Status of Emergent Spacetime from Quantum Information Geometry: Regime-Dependent Einstein Coupling at $L = 3$

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Abstract

We revisit and update the numerical status of the "Quantum Information Geometry" (QIG) program, in which a spacetime-like metric emerges from the quantum Fisher information of a many-body system and satisfies an Einstein-like relation $G_{ij} \approx \kappa T_{ij}$ at the lattice level. Earlier drafts and internal notes reported a continuum extrapolation $\kappa_\infty \approx 4.1 \pm 0.2$ based on a three-point finite-size fit across nominal system sizes $L = 2, 3, 4$. A complete audit of the code and data, together with a new suite of diagnostics, has revealed that this claim is not supported by the validated pipeline.

In this status update we present our most reliable result to date: a statistically robust measurement of the Einstein-like coupling at system size $L = 3$ in the transverse-field Ising model (TFIM). Using one-sided local field perturbations in an intermediate "geometric" regime, and fitting the average diagonal Einstein tensor against the corresponding stress-energy trace with a free intercept, we obtain $\kappa_3^{\text{geo}} = 41.09 \pm 0.59$ from an ensemble of three random seeds and fifty perturbations per seed, with $R^2 > 0.99$ and a coefficient of variation of $\approx 2.5\%$. At the same time we find strong dependence of the effective coupling on the perturbation regime, and clear evidence that $L = 2$ is contaminated by boundary artefacts.

We therefore *withdraw* the previous specific claim $\kappa_\infty \approx 4.1$ and instead treat κ as a regime- and size-dependent effective coupling on the lattice. We outline the diagnostic infrastructure that led to this reassessment and set out a concrete next milestone—a full $L = 4$ ensemble with null and control experiments—that we regard as mandatory before making any further claims about continuum values or phenomenological constants.

1 Introduction

The QIG program proposes that classical spacetime geometry and Einstein-like dynamics can emerge from the information geometry of quantum many-body systems. Concretely, given a family of quantum states $\rho(\boldsymbol{\theta})$ with parameters $\boldsymbol{\theta}$, one can define a quantum Fisher information (QFI) metric $F_{\mu\nu}(\boldsymbol{\theta})$ and, after suitable regularisation, treat

$$g_{\mu\nu}(\boldsymbol{\theta}) = \frac{1}{4} F_{\mu\nu}(\boldsymbol{\theta}) + \varepsilon \delta_{\mu\nu}, \qquad \varepsilon \ll 1 \tag{1}$$

as an effective spacetime metric. From this one constructs discrete Christoffel symbols, curvature tensors, and an Einstein tensor G_{ij} on a spatial lattice. Local expectation values of a Hamiltonian density define a stress-energy tensor T_{ij}, and one can ask whether an emergent relation of the form

$$G_{ij}(\mathbf{x}) \approx \kappa\, T_{ij}(\mathbf{x}) \tag{2}$$

holds, with some effective coupling κ.

Earlier versions of this work reported encouraging evidence for Eq. (2) in a two-dimensional transverse-field Ising model (TFIM), including statements that a finite-size scaling analysis across $L = 2, 3, 4$ supported a continuum coupling $\kappa_\infty \approx 4.1$. However, those statements predated the current diagnostic infrastructure. A subsequent audit has revealed a mixture of issues:

- use of zero-intercept fits and mixed perturbation regimes, which can artificially compress κ,
- strong finite-size and boundary artefacts at $L = 2$,
- and the absence of fully validated $L = 4$ data in the current code path.

As a result we no longer regard the specific numerical value $\kappa_\infty \approx 4.1$ as trustworthy.

The purpose of this paper is two-fold:

1. to present a clean, fully documented measurement of the Einstein-like coupling at $L = 3$ in a well-defined perturbation regime, and
2. to clearly separate what is currently well-supported numerically from what remains speculative, including a concrete roadmap for the next validation milestone.

2 Model and Information-Geometric Construction

We work with the two-dimensional TFIM on an $L \times L$ lattice,

$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x, \tag{3}$$

with J set to unity and h chosen near the critical region. The parameter manifold is spanned by local field perturbations $\delta h(\mathbf{x})$ which act as sources, so that

$$H(\boldsymbol{\theta}) = H_0 + \sum_{\mathbf{x}} \theta_{\mathbf{x}}\, \sigma^x_{\mathbf{x}}, \qquad \theta_{\mathbf{x}} \equiv \delta h(\mathbf{x}). \tag{4}$$

We compute ground states $|\psi(\boldsymbol{\theta})\rangle$ using variational tensor network methods (DMRG / PEPS depending on L), then construct the QFI Hessian

$$F_{\mu\nu} = 4\,\mathrm{Re}\Big[\, \langle \partial_\mu \psi | \partial_\nu \psi \rangle - \langle \partial_\mu \psi | \psi \rangle \, \langle \psi | \partial_\nu \psi \rangle \,\Big], \tag{5}$$

with indices μ, ν labeling spatial sites and perturbation directions.

Adding a small regulator ε to stabilise inversions, we define the effective metric as in Eq. (1), then compute discrete Christoffel symbols, curvature, and Einstein tensor G_{ij} on the lattice via standard finite-difference approximations. Local energy densities from H

are combined into an effective stress-energy tensor T_{ij}; for the purposes of this paper we focus on their traces and diagonal components, which are the most numerically stable.

A more detailed description of the geometric discretisation, including Regge-like constructions and Gauss–Bonnet consistency checks on the torus, can be found in the longer QIG manuscript. Here we focus on the validated numerical behaviour of κ.

3 Numerical Pipeline and Diagnostics

The current codebase implements a multi-phase diagnostic pipeline:

Phase 0 (audit). Historical measurements of κ are re-run or re-analysed under a common interface. We track the influence of:

- fit choices (free vs. zero intercept),
- perturbation regimes (small linear, intermediate "geometric", large breakdown),
- spatial sampling (centre vs. edge vs. corner sites).

This produces a "κ audit" document summarising where past values came from and which are superseded.

Phase 1 ($L = 3$ ensemble). For a fixed $L = 3$ system, we perform an ensemble of runs:

- three random seeds for tensor-network initialisation,
- $N_{\text{perts}} = 50$ perturbations per seed,
- one-sided perturbations $\delta h > 0$ in a chosen regime,
- and linear fits of ΔR vs. ΔT with a free intercept.

We record κ and R^2 for each run and compute ensemble statistics.

Phase 2 (infrastructure for $L = 4$). We develop and test DMRG-based infrastructure for $L = 4$ that avoids the combinatorial cost of naive Hamiltonian construction. At this stage the focus is on code correctness and runtime profiling; a full $L = 4$ ensemble is reserved for a dedicated future milestone.

All phases include automated logging, residual analysis, and unit tests on geometry (e.g. approximate satisfaction of discrete Bianchi identities and baseline flatness when no defects are present).

4 Results at $L = 3$: Regime Dependence and Main Coupling

Within this pipeline we find that the *existence* of a linear relation of the form Eq. (2) at $L = 3$ is robust, but the inferred coupling κ depends strongly on the perturbation regime.

We work with the trace-like quantities

$$\Delta R = \frac{1}{d} \sum_{i=1}^{d} \Delta G_{ii}, \qquad \Delta T = \frac{1}{d} \sum_{i=1}^{d} \Delta T_{ii}, \tag{6}$$

and fit

$$\Delta R = \kappa\,\Delta T + b \tag{7}$$

with a free intercept b.

4.1 Geometric regime (primary result)

In an intermediate "geometric" window of perturbations,

$$0.45 \lesssim \delta h \lesssim 0.7, \tag{8}$$

we obtain our most stable measurements. Across three seeds and fifty perturbations per seed, all fits in this regime satisfy

$$R^2 > 0.99, \tag{9}$$

and the ensemble-averaged coupling is

$$\kappa_3^{\mathrm{geo}} = 41.09 \pm 0.59, \tag{10}$$

with a coefficient of variation CV $\approx 2.48\%$. This is the number we regard as the current "headline" result: the Einstein-like relation is clearly present and highly reproducible at $L = 3$ in this perturbation window, with a coupling of order 40.

4.2 Linear and breakdown regimes

The same pipeline reveals different couplings (or a breakdown of the relation) in other regimes:

- **Linear regime** ($\delta h \lesssim 0.3$): Here we find a smaller effective coupling, $\kappa_3^{\mathrm{lin}} \sim \mathcal{O}(10)$, with good but weaker correlations (R^2 typically in the range 0.95–0.97). This regime is closer to a perturbative response.
- **Breakdown regime** ($\delta h \gtrsim 0.8$): In this region, fits can yield negative κ and poor R^2. Rather than interpreting such values literally, we regard this as a regime where the simple linear Einstein-like relation fails and higher-order or nonlocal effects dominate.

4.3 Component robustness

Within the geometric regime we have checked that fitting individual tensor components (e.g. G_{xx} vs. T_{xx}, G_{yy} vs. T_{yy}) yields consistent values of κ within the statistical uncertainties. This provides a basic internal check of approximate isotropy in the emergent geometry at $L = 3$.

5 Finite-Size Effects and the Status of κ_∞

Earlier drafts invoked a three-point finite-size ansatz

$$\kappa(L) = \kappa_\infty + \frac{c}{L^\alpha}, \tag{11}$$

with nominal values at $L = 2, 3, 4$ and a best-fit $\kappa_\infty \approx 4.1$. The current audit has revealed several problems with this picture:

1. The $L = 4$ data used in those fits were generated with an older, partially diagnosed pipeline that is not directly comparable to the current code path.

2. The $L = 2$ value is strongly affected by boundary and discretisation artefacts; under some choices of fit and sampling it can be as small as $\kappa_2 \approx 1.5$ or even negative.

3. Mixed-regime fits and enforced zero-intercept constraints can artificially drive κ towards small values.

Given these issues, we no longer regard the specific number $\kappa_\infty \approx 4.1 \pm 0.2$ as trustworthy. Instead we adopt the following conservative stance:

- **At present, only $\kappa_3^{\rm geo}$ is numerically solid.** It is derived from a well-documented ensemble with strong internal diagnostics.

- **Finite-size scaling is an open problem.** We continue to use $\kappa(L) = \kappa_\infty + c/L^2 + \cdots$ as a theoretically motivated template, but we leave κ_∞ undetermined until a full $L = 4$ ensemble is available.

- **Regime dependence is physical.** The difference between $\kappa_3^{\rm lin}$ and $\kappa_3^{\rm geo}$ is not a "bug"; it reflects genuine structure in how the QFI geometry responds to different classes of perturbations.

6 Phenomenological Interpretation and Caution

The broader QIG program connects the emergent coupling κ to effective gravitational physics through relations of the schematic form

$$8\pi G_N \sim \kappa a^2, \tag{12}$$

where a is an effective lattice spacing or microscopic cutoff. In this context, our current results support the existence of an Einstein-like relation on the lattice with κ in the range $\mathcal{O}(10–40)$, depending on regime and size.

However, we emphasise:

- We do *not* claim a precise value of κ_∞ at this stage.

- Any phenomenological prediction that depends sensitively on the exact value of κ (e.g. precise Yukawa strengths at a given scale) should be expressed as a band spanning the plausible range of κ and a, not as a single sharp number.

- The falsifiability of QIG-inspired experimental proposals should ideally be robust to $\mathcal{O}(1)$ variations in these parameters.

7 Next Milestone: $L = 4$ and Hostile-Reviewer Diagnostics

We regard the following as the critical next milestone before any further publication claims:

$L = 4$ ensemble. Run a statistically meaningful $L = 4$ ensemble in the same geometric regime (e.g. two or three seeds, ~ 20 perturbations per seed). Extract κ_4^{geo} and compare to κ_3^{geo}, examining trends and error bars.

Null and control experiments. Demonstrate that the Einstein-like relation fails where it should:

- product states and trivial thermal states (expect R^2 low, κ unstable),
- mismatched stress-energy (e.g. using a "wrong" Hamiltonian to define T_{ij}),
- deep trivial phases where curvature and stress-energy are nearly zero.

Internal consistency checks. Extend component- and site-resolved checks, and verify that the geometric discretisation satisfies basic consistency conditions (approximate Bianchi identities, Gauss–Bonnet on the torus, etc.) in the new ensembles.

Only once this "hostile reviewer" suite is complete will we be in a position to make stronger claims about finite-size scaling, continuum values, or universal couplings across different models.

8 Conclusions

The main message of this status update is that the QIG program has reached a new level of numerical maturity. We have:

- established a robust Einstein-like relation at $L = 3$ in a well-defined geometric regime, with $\kappa_3^{\mathrm{geo}} = 41.09 \pm 0.59$ and $R^2 > 0.99$,
- identified and quantified strong regime and finite-size dependence of the effective coupling,
- audited and superseded earlier claims of a precise continuum value $\kappa_\infty \approx 4.1$,
- and developed infrastructure for $L = 4$ and for systematic null and control experiments.

The next steps are clear and technically feasible. Whether or not QIG ultimately yields a viable route to emergent gravity, the existence of these robust lattice-scale Einstein-like relations—and their regime dependence—is already an interesting phenomenon in its own right.