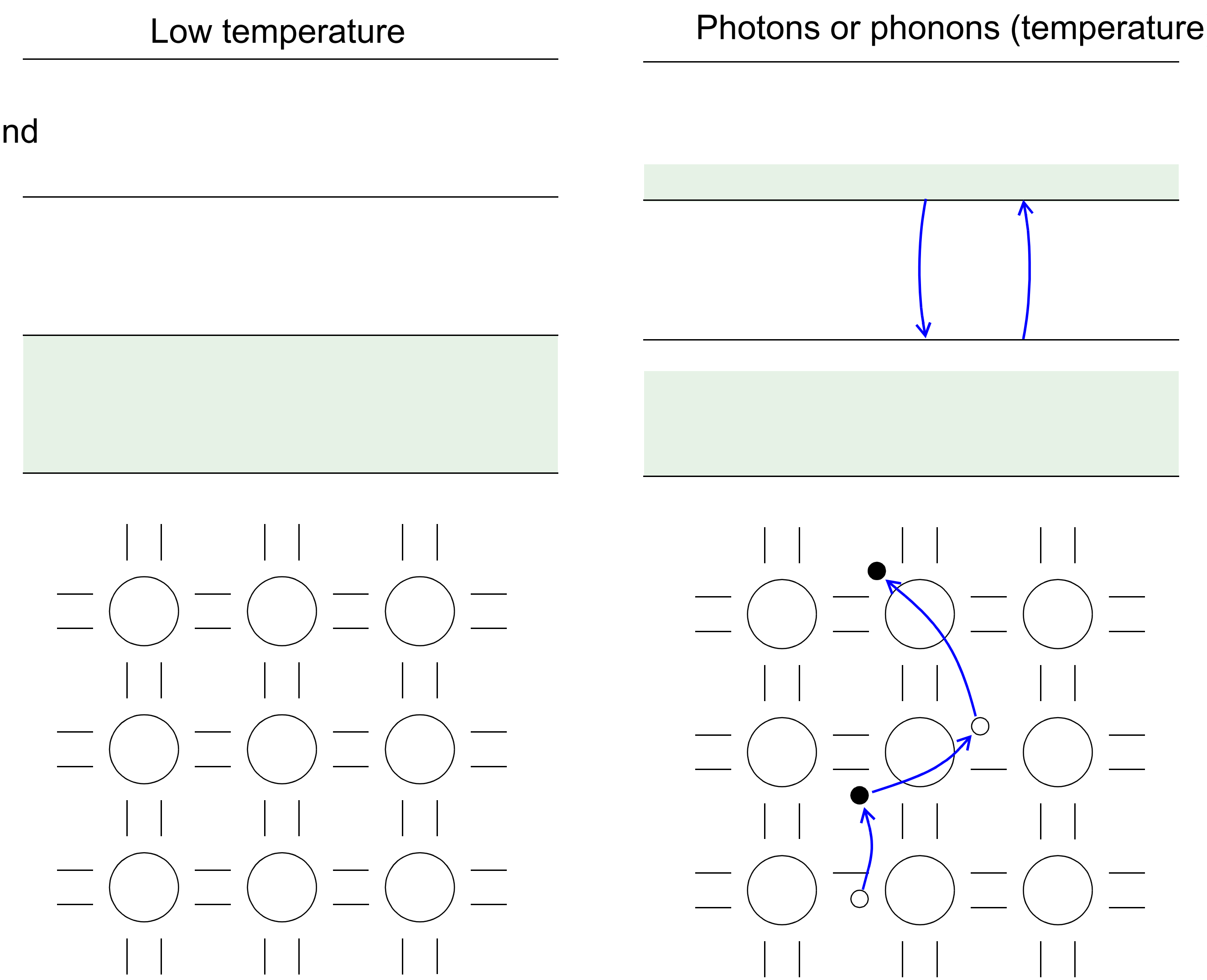
Semiconductors and Charge Carriers

Conduction band

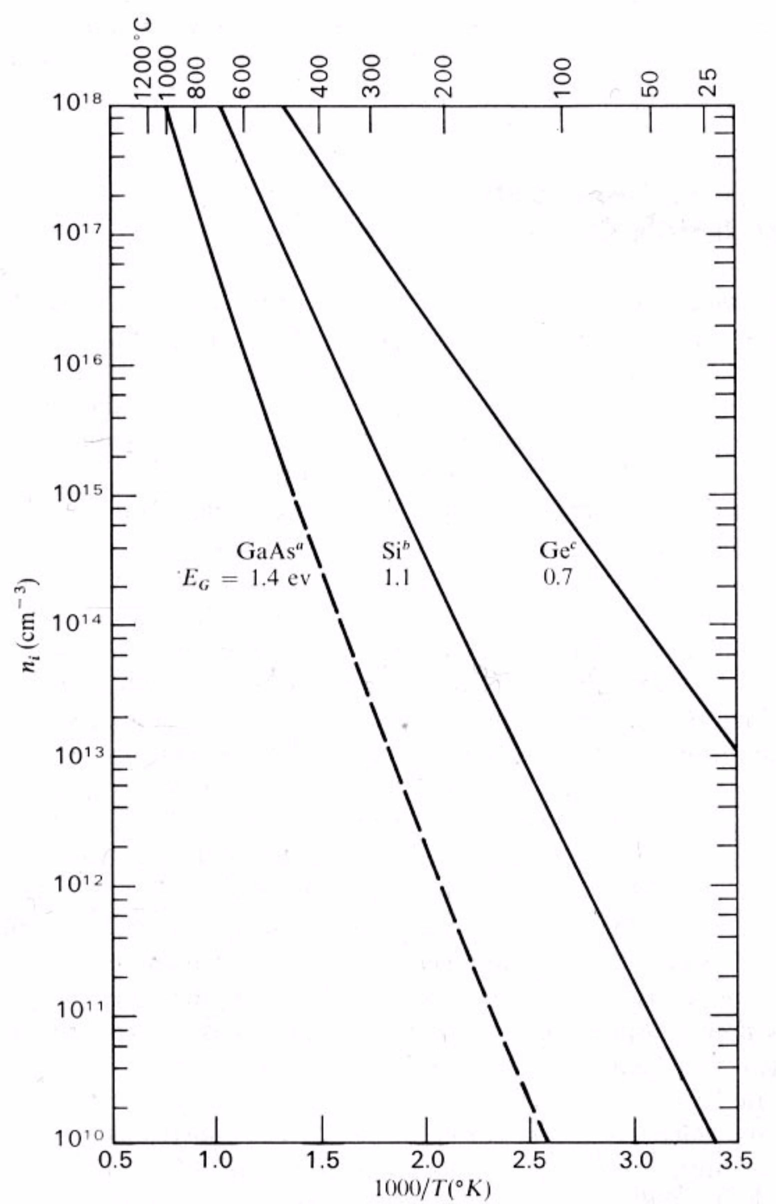
Valence band



Covenant bands gives filled valence band.

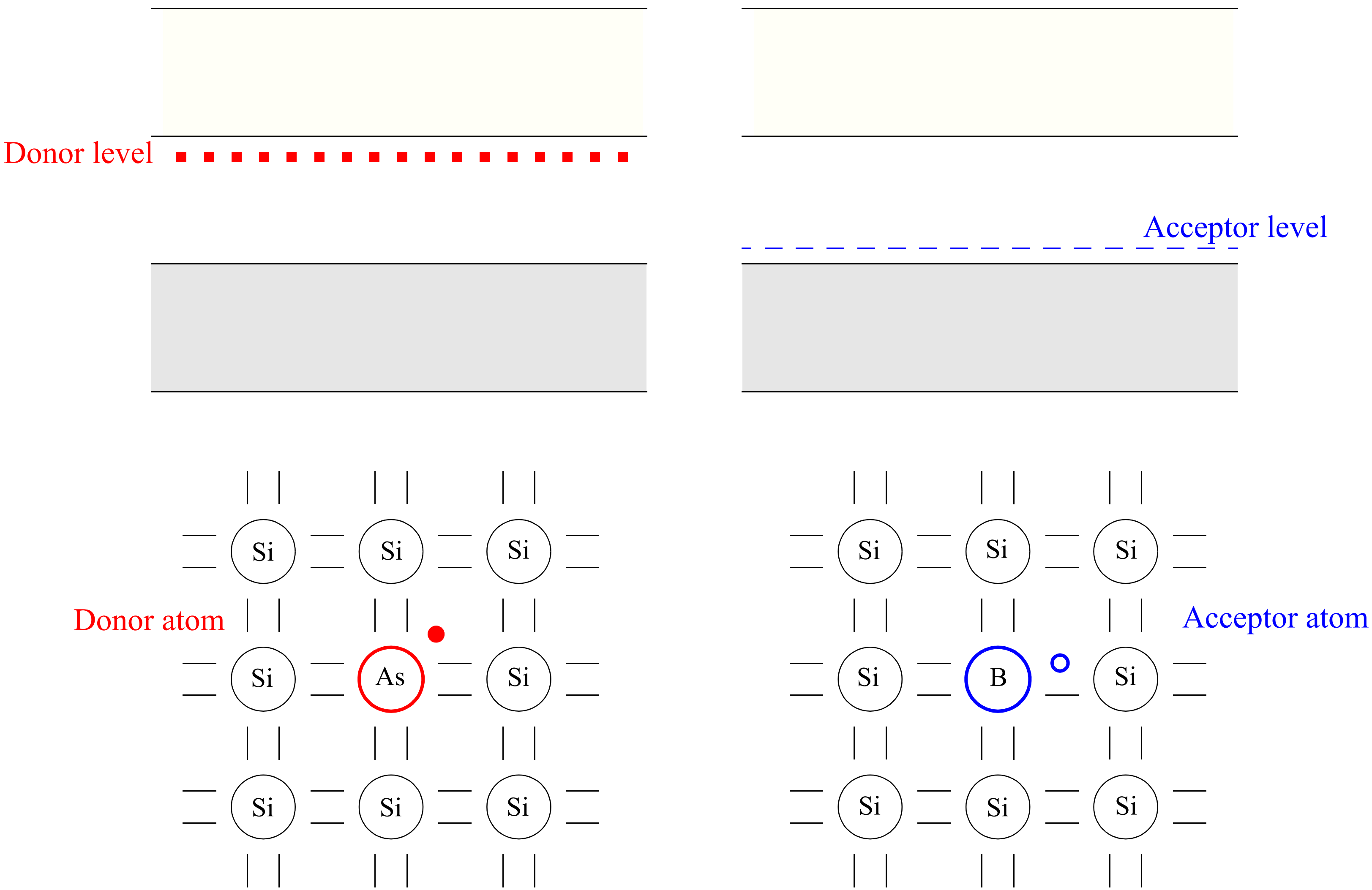
8 electrons in outer shell.

Concentration of free charge carriers
in intrinsic semiconductor.

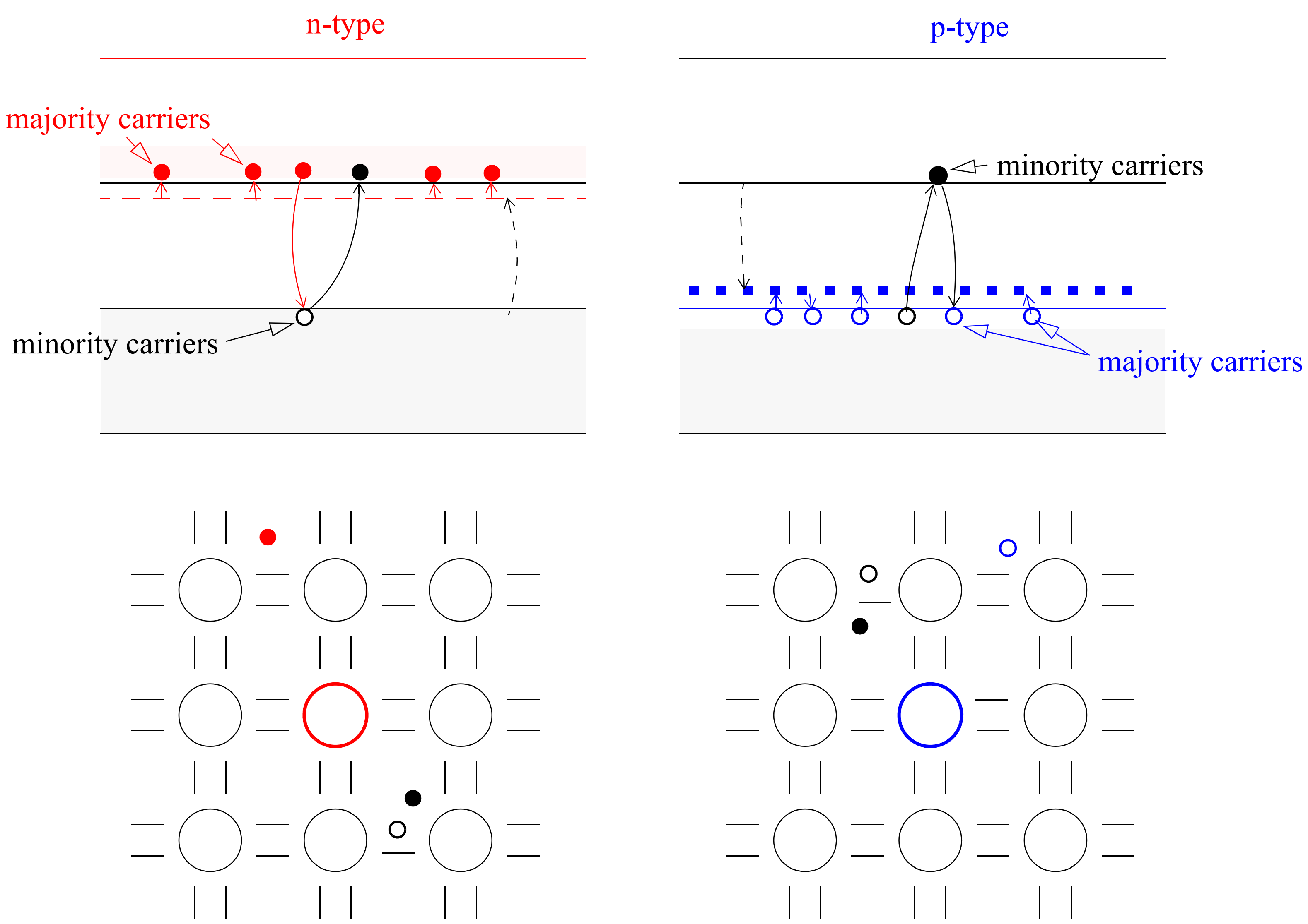


Ref: Grove

Adding impurities: Doped semiconductor



Donor densisty: N_d Acceptor densisty: N_a


n-type
p-type
majority carriers
minority carriers
minority carriers
majority carriers

Fermi Energy

(Fermi Level)

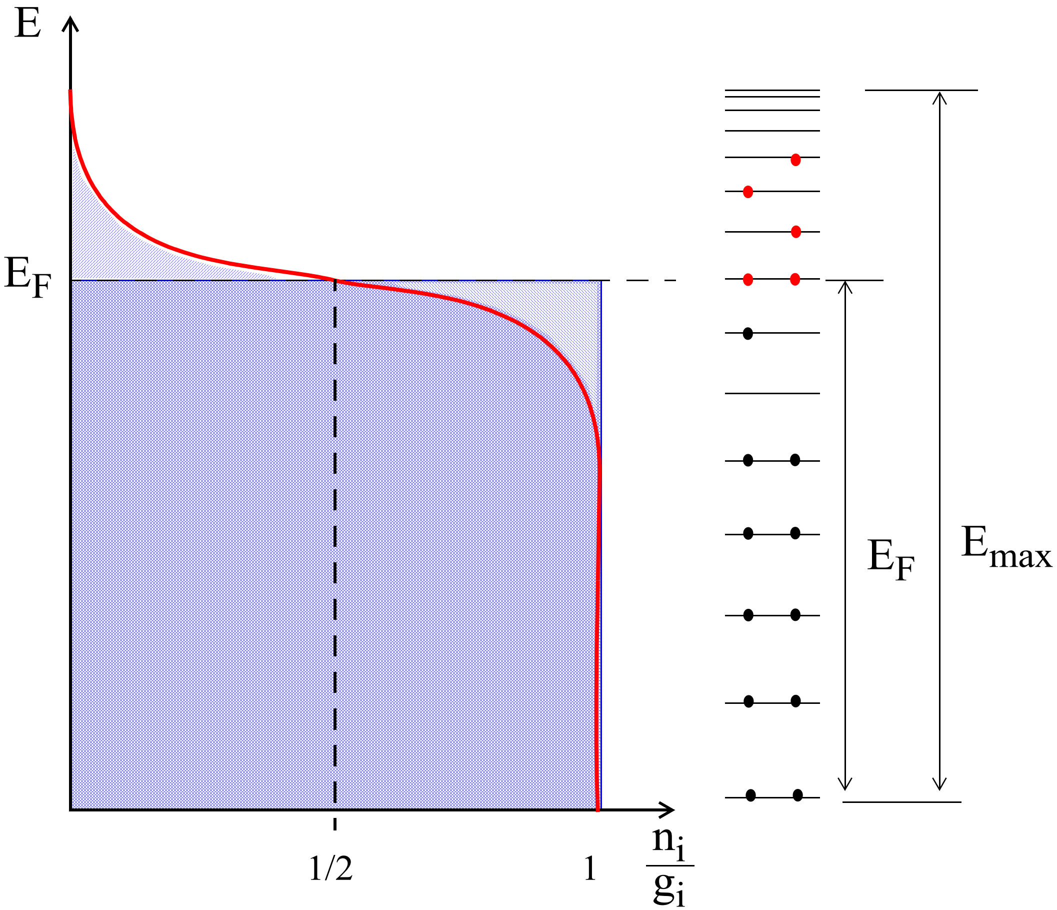
Number of electrons n_i < number of states g_i

Metal in its ground state, T=0 ºK:

The electrons fill the states below the Fermi level E_F = Fermi-energy.

At higher temperatures:

The electrons are excited to higher levels.



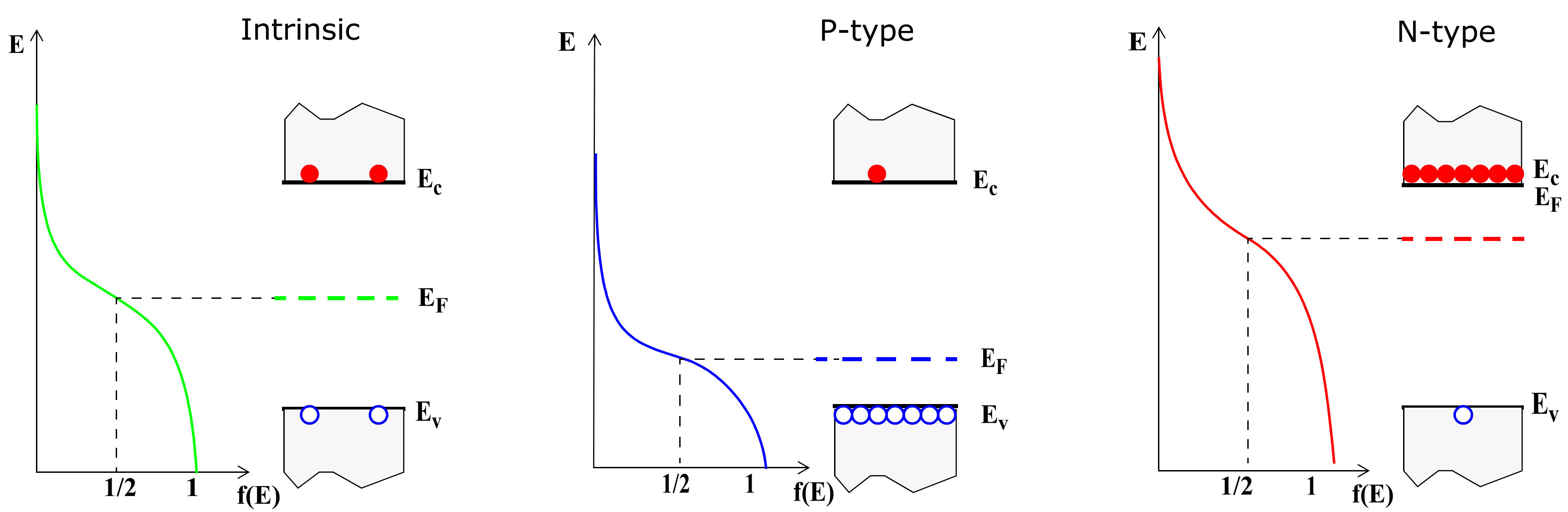
> At equilibrium, the Fermi level will be constant throughout the material.

Fermi-Dirac statistics

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \qquad f(E_F) = \frac{1}{2} \tag{1.1}$$

f(E): A distribution function, the probability of a energy level being occupied by an electron. Boltzmann constant: k= 1.3805 x 10^{-23} J/ ºK,T is absolute temperature. The probability is equal to 1/2 at the Fermi Level.

Fermi Level in undoped (intrinsic) semiconductor must be in the middle of the gap because the number of energy states is the same in the valence band and conduction band, and the number of holes in the valence bans equals the number of electrons in the conduction band. In a N-type and P-type semiconductor the energy states are unchanged. In N-type, the concentration of electrons in conduction band is increased. In P-type, the concentration of holes (missing electrons) in the valence band is increased. Thus the Fermi level and the hole distribution is shifted up in N-type and down in P-type semiconductor



Ref: Grove

N-type:

E_F is higher than E_i. At actual dopant concentration:

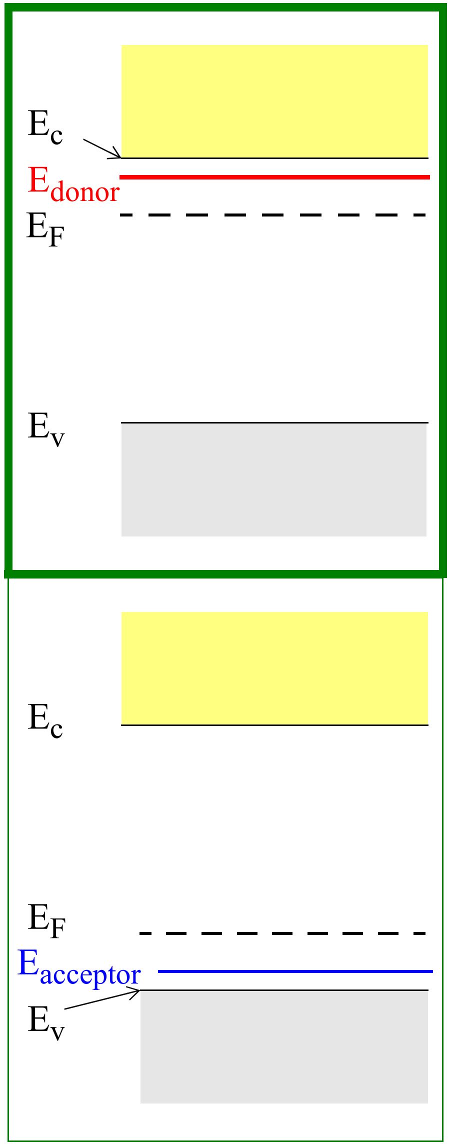
$$(E_C - E_F) > \text{a few } kT; \quad e^{(E_C - E_F)/(kT)} \gg 1$$

The probability of an electron being excited to the conduction band E_c:

$$f(E_c) = \frac{1}{1 + e^{(E_C - E_F)/(kT)}} \approx e^{-(E_C - E_F)/(kT)} \tag{1.2}$$

Multiplied with the density of energy states N_c, the concentration of free electrons in conduction band (majority carriers) can be written as:

$$n = N_c e^{-(E_C - E_F)/(kT)} \tag{1.3}$$



Ref: Grove

P-type:

E_F is lower than E_i and (E_v is lower than E_F).
(E_V - E_F)= - (E_F - E_v). At actual dopant concentrations:

$$|E_V - E_F| > \text{a few } kT; \quad e^{-(E_F - E_V)/(kT)} \ll 1$$

The probability of an electron in the valence band E_v:

$$f(E_v) = \frac{1}{1 + e^{-(E_F - E_V)/(kT)}} \approx 1 - e^{-(E_F - E_V)/(kT)} \tag{1.4}$$

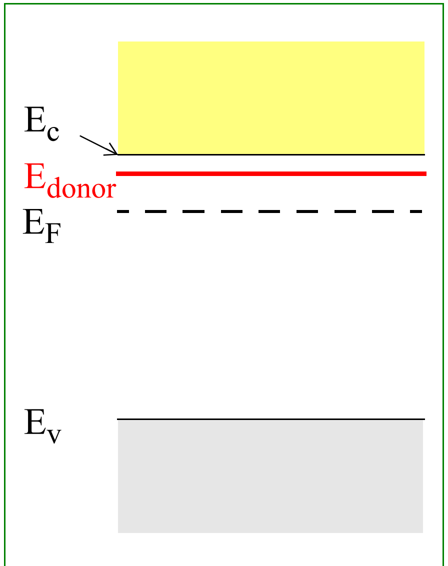
The approximate expression is obtained by using the first two terms in the series:

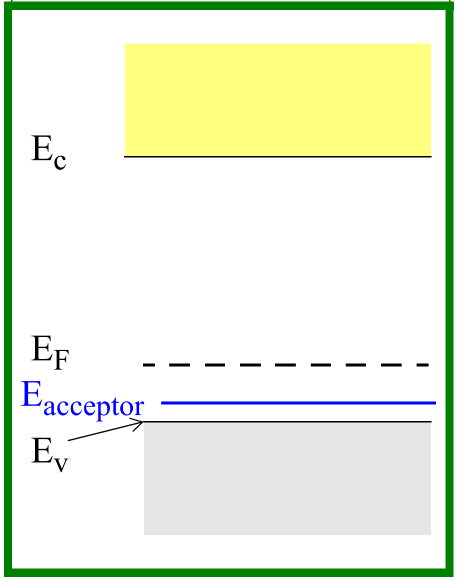
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{where} \quad x = -e^{-(E_F - E_V)/(kT)}$$

The probability of a hole in the valence band is then 1-f(E_V), and multiplied by the density of energy states in E_v, gives the concentration of holes (majority carriers):

$$p = N_v e^{-(E_F - E_V)/(kT)} \tag{1.5}$$

Or by using $n_i = N_c e^{-(E_C - E_i)/(kT)}$ **and** $n_i = N_v e^{-(E_i - E_V)/(kT)}$





Ref: Grove

Where E_i is the Fermi level for intrinsic semiconductor:

$$n = N_c e^{-(E_C - E_F)/(kT)} = n_i e^{(E_C - E_i)/(kT)} e^{-(E_C - E_F)/(kT)} = n_i e^{(E_F - E_i)/(kT)} \quad (1.6)$$

$$p = N_v e^{-(E_C - E_F)/(kT)} = n_i e^{(E_i - E_V)/(kT)} e^{-(E_F - E_V)/(kT)} = n_i e^{(E_i - E_F)/(kT)} \quad (1.7)$$

and therefore

$$pn = n_i^2 e^{(E_i - E_F)/(kT)} e^{(E_F - E_i)/(kT)} = n_i^2 = N_c N_v e^{-(E_G)/(kT)} \quad (1.8)$$

where $E_G=E_C-E_V$

$$\boxed{pn = n_i^2} \quad (1.9)$$

Ref: Grove

Charge neutral, doped semiconductor

$\rho = q\,(p - n + N_D - N_A) = 0$ implies that $p - n = N_A - N_D$

Net density of free carriers = Net dopant concentration

N-type:

Majority carriers

$$n_n = N_D - N_A + p = N_D - N_A + \frac{n_i^2}{n_n}$$

$$n_n^2 - n_n[N_D - N_A] - n_i^2 = 0$$

$$n_n = \frac{1}{2}\left[N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right]$$

$$n_n \approx N_D - N_A \qquad \text{for} \quad N_D - N_A \gg n_i$$

Minority carriers

$$p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D - N_A} \qquad (1.10)$$

P-type:

Majority carriers

$$p_p = N_A - N_D + n = N_A - N_D + \frac{n_i^2}{p_p}$$

$$p_p^2 - p_p[N_A - N_D] - n_i^2 = 0$$

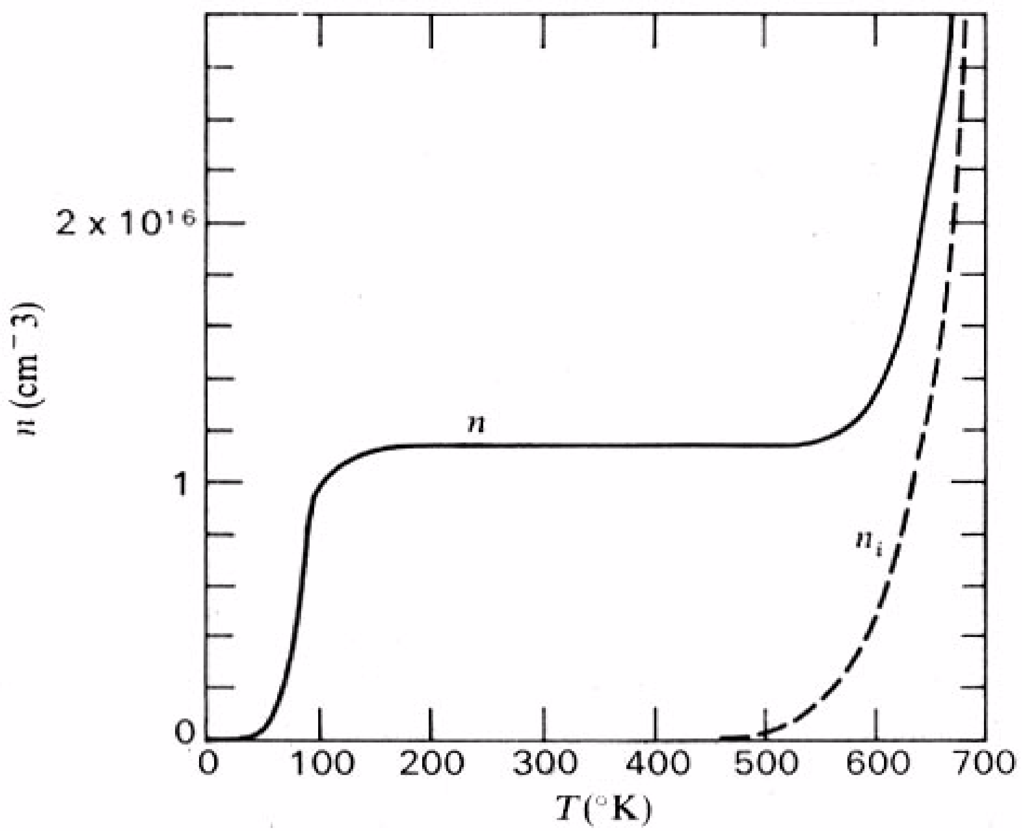
$$p_p = \frac{1}{2}\left[N_A - N_D + \sqrt{(N_A - N_D)^2 + 4n_i^2}\right]$$

$$p_p \approx N_A - N_D \qquad \text{for} \quad N_A - N_D \gg n_i$$

Minority carriers

$$n_p = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A - N_D} \qquad (1.11)$$

Ref: Grove



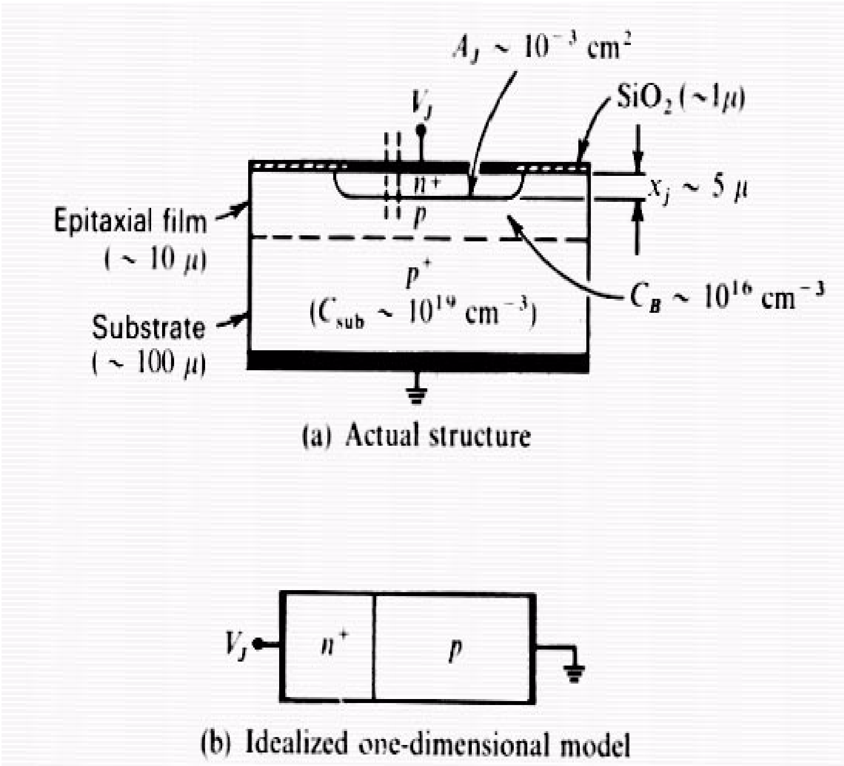
Concentration of majority carriers in N-type semiconductor vs. temperature.

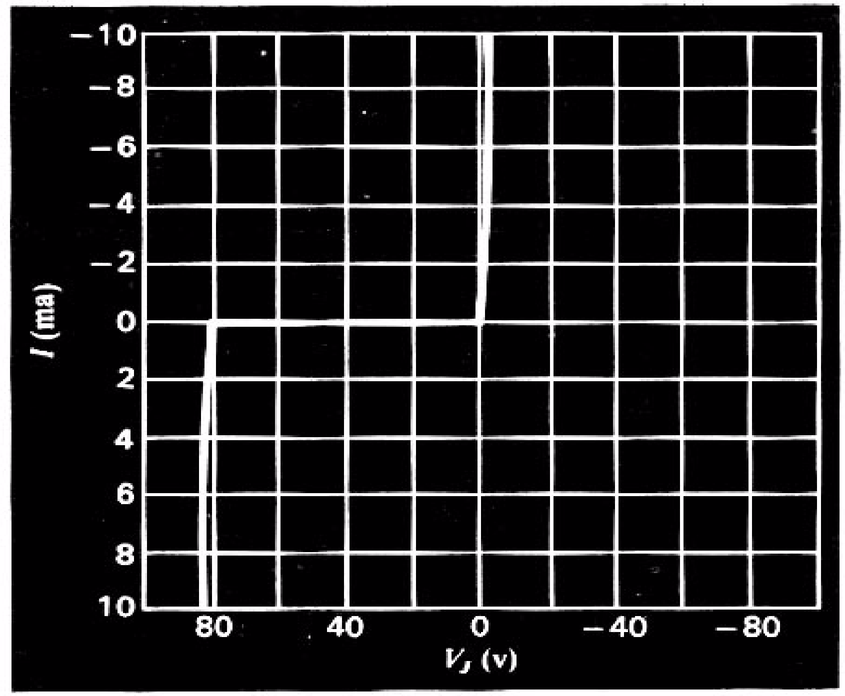
Ref: Grove

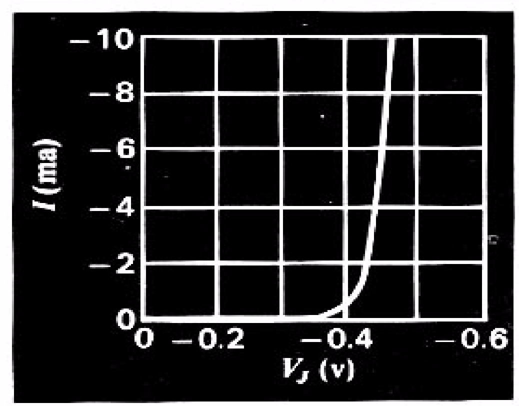
PN JUNCTION

DIODE

DIODE

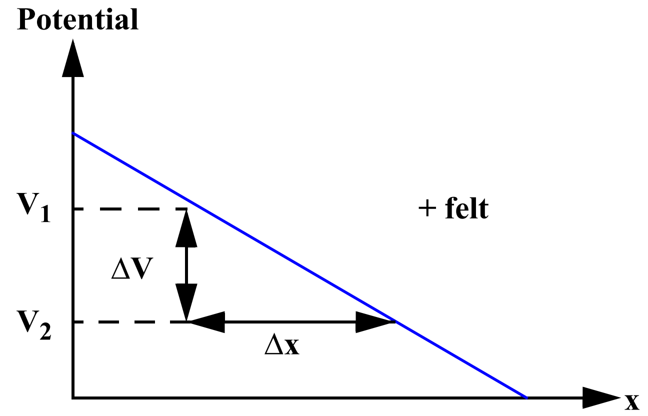






Ref: Grove

Static electrical field



Field:

Causes a force $q\mathcal{E}$ on the elementary charge q.

Negative potential difference per distance.
Field in a point: Negative potential difference across a infinite small distance.

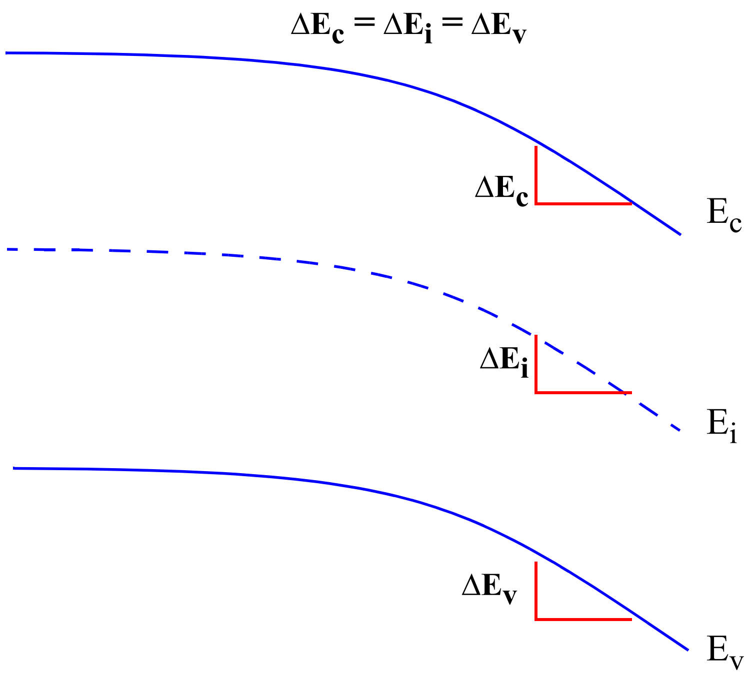
$$\mathcal{E} = -\frac{\Delta V}{\Delta x} = -\frac{dV}{dx}$$

The electrostatic potential ϕ is defined such that the negative gradient = field:

$$-\frac{d\phi}{dx} = \mathcal{E} \tag{1.12}$$

The gradient, and force on a carrier, is equal at all energy levels E_C, E_V and E_i since the band gap is constant.
We can use E_i to determine the electrostatic potential for an electron (-q) relatively to a position x (=0).

$$\phi(x) = \frac{E_i(x)}{-q} \qquad \phi(x)(-q) = E_i(x) \tag{1.13}$$



Poisson's equation:

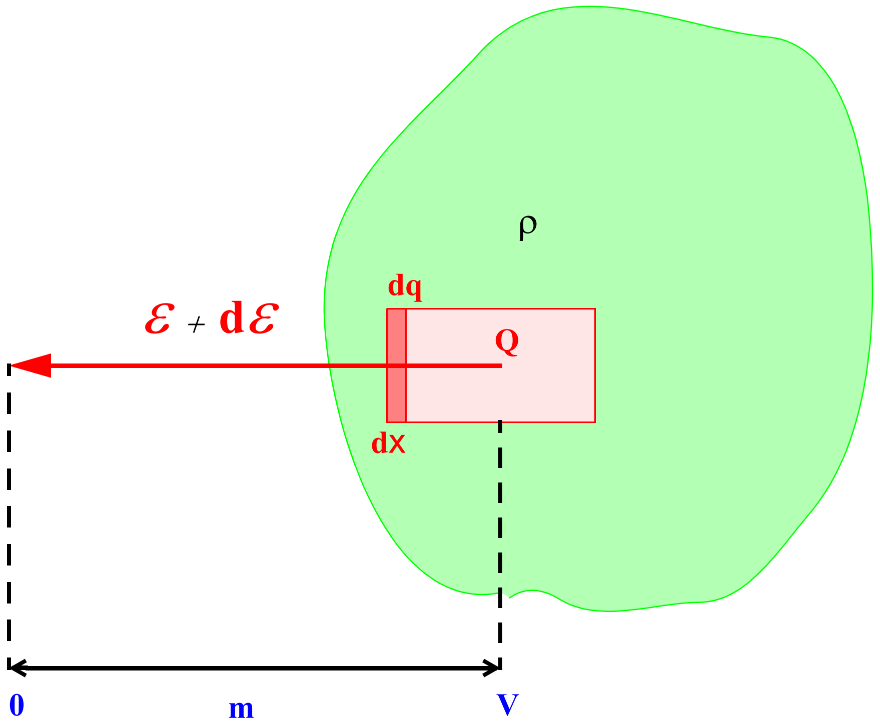
$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_r \varepsilon_0}$$

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon_r \varepsilon_0} \qquad (1.14)$$

$$\frac{d^2 E_i}{dx^2} = q\frac{N}{\varepsilon_r \varepsilon_0}$$

ϕ is the potential,
ρ is charge density
ε_r is dielectric constant,
ε_0 is vacuum permittivity,
q is elementary charge.

The rectangular with the charge Q contributes to the total field by $\mathcal{E}$ like dq contribute by $d\mathcal{E}$.
Potential difference between two points separated by the distance m is V.



The interface between N-type and P-type semiconductor, a PN- junction.

- P-type semiconductor has high concentration of holes (free electron states in the valence band)
- N-type semiconductor has high concentration of electrons (free electrons in the conduction band).
- Electrons and holes diffuse across the interface and recombine.
- Diffused carriers leave a space charge (Donor atoms and Acceptor atoms)
- Charged areas creates an positive electrical field, from the positively charged area to negatively charged area, which drives the carriers in opposite direction and balance the diffusion current.
An equilibrium is established.

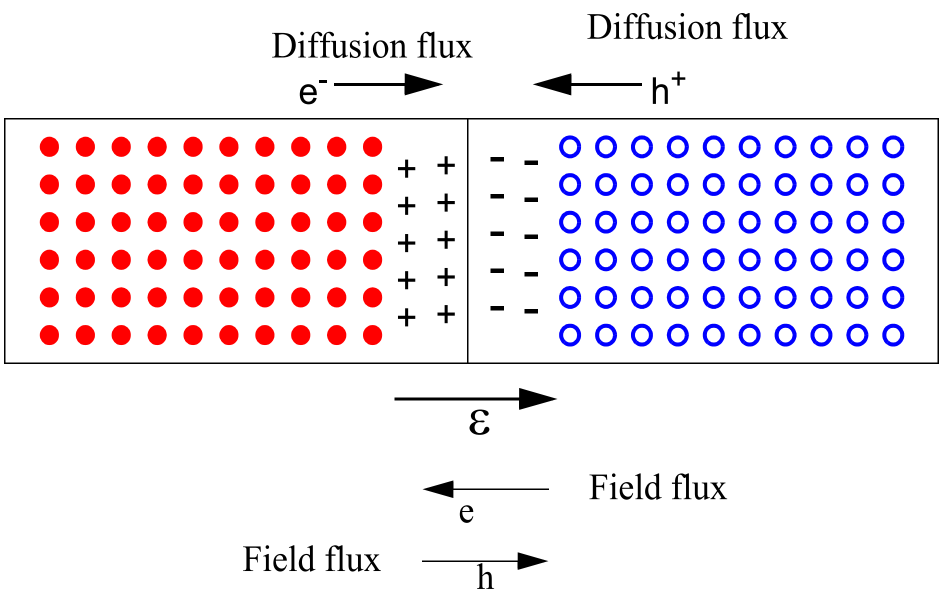
Holes:

Diffusion flux: $-D_p \frac{dp}{dx}$ where D ≡ diffusion constant.

Diffusion constant can be written as: $D = \mu \frac{kT}{q}$

Field flux: $\mu_p \mathcal{E}$ where μ ≡ mobility of carriers.

Net flux of holes: $F_p = -D_p \frac{dp}{dx} + \mu_p \mathcal{E}$ (1.15)



Electrons (similar):

Net flux of electrons: $F_n = D_n \frac{dn}{dx} - \mu_n \mathcal{E}$ (1.16)

Ref: Grove

Poisson's equation:

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon_r\varepsilon_0}$$

$$\frac{d^2E_i}{dx^2} = q\frac{N}{\varepsilon_r\varepsilon_0} \qquad (1.17)$$

$$d\mathcal{E} = \frac{dQ}{\varepsilon_r\varepsilon_0}$$

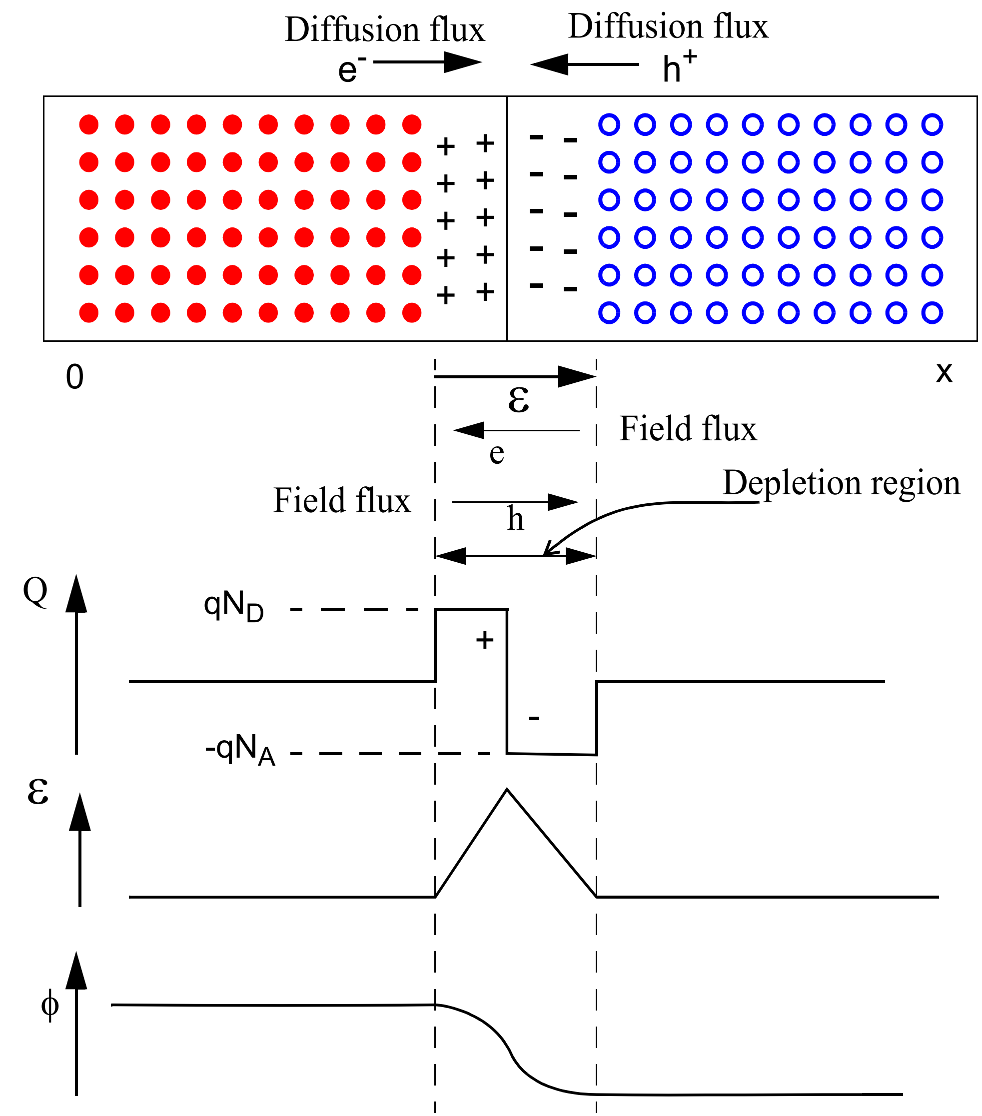
ϕ is the potential,
ρ is the charge density,
ε_r is the dielectric constant,
ε_0 is the vacuum permittivity,
q is the elementary charge and
N is the density of dopant atoms.

The field:

$$\mathcal{E} = \int_0^x \frac{\rho}{\varepsilon_r\varepsilon_0}dx \qquad (1.18)$$

The potential:

$$\phi = -\int_0^x \mathcal{E}dx \qquad (1.19)$$



Charge neutrality requires equal total charge on each side of the interface:

$$N_D x_n = N_A x_p$$

Maximum electrical field is found by using Poisson's equation. Integration of charge (the area under Q):

$$\mathcal{E}_{max} = \frac{qN_D x_n}{\varepsilon_r \varepsilon_0} = \frac{qN_A x_p}{\varepsilon_r \varepsilon_0} \qquad (1.20)$$

Second integration to find the potential difference. (by geometry):

$$|\phi_B| = \frac{1}{2}\mathcal{E}_{max}(x_n + x_p) = \frac{1}{2}\mathcal{E}_{max} W \qquad (1.21)$$

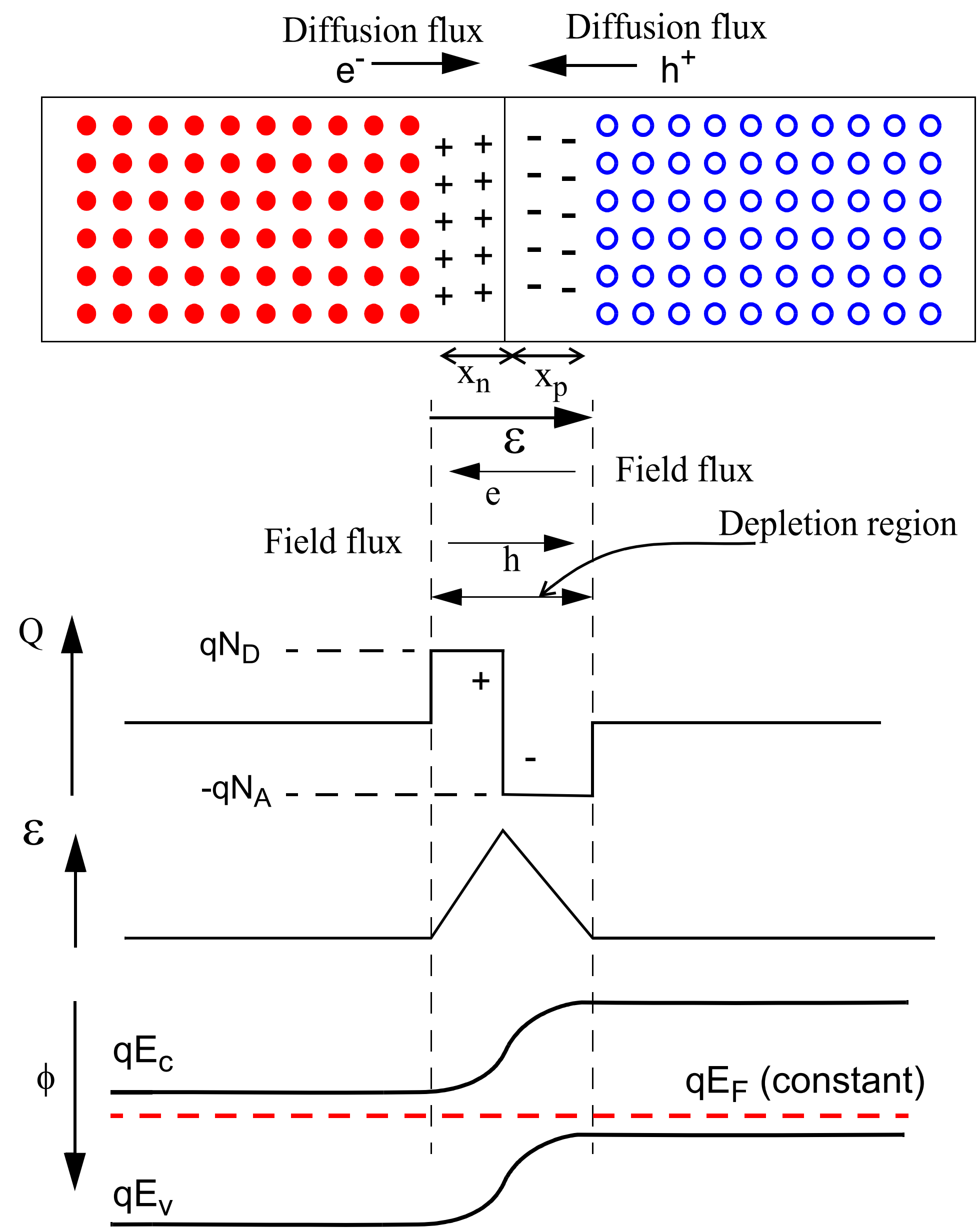
where total depletion region $W = x_n + x_p$ (1.22)

Combining (1.20), (1.20) and (1.22):

$$W = \mathcal{E}_{max}\frac{\varepsilon_r \varepsilon_0}{qN_D} + \mathcal{E}_{max}\frac{\varepsilon_r \varepsilon_0}{qN_A} = \frac{2\phi_B}{W}\left(\frac{\varepsilon_r \varepsilon_0}{qN_D} + \frac{\varepsilon_r \varepsilon_0}{qN_A}\right)$$

$$W = \sqrt{2\frac{\varepsilon_r \varepsilon_0}{q}\frac{N_A + N_D}{N_A N_D}\phi_B} \qquad (1.23)$$

Ref: Grove



Using (1.7) to express the derivative of hole concentration $\frac{dp}{dx} = \frac{p}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right)$ and that $D = \mu\frac{kT}{q}$,

Net hole flux (1.15) can be written as: $$F_p = \frac{p}{kT}D_p\frac{dE_F}{dx} = \mu_p\frac{p}{q}\frac{dE_F}{dx} \quad (1.24)$$

Similar for net electron flux: $$F_n = \frac{p}{kT}D_n\frac{dE_F}{dx} = \mu_n\frac{n}{q}\frac{dE_F}{dx} \quad (1.25)$$

Net flux for a PN junction in equilibrium is equal to zero.

$$F_n = -\mu_n\frac{n}{q}\frac{dE_F}{dx} = 0 \qquad F_p = \mu_p\frac{p}{q}\frac{dE_F}{dx} = 0 \qquad \text{krever at} \qquad \frac{dE_F}{dx} = 0$$

That is:

The Fermi level in a PN junction in equilibrium is constant

Ref: Grove

$$\phi_{Fp} \equiv -\left.\frac{E_F - E_i}{q}\right|_{p-region} \qquad \phi_{Fn} \equiv -\left.\frac{E_F - E_i}{q}\right|_{n-region}$$

$$N_A \approx p = n_i e^{(E_i - E_F)/kT} \quad \Rightarrow \quad E_i - E_F = kT\,\mathbf{ln}\frac{N_A}{n_i}$$

$$N_D \approx n = n_i e^{(E_F - E_i)/kT} \quad \Rightarrow \quad E_F - E_i = kT\,\mathbf{ln}\frac{N_D}{n_i}$$

$$\phi_{Fp} = \frac{kT}{q}\,\mathbf{ln}\frac{N_A}{n_i} \qquad (1.26)$$

$$\phi_{Fn} = -\frac{kT}{q}\,\mathbf{ln}\frac{N_D}{n_i} \qquad (1.27)$$

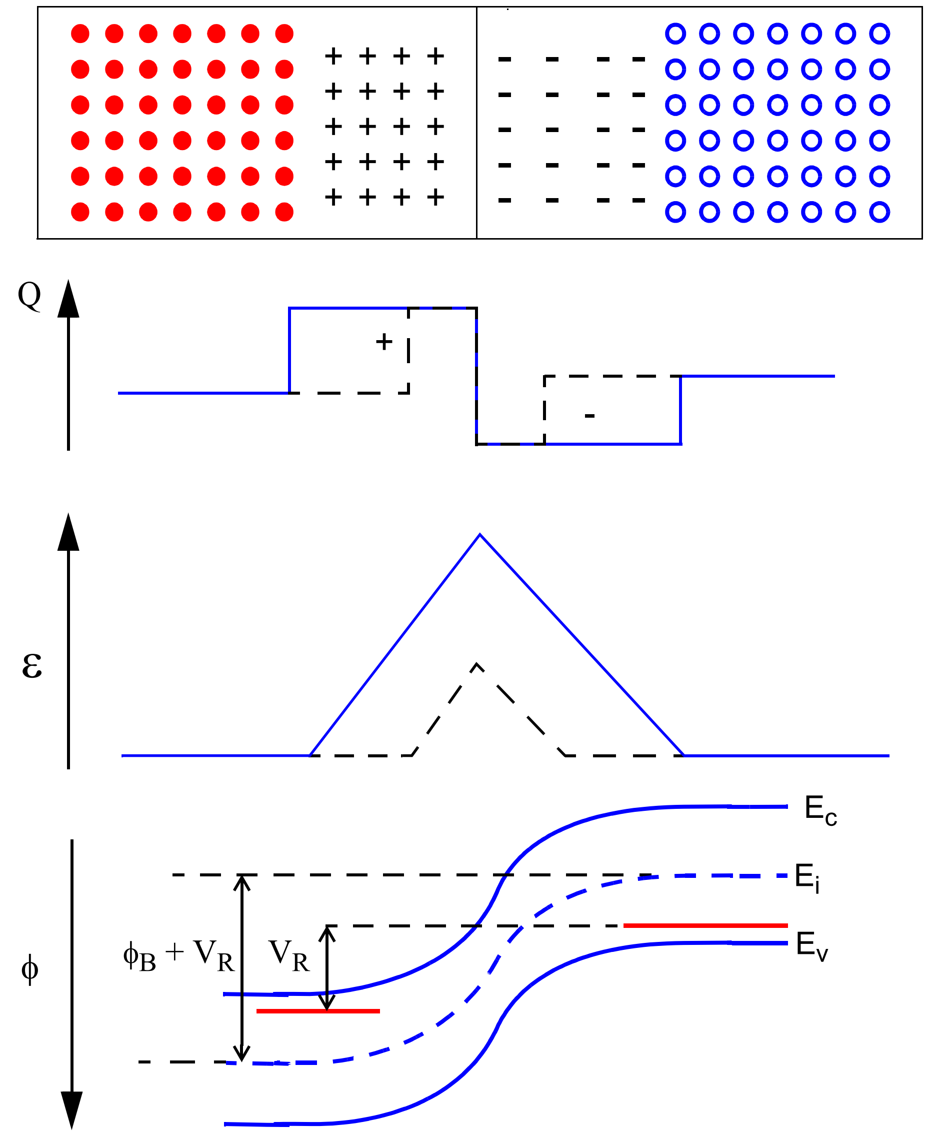
"Build-in-voltage" of the PN junction, the bending of E_i:

$$\phi_B = \phi_{Fp} + |\phi_{Fn}| \qquad (1.28)$$

Ref: Grove

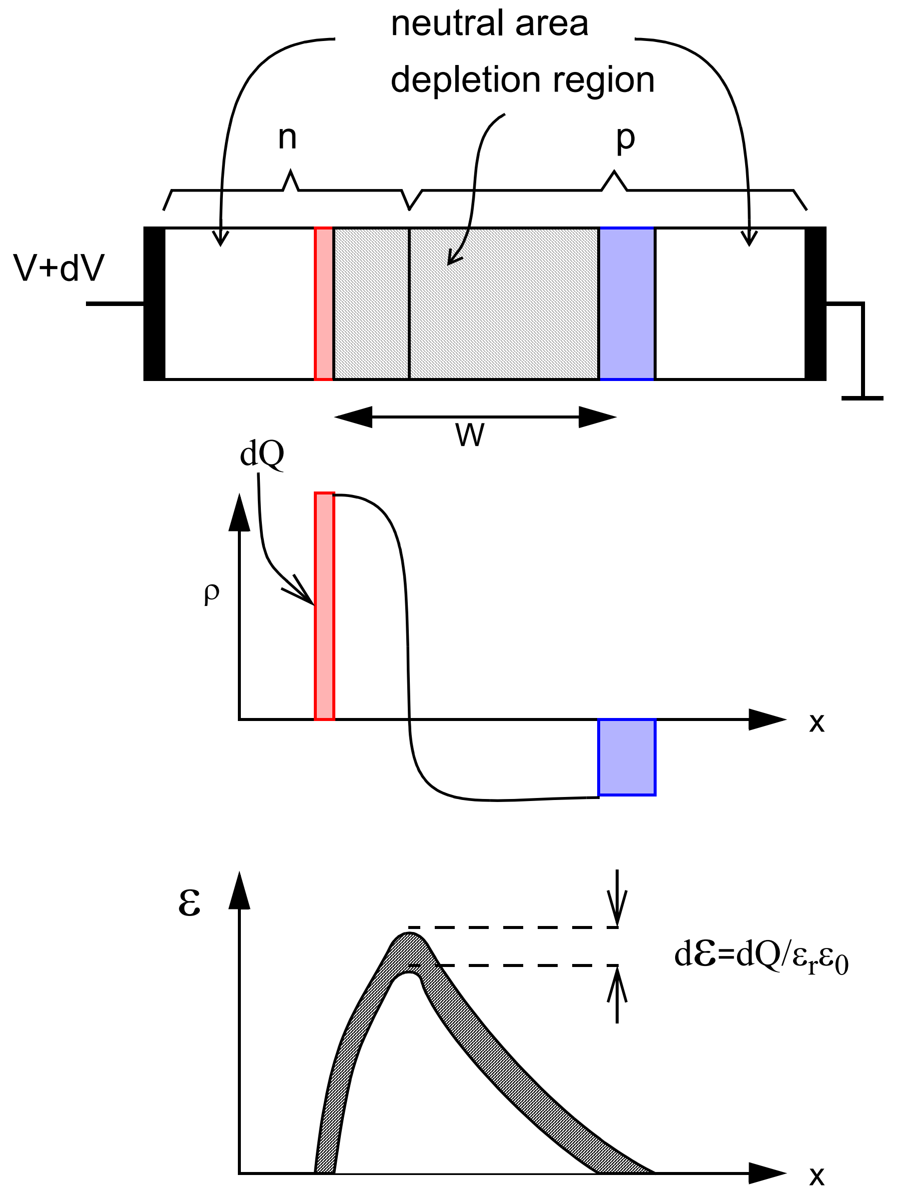
PN- junction reverse biased

- Field increases
- Free charge carriers p and n are repelled further away.
- Depletion region widens.
- The space charge corresponds to the field.
- Band bending corresponds to the field.



Capacitance - Voltage Characteristics

$$C \equiv \frac{dQ}{dV} \tag{1.29}$$



Increased field due to V_R:

$$d\varepsilon = \frac{dQ}{\varepsilon_r \varepsilon_0}$$

$$dV = (d\varepsilon)W = \frac{dQ}{\varepsilon_r \varepsilon_0}W$$

Capacitance:

$$C = \frac{dQ}{dV} = \frac{\varepsilon_r \varepsilon_0}{W}$$

Combined with the expression for W and total potential difference: $\phi_B + V_R$:

$$C = \sqrt{\frac{q\varepsilon_r\varepsilon_0}{2} \frac{N_A N_d}{(N_A + N_d)} \frac{1}{(\phi_B + V_R)}} \tag{1.30}$$

Ref: Grove

References

Grove

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