

On the Einstein Coupling in Quantum Information Geometry: Erratum and Status Update (November 2025)

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Summary

Earlier versions of the “Quantum Information Geometry” (QIG) manuscript and associated internal documents reported a continuum Einstein-like coupling

$$\kappa_\infty \approx 4.1 \pm 0.2, \tag{1}$$

based on a finite-size scaling analysis of an emergent relation $G_{ij} \approx \kappa T_{ij}$ in a transverse-field Ising model. The purpose of this note is to withdraw that specific numerical claim and to summarise the current status of the coupling κ after a complete audit of the code and data.

What has changed

Since the original draft, the numerical pipeline has been significantly extended:

- A comprehensive “ κ audit” re-ran or re-analysed all historical measurements under a unified, better-diagnosed code path.
- New diagnostics were added for fit quality, perturbation-regime dependence, spatial sampling, and geometric consistency (e.g. baseline flatness, approximate Bianchi identities).
- A clean $L = 3$ ensemble was generated with three seeds and fifty perturbations per seed in a well-defined “geometric” perturbation regime, using one-sided perturbations and free-intercept fits.

Current reliable result

The most robust statement we can make at present is:

In an intermediate geometric regime of perturbations at system size $L = 3$, the lattice Einstein-like relation $G_{ij} \approx \kappa T_{ij}$ holds with high statistical quality ($R^2 > 0.99$), and the ensemble-averaged coupling is $\kappa_3^{\text{geo}} = 41.09 \pm 0.59$ with a coefficient of variation of $\approx 2.5\%$.

At the same time, the coupling shows strong dependence on the perturbation regime: a smaller value in a more linear regime and a breakdown of the relation for large perturbations.

Why $\kappa_\infty \approx 4.1$ is withdrawn

The previous estimate $\kappa_\infty \approx 4.1$ came from a three-point fit across nominal system sizes $L = 2, 3, 4$. The audit revealed that:

1. The $L = 4$ data used in that fit were not generated with the current, fully diagnosed pipeline.
2. The $L = 2$ value is strongly contaminated by boundary artefacts and is not a reliable indicator of bulk behaviour.
3. Mixed perturbation regimes and enforced zero-intercept fits can artificially push κ towards smaller values.

Given these issues, the numerical evidence no longer supports a precise continuum value of κ at the level previously claimed.

Conservative interpretation

We therefore adopt the following conservative interpretation:

- The existence of an Einstein-like relation on the lattice is supported by the $L = 3$ ensemble.
- The effective coupling κ is regime- and size-dependent, and is currently known to lie in the range $\mathcal{O}(10\text{--}40)$ for the parameter choices studied.
- No specific continuum value κ_∞ should be quoted until at least one additional system size (e.g. $L = 4$) has been studied with comparable statistical quality and diagnostics.

In particular, any phenomenological predictions that previously depended on $\kappa_\infty \approx 4.1$ should be interpreted instead as order-of-magnitude estimates over a band of plausible values of κ and the associated microscopic cutoff.

Next steps

Work is underway on:

- a full $L = 4$ ensemble in the same geometric regime,
- null and control experiments (product states, mismatched stress-energy, trivial phases) to demonstrate where the relation fails,
- and an updated phenomenological analysis that presents predictions as parameter bands rather than single numbers.

A separate, more detailed status paper presents the full methodology and results.

Any reader of the original QIG manuscript should treat the specific numerical claim $\kappa_\infty \approx 4.1 \pm 0.2$ as superseded by the present note and by the forthcoming status update.