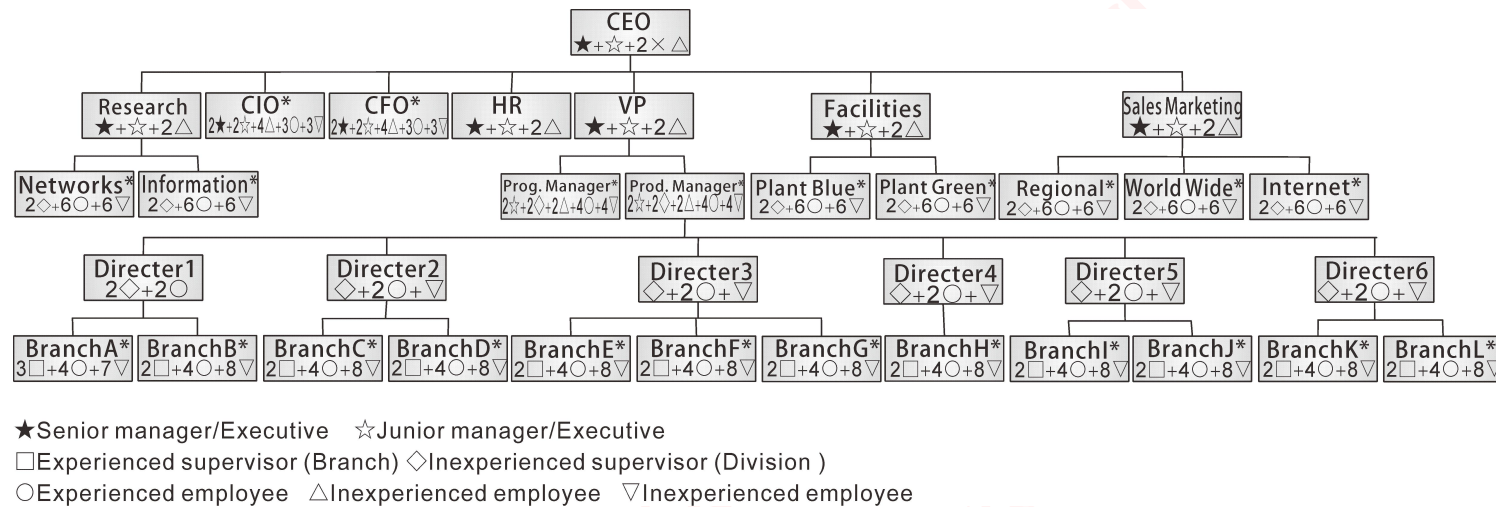


Figure 1: Staff distribution structure in each division | 每个部门的员工分布结构

2.2.2 The definition of node and edge | 节点和边的定义

Taking the reference to the data of the structure of the company and the level of employee given in the question, we know that the network we build need to reflect several internal relationships. So it is necessary to build a network divided by different factors and then make the assignment to nodes and edges. Building the network just means defining nodes and edges in the network. The relationships in the company mainly have affiliation and the cooperative relationships: affiliation refers to the leading and being led relationships in divisions or between the division and its lower division; cooperative relationships refers to the cooperation of internal members in the same division, so we may as well regard that any two members have cooperative relationships in the same division. To simplify the problem, we consider the affiliations between divisions just the affiliation between two department managers. Considering that affiliation and the cooperative relationships exist simultaneously in the company, but the mutual correlation isn't very osculating.

参考问题中所给出的公司结构和员工层级的数据, 我们意识到我们所要构建的网络需要反映一些内部关系. 因此, 有必要构建一个按不同的因素分类的网络, 并将这些因素分配给节点和边. 构建这样的网络仅仅意味着给出网络中节点和边的定义. 公司中的关系主要分为附属关系和合作关系两种: 附属关系是指部门之间或者部门与其下级部门间的领导与被领导的关系; 合作关系是指同一部门内成员间的合作, 因此我们不妨认为同一部门的任意两位员工都存在合作关系. 为了简化这一问题, 我们将不同部门的附属关系仅仅看作是部门经理之间的附属关系. 考虑到合作关系和附属关系在公司内部虽同时存在, 但相互关联并不十分密切. 借鉴参考电场中的叠加原理, 我们提取出两个独立的网络关系.



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Using reference on superposition principle in circuit field, we abstracted the two relationships networks independently.

- **Static affiliation network in ICM**

Taking the aeolotropism of affiliation into consideration, this network belongs to typical directed network; meanwhile, there is no Intensity difference in affiliation, so it can be regarded as unweighted network. This kind of network $G = (V, E)$ can be formed by a node set V and a edge set E . The simplest way of showing affiliation is to represent direct connection in a matrix with a size of 370×370 .

If there is affiliation in position, we define the manager as source and people being managed as target. There are total three conditions: when the source points to the target, we record 1 in certain place in the matrix; when the target points to the source, we record -1; when there is no affiliation, we record 0 in certain place in the matrix.

The following Figure 2 is the affiliation network structure. In order to better reflect the main characteristic in each position, we count the degree of each position and directly reflect it in the radius of each point. The value of the degree reflects the importance of this point in the whole network. The relationship is shown in Figure 2.

- **Static cooperation relationships network in ICM**

For cooperation relationships in divisions or offices, since that there only exits/exist four-team and fourteen-team two types, we can build a small network with a degree of one (there are edges between any two points) for discussion.

Considering that cooperation relationships has isotropy, this network belongs to typical undirected network; meanwhile, there are distinct Intensity differences in the network, so we regard it as weighted network. The intensity of cooperation relationship δ_{ij} between position i and position j is affected by the intensity of affiliation α_{ij} and the intensity friendship relationship. Similarly to the former method, these are record in a matrix with a size of 370×370 . Here, we define the intensity of affiliation α_{ij} is related to the shortest

- **ICM 中的静态附属关系网络**

考虑到附属关系具有有向性的特点, 因此该网络属于典型的有向网络; 同时附属关系没有强度差异, 也就是说它可以看作是非加权网络. 这种类型的网络 $G = (V, E)$ 可由一个节点集 V 和一个边集 E 构成. 表示附属关系最简单方法是将直接联系表示在一个 370×370 的矩阵中.

如果职位中有附属关系, 我们记管理者为头, 被管理者为尾. 这共有三种情况: 如果从头指向尾, 我们在矩阵特定位置记为 1; 如果从尾指向头, 我们记为 -1; 如果没有附属关系的, 我们则在矩阵特定位置记为 0.

下图2所示的是附属关系网络的结构. 为了更好地反映每个位置的主要特征, 我们计算每个位置的度并且直接将它反映在每个节点的半径大小上. 度值的大小反映了这个点在整个网络中的重要性. 附属关系如图2所示.

- **ICM 中的静态合作关系网络**

对于部门或办公室的合作关系, 这里仅存在 4 人团队和 14 人团队两种类型, 因此我们可以建立一个度为 1 的小型网络来讨论 (任意两点之间都存在边).

考虑到合作关系具有各无向性的特点, 合作网络属于典型的无向网络; 同时在网络中有强度大小的区分, 所以我们可以将其看作是加权网络. 职位 i 和职位 j 之间合作关系的强度 δ_{ij} 受附属关系强度 α_{ij} 和友谊强度的影响. 类比于前面的方法, 仍用一个 370×370 的矩阵来记录关系. 这里, 我们定义附属关系强度 α_{ij} 为附属关系网络中两个位置的最短距离 $d_{ij, \min}$ 有关, 计算公



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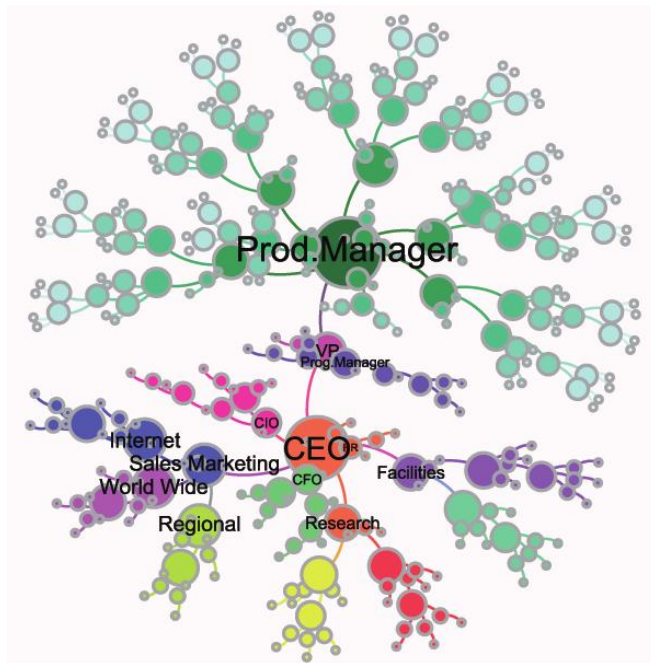


Figure 2: Static affiliation network in ICM. | ICM 静态附属关系网络

length $d_{ij,min}$ of the two position in the affiliation network, the equation can be shown as follows:

$$\alpha_{ij} = \frac{d_{\max} - d_{ij,min}}{d_{\max}}$$

Where, d_{\max} is the maximum of the shortest length in the network. For example, as is shown in Figure 3, the shortest length between employee3 and Exp.employee6 is 4. And the maximum length of shortest way is 5 so the relatively relationship intensity is 0.2. The intensity friendship relationship value is fixed at the beginning, according the request, it will be affected by the churn of employee so that it may decrease. The equation of the intensity

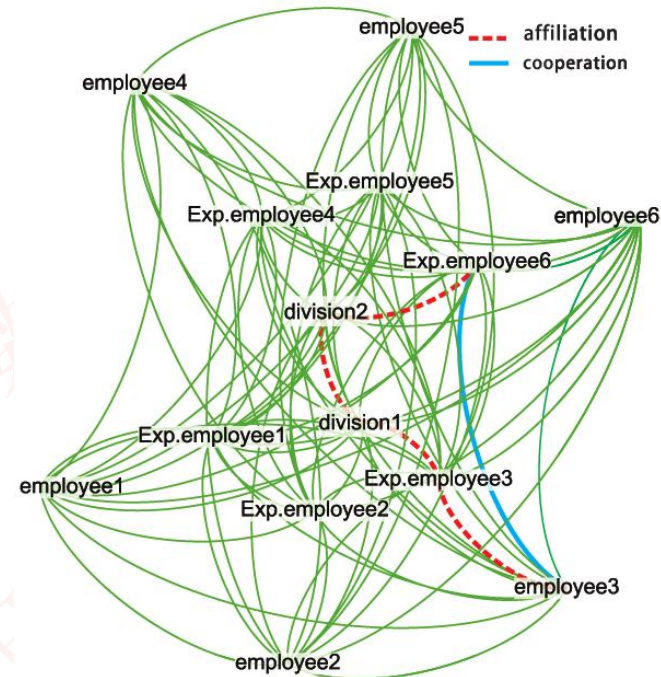


Figure 3: Static cooperation relationships network in ICM. | ICM 静态合作关系网络

式如下:

$$\alpha_{ij} = \frac{d_{\max} - d_{ij,min}}{d_{\max}}$$

其中, d_{\max} 表示网络中最短路径长度的最大值. 例如, 如图3中所, 员工 3 与员工 6 间的最短路径长度为 4, 而最短路径长度的最大值为 5, 因此附属关系强度为 0.2. 友谊强度在初始时刻就被固定下来, 根据要求, 它受员工流失的影响可能下降. 合作关系强度 δ_{team1} 的计算公式如下:



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of cooperation relationship team1 is shown as follows:

$$\delta_{\text{team1}} = \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} \times \beta_{i,j}$$

where, n is the total number of nodes.

$$\delta_{\text{team1}} = \sum_{i=1}^n \sum_{j=1}^n \alpha_{ij} \times \beta_{i,j}$$

其中 n 表示节点总数.

2.2.3 Node attributes in the network | 节点对网络的贡献

In the following paper, our model is start from the perspective of HR office, so it is necessary to define the attribute of each node(position). Our processing method is:

后文中, 我们的模型将从人力资源部门的角度考虑问题, 因此定义每个节点 (职位) 对网络的贡献是有必要的, 我们的处理方法如下:

- Set the number of ICM employees in descending order according to seven levels: From 1 to 370.
- Establish a Post Archives Matrix with 370 edges, and each edge reflects the working attributes in the position. The position attributes that will be used includes: division or offices the employee is in, the level of the position, salary, training cost, recruitment cost, churn rate, the number of changing times, working time in the position, working enthusiasm (working efficiency), basic productivity in the position, on-job condition and request for departure, etc. In these attributes, some will be refreshed because of the change of employee (such as the working time in the position, working enthusiasm (working efficiency), basic productivity in the position, on-job condition and request for departure) while others will stay the same. In particular, the number of changing times will continue to add one each time there is a position change to record the position condition in details. Besides, working enthusiasm is quantized through the number vary from 0 to 1.
- When there is change in position, we need to make some change in certain place in the Post archives matrix. We make the regulation: when the employee is promoted to new position, the original working time will not be retained. In other words, the working time will be record anew and this is in accord with the phenomenon that the employee will study some skills in
- 根据七个层级对 ICM 组织的员工按层级从高到底编号: 从 1 到 370.
- 建立有 370 条边的档案矩阵, 每条边要反映职位的工作属性. 将会用到的职位属性要包括: 员工所在的部门或办公室, 职位的层级, 薪水, 培训费用, 招聘费用, 流失率, 改变次数, 在职工作时间, 工作积极性 (工作效率), 岗位基础生产能力, 是否在岗, 离职需求等等. 在这些属性中, 一些属性会随员工的改变而不断更新 (比如在职工作时间, 工作积极性 (工作效率), 岗位基础生产能力, 是否在岗, 离职需求), 而其他一些属性将保持不变. 特别的, 每次当有职位变化时改变次数增加 1, 用以详细记录该职位情况. 此外, 通过 0 到 1 之间变化的数字来量化工作效率.
- 当有有职位发生变化时, 我们就需要在档案矩阵的特定位置做出一些更改. 我们规定: 当员工晋升到新的职位时, 他原来的工作时间将不会被保留, 换句话说, 他的工作时间将被重新记录, 这符合员工将学习一些新技能以适应新职位的现象.



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order to adapt to the new position.

- The effect caused by training is extremely important. The company spends a certain money for training, we think that this kind of training will bring two aspects of benefits: on the one hand, training can promote the level of productivity in present position, which provides theoretical support when calculate productivity when consider the effect of working time; on the other hand, training can facilitate the recovery of the employee's efficiency τ and restraint the rise of churn rate ν . In each time period, the equation is:

$$\tilde{\nu} = \nu - \xi_2(\nu - 18\%)$$

$$\tilde{\tau} = \tau + \xi_1(1 - \tau)$$

Where, ξ_1 and ξ_2 are the recovery coefficient which indicates changing rate of the relatively ideal value of employee's efficiency per time unit and the churn rate. 18% is the initial churn rate ascertained in the problem, and this value can be adjusted according to different conditions.

- 员工培训具有重要的影响。公司投入一定量资金对员工进行培训，我们认为这种培训将会带来两方面的好处：一方面，培训会提升员工在当前职位的生产力水平，这为计算公司业绩时考虑工作时间的效果提供了理论依据；另一方面，培训能够促进员工工作效率 τ 的恢复，以及控制流失率 ν 的上升。在每一个时间阶段内，工作效率的恢复以及流失率的控制公式如下：

$$\tilde{\nu} = \nu - \xi_2(\nu - 18\%)$$

$$\tilde{\tau} = \tau + \xi_1(1 - \tau)$$

其中， ξ_1 和 ξ_2 为恢复系数，表示在一个单位时间内员工工作效率和流失率相对理想值的改变率。赛题中表明初始流失率为 18%，这个值可以根据不同的情况可以调整。

2.2.4 The input and initialization of static network model | 静态网络模型的输入和初始化

Based on the requirements and data given, it is quite convenient for us to initialize the main inputs. But there are still some phenomena and algorithms that only described qualitatively, so we need to make some reasonable **assumptions** for them.

题目要求和所给数据为我们初始化主要输入提供了方便，但是仍然存在一些只被定性描述的现象和规则，因此我们为此需要做出一些合理的假设。

- **Working rate:** As is stated in the task, the present working rate is 85% and middle positions cannot be vacant, so we place the vacant position evenly in low level positions.
- **Churn rate:** the given initial churn rate is 18% in the task, so we suppose that the churn rates in different levels are the same. In other words, the possibility of churn for each employee is 18% in a year.
- **Recruitment rate:** the determined recruitment rate is set approximately as 2/3 of the vacant position, so that it has the negative feedback effect: when the number of churn employees rise, the number of recruitment employee

- **在职率:** 正如赛题任务中所述，当前在职率为 85%，而且中层级不能有职位空缺，所以我们把空缺职位均匀的分布在低层级职位上。
- **流失率:** 赛题任务中给定的初始流失率为 18%，因此我们假设不同层级员工的流失率相同，换句话说，每一位员工在一年中流失的可能均为 18%。
- **招聘率:** 给定的招聘率被设置约为空缺职位的 2/3，因此这会存在负反馈效应：当流失员工数量增加，招聘人数也会同时增加。此外，招聘过程具有延迟效应，具体



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will rise simultaneously. Besides, there is delaying effect in the process of recruitment and the specific algorithm will be illustrate in the following paper.

2.2.5 Output of static network model | 静态网络模型的输出

Focused on human capital static network model, the main output can reflect the running condition of the company. We select the following datum as measurement index. The calculation of organization's productivity: The company's productivity is the sum of each employee's productivity. Our performance is based on the following equations:

$$\varphi = \vec{a}(\vec{b} + \mu\vec{c})$$

Where, \vec{a} is the row vector that record the employee's efficiency in Post Archives Matrix; \vec{b} is the column vector that record the employee's basic productivity in Post Archives Matrix; \vec{c} is the column vector that record the employee's working time in Post Archives Matrix; μ a fixed coefficient that reflects the rise in employee's productivity per time unit.

Based on the calculation method above, we can find that the company's productivity is affected by the churn directly and indirectly: on the one hand, the direct effect is that because there is adjustment in position, the working time will change and this is often regarded as the loss of working experience in the position in our real life; on the other hand, there are some indirect effects such as the decline in cooperation relationships and working enthusiasm caused by churn that can't be ignored. At the same time, the churn rate will also rise. All these analyzed above will cause bad effect to the company.

2.3 Three evolution driving algorithm | 三大演进算法

After building the static network model, considering that our network model has the evolution characteristic in time dimension, we need to use evolution driving force to make the static network model develop and change. This is the source of churn in the organization.

There are three main drivers of evolution: the promotion of employees, the churn of employees and the recruitment of new employees externally. In the following section we will describe how these driving forces affect the evolution process

的算法将会在后文中说明.

对于人力资本静态网络模型, 其主要的输出可以反映公司的运行情况. 我们选取以下基准作为衡量指标. 公司生产力 φ 的计算: 公司的生产力为每位员工生产力的总和. 我们是基于下面的公式实现的:

$$\varphi = \vec{a}(\vec{b} + \mu\vec{c})$$

其中: \vec{a} 为行向量, 记录了档案矩阵中员工的工作效率; \vec{b} 为列向量, 记录了档案矩阵中员工的基础生产力. \vec{c} 为列向量, 记录了档案矩阵中员工的工作时间. μ 是一个固定的常数, 反映了单位时间内员工生产力的增加.

根据上述计算方法, 我们可以发现公司的业绩受到员工流失的直接和间接影响: 一方面是直接影响, 因为随着职位的调整, 工作时间也会改变, 在现实生活中这通常被看作是工作经验的流失; 另一方面, 间接影响也不容忽视, 例如员工流失引起的合作关系下降, 以及员工流失对工作积极性的影响. 同时, 流失率也会上升. 上述这些分析都会对公司产生消极的影响.

在建立静态网络模型后, 考虑到模型有随着时间发展的特点, 我们需要利用演进驱动力使得静态网络模型发展变化. 这也是组织中员工流失的缘由.

系统中存在三大主要的演进驱动机制: 员工的晋升, 员工的流失和新员工的外部招聘. 在接下来的这一节中, 我们将详细描述这些驱动力是如何影响网络的演进过程, 然后把



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of our network in details and then put it into operable and quantizable algorithm. Since the model is based on a series of bold assumptions, we will first propose our assumptions.

2.3.1 Employee dismissal algorithm | 员工离职算法

Just as the CEO is panicked when hearing that the current churn rate is 18% per year, the toughest problem the company faces is the extremely high churn rate. So the churn of employee has become the primary driving force, thus must be given priority to consideration. On the one hand, because employees in high-middle-level positions often has he high working ability, so if they churn, the company will suffer direct effects of the lower of productivity in certain position as well as the rising of recruitment cost. On the other hand, apart from direct effects, there are still some indirect effects that can't be ignored such as the rising of churn rate or the lower of working enthusiasm in other divisions or offices. What's more, if the vacant position needs to be filled through internal promotion, there will be a series of changes in position, which can badly affect the benefit of company.

Aiming at ICM company, the employee churn algorithm is based on the following **assumptions**:

- The churn of employee is random, but throughout the year, the present churn rate is 18%(as is the case in issue 7), so we suppose the churn of employee is even throughout the year.
- If one employee put forward departure, it will cause bad effect to other employees in his division or office(as is the case in issue 2).
- The position change, including promotion, churn and recruitment is regarded to take place once a month(the data given with the unit of per month); this assumption is always valid in the following statement.
- As is stated in the task, the mid-level positions are critical one that need to be filled all the time. So we assume that employees in middle-low-level positions can leave the position directly but employees in middle-high-level positions must wait until there are new suitable substitutes.

Now we start to illustrate our employee dismissal algorithm in detail. The

它变为可操作可量化的算法. 由于模型是基于一系列大胆的假设, 我们将首先提出我们的假设.

正如 CEO 听到当前员工年流失率为 18% 时很恐慌一样, 非常高的员工流失率是公司面临的最棘手的问题. 因此员工流失成为主要的驱动力, 必须优先给予考虑. 一方面, 因为高中层级的员工具有较高的工作能力, 所以如果他们流失, 公司将受到特定职位生产力降低的直接影响, 还有上升的招聘成本. 另一方面, 除了直接影响, 一些间接影响也不能被忽视, 例如员工流失会导致其他部门或办公室员工流失率增加或者工作积极性减弱. 更要的是, 如果空缺职位需要通过内部晋升填补, 那么职位就会有一系列的变化, 这会对公司的效益产生消极影响.

针对 ICM 组织, 员工离职算法基于以下假设:

- 员工的离职是随机的, 但是在现在这一年中, 当前的流失率为 18% (正如问题 7 中所描述), 所以我们假设员工的流失在全年中分布均匀.
- 如果一个员工提出离职, 会对所在部门或办公室的其他员工产生消极影响 (正如问题 2 中所描述).
- 我们假设职位的改变每月发生一次 (也就是以月为单位时间给出数据), 职位的改变包括晋升, 流失和招聘; 该假设适用于后文的陈述.
- 正如在任务中所述, 中层级职位是重要的, 任何时候都不能有空缺. 所以我们假定中下层员工可以直接离职, 但是中上层员工离职必须等到有合适的员工接替时才能离职.

现在我们将详细地说明员工离职算法. 员工的流失是随



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churn of employee is random; here we use Monte Carlo Algorithm to simulate whether the employee will churn. Monte Carlo Algorithm is a method that uses random numbers to do the simulation experiment. It can be used to do the random observation of sampling in system we are studying, then we can do the observation statistics of the sample to obtained the possible development of system in the simulation condition.

机的; 这里我们使用蒙特卡洛算法来模拟员工是否会离职.. 蒙特卡洛算法是一种利用随机数做模拟试验的方法, 它可以实现我们研究系统的随机抽样观察, 然后我们对观察样本做统计就可以获得系统在模拟条件下的可能发展趋势.

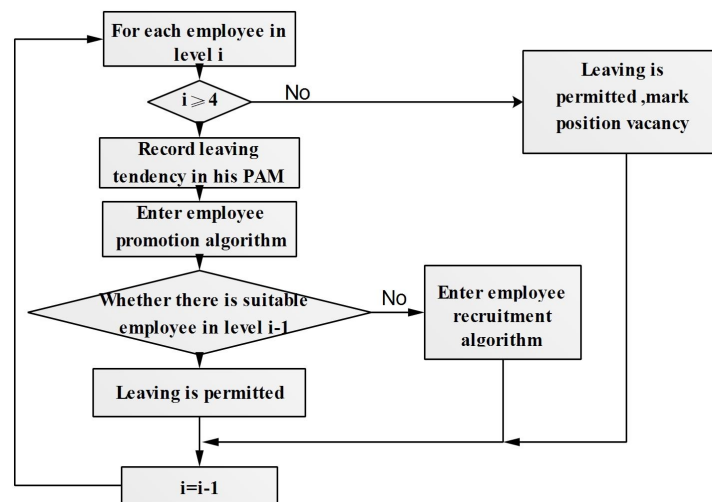


Figure 4: Employee dismission algorithm | 员工离职算法

In every month, we use Monte Carlo Algorithm to simulate whether an employee will churn and the expected churn rate is determined by the specific request in the task. Figure 4 is employee churn algorithm, the mainly steps are as follows:

- **Step 1:** Traverse every employee from high level to low level to set the churn probability according to the requirement of the question. Use Monte Carlo Algorithm to simulate the churn condition. If he churns, enter step 2; if not, repeat step 1 to test next employee.
- **Step 2:** For churned employee, first call Post Archives Matrix to read his

在每个月, 我们都使用蒙特卡洛算法模拟判断某个员工是否会离职, 并且预期流失率是根据任务中特定要求确定的. 图4是员工流失算法流程图, 其主要步骤如下:

- **第 1 步:** 根据问题要求从高层级到低层级遍历每一个员工设置其流失概率, 利用蒙特卡洛算法模拟流失情况. 如果该员工离职, 进入第 2 步; 否则, 重复第 1 步测试下一个员工.
- **第 2 步:** 对于离职的员工, 首先从档案矩阵中读取有



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level and record that he has churn tendency. Judge his level: if he is in low level then churn is permitted and return to step 1; if he is in high-middle level, then enter step 3.

- **Step 3:** Given priority to promotion algorithm for the post mobilization in high-middle level position: if internal promotion can realize then churn is permitted meanwhile record vacancy in pre-promotion position; if not, call employee recruitment algorithm and the employee can only leave till meet the median of the certain recruitment time. After that, the position is filled with new worker, the archive is refreshed.

To make further illustration, the analysis above contains the record of Post Archives Matrix: when churn request is put forward, position records as churn put forward; when churn happens, the number of position changing time add one and others return to initial values. Besides, if there exist the changing in other positions caused by churn, similar adjustment should be made in original position.

2.3.2 Employee promotion algorithm | 员工晋升算法

Every company will give priority to promote internal employee to higher position, because this can directly promote employees' production enthusiasm. However, we must consider that only the employee with certain ability that required in a higher level position can be promoted, so that we can avoid undermining the overall strength of the employees in the company.

The promotion of employees is based on the following **assumptions**:

- When one position is in vacancy, the HR office will first consider the promotion of internal employee rather than the external recruitment.
- Whether the employee will be promoted will be affected by two conditions, which are indispensable: for one thing, the employee's synthetic ability meets the promotion standard; for another, there is vacancy in high level position. Once the employee meets these conditions, we consider that the employee is always willing to be promoted to higher position.
- The employee's promotion is strictly according to the level (there are seven levels in total) given in the question, so there is no grade-skipping promotion. The employee's is never restricted by present position, so they can be

关他所在的层级并记录其离职意向. 判断他的层级: 如果他在低层级那么允许直接离职并返回 textbf 第 1 步; 如果他在中高层级, 则进入第 3 步.

- **第 3 步:** 对于高层级员工优先考虑晋升算法: 如果内部晋升是可行的, 那么允许当前职位的员工离职, 并且同时记录预晋升职位的空缺; 否则调用员工招聘算法, 当满足等到确定的招聘时间的中值时员工才能离职. 之后, 新员工填补空缺职同时更新档案.

为了进一步说明, 以上分析也包含了档案矩阵的记录: 当员工提出离职请求时, 在相应的职位做记录; 当员工已经离职, 职位改变次数增加 1, 其他返回到初始值. 此外, 如果离职引起了其它职位的变化, 需要在原来的职位上做类似调整.

每个公司都会优先考虑内部员工晋升到更高的职位, 因为这直接提升了员工的工作热情. 但是我们必须确保只有达到更高层次职位所要求的工作能力的员工才会被晋升, 这样我们可以避免降低公司员工的整体实力.

员工的晋升基于以下假设:

- 当有某个职位空缺时, 人力资源办公室首先会考虑内部员工的晋升而不是从公司外部招聘.
- 员工是否能晋升受到两个必不可少的条件影响: 一是, 员工的综合能力达到晋升的标准; 二是高层次的职位有空缺. 一旦员工达到这些条件, 我们认为员工总是愿意被提拔到更高职位的.
- 员工的晋升是严格按照问题所给的职位层级 (总共有七个层级), 因此没有越级晋升. 员工永远不会被当前的职位所限制, 所以他们可以被提拔到其他部门和职



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promoted to other divisions and positions.

Next, we will discuss the employee's promotion strategy in details. Taking the promotion condition in assumption 2, we need to traverse the present position every month in order to promote the employee who meets the promotion condition to higher position to facilitate employees' working enthusiasm. In view of the fact that the company has seven position levels, so we must first fill the position in higher level. In other words, we need to test the vacant position in descending order. The promotion has the priority that the internal promotion has preference to external recruitment. Since the promotion the caused by employee's churn has been discussed in employee churn algorithm, here we just consider promotion mechanism when there are vacant position in our promotion algorithm. Figure 5 is the employee promotion algorithm, the mainly steps are as follows:

- **Step 1:** Start from the highest position to check whether it is vacant. If it is vacant, enter the next level.
- **Step 2:** In the lower level, rank the time of working time of employees who are still at position and have not put forward departure. Then select the maximum value and judge whether he meets the promotion condition. Promotion condition is regard as the employee's maximum productivity meets the initial value in higher level.
- **Step 3:** If meets the condition, then promotion is permitted and record vacancy in the present position; otherwise, promotion isn't permitted so the position is still vacant. Then return to step 1 to repeat the same step in next position.

To make further illustration, the Post Archives Matrix must be record timely when there exit promotion: when meets the promotion condition, the vacant position is filled and the employee-changing number adds one, other numbers return to the initial value; In the meantime, there will be new vacant position in lower level, which will be tested whether this position can be filled timely.

2.3.3 Employee recruitment algorithm | 员工招聘算法

When there is vacant position but no internal employee can be promoted to

位.

接下来,我们将详细讨论员工的晋升策略. 为了促进员工的工作积极性,根据假设 2 中的晋升条件,每个月我们需要遍历当前所有职位,以提拔那些达到高职位晋升条件的员工. 鉴于公司有七个职位层级的实际情况,所以我们必须保证较高层级中职位没有空缺. 也就是说,我们需要以降序方式检查空缺职位. 员工的晋升是优先的,也就是说员工的内部晋升要优于外部招聘. 由于员工流失引起的职位晋升已经在员工流失算法中讨论过,这里,在我们的晋升算法中仅考虑当有职位空缺时的晋升机制. 图5是员工晋升的流程图,主要步骤如下:

- **第 1 步:** 从最高职位开始检查是否有职位空缺,如果有,进入下一个层级.
- **第 2 步:** 在较低层级,对那些仍然在岗并且没有提出离职的员工工作时间进行排序. 然后选出最大值,并判断该员工是否达到了晋升条件,晋升条件即为员工的最大生产力达到了较高层级生产力的初始值.
- **第 3 步:** 如果达到晋升条件,那么该员工的晋升是允许的,同时记录当前的空缺职位;否则该员工不允许晋升,那么较高职位仍然空缺. 这时返回步骤 1 对下一个职位重复上述步骤.

为了进一步说明,当有职位晋升时档案矩阵要及时记录:当员工达到晋升条件,空缺职位会被填补,员工变化数量加 1,其他量返回到初始值;同时,较低职位就会有新的空缺,要检查这个空缺职位是否能被及时的填补.

当存在空缺职位,却没有可以晋升到此职位的内部员工,



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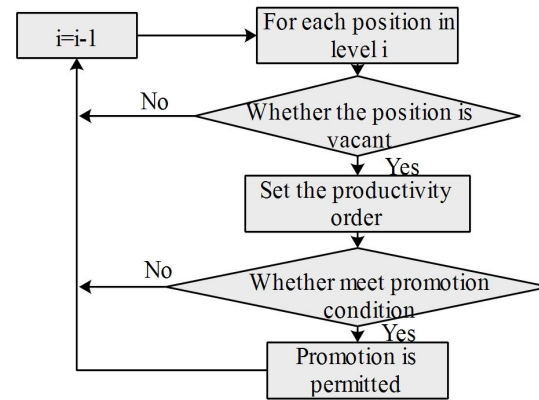


Figure 5: Employee promotion algorithm | 员工晋升算法

this position, company must take external recruitment strategy. In the question, the recruitment of employees in different levels needs different time, which is just the delaying effect of recruitment.

The recruitment is based on the following **assumptions**:

- Since the median of recruitment time for employees in each level has been given, we consider that the company can always recruit suitable employees in certain time after the sending out of recruitment notice.
- The employee who is recruited should be with the lowest productivity in certain level.
- The number of the position that need recruitment should be in direct proportion to the number of present vacant position.

Next, we will illustrate the recruitment algorithm in details. Based on the churn and promotion algorithm, we finally take the recruitment algorithm. For high-middle level position, because that it isn't permitted to be vacant, so if there is no one meets the promotion condition we will start the recruitment directly, which has been illustrated in churn algorithm. So here we will mainly illustrate recruitment algorithm in low level:

- **Step 1:** After doing the churn and promotion algorithm, we sum up the

公司必须采取从外部招聘的策略. 在问题中, 招聘不同层级员工所需的时间不同, 这只是招聘的延迟效应.

招聘算法基于以下**假设**:

- 由于每个层级员工招聘时间的中值已经给出, 我们认为公司在发出招新通知后总能在一定时间内招聘到合适的员工.
- 招聘到的员工应当具有相应层级的最低生产力.
- 需要招聘的职位数量应当直接正比于当前空缺职位的数量.

接下来, 我们将详细阐明员工招聘算法. 在员工流失算法和晋升算法的基础上, 我们最后给出招聘算法. 对于中高层级职位, 因为不允许有空缺, 所以如果没有内部员工达到晋升条件, 我们将直接启动招聘程序, 这在员工流失算法中已经给出了说明. 所以这里我们主要说明低层级职位中的招聘算法:

- **第 1 步:** 在使用员工流失算法和招聘算法后, 我们汇总



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vacant position condition in the company.

- **Step 2:** Make use of the given recruitment rate to calculate the number of employees needed in different levels and then record it respectively in time vector which reflects the recruitment time. Each month, reduce one in this vector when there is recruitment plan in certain position and record the time value of the number of employee needs to be recruited in this month.
- **Step 3:** Make initialization in Post Archives Matrix when the recruitment time reduces to zero, that means the position obtained new employee.

2.4 Simulation results of the model | 模型模拟结果

2.4.1 Prediction for ICM's budget requirements | 预测 ICM 的预算需要

Using our human capital evolution network model, we can commendably predict the budget requirements in recruitment and training in two years.

- Parameters identification

The mainly parameters for the model has been given in the problem, so here we just identify a few parameters that are hard to quantify. We define these parameters to reflect the changes of employees' efficiency and churn rate. According to our model, when $\xi_1 = 1/3$, $\xi_2 = 1/3$, $\varepsilon\% = 2\%$, we can simulate the on-the-job rate curve in next 100 months, as is shown in Figure 6. In the figure, it's quite clearly that the working rate fluctuates around 85% at the beginning six months. Later there are some changes and finally becomes stable at 80%. This result is in accord with the real situation of ICM company.

- Model results

Because we use the Monte Carlo algorithm, it is inevitable that there are some fluctuations. In order to deal with this problem, we simulate model 10 times to predict the cost of recruitment and training cost per month, as is shown in Figure 7, Figure 8. Calculate the average of ten simulation results, we obtain the conclusion that the budget requirements for recruitment is 26.22σ and the budget requirements for training is 140.15σ .

公司的职位空缺情况.

- **第 2 步:** 利用所给招聘率计算不同层级需要招聘的员工数量, 然后分别记录在反映招聘时间的时间向量中. 每个月, 当某些职位上有招聘计划时, 在相应的时间向量中减 1, 并记录这个月需要招聘员工的这些时间值.
- **第 3 步:** 当招聘时间减少到 0 时, 我们就初始化档案矩阵, 这意味着, 该职位有了新员工.

利用我们的人力资本演进网络模型, 可以很好地预测出公司在未来两年内的招聘和培训成本预算.

- 参数识别

模型的主要参数在问题中已经给出, 因此这里只需确定几个难以量化的参数. 我们定义这些参数是反映员工工作效率和流失率变化的. 根据我们的模型, 当 $\xi_1 = 1/3$, $\xi_2 = 1/3$, $\varepsilon\% = 2\%$ 时, 我们可以模拟出接下来 100 个月的在职率曲线, 如图6所示. 在图中, 非常清晰地看到, 在最初的六个月在职率围绕 85% 波动, 随后有一些改变, 最后稳定在 80%, 这与 ICM 公司的实际情况相符.

- 模型结果

因为我们使用了蒙特卡洛算法, 所以有些波动是不可避免的. 为了解决这个问题, 我们做了 10 次模拟来预测每个月员工的招聘和培训费用, 如图7和图8所示. 计算 10 次模拟结果的平均值, 我们得到结论: 招聘预算为 26.22σ , 培训预算为 140.15σ .



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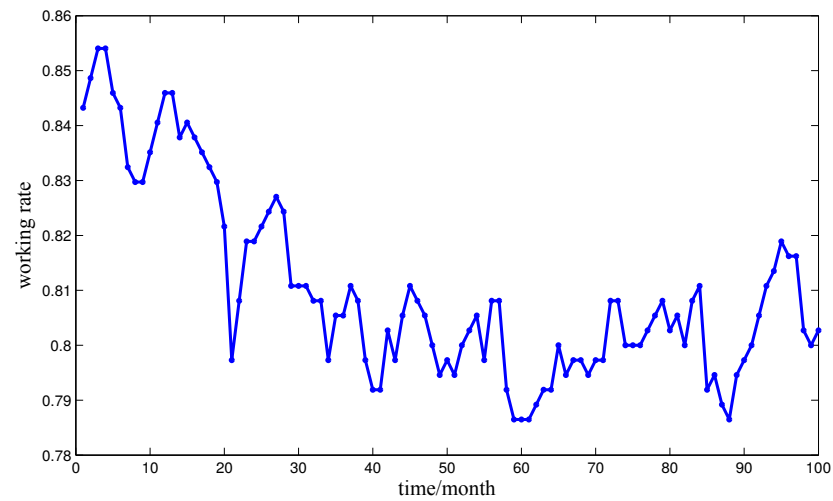


Figure 6: The prediction of working rate | 在职率预测

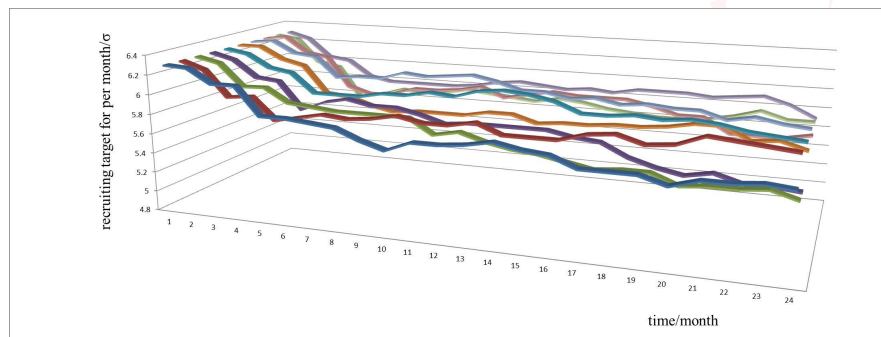


Figure 7: The cost of recruitment per month | 每个月员工招聘费用

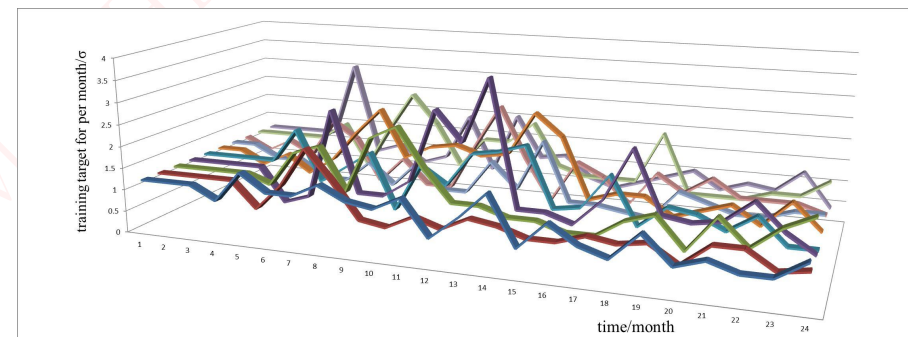


Figure 8: The cost of training per month | 每个月员工培训费用

- Rationality Analysis
According to simulation results in recruitment cost, we find the cost is rela-

- 合理性分析
根据模拟结果中的招聘成本, 我们发现它在开始的三个



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tively stable in the beginning three months; this is affected by the hysteresis effect of low-level recruitment. Later, from the fifth month to the fifteenth month, recruitment cost fluctuates drastically, the standard deviation is 0.955 and the highest cost per month even reached to 2.9σ . We analysis that during this period the churn effect is remarkable and the system is transiting from a stable condition to another stable condition. After the fifteenth month, because the recruitment cost gradually becomes stable, the system enters a stable condition and the standard deviation is 0.156. It's obvious from the simulation results that the training cost declines. We think that this is because every employee should be trained, so the working rate declines thus the training budget declines gradually and then reaches a stable value.

2.4.2 Effects of the changing churn rate | 改变流失率的影响

We know from the background, ICM suffers a lot from the increasing churn rate. On the one hand, the churn of employee will directly cause the declination of company's performance; On the other hand, it can also cause indirect effects such as other employees' working efficiency and churn rate which may cause the continue declination of the whole company performance. Then we suppose the churn rate reaches to 25% and 35% respectively to simulate the possible results to quantify the effects.

- **Parameter idendification**

The initialization parameter is the same with the former one, we only need to make some adjustments for ICM's churn rate to let it to 25% and 35% respectively.

- **Model results**

In order to describe the costs of these higher turnover rates and indirect effects of these high churn rates, we select three indicators: working rate, company performance and average enthusiasm of the employee. Then we use our model to simulate the changes of these indicators in the following 100 months and the results are as follows:

月相对稳定, 这是由于受到招纳低层级员工滞后效应的影响. 随后, 从第五个月到第十五个月招聘成本大幅波动, 数据标准差为 0.955, 最高成本甚至达到一个月 2.9σ . 我们分析认为这段时间员工流失产生的影响是显著的, 系统正从一个稳定状态过渡到另一个稳定状态. 第十五个月后, 因为招聘成本逐渐变的稳定, 因此系统也进入稳定状态, 标准差为 0.156. 从模拟结果可以很明显看出培训成本在降低. 我们认为这是因为每一个员工都要进行培训, 因此在职率的降低会引起培训预算逐渐下降, 然后达到一个稳定值.

我们从问题的背景知道, 日益增长的流失率十分困扰 ICM. 一方面, 员工流失将直接导致公司业绩下降; 另一方面, 它也带来间接影响, 诸如影响其他员工的工作效率和流失率, 这将可能导致整个公司业绩持续下降. 然后我们假设流失率分别达到 25% 和 35% 时, 模拟可能的结果以量化其影响.

- **参数识别**

参数初始化值与之前相同, 我们只需对 ICM 组织的流失率做出某些调整, 使之分别为 25% 和 35%.

- **模型结果**

为了描述这两种较高流失率下的成本和间接影响, 我们选择三个描述指标: 在职率, 公司业绩和员工的平均工作积极性. 然后我们利用本文的模型去模拟接下来的 100 个月内这些指标的变化, 结果如下:



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When the churn rate is 25%, the working rate curve figure in the following 100 months is Figure 9, the scatter diagram of total company performance per month is Figure 13 and the working efficiency curve figure of ICM's employee Figure 11.

When the churn rate is 35%, the working rate curve figure in the following 100 months is Figure 10, the scatter diagram of total company performance per month is Figure 12 and the working efficiency curve figure of ICM's employee is Figure 14.

When the churn rate is 25%: ICM's working rate declines gradually. In the twelfth month, the working rate drops to below 80% and ultimately reaches to around 75%. The company performance is continues in adjacent month, there is obvious fluctuation at the beginning and gradually becomes stable. Ultimately, company performance reaches to around 179σ . In our model, employees' enthusiasm is a parameter with the value between 0 to 1 and the higher the value, the effective the employee. So this is an indirect indicator of company performance. The result of our simulation is that employees' working efficiency reaches to 0.87.

When the churn rate is 35%: ICM's working rate declines faster than that of 25%. In the fourth month, the working rate drops to below 80% and ultimately reaches to around 67%. The company performance is continues in adjacent month, there is obvious fluctuation at the beginning and gradually becomes stable. Ultimately, company performance reaches to around 153σ . The result of our simulation is that employees' working efficiency reaches to 0.81, which is lower than the former result.

Based on what we have discussed above, we can draw the conclusion: with the increasing of churn rate, the working rate declines, the whole company performance slips down and the employee's enthusiasm drops down correspondingly. To be more distinct, the direct effect is that benefits of certain position declines and the indirect effect is that churn rate increases and the working enthusiasm declines in other divisions and offices. Caused by direct and indirect effects, the company performance continually declines. In other words, churn rate has positive feedback effect that can accelerate the declination of company performance.

当流失率为 25% 时, 随后 100 个月的在职率曲线如图9所示, 图13为每月公司总业绩的散点图, ICM 员工的工作效率曲线如图11所示。

当流失率为 35% 时, 随后 100 个月的在职率曲线如图10所示, 每月公司总业绩散点图如图12所示, 图14表示 ICM 员工的工作效率曲线。

当流失率为 25% 时: ICM 的在职率逐渐降低, 在第十二个月时, 在职率跌至 80% 以下, 最终达到 75% 左右; 公司业绩在起初的几个月有明显地波动并逐渐变得稳定, 最终公司业绩约为 179σ 。在我们的模型中, 员工的积极性是一个取值在 0 到 1 之间的参数, 值越大表示员工的工作效率越高, 因此这是反映公司业绩的一个间接指标。我们模拟出来的结果是: 员工的工作效率可达 0.87。

当流失率为 35% 时: ICM 的在职率下降速度快于 25% 时。在第四个月时, 在职率下跌至 80% 以下最终达到 67% 左右; 公司业绩在起初的几个月有明显地波动并逐渐变得稳定, 最终公司业绩约为 153σ 。我们的模拟结果得到员工的工作效率为 0.81, 低于先前的模拟结果。

基于我们以上的讨论, 我们可以得出结论: 随着员工流失率的增加, 员工在职率下降, 公司总业绩下滑, 相应地, 员工工作积极性也降低。为了更明显, 直接影响是确定职位的收益下降, 间接影响是流失率的增加以及其他部门和办公室员工工作积极性地下降。由于直接和间接的影响, 公司业绩持续下降。也就是说, 员工流失率具有负反馈作用, 它可以加快公司业绩的下降。



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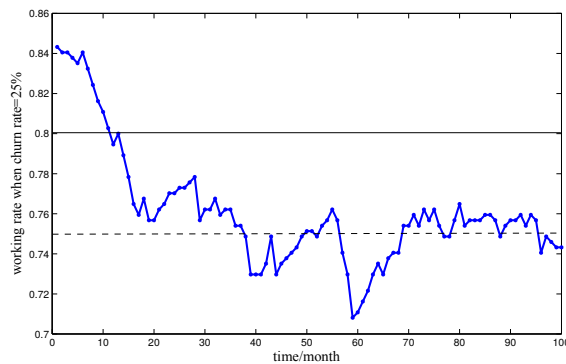


Figure 9: Working rate when churn rate is 25% | 离职率为 25% 下的在职率

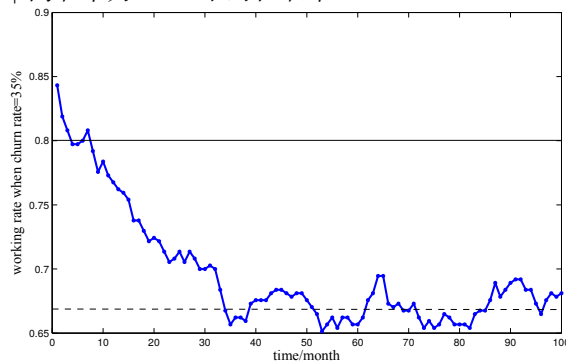


Figure 10: Working rate when churn rate is 35% | 离职率为 35% 下的在职率

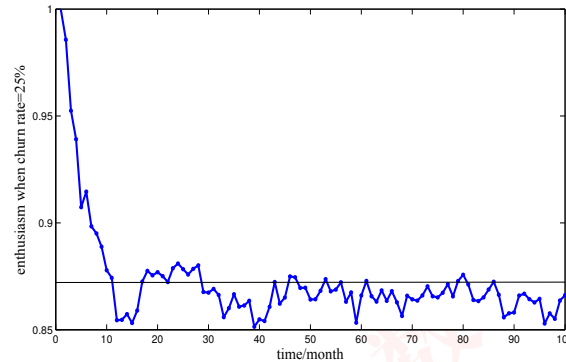


Figure 11: Working efficiency when churn rate is 25% | 离职率为 25% 下的工作效率

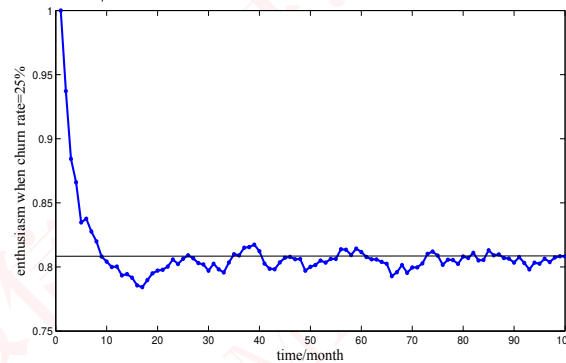


Figure 12: Working efficiency when churn rate is 35% | 离职率为 35% 下的工作效率

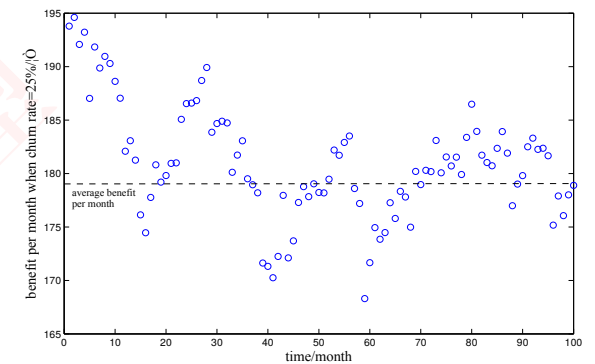


Figure 13: Company performance when churn rate is 25% | 离职率为 25% 下的公司业绩

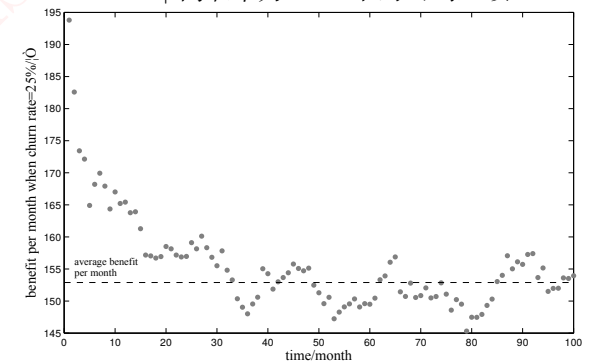


Figure 14: Company performance when churn rate is 35% | 离职率为 35% 下的公司业绩

• Rationality analysis

The results we get is basically in accord with actual conditions in the company: under the condition that the present management method is unchanged, the more the employee churn, the faster the declination is the company performance. Only when consider the employee's enthusiasm, we initialize it as 1 at the beginning in our model and this can be a cause of fast

• 合理性分析

我们得到的结果与公司实际情况基本一致: 当前管理方式不变的条件下, 员工会流的越多, 公司业绩下滑速度越快. 只有当考虑了员工的工作积极性, 一开始在模型中我们设定其初始值为 1, 这可能是普通员工工作积极性在一开始快速下降的原因.



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declination of average employee enthusiasm at the beginning.

2.4.3 Effects of no external recruitment of middle-level employees | 没有外部招聘对中层员工的影响

We know from the background, ICM suffers a lot from the increasing churn rate. On the one hand, the churn of employee will directly cause the declination of company's performance; On the other hand, it can also cause indirect effects such as other employees' working efficiency and churn rate, which may cause the continue declination of the whole company performance. Then we suppose the churn rate reaches to 25% and 35% respectively to simulate the possible results to quantify the effects. The ICM HR supervisor presents us higher request for our human capital network model: in two years, there is no external recruitment of middle-level employees and we just consider internal promotion. In this way, we must make some adjustment in churn algorithm, promotion algorithm and recruitment algorithm to adapt to new regulation.

- Improvement for original algorithm

1. In churn algorithm, there is no extra effect for low-level employees. However, for middle-level employees, they must stay until the low-level employees are able to be promoted. In order to be accord with real case, middle-level employees' efficiency $\varepsilon\%$ will decline to 2%, if they have leaving tendency.
2. In recruitment algorithm, the company just recruit low-level employee. Under the condition that the recruitment employee is 2/3 of the vacant position, the recruitment of low-level employees will increase.

- Parameter identification

The Initialization parameter is the same with the former one, we only need to make some adjustments for ICM's churn rate, to let churn rate of middle-level employees to be 30% and others' churn rate still maintains at 18%.

我们从问题的背景知道, 日益增长的流失率十分困扰 ICM. 一方面, 员工流失将直接导致公司业绩下降; 另一方面, 它也带来间接影响, 诸如影响其他员工的工作效率和流失率, 这将可能导致整个公司业绩持续下降. 然后我们假设流失率分别达到 25% 和 35% 时, 模拟可能的结果以量化其影响. ICM 组织人力资源主管对我们的人力资本网络模型提出了更高的要求: 在两年内, 对中层级员工没有外部招聘, 我们只考虑内部晋升. 这样, 我们必须对员工流失算法, 晋升算法和招聘算法做出适当的调整以适应新的要求.

- 改进原算法

1. 在员工流失算法中, 低层级员工的外部招聘不受影响. 然而, 对于申请离职的中层级员工, 他们必须留在岗位直到低层级员工能够被提拔. 为了与实际情况相符, 中层员工如果有离职的倾向, 那么他们的工作效率 $\varepsilon\%$ 下调为 2%.
2. 在员工招聘算法中, 公司只招聘低层级员工. 在招聘人数为空缺职位 2/3 的情况下, 低层级员工招聘人数将会增加.

- 参数识别

参数初始化值与之前相同, 我们只需对 ICM 组织的流失率做出某些调整, 使得中层级员工的流失率变为 30% 其他仍保持 18%.



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- Model results

What are the effects for the HR health of the organization if we change the model? Here, we will describe the HR health in four indicators: working rate, company performance and average enthusiasm of middle-level employees and average change times of middle-level employees.

In order to make our simulation results directly reflect the effects of whether middle-level recruitment is permitted, we use control-variate method: that is we only change the recruitment method for middle-level position and other parameters such as churn rate stay the same. After ten times simulation, we calculate the average value of these main indicators per month to draw the graph in order to analysis these indicators respectively.

Working rate: the working rate of two different recruitment methods in two years is shown in Figure.15. Comparing the two curves, we find that the working rate declines gradually from 85% and the working rate is higher if there is no external recruitment. From this result, we can draw the conclusion that taking the only-promotion method will decline the churn rate and this can do good to HR health.

Company performance: the company performance of two different recruitment methods in two years is shown in Figure.16. Comparing the two curves, we find that the company performance will gradually decrease and then become flat; there is no distinct difference in the company performance: there is just a little increase if there is no external recruitment. So if we use the recruitment method that middle-level employees only promoted internally, we can slightly improve the company performance and this is good to the HR health.

Average enthusiasm of middle-level employees: the average enthusiasm of middle-level employees of two different recruitment methods in two years is shown in Figure.17. Comparing the two curves, we find that the average enthusiasm is quite close in the first ten months; after ten months, the average enthusiasm is higher if we hire the middle-level employees externally. This shows that if we use no-external recruitment method of middle-level employees, the average enthusiasm will decline. So HR office should pay attention to it.

- 模型结果

如果改变我们的模型会给公司的人力资源健康发展带来什么影响呢? 这里, 我们将用四个指标来描述人力资源健康: 在职率, 公司业绩, 中层员工的平均工作积极性和中层员工的平均改变时间.

为了使我们的模型能直接地反映是否允许招纳中层级员工, 我们采用控制变量法: 也就是我们只改变中层级职位的招聘办法, 其他参数如流失率等保持不变. 通过十次模拟, 我们计算出这些主要指标的月平均值, 并分别绘图以便分析.

在职率: 图15显示了在两年内两种不同招聘方式的在职率. 比较两条曲线, 我们发现在在职率从 85% 开始逐渐下降, 如果没有外部招聘在职率会更高一些. 从这些结果, 我们可以得出这样的结论: 只有一种晋升方式可以降低流失率并且有益于人力资源的健康发展.

公司业绩: 图16显示的是两年内两种不同招聘方式下公司业绩. 比较两条曲线, 我们发现公司业绩逐渐下降然后变得平坦; 两种不同招聘方式下公司业绩没有显著的差别: 如果没有外部招聘, 那么业绩会有那么一点点地提高. 因此如果我们采用只通过内部晋升的方式来招聘中层级员工, 我们能够略微提高公司的业绩, 这也利于公司的人力资源健康发展.

中层员工的平均工作积极性: 图17给出了两年内两种招聘方式下中层级员工平均工作积极性. 比较两条曲线, 我们发现在最初的 10 个月两条平均工作积极性曲线非常接近; 十个月之后, 采取外部招聘中层级员工的方式, 员工平均工作积极性较高. 这表明, 如果我们采用不对外招聘中层级的员工的方式 (只采用内部晋升的方式), 平均工作积极性会降低, 所以人力部门要注意这一点.



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Average change times of middle-level employees: the average enthusiasm of middle-level employees of two different recruitment methods in two years is shown in Figure.18. Comparing the two curves: if we permit external recruitment of middle-level employees, the average enthusiasm of middle-level employees will increase. We consider that this is because if we use external method, it is more convenient for employees to transfer the position. From the perspective of HR health, it will better improve the stability of the position if we permit external recruitment for middle-level employees.

Based on what have discussed above, we can draw the conclusion: from several indicators such as working rate, company performance and average enthusiasm of middle-level employees and average change times of middle-level employees, we think that it is more beneficial for HR health if we use external method. Besides, although it may decrease the enthusiasm of some middle-level employee, it can greatly increase the enthusiasm of low-level employee and the stability of middle-level group.

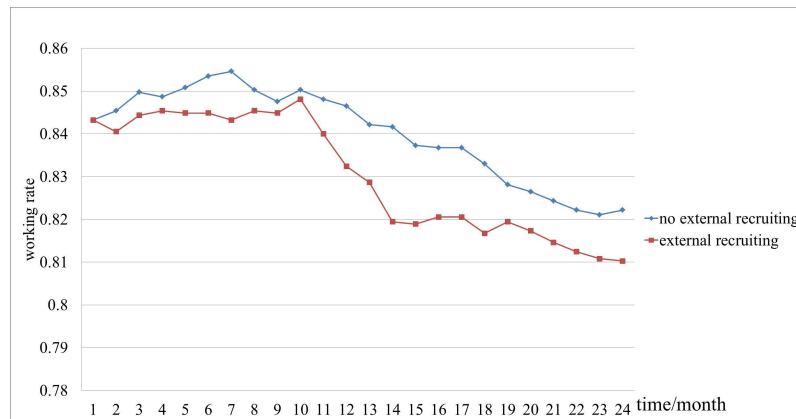


Figure 15: Working rate of two recruitment methods | 两种招聘方式下的在职率

- Rationality Analysis

中层员工的平均改变次数: 图18显示的是两年内两种不同招聘方式下中层员工的平均改变次数. 比较两条曲线: 如果我们允许从外部招聘中层级员工, 中层员工的平均改变次数会增加. 我们认为这是因为如果我们使用从外部招聘, 这将方便员工更换职位, 从人力资源健康的角度来看, 如果在中层级职位我们招纳外部员工, 这将有利于提高员工在职稳定性.

基于以上讨论, 我们得出结论: 根据一些指标如工作速率, 公司业绩, 中层员工的平均工作积极性和中层员工的平均改变次数, 我们认为采用外部招聘策略将更有利于人力资源健康. 此外这可能降低一些中层级员工的工作热情, 但却能很好的提高低层级员工的工作热情和中等群体的稳定性.

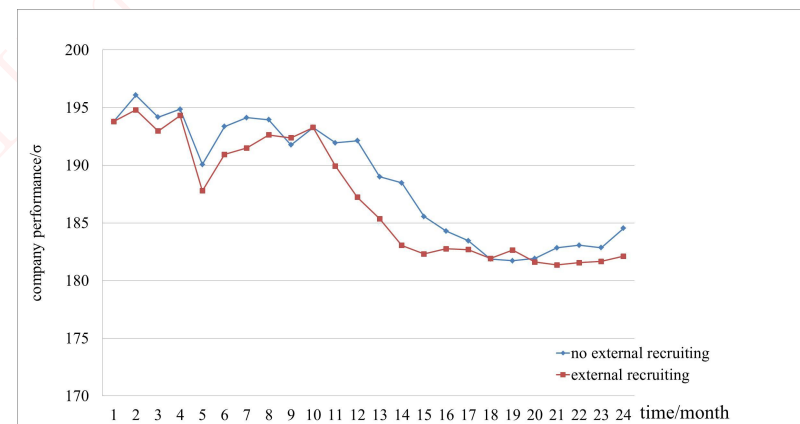


Figure 16: Company performance of two recruitment methods | 两种招聘方式下的公司业绩

- 合理性分析



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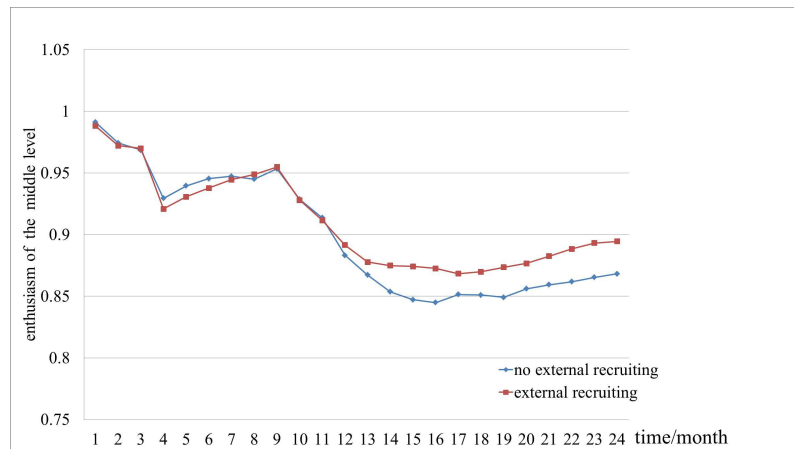


Figure 17: Average enthusiasm of middle-level employees of two recruitment | 两种招聘方式下中层员工的平均工作积极性

The results is basically in accord with the real case. What's more, the study in human capital fields also provide us following conclusions: if we use internal promotion method for middle-level employees, it will do good to personnel decision; the employee who has been working in an organization for some time can get involved in the organizational culture so that they have higher integrating degree with the organization; internal promotion can provide opportunities for each employee and this can increase the working enthusiasm; it can reduce the cost.

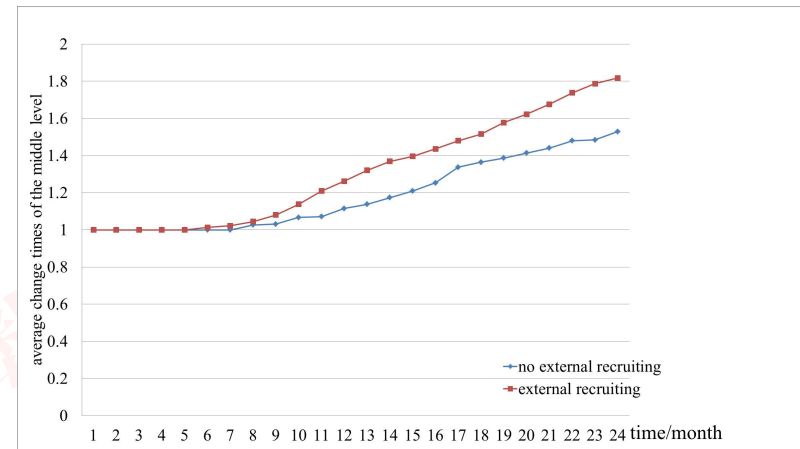


Figure 18: Average change times of middle-level employees of two recruitment | 两种招聘方式下中层员工的平均改变次数

结果基本符合实际情况. 而且人力资本研究领域也为我们提供了以下结论: 如果我们采取内部晋升的方式来招聘中层级员工, 这将有助于个人的决断; 员工如果在组织中工作过一段时间, 那么他可以融入到组织文化当中, 因而能更好的与组织一体化; 内部晋升可以为每个员工提供机会, 这样可以提高工作积极性; 还可以降低成本.

3 Multilayer Network Model | 多层网络模型

3.1 The multilayer network for ICM | ICM 的多层网络

The single layer network can only include one aspect of a problem. However, the team performance is not only influenced by working relationship but many relationships in common life. The Human Capital network is a one-layer network reflecting the relationship between higher level and lower level. We add the friend-

单层网络只能包含问题的一方面. 然而一个团队的表现不仅受到工作关系的影响而且受到生活中多种关系的影响. 人力资本网络是一个反映高层级与低层级之间关系的单层网络. 我们加入了友谊网络和竞争网络, 这样就形成了多层网



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ship network and the competition network to the human capital network. Thus we get a multi-layer network.

The multilayer network is based on the Networks office in ICM, which consists of two divisions of 7 people. The reason why not based on the whole company is the complexity is too huge, that the number of possible edges grows as $O(n^{2(d+1)})$ [7].

The Human Capital network in this office has been established above. And the relationship in it won't change anytime. For the friendship network, the edges between two nodes i, j stand for that i and j are good friends so they will attract each other. In other words, if i wants to leave ICM, j is one reason can hold i back. In the competition network, we assume the major competitions happen in peer group and all competitions are well to promote everyone to improve. The edges between two nodes i, j stand for that i and j compete with each other. Obviously, proper competitions can improve team performance which will also hold employees to stay.

络.

多层网络是基于 ICM 的办公室网络, 它由两个每组 7 个人的部门组成. 我们只选取两个部门原因是由于整个公司的复杂性过大, 使得网络的边数会以 $O(n^{2(d+1)})$ 数量级增长 [7].

前面我们已经建立了人力资本网络, 其中的关系不会一直变化. 对于友谊网络, 连接两个节点 i 和 j 的边代表 i 和 j 是好朋友, 他们相互吸引对方. 换句话说, 如果 i 想离开 ICM 组织, j 是让 i 继续留在公司的一个原因. 在竞争网络中, 我们假设主要的竞争发生在实力相当的两个小组, 所有的竞争有利于每个人晋升. 节点 i 和 j 之间的边代表两者之间有竞争. 很明显, 适当的竞争可以提高团队业绩并且防止员工离职.

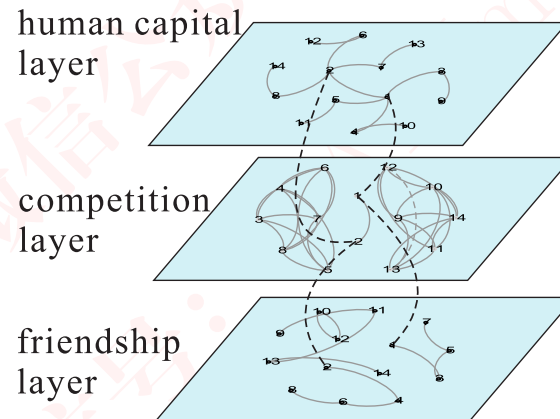


Figure 19: The multilayer network for Networks office | 办公网络的多层网络

Figure 19 shows the whole multilayer network structure, defined as $M = (V_M, E_M, V, L)$. The network M has a total of 14 nodes, every node represent a worker, $V = 1, 2, \dots, 14$. Three layers are connected together, so the $L = (L_1, L_2, L_3)$. We

图19显示的是整个多层网络的结构, 定义为 $M = (V_M, E_M, V, L)$, 网络 M 共有 14 个节点, 每一个节点代表一名员工, 用 $V = 1, 2, \dots, 14$ 表示. 网络的每个层级相互连接,



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represent the Human Capital layer as L_1 . the friendship layer as L_2 and the competition layer as L_3 . Each layer contains whole 14 nodes of V . The set V_M contains the whole nodes of the multilayer network. In this place it has 42 nodes. To distinguish every node in every layer, we use (i, L_j) to name the people i in the layer L_j . E_M show the set which contains whole edges in M . We show the edges that remain inside of a layer as solid lines and the edges that cross layers as dotted lines. The real meaning of the solid lines has been given above. And the cross-layers-edges only couple the same people in different layers. Which means two different people can't "communicate" in different layers. Not every cross-layers-edge is drawn in the Figure 19 just to show the meaning. To define every edge, we use $((i, L_m), (j, L_n))$ to stand for the edge between node (i, L_m) and node (j, L_n) . In ordinary graphs, adjacency matrices are often used to describe network. There are two methods to describe multilayer networks, the adjacency tensor and the supra-adjacency matrices [7]. The adjacency tensor is named as $A_{ijL_mL_n}$. For every person i in the layer L_m and person j in the layer L_n , $A_{ijL_mL_n}$ has a value of 1 if and only $((i, L_m), (j, L_n)) \in E_M$, otherwise the value is 0. And to get the supra-adjacency matrix, we should use tensor flattening to combine all aspects into one aspect. Then the supra-adjacency is just like normal adjacency matrices of ordinary graphs. In this paper, we define G_1, G_2, G_3 as the adjacency matrices of every layer and G_M as the supra-adjacency matrices of the multilayer network.

3.2 The analysis of multilayer network | 多层网络的分析

Some important basic characters such as node degrees, clustering coefficients, walks can be pretty helpful for analyzing the graph in general single layer network. And these characters can also be used in multilayer by aggregating data from the different layers into a monolayer network. According to the reference given by the question, team performance mainly affected by shared cognition. And shared cognition has positive correlation with the complexity of every aspect of the multilayer network. To explain this, we consider that:

- In the Human Capital layer, more edges or bigger degrees means one person needs to work with more people. It can make people get more training to

即 $L = (L_1, L_2, L_3)$. L_1 代表人力资本层, L_2 和 L_3 分别为其友谊层和竞争层, 每个层中都包含 V 中的 14 个节点. 集合 V_M 包含多层级网路中的所有节点. 在这里共有 42 个. 为了区分每个层级中的节点, 我们用 (i, L_j) 表示员工 i 在 L_j 层中. E_M 为包含了 M 中所有边的集合. 我们用实线表同一层级之间的边, 用虚线表示交叉层之间的边. 实线的真正含义上面已经给出. 不同层级之间也只相同的人才会有边相连. 这意味着不同层级之间的两个人不能“交流”. 图19只是作说明, 而并没有画出所有的交叉层的边. 为了定义每一条边, 我们用 $((i, L_m), (j, L_n))$ 来表示节点 (i, L_m) 和节点 (j, L_n) 之间的边. 对于普通图, 通常用邻接矩阵来描述网络. 对于多层网络, 存在邻接张量和超邻接矩阵两种方法描述 [7]. 邻接张量被命名为 $A_{ijL_mL_n}$, 对于 L_m 层中的每一个员工 i 和 L_n 层中的每一个员工 j , 当且仅当 $((i, L_m), (j, L_n)) \in E_M$ 时, $A_{ijL_mL_n}$ 的值为 1, 其他情况值为 0. 对于超邻接矩阵, 我们应当用张量将多个因素压缩成一个因素, 这样超邻接矩阵就如同图论中的普通邻接矩阵. 在文中, 我们定义 G_1, G_2, G_3 分别为不同层级的邻接矩阵, G_M 为多层级网路的超邻接矩阵.

一些基本的重要特性如节点度, 聚集系数, 网络都可以很好的帮助分析单层网络中的图. 通过将不同层网络数据整合到单层网络, 这些特性同样可以被用到多层网络中. 根据问题中所给出的文献, 团队业绩主要受到共享认知的影响. 而且共享认知和多层级网路中每一个方面的复杂度呈正相关. 为了解释这一点, 我们认为:

- 在人力资本层, 边越多或是节点的度越大, 表示一个员工要与更多的人一起工作. 这可以使人获得更多的训



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cooperate with others so that the team performance can improve quickly.

- In the friendship layer, bigger degree for a node means this person has great relations with people which will make him/her in well mood and not want to leave.
- In the competition layer, proper competition is the best policy to promote every teammate to work efficiently. Besides, cooperation walks along with competition, so that people can learn a lot about teamwork.

练,能更好的与他人合作,这样团队的业绩会迅速提高.

- 在友谊层,节点的度越大意味着这个人与周围人关系融洽,这使他有好心情,不想离开职位.
- 在竞争层,适当的竞争是促使每一个员工高效工作的最佳策略,此外,竞争伴随着合作,因此,人们可以学会团队协作.

Table 1: Main characters for networks | 网络的主要特征

The layer type	Average degree	Average cluster	Average path length	Graph density
The Human Capital layer	0.929	0	2.978	0.143
The friendship layer	0.86	0	1.632	0.11
The competition network	2.214	1	1	0.341
The multilayer network	3.214	0.638	1.571	0.484

Table 1 gives out the main character conclusions of three monolayer networks and the multilayer network. We can find that every character of the multilayer network shows more detail information than monolayer networks. Using multilayer network can simulate the social relations more clearly than only using one monolayer network. We draw conclusions:

- The HR office should use more detail data in this multilayer network to simulate the influence of churn.
- The competition network is important in teams, the company can divide divisions more scientifically to make teams in well competition.

表1给出了三个单层网络和多层网络的主要特征数据. 我们发现, 每一个多层网络的特征数据比单层网络显示更多详细信息. 利用多层级网路可以比单层网络更清晰地模拟社会关系. 我们得出结论:

- 人力资源办公室应当在多层网络中使用更详细的数据来模拟员工流失的影响.
- 竞争网络在团队中非常重要, 公司可以通过科学划分部门使其合理竞争.

4 Sensitivity Analysis | 灵敏度分析

Our algorithm use Monte Carlo random numbers and some coefficients determined by experience. This may bring to uncertainty effects and subjective effects. To analyze this influence we see these matters as independent variable and observe some important outcomes.

我们的算法使用了蒙特卡洛随机数和一些由经验确定的系数. 这些将带来不确定的影响和主观影响. 为了分析这些影响, 我们将它们作为独立变量, 观察其导至的重要结果.



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4.1 Uncertain effects produced by stochastic simulation | 随机模拟产生的不确定因素

To observe the stochastic error produced by stochastic simulation. We use the working rate as the output and get the output in same situation by 6 times. Then take 6 groups $(a_1, a_2, a_3, a_4, a_5, a_6)$ into the formula.

$$w = \frac{(a_1 + a_2 + a_3) - (a_4 + a_5 + a_6)}{3}$$

While the w represent the stochastic error. Figure 20 show us the stochastic error produced by the random process. The maximum of the error is under 0.015 which is small enough. So the stochastic error is under the limited range that we can assume it wonnot affect the output too much.

为观察由随机模拟产生的随机误差. 我们将在职率作为输入, 在相同条件下做 6 次重复模拟, 得到 6 组值 $(a_1, a_2, a_3, a_4, a_5, a_6)$ 带入公式;

其中 w 表示随机误差, 图20给出了随机过程产生的随机误差. 误差最大值足够小, 小于 0.015. 因此误差在允许范围内, 我们猜想它不会过多地影响输出值.

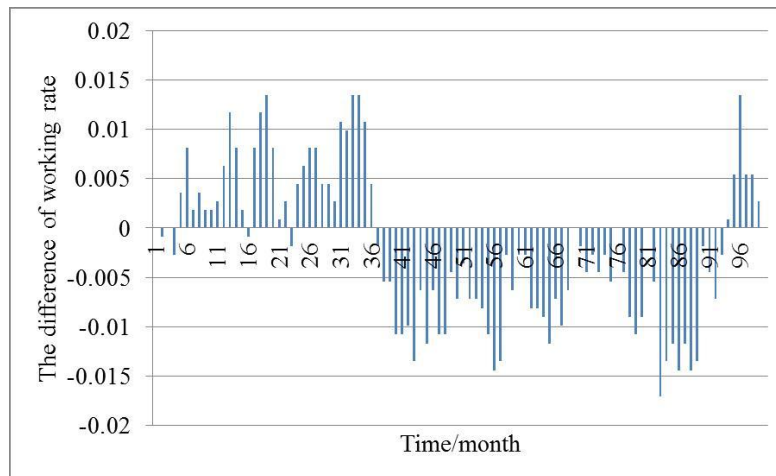


Figure 20: The stochastic error | 随机误差

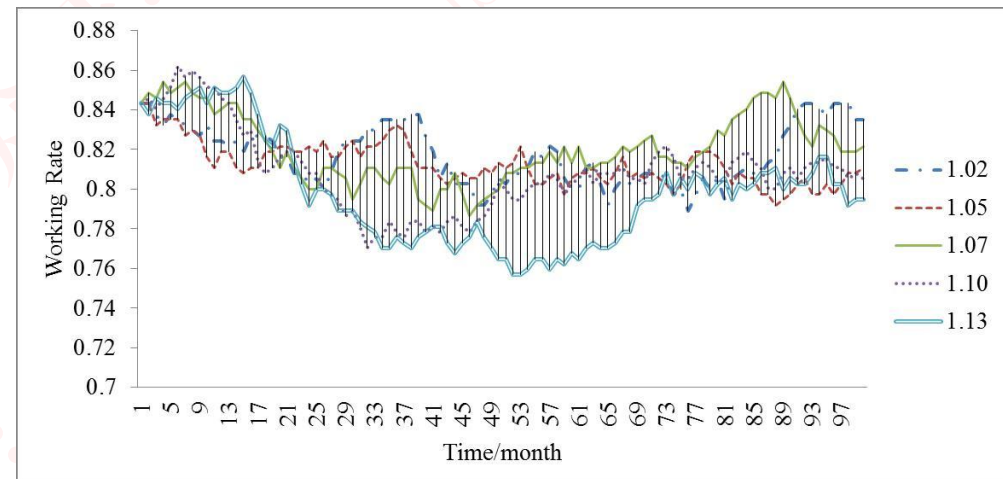


Figure 21: The working rate under different growth coefficient | 不同增长系数下的在职率

4.2 Subjective effects produced by some coefficients | 一些系数产生的主观影响

Because of the limited data some coefficients are determined by experience. In order to analyze the influence of these subjective effects, we choose two important

因为所给数据的限制, 一些系数是由经验确定的. 为了分析这些主观因素的影响, 我们选择了两个重要的系数稍作变



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coefficients to change slightly and observe what happen.

- Change the attenuation coefficient produced by unsuccessful promotion of job When a worker is full filled with the condition of promotion, his working efficient will decline if he can't get promoted. The attenuation coefficient is set as 0.95 by experience. We change this value into 0.90, 0.93, 0.95 and 0.97 to observe what happened to company performance. From the result we find that when the value is more than 0.93 the company performance didn't change obviously.
- Change the growth coefficient of turnover rate produced by churn When a people leave company will lead to a churn, which will promote some people surrounding to have more possibility to leave. The growth coefficient is set as 1.02 by experience. We change this value into 1.05, 1.07, 1.10 and 1.13 to observe what happened to the working rate of company. A graph is given to describe the result. Figure 21 shows the change of working rate under different coefficient value. The values 1.02, 1.05, 1.07, 1.10 and 1.13 stand for the different number of the growth coefficient. We can find that when the value is more than 1.07 the output didn't change obviously. That means the value set as 1.02 wonnot cause too much influence on the outcomes. So it is proper to set this value as 1.02.

5 Conclusion | 结论

As an indispensable part of team science, managing human capital in organizations directly determines the company's productivity. This paper tries to combine network model with team science. **Firstly**, we use ICM's 370 positions as nodes, the affiliations and cooperative relationships as edges to build a human capital static network model. In our model, we establish Post Archives Matrix to record the internal attributes of each position so that we can quantize some abstract concepts such as cooperative relationships reasonably. **Secondly**, we regard turnover, promotion and recruitment as main causes of network evolution and internal churn in the company. Then we design three evolution driving algorithms respectively which make the network change with certain initial conditions as well as the de-

动, 并观察产生的后果.

- 改变衰减系数, 它是由一个员工满足晋升条件而没有成功升迁产生的, 如果不能被提拔他的工作效率会降低. 根据经验衰减系数设置为 0.95. 当设置为 0.90, 0.93, 0.95, 0.97 时, 分别观察公司业绩发生的变化. 从结果来看, 我们发现当值大于 0.93 时公司业绩不再有显著变化.
- 改变流失率的增长系数, 它是由一个员工离开公司就会导致流失产生的, 这将使得促使周围人有更大可能离职. 根据经验增长系数设置为 1.02, 当分别改变该值为 1.05, 1.07, 1.10 和 1.13 时, 观察公司的在职率有何变化. 给出了一个数据图来描述所得结果. 图21给出了不同系数值下在职率的变化. 1.05, 1.07, 1.10, 1.13 这些值代表了增长系数的不同值. 我们发现当值大于 1.07 时, 输出不再有显著变化. 这意味着该值设置为 1.02 不会对结果产生太大影响, 所以设置为 1.02 是合理的.

作为团队科学不可缺少的一部分, 人力资本管理直接决定了公司的绩效. 这篇文章试图将网络模型与团队科学结合. **第一**, 我们利用 ICM 组织的 370 个职位作为节点, 它们之间的附属和合作关系作为边建立人力资本网络静态模型. 模型中我们建立了档案矩阵来记录每个职位的内在属性, 这样我们可以合理地量化一些抽象的概念, 如合作关系. **第二**, 我们把员工流失, 晋升和招聘作为网络演进和公司内部员工流失的主要起因. 随后, 我们分别设计了三大演进算法, 使得网络在特定的初始条件下变化, 也使得算法在时间维度上推进. **第三**, 我们关注于主管需要我们解决的问题, 我们修正模型



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velopment of algorithm in time dimension. **Thirdly**, focused on the issues that the supervisor need us to deal with, we revise our model's initial parameters and algorithms. Based on our simulation results, we obtain our predictions for these issues and analyze effects. **Fourthly**, after studying the theory of multilayer network, we regard friendships, competition and affiliation as three network layers to describe the connection compactness in the whole team. We also find that multilayer network can reflect internal connections more precisely and comprehensively than human capital network. **Finally**, we do the sensitivity analysis for simulation methods and some parameters.

5.1 Strengths | 优点

- We build a Post Archives Matrix to show the internal attributes of nodes. For one thing, we can use it to describe each position; For another, when an employee transfer from one position to another, he can bring the archives them him and initialize the original position.
- We consider the affiliations and cooperative relationships as edges to connect positions. Affiliations are the bridges for the company's operation and cooperative relationships are the roll booster for team benefits. In the model, we combine these to relationships together and derive the equation of team work to qualify the connection compactness for a team.
- Network evolution is based on the connection and effects of three algorithms. There is an order in employees' demission, promotion and recruitment. Based on the requirements, we determine a set of strategies for the real churn in ICM company and then use it solve the requests in the task.
- The details in algorithms are in accord with real conditions. We use it realize the reasonable quantification of abstract factors. For example, in order to describe the bad effects caused by churn employees, we quantify employees' working enthusiasm and the changing churn rate. There is another example: we consider the effect of training and find that training can increase the working enthusiasm and at the same time decrease the demission possibility.

的初始参数和算法. 根据模拟结果, 我们得到了这些问题的预测, 并分析影响. **第四**, 通过研究了多层级网络的相关理论, 我们将友谊, 竞争和附属关系作为描述整个团队紧密联系的三层网络. 我们发现多层网络比人力资本网络能更精确全面地反映内部联系. **第五**, 我们对模拟方法和一些参数做了灵敏度分析.

- 我们建立了档案矩阵来表明每个节点的内在属性. 一方面, 我们可以用它描述每一个职位; 另一方面, 当员工从一个职位调到另一个职位时, 他可以带走他的档案属性, 然后初始化原来的位置.
- 我们将附属关系和合作关系作为连接职位的边. 附属关系是公司运作的桥梁, 合作关系是增加团队效益的助推器. 在模型中, 我们把这些关系结合在一起, 并且推导出团队协作的公式来量化团队间紧密的联系.
- 网络演进是基于三大算法的联系和影响. 员工的离职, 晋升和招聘之间有一个顺序. 根据要求, 对 ICM 组织员工的实际流失我们制定了一套策略, 然后用它来解决任务中要求解决的问题.
- 算法中的细节与实际情况相符. 我们用它来合理地量化抽象因素. 例如, 为了描述员工流失带来的不良影响, 我们量化了员工工作积极性和不断变化的流失率. 再如, 我们考虑员工培训的影响, 并发现培训能提高工作积极性同时降低离职的可能性.



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5.2 Weaknesses | 缺点

- Different divisions' effects and operation in ICM company should be regarded as another network to study. Considering the lacking of conditions and the time for us is limited, we don't make further study in this aspect.
- Our model doesn't consider the effects of the employees who retire or leave their position temporarily.
- ICM 组织的不同部门间的影响和运作应当被看作是另一个网络来研究，考虑到缺少相应条件和时间限制，我们在这方面没有做进一步研究。
- 我们的模型没有考虑员工退休或是暂时离开职位的影响。

References

- [1] John G Richardson. *Handbook of Theory and Research for the Sociology of Education*. Greenwood Publishing Group, 1986.
- [2] Michael Spence. Job market signaling. *The quarterly journal of Economics*, pages 355–374, 1973.
- [3] Paolo Magrassi. A taxonomy of intellectual capital. *Wikimedia Foundation, Inc*, 2002.
- [4] Stephen R Proulx, Daniel EL Promislow, and Patrick C Phillips. Network thinking in ecology and evolution. *Trends in Ecology & Evolution*, 20(6):345–353, 2005.
- [5] Stephen P Borgatti, Ajay Mehra, Daniel J Brass, and Giuseppe Labianca. Network analysis in the social sciences. *science*, 323(5916):892–895, 2009.
- [6] Eduardo Salas, Nancy J Cooke, and Michael A Rosen. On teams, teamwork, and team performance: Discoveries and developments. *Human Factors: The Journal of the Human Factors and Ergonomics Society*, 50(3):540–547, 2008.
- [7] Mikko Kivelä, Alex Arenas, Marc Barthélemy, James P Gleeson, Yamir Moreno, and Mason A Porter. Multilayer networks. *Journal of Complex Networks*, 2(3):203–271, 2014.
- [8] Tim Hellmann and Mathias Staudigl. Evolution of social networks. *European Journal of Operational Research*, 234(3):583–596, 2014.
- [9] Qingqi Long. Three-dimensional-flow model of agent-based computational experiment for complex supply network evolution. *Expert Systems with Applications*, 42(5):2525–2537, 2015.
- [10] Susana García-Herrero, MA Mariscal, Jose M Gutiérrez, and Dale O Ritzel. Using bayesian networks to analyze occupational stress caused by work demands: Preventing stress through social support. *Accident Analysis & Prevention*, 57:114–123, 2013.



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