ELEC5470 - Convex Optimization, Fall 2020-21

Homework Set #4

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Justify all the answers. You must provide complete solutions with all steps described in detail. Submission will be done via Canvas by submitting a `ZIP` file, containing:

1) **Homework solution** (with filename `solution`): preferably in PDF format,
2) **Source code and a README file** (with filename `code`): all necessary code for running your program as well as a brief user guide for the TA to run the programs easily to verify your results, all compressed into a single `ZIP` or `RAR` file.

Below are the problems:

1) (50 points) *Barrier method for LASSO*. Implement a barrier method for solving the following problem

$$\underset{\boldsymbol{\beta}\in\mathbb{R}^p}{\text{minimize}} \quad \|\mathbf{y}-\mathbf{X}\boldsymbol{\beta}\|_2^2+\lambda\|\boldsymbol{\beta}\|_1, \tag{1}$$

where $\mathbf{X}$ is of size $n\times p$ and $\lambda>0$ is a constant (use $\lambda=0.2$ in the code).

a) Give the mathematical expressions of the gradient and the Hessian you used in your designed barrier method.

b) Plot one figure showing the value of the objective function versus the number of iterations and another showing the duality gap bound versus Newton steps.
Matlab code: *randn('seed',1); beta = zeros(10,1); beta(3) = 1; beta(5) = 7; beta(10) = 3; n = 100; p = 10; X = randn(n,p); y = X*beta + 0.1*randn(n,1); lambda = 0.2;*

2) (50 points) *MM method for designing unimodular sequence*. Consider the following unimodular sequence designing problem

$$\begin{array}{ll}\underset{\mathbf{x}\in\mathbb{C}^n}{\text{minimize}} & \mathbf{x}^H\mathbf{M}\mathbf{x}\\ \text{subject to} & |x_i|=1, i=1,\ldots,n,\end{array} \tag{2}$$

where $\mathbf{M}$ is a complex $n\times n$ Hermitian matrix (may not be positive-semidefinite).

a) Determine whether the problem (2) is convex. If not, please write down a majorization-minimization (MM) algorithm to solve it. Note that you are expected to give a closed-form solution in each iteration.
Hint: If $\mathbf{A}\in\mathbb{C}^{n\times n}$ is a positive-semidefinite Hermitian matrix, then $\mathbf{w}^H\mathbf{A}\mathbf{w}\geq 0$ holds for any $\mathbf{w}\in\mathbb{C}^n$.

b) Implement your designed algorithm. Plot one figure showing the objective function value versus the number of iterations.
Matlab code: *rng(1); n = 5; Y = complex(normrnd(0, 1, n, n*2), normrnd(0, 1, n, n*2)); M = Y*Y';*