# Neuarl Network Abduction Jun 13, 2025

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**1 author:**

Gary Nan Tie

Mu Risk LLC

# Neural Network Abduction

## Gary Nan Tie, June 13, 2025

### Abstract
For hypotheses whose effect is manifested by observations, abduction seeks to explain a given observation by finding a hypothesis whose effect is that observation. Abductive reasoning has a fibration semantics that can be implemented by neural networks; a step towards artificial general intelligence.

A natural application is in making a medical diagnosis. Fibration structure and properties enable coherency and consistency in this process. Moreover being machine learnable, can help physicians uncover diagnostic insights in large datasets.

Welcome Sher-bot Holmes!

An arrow $p\downarrow$ from $K$ to $V$ in a category is said to be

$$\begin{tikzcd} K \arrow[d, "p"'] \\ V \end{tikzcd}$$

a <u>fibration</u> with respect to a class of arrows $\mathcal{M}$

when for any $m \in \mathcal{M}$ and solid commutative diagram:

$$\begin{tikzcd} B \arrow[r, "f"] \arrow[d, "m"'] & K \arrow[d, "p"] \\ Q \arrow[r, "g"'] \arrow[ur, dashed, "\sigma"] & V \end{tikzcd}$$

i.e. $p \circ f = g \circ m$

there exists dotted lifting $\sigma_{m,f,g} : Q \longrightarrow K$

such that $p \circ \sigma = g$ and $\sigma \circ m = f$.

Liftings are said to be <u>compatible</u> if for any

solid commutative diagram:

$$\begin{tikzcd} D \arrow[r, "j"] \arrow[d, "n"'] & B \arrow[r, "f"] \arrow[d, "m"] & K \arrow[d, "p"] \\ C \arrow[r, "k"'] \arrow[urr, dashed, "\sigma"] & Q \arrow[r, "g"'] \arrow[ur, dashed, "\tau"'] & V \end{tikzcd}$$

$p$ a fibration wrt $\mathcal{M}$

$n, m \in \mathcal{M}$

$\sigma = \sigma_{n,\, f\circ j,\, g \circ k}$

$\tau = \sigma_{m, f, g}$

we have $\tau \circ k = \sigma$.

Suppose $p \big\downarrow$ is a fibration wrt $\mathcal{M}$

$$\begin{tikzcd} K \arrow[d, "p"'] \\ V \end{tikzcd}$$

and $m \in \mathcal{M}$

$$\begin{tikzcd} B \arrow[r, "f"] \arrow[d] & K \arrow[d, "p"] \\ Q \arrow[r, "g"'] & V \end{tikzcd}$$

commutes.

If $g \circ m \in \mathcal{M}$ then

$$\begin{tikzcd} B \arrow[r, "1"] \arrow[d, "m"'] & B \arrow[r, "f"] \arrow[d, "g \circ m"] & K \arrow[d, "p"] \\ Q \arrow[r, "g"'] \arrow[urr, dashed, "\sigma"] & V \arrow[r, "1"'] \arrow[ur, dashed, "\tau"'] & V \end{tikzcd}$$

i) lifting $\sigma$ satisfies $p \circ \sigma = g$ and $\sigma \circ m = f$

ii) lifting $\tau$ satisfies $p \circ \tau = 1_V$ and $\tau \circ (g \circ m) = f$

iii) and by compatibility $\tau \circ g = \sigma$.

Upshot: Suppose effect $p : K \to V$ is a fibration wrt $\mathcal{M}$

and

$$\begin{tikzcd} B \arrow[d, "m \in \mathcal{M}"'] \arrow[r, phantom, "\overset{(f,g)}{\Longrightarrow}"] & K \arrow[d, "p"] \\ Q & V \end{tikzcd}$$

is an attention 2-cell for analogy $m$.

If $g \circ m \in \mathcal{M}$ then $p \circ \tau = 1_V$

that is $\tau$ is an abduction for effect $p$.

# Neural Network Abduction

For effect $p \; \begin{matrix} K \\ \downarrow \\ V \end{matrix}$ and analogy $m \; \begin{matrix} B \\ \Downarrow \\ Q \end{matrix}$

suppose we have data from $K$, $V$, $B$ and $Q$.

- learn or define effect $p$ from $\{(k,v)\}$
- learn or define analogy $m$ from $\{(b,q)\}$
- learn or define attention 2-cell $(f,g)$

  from $\{(b,k)\}$ for $f$ and from $\{(q,v)\}$ for $g$

  such that $p \circ f \approx g \circ m$
- learn lifting $\sigma$ from $\{(q,k)\}$

  such that $p \circ \sigma \approx g$ and $\sigma \circ m \approx f$
- learn abduction $\tau$ from $\{(v,k)\}$

  such that $\tau \circ g \approx \sigma$

$$
\begin{tikzcd}
B \arrow[r, "1"] \arrow[d, "m \in M"'] & B \arrow[r, "f"] \arrow[d, "g \circ m"', "\in M"] & K \arrow[d, "p \text{ fibration}"] \\
Q \arrow[r, "g"'] \arrow[urr, dashed, "\sigma"] & V \arrow[r, "1"'] \arrow[ur, dashed, "\tau"'] & V
\end{tikzcd}
$$

## References

'Fibrations explain all you need!'
- Fibrations, Abduction and Attention
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