Judges' Commentary: The Dam Problem

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Overview of the Problem

Students who explored Problem A in this year's Mathematical Contest in Modeling examined options to address concerns over the potential instability of the Kariba Dam on the Zambezi River basin between Zambia and Zimbabwe. This potential instability was brought about by the erosion of bedrock at the base of the dam caused by years of water spilling over the spillway.

This commentary includes a brief overview of the problem statement. That is followed by a list of some of the common themes and more detailed points that emerged as the judging proceeded, including a list of some of the common approaches adopted by the students. Finally, we offer a summary of the key discriminators used by judges to compare student submissions.

The Problem Requirements

There were two main requirements:

- To provide a brief assessment of three options to address the deteriorating conditions of the Kariba Dam: repairing the dam, rebuilding the dam, or removing and replacing the dam with a series of 10 to 20 smaller dams. This assessment was to be limited to one or two pages and was expected to include an overview of the potential costs and benefits of each option.
- To provide a detailed analysis of the option to remove and replace the dam with a number of smaller dams. This analysis was not to exceed 20 pages.

The UMAP Journal 38 (3) (2017) 289–296.

In addition to the written analysis addressing the two requirements, a single-page summary sheet was expected.

The Option Analysis Requirement

The goal for this requirement was to identify the costs and benefits associated with each option for addressing the worsening situation with the Kariba Dam, and to combine them in a meaningful way to provide a supportable recommendation.

Successful teams considered not only dam demolition, construction, and repair costs, but also costs of lost revenue related to electricity production, drinking water, irrigation, fishing, and recreation. Successful teams also incorporated the time required to implement each option into their analysis, and provided a relative comparison of the safety aspects of each option, as well. Team recommendations varied based on the relative importance each team placed on the costs versus the benefits, and whether they focused on the long-term or short-term costs and benefits.

In evaluating this requirement, judges were particularly looking for consistency between statements that teams made about the relative importance of the factors affecting their decision and their ultimate conclusion. Teams that simply provided a discussion of costs and benefits without a recommendation, or a recommendation without supporting rationale, were not considered successful with this requirement.

The Remove and Replace Option Requirement

As indicated by the page limit imposed on the analysis of the two requirements, the primary focus of the problem was to provide an analysis of the option to remove the Kariba Dam and replace it with smaller dams. Successful teams clearly addressed the following questions:

- How many dams are required?
- Where should the dams be placed?
- How should the flow of water be modulated through the dams to balance safety and costs during prolonged flooding?
- How should the flow of water be modulated through the dams to balance safety and costs during prolonged drought?
- What are the restrictions on locations that should be exposed to detrimental effects of extreme water flow conditions and the lengths of time of that exposure?

For this requirement, teams typically identified a number of criteria to be applied to identify candidate locations for dam placement. These criteria often included

- the width of the river,
- the slope of the neighboring terrain,
- the proximity of population centers, and
- the geology of both the dam site itself as well as the area to be occupied by the reservoir.

Some teams simply chose locations equally spread out along either the length or the elevation of the river.

After identifying candidate sites, teams typically applied constraints to ensure that the replacement dams provide the same capability and functionality as the Kariba Dam. Such constraints included overall water storage capacity as well as electrical energy-generation potential. Teams then mathematically modelled these constraints and the overall costs associated with each dam in order to minimize the overall cost of the Zambezi River water management system.

Most good papers offered logically constructed models with concise and understandable descriptions. The judges expected the students to clearly describe the objectives and evaluate their models in a manner consistent with the stated objectives. Specific results and conclusions were presented in higher-ranked papers in a clear and concise manner.

The Water Modulation Requirement

In addition to identifying the recommended number and location of dams to support efficient management of water in the Zambezi River Basin, teams were asked to include a strategy for modulating the water flow through the multiple dam system that balanced safety and costs.

To achieve this, teams had to model ways to quantify both costs and safety, and develop a simulation or analytical model to "balance" the two. These models needed to consider

- the water held in the reservoir behind each dam,
- the amount of water flowing from one dam to another,
- the amount of water from rain and runoff,
- the amount of water siphoned off for human consumption and irrigation, and
- water loss due to evaporation.

These models needed to be applied to the extremely high and extremely low values of water flow due to flooding and drought.

The Summary Requirement

An essential requirement was to write a standard one-page summary sheet. Successful papers included a brief overview of the technical approach to satisfying the problem requirements, along with a summary of the results. At a minimum, successful papers included the recommended option for addressing the Kariba Dam foundation issues and the number and location of recommended replacement dams.

Condensing the substance of the team's analysis into a single page was a difficult task for the teams. The student teams that managed to convey a sense of the basic models, the underlying assumptions, and the limitations of their models tended to make a stronger impression.

The Modeling Effort

With the quantity and availability of information available on the Internet today, many teams were able to find an example of a solution to this problem or one similar to it and modify it for use in this competition. The most successful teams, however, built cohesive mathematical models and described their derivation in sufficient detail to explain both the underlying logic as well as the results and conclusions drawn from them.

The organization and consistency of papers was also a discriminator. Successful teams

- clearly defined the variables that impacted the problem,
- documented their data collection effort,
- stated any assumptions made to either compensate for the lack of data availability or to support the development of the model,
- expressed the relationships between the variables with mathematical equations, and then
- used suitable approaches or appropriate techniques to optimize or analyze these relationships to support a conclusion and recommendation.

Variable Definition

Key to the modeling effort was the definition of variables used: Judges expected each variable used in the model to be explained. Additionally, each variable should be associated with some type of unit, if appropriate. Units of variables should be chosen to ensure that a dimensional analysis of all derived equations indicates the same fundamental quantities on both sides. Papers that included models involving generic "factors," "constants," or "coefficients" without a detailed explanation of how the variable was derived, or specifically why it was used, were not as successful as those that provided more detailed explanations. This was very often the case when a team leveraged work from an Internet source without adequate description. Many teams chose to define all of the variables that they used up front in their paper, in a table dedicated to just that purpose. Some of those teams either failed to include all of the variables they used, or included variables that were not used elsewhere in the paper.

The more successful teams defined each variable as it was introduced in the paper. This way, the variables were described within the context of the modeling effort, making it much easier for the judges to follow both what the variable was meant to represent and how it was to be used in the model. Another observation concerned the number of variable definitions used. Statistician George Box is often quoted for his saying, "All models are wrong but some are useful" [Box 1979, 202]. What he meant by this is that, try as we may, models cannot exactly represent a system in the real world (with emphasis on the word "exactly"). Given that we cannot precisely model such things, Box felt that "overparameterization is often the mark of mediocrity" [Box 1976].

Many of the models presented to address the requirements of the Kariba Dam problem had a large number of variables, were overly complex, and provided the teams much difficulty with conducting meaningful sensitivity analysis. The most effective papers constrained their models to the variables most pertinent to the problem at hand, and lent themselves well to sensitivity analysis.

Data Collection

In some cases, data availability was a driving force in the derivation of the model itself. Clearly, it makes no sense to develop a mathematical model that requires input data that are unobtainable. On the other hand, the source of the data also affects the credibility of the modeling effort. Successful teams not only developed models, but collected data to support them and documented the sources of the data, as well.

Assumptions

Most teams included a section about the assumptions made during the course of model development. Successful teams also included some rationale as to why they were making the assumption, how the assumption ultimately influenced the model, and why the assumption was reasonable to make. The most successful teams clearly limited the assumptions that they documented to ones that were necessary. In other words, the most successful teams only made assumptions pertinent to the model—ones that if, in the event that they turned out to be wrong, would likely alter the model itself or any conclusions drawn from it.

As with variable definitions, some teams chose to dedicate a section of their paper to assumptions and justifications. While this technique ensured that judges could clearly see what assumptions were made by the team, it did not always convey the rationale or the need for the assumptions in the model development. Teams that included their assumptions in-line with their model derivation were generally more successful in conveying not only why the assumption was being made, but how it was applied in the model as well.

Solution Approaches

Once variables were identified, then data were collected, assumptions were made, and mathematical models were presented. Approaches to satisfy the requirements of the problem varied. The time to address this problem was limited, and nothing in the problem statement suggested any particular direction with respect to solution approaches.

So the judges did not expect extensive data collection efforts and elaborate sophisticated models. They did expect a strong analysis that was consistent with any models that were developed. Below is a summary of some of the common approaches used. If standard methods developed by others for dealing with certain aspects of the problem were used, appropriate citations were expected. Proper validation of the models used against similar problems or situations that are documented in the literature was also a discriminator.

Analytic Hierarchy Process

Many teams derived costs and benefits for the three options for addressing the Zambezi River situation and then used an Analytic Hierarchy Process for rank-ordering the options.

While the Analytic Hierarchy Process is widely used to support group decision-making, it is also extremely useful in solving problems like the Kariba Dam where alternatives are to be rank-ordered based on a set of evaluative criteria.

Genetic Algorithm

Many teams used a heuristic genetic algorithm to optimize the number and location of dams required for the option to replace the Kariba Dam with smaller dams. The genetic algorithm is a common technique for solving optimization and search problems, particularly if the modeled objective function is not linear. This approach typically required the use of some sort of computer program to iteratively "evolve" a solution.

Integer Linear Programming

In addressing the problem of replacing the Kariba Dam with multiple smaller ones, some teams expressed restrictions on water storage capacity, electricity generation, and other economic factors as linear constraint equations and then used linear integer programming methods to determine the least cost set of dam sites to satisfy the constraints. Teams that took this approach also made use of existing commercial integer programming software packages.

Simulation

Many teams used simulation in their analysis, particularly to address the water modulation requirement. Successful teams provided an explanation of the statistical significance of the simulation results. Most teams that used simulation included the source code used in an appendix to the paper. Yet the most successful teams explained their simulation process in detail; some even included flow diagrams.

Sensitivity Analysis

Not all teams had time to perform sensitivity analysis. Of those that did, some simply varied selected variables and produced graphs indicating the sensitivity of their model results to those particular variables. More successful papers not only labeled their graphs appropriately, but actually explained the implications of these graphs, along with recommendations for further analysis or data collection if warranted.

The most successful teams, however, focused their sensitivity analysis on the assumptions that they made during the course of modeling. This provided consistency between the sensitivity analysis and the modeling process, and either enhanced the credibility of the team's recommendations (if the model results were not sensitive to the assumptions made) or suggested areas for further analysis.

Summary

This problem was a difficult one to address in the time given in the competition. Many teams were sidetracked by delving too deeply into one aspect of the problem or another. The most successful teams stayed focused, managed their time wisely, addressed all of the requirements of the problem and included:

- pertinent variables that were well defined (with units),
- data collection of the pertinent variables from reliable sources,

- assumptions made either to compensate for the lack of data, or to explain mathematical relationships between the pertinent variables,
- an appropriate mathematical technique or approach to analyze the mathematical relationships and form a recommendation, and
- sensitivity analysis to address any assumptions made, or address anticipated changes in the pertinent variables used.

References

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About the Author

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