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Neural Network Abduction

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Abstract

For hypotheses whose effect is manifested by observations, abduction seeks to explain a given observation by finding a hypothesis whose effect is that observation. Abductive reasoning has a fibration semantics that can be implemented by neural networks; a step towards artificial general intelligence.

A natural application is in making a medical diagnosis. Fibration structure and properties enable coherency and consistency in this process. Moreover being machine learnable, can help physicians uncover diagnostic insights in large datasets.

Welcome Sher-bot Holmes!

An arrow $\begin{array}{c} K \\ p \downarrow \\ V \end{array}$ in a category is said to be a <u>fibration</u> with respect to a class of arrows $\mathcal{M}$ when for any $m \in \mathcal{M}$ and solid commutative diagram:

$$\begin{tikzcd} B \arrow[r, "f"] \arrow[d, "m"'] & K \arrow[d, "p"] \\ Q \arrow[r, "g"'] \arrow[ur, dashed, "\sigma"] & V \end{tikzcd}$$

i.e. $p \circ f = g \circ m$

there exists dotted lifting $\sigma_{m,f,g} : Q \longrightarrow K$ such that $p \circ \sigma = g$ and $\sigma \circ m = f$.

Liftings are said to be <u>compatible</u> if for any solid commutative diagram:

$$\begin{tikzcd} D \arrow[r, "j"] \arrow[d, "n"'] & B \arrow[r, "f"] \arrow[d, "m"] & K \arrow[d, "p"] \\ C \arrow[r, "k"'] \arrow[urr, dashed, "\sigma"] & Q \arrow[r, "g"'] \arrow[ur, dashed, "\tau"'] & V \end{tikzcd}$$

p a fibration wrt $\mathcal{M}$

$n, m \in \mathcal{M}$

$\sigma = \sigma_{n,\, f \circ j,\, g \circ k}$

$\tau = \sigma_{m,f,g}$

we have $\tau \circ k = \sigma$.

Suppose $\begin{matrix} K \\ p \downarrow \\ V \end{matrix}$ is a fibration wrt $\mathcal{M}$

and $m \in \mathcal{M}$
$$\begin{tikzcd} B \arrow[r, "f"] \arrow[d] & K \arrow[d, "p"] \\ Q \arrow[r, "g"'] & V \end{tikzcd}$$
commutes.

If $g \circ m \in \mathcal{M}$ then

$$\begin{tikzcd} B \arrow[r, "1"] \arrow[d, "m"'] & B \arrow[r, "f"] \arrow[d, "g \circ m"] & K \arrow[d, "p"] \\ Q \arrow[r, "g"'] \arrow[urr, dashed, "\sigma"] & V \arrow[r, "1"'] \arrow[ur, dashed, "\tau"] & V \end{tikzcd}$$

i) lifting σ satisfies $p \circ \sigma = g$ and $\sigma \circ m = f$

ii) lifting τ satisfies $p \circ \tau = 1_V$ and $\tau \circ (g \circ m) = f$

iii) and by compatibility $\tau \circ g = \sigma$.

<u>Upshot:</u> Suppose effect $p : K \to V$ is a fibration wrt $\mathcal{M}$

and
$$\begin{tikzcd} B \arrow[d, "m \in \mathcal{M}"'] \arrow[r, phantom, "\overset{(f,g)}{\Longrightarrow}"] & K \arrow[d, "p"] \\ Q & V \end{tikzcd}$$
is an attention 2-cell for analogy m.

If $g \circ m \in \mathcal{M}$ then $p \circ \tau = 1_V$

that is τ is an abduction for effect p.

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For effect $p: K \to V$ with analogy $m: B \to Q$

suppose we have finite data from K, V, B and Q.

1. - learn or define effect p from $\{(k,v)\}$

2. - learn or define analogy m from $\{(b,q)\}$

3. - learn or define attention 2-cell (f,g),

   from $\{(b,k)\}$ for f and from $\{(q,v)\}$ for g

   such that $p \circ f \approx g \circ m$

4. - learn lifting σ from $\{(q,k)\}$

   such that $p \circ \sigma \approx g$ and $\sigma \circ m \approx f$

5. - learn 'abduction' τ from $\{(g(q), \sigma(q))\}$

   such that $\tau \circ g \approx \sigma$, then $(p \circ \tau) \circ g \approx g$.

   When g is invertible, $p \circ \tau = 1_V$.

References

'Fibrations explain all you need!'
- Fibrations, Abduction and Attention
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