# Supplement:

Let $H(K)$ be a RKHS on $X$ with reproducing kernel

$K: X \times X \to \mathbb{C}$ and $f: X \to \mathbb{C}$ a function.

1. $f \in H(K) \Leftrightarrow \exists c > 0$ such that $c^2 K(x,y) - f(x)\overline{f(y)}$ is a kernel.

2. If $f$ is a nonzero function on $X$ then $K(x,y) = f(x)\overline{f(y)}$ is a kernel.

3. $H(K_1) \subseteq H(K_2) \Leftrightarrow \exists c > 0$ such that $c^2 K_2 - K_1$ is a kernel.

4. $K_1$ and $K_2$ kernels $\Rightarrow$ $K_1 + K_2$ is a kernel.

<u>Prop</u> Let $K: X \times X \to \mathbb{C}$ be a kernel and $f: X \to \mathbb{C}$ a nonzero function

then $\exists$ kernel $\tilde{K}$ such that $H(K) \subseteq H(\tilde{K})$ and $f \in H(\tilde{K})$.

<u>Pf</u>

Let $\tilde{K}(x,y) \triangleq f(x)\overline{f(y)} + K(x,y)$

$\tilde{K}(x,y) - K(x,y) = f(x)\overline{f(y)}$ a kernel $\Rightarrow H(K) \subseteq H(\tilde{K})$

$\tilde{K}(x,y) - f(x)\overline{f(y)} = K(x,y)$ a kernel $\Rightarrow f \in H(\tilde{K})$

$\square$

<u>Cor</u> Given a kernel $K: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ and a projection

function $\pi_i : \mathbb{R}^n \to \mathbb{R}$, $(x_1, \dots, x_n) \mapsto x_i$. then $\pi_i \in H(\tilde{K})$ and $H(K) \subseteq H(\tilde{K})$

For $\tilde{K}(x,y) = x_i y_i + K(x,y)$.