Coco-GAN Jan 1, 2025

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CoCo-GAN :

Cooperative-Competitive game regularization

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Abstract

Consider finite two-person incomplete-information strategic games with transferable utility. Folk theorems tell us that games with contracts and voluntary commitments possess large sets of Nash equilibria. Can an arbitrator bring about an efficient and fair outcome by suggesting a "natural" equilibrium payoff of a noncooperative arbitration game that allows for commitments?

A game has a natural cooperative/competitive decomposition into a cooperative team component in which the interests of the players are fully aligned and a fully competitive zero-sum game in which the player's interests are in direct conflict. Game theory has appealing solutions to these two types of games: the maxmax for cooperative team games and the minmax for zero-sum games. There is a certain combined payoff called the coco-value in an arbitration scheme: the players play the maxmax strategies of the cooperative component to yield equal (maximal) payoffs, which are efficient in this component of the game. But then the equal payoffs are adjusted by a zero-sum compensating transfer from the strategically weaker player to the stronger one.
The coco value is the only value that satisfies the axioms of Pareto efficiency, Shift invariance, Monotonicity in actions, Payoff dominance, Invariance to redundant strategies, and Monotonicity in information [Kalai, Kalai, 2013], thus identifying a natural equilibrium payoff.

We utilize the concept of a coco-value in the context of a Wasserstein GAN, for the purpose of regularization, to create a new generative adversarial network, that we dub CoCo-GAN. Moreover, we enable gradient descent by using amortized Kantorovich duality [Dam et al, 2019] in our generator-critic formulation.

Notation

ambient data space $\mathcal{X}$ with distribution μ

latent space $\mathcal{Z}$ with distribution ν

generator $g_\theta : \mathcal{Z} \to \mathcal{X}$, want $g_{\theta\#}\nu \approx \mu$

encoder $f_\varphi : \mathcal{X} \to \mathcal{Z}$, want $f_{\varphi\#}\mu \approx \nu$

lower semicontinuous cost function $c : \mathcal{X} \times \mathcal{X} \to [0, +\infty]$

Wasserstein distance $W(\mu, \nu) \triangleq \min\limits_{\pi \in \Pi(\mu,\nu)} \mathbb{E}_{(x,y)\sim\pi} [\, c(x,y) \,]$

c-transform $f^c(x) \triangleq \min\limits_y \{ c(x,y) - f(y) \}$

then $W(\mu, \nu) = \max\limits_f \{ \mathbb{E}_{x\sim\mu}[f^c(x)] + \mathbb{E}_{y\sim g_{\theta\#}\nu}[f(y)] \}$

Let $\alpha(x) \triangleq \arg\min\limits_y \{ c(x,y) - f(y) \}$

then $f^c(x) = c(x, \alpha(x)) - f(\alpha(x))$

Approximate minimizer $\alpha(x)$ by a learnable neural network

$h_\psi(x)$ that optimizes $\tilde{\alpha} = \min\limits_\psi \mathbb{E}_{x\sim\mu} [\, c(x, h_\psi(x)) - f(h_\psi(x)) \,]$

Note that $\mathbb{E}_{x\sim\mu}[f^c(x)] \leq \tilde{\alpha}$

<u>Game theoretic regularization</u>

Suppose $\tilde{f}(x) + f(y) \leq c(x,y) \quad \forall x,y \in \mathcal{X}$

let $\varphi^A(\tilde{f}, f) = \mathbb{E}_{x\sim\mu}[\tilde{f}(x)]$ and $\varphi^B(\tilde{f}, f) = \mathbb{E}_{y\sim g_{\theta\#}\nu}[f(y)]$

Define cooperative payoff

$$\Phi^{\#}(\tilde{f}, f) = \frac{\varphi^A(\tilde{f}, f) + \varphi^B(\tilde{f}, f)}{2}$$

and competitive payoff

$$\Phi^{b}(\tilde{f}, f) = \frac{\varphi^A(\tilde{f}, f) - \varphi^B(\tilde{f}, f)}{2}$$

Note that $\varphi^A = \Phi^{\#} + \Phi^{b}$ and $\varphi^B = \Phi^{\#} - \Phi^{b}$

Observe $V^{\#} \triangleq \max\limits_{\substack{\tilde{f}\, f \\ \tilde{f}(x)+f(y)\leq c(x,y)}} \{ \varphi^A(\tilde{f}, f) + \varphi^B(\tilde{f}, f) \} = W(\mu, \nu)$

is the largest cooperative payoff shared by players A and B,

and $\quad V^{b} \triangleq \max\limits_{\tilde{f}} \min\limits_{f} \varphi^{b}(\tilde{f}, f) = \max\limits_{\tilde{f}} \min\limits_{f} \dfrac{\mathbb{E}_{x\sim\mu}[\tilde{f}(x)] - \mathbb{E}_{y\sim g_{\theta\#}\nu}[f(y)]}{2}$

is the largest competitive payoff of a zero-sum

game between A and B.

$$\text{coco-value}(A,B) \triangleq \left(\frac{V^{\#}}{2} + V^{b}, \frac{V^{\#}}{2} - V^{b} \right)$$

CoCo-GAN

Reformulate the condition $\tilde{f}(x) + f(y) \leq c(x,y)$

by using the c-transform of f, $f^c(x) = \min_y \{c(x,y) - f(y)\}$,

then $$\Phi^{\#}(f) = \frac{\varphi^A(f^c) + \varphi^B(f)}{2}$$

and $$\Phi^{b}(f) = \frac{\varphi^A(f^c) - \varphi^B(f)}{2}$$

CoCo-GAN solves the regularized optimization problem:

$$\min_\theta \max_\varphi \min_\psi \left\{ \Phi^{\#}(f_\varphi) - \lambda \left| \Phi^{b}(f_\varphi) \right| \right\}, \quad \lambda > 0$$

where $f^c(x)$ is replaced by $c(x, h_\psi(x)) - f_\varphi(h_\psi(x))$.

Loosely speaking, we target $\frac{V^{\#}}{2}$ and penalize large V^b,

to learn generator g_θ, critic f_φ and mover h_ψ.

CoCo-GAN regularizes a Wasserstein-like GAN by a cooperative-competitive game. Generator g_θ and critic f_φ are chosen to optimize cooperative payoff, regularized by competitive payoff. Lipschitz conditions are avoided by using amortized duality, thereby enabling gradient descent.

References

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