ELEC5470 HW2

TANG Jiawei 20672550

Problem1

$$\underset{\boldsymbol{x}>0}{\text{minimize}} \quad \frac{1}{2}\boldsymbol{x}^\top \Sigma \boldsymbol{x} - \boldsymbol{b}^\top \log(\boldsymbol{x})$$

Analysis

As both quadratic function, $\frac{1}{2}\boldsymbol{x}^\top \Sigma \boldsymbol{x}$, and minus log function, $-\log(\boldsymbol{x})$, are convex function in $\mathbb{R}_+^n$. This RPP problem is convex.

Code

```
sigma = [1 0.0015 -0.02;
         0.0015 1 -0.1;
         -0.02 -0.1 1];
b = [ 0.1594, 0.0126, 0.8282]';
n = 3;

cvx_begin
    variable x(n)
    minimize(0.5 * x' * sigma * x - b'*log(x))
cvx_end

w = x/(ones(1,n)*x)
```

with answer: $x^* = [0.4085, 0.1671, 0.9226]^T$, $w = [0.2726, 0.1115, 0.6158]^T$

Problem2

$$\begin{aligned} \underset{w}{\text{minimize}} \quad & \boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w} - \lambda \boldsymbol{\mu}^\top \boldsymbol{w} \\ \text{subject to} \quad & \boldsymbol{w} \geq \mathbf{0}, \boldsymbol{w}^\top \mathbf{1} = 1 \end{aligned}$$

Analysis

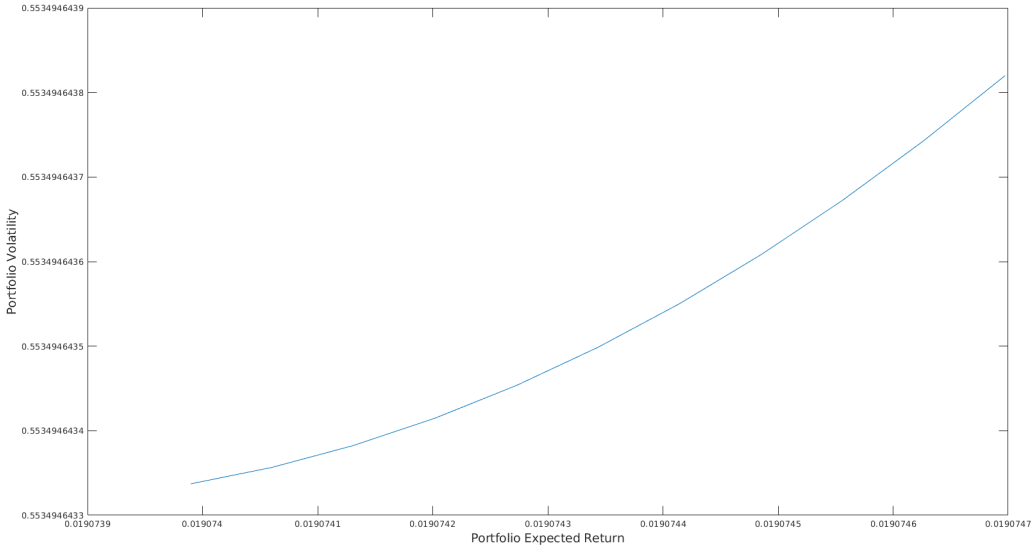
Asboth quadratic function, $\frac{1}{2}\boldsymbol{w}^\top \Sigma \boldsymbol{x}$, and afine function, $-\lambda \boldsymbol{\mu}^\top \boldsymbol{w}$, are convex function. And the constraint $\boldsymbol{w} \geq \mathbf{0}, \boldsymbol{w}^\top \mathbf{1} = 1$ are convex set. This is a convex problem.

Code

```
sigma = [1 0.0015 -0.02;
         0.0015 1 -0.1;
         -0.02 -0.1 1];
mu = [0.001,0.05,0.005]';
n = 3;

lambda  = zeros(11,1);
for i = 1:11
    lambda(i) = (i-1)*10^-4;
end
```

```
w_optimal = zeros(11,3);
for i = 1:11
    cvx_begin
        variable w(n)
        minimize(w'*sigma*w - lambda(i)*mu'*w)
        subject to
            w >= 0
            w'*ones(n,1) == 1
    cvx_end
    w_optimal(i,:) = w';
end
expected_return = w_optimal*mu;
volatility = zeros(11,1);
for i = 1:11
    volatility(i)=sqrt(w_optimal(i,:)*sigma*w_optimal(i,:)');
end
plot(expected_return,(volatility))
xlabel('Portfolio Expected Return', 16)
ylabel('Portfolio Volatility','Interpreter','latex','fontsize', 16)
```

Figure



Problem3

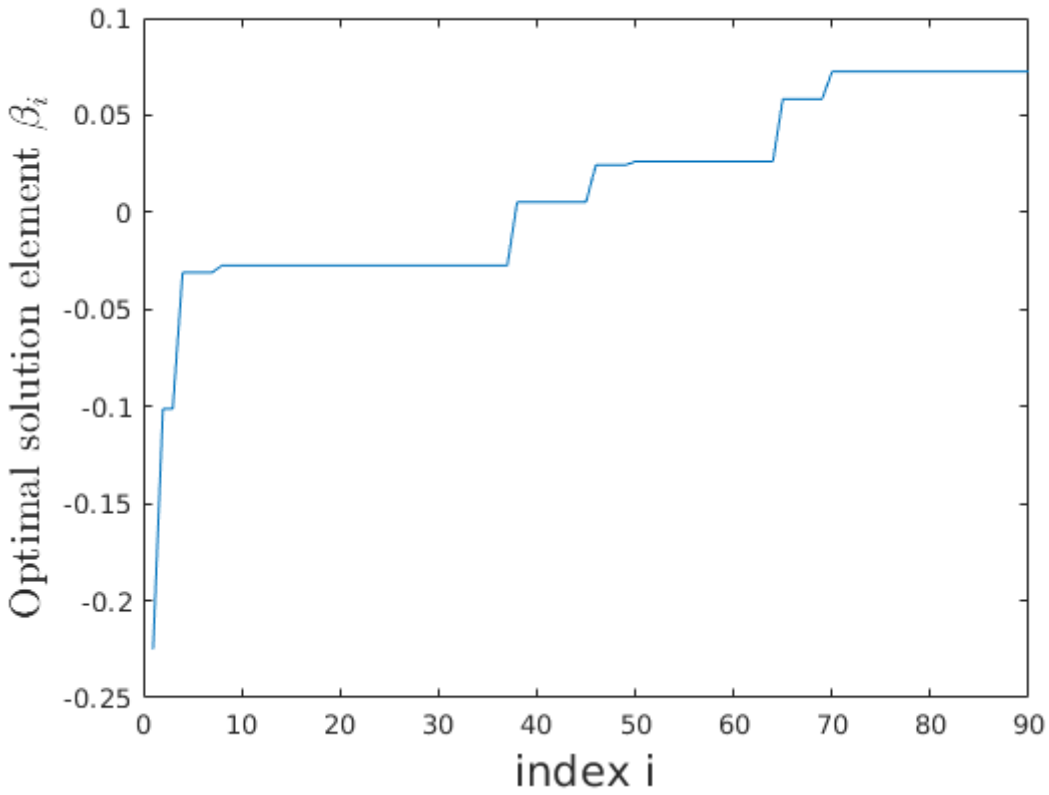
Code

```
n = randi([1,100],1);
m = randi([1,100],1);
X = randi(m,n);
y = randi(m,1);
cvx_begin
    variable x(n)
    minimize( norm( y-X*x, 2 )^2 )
    subject to
        for i = 1:n-1
            x(i) <= x(i+1)
        end
cvx_end
solution = cvx_optval;
```

```
plot(x)
xlabel("index i",'fontsize', 16)
ylabel('Optimal solution element ${\beta_{i}}$','Interpreter','latex','fontsize',
16)
```

For $n = 90, m = 39$, the optimal value is 3.3471e+04

Figure of optimal solution β^*



Problem 4

Code

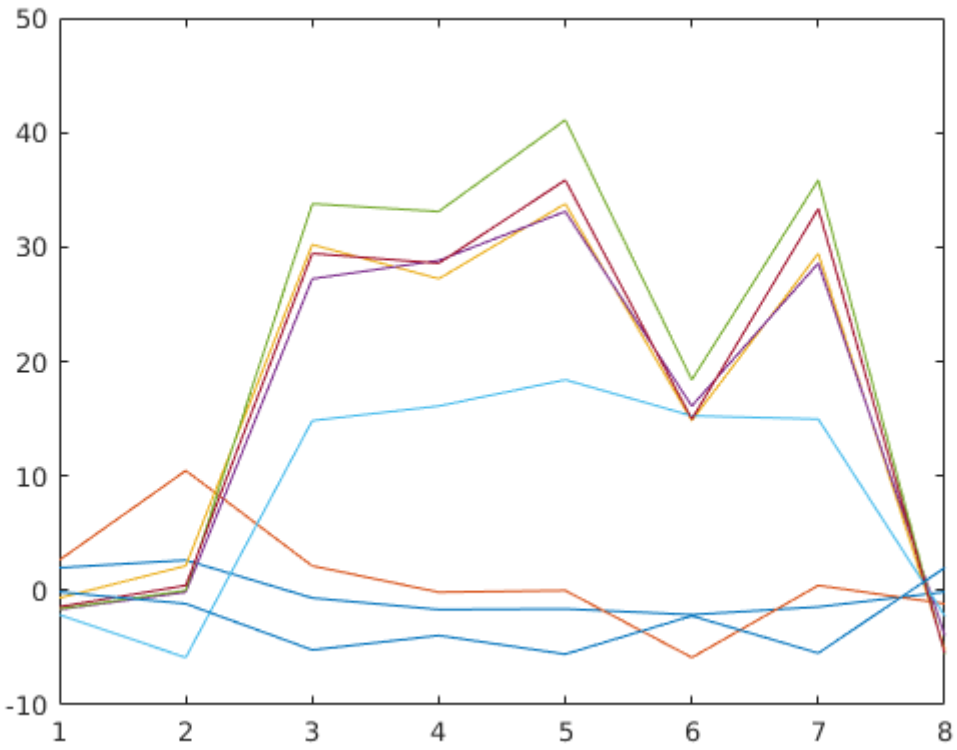
```
clear;clc;
% n = randi([1,100],1);
% p = randi([1,100],1);
n = 10;
p = 8;
X = zeros(n,p);
cMatrix = eye(n);

for i = 1:p
    noiseVec = randn(n,1);
    noiseVec = cMatrix*noiseVec;
    X(:,i) = noiseVec;
end
S = 1/n * (X')*X;

num = 100;
theta_1_norm = zeros(num,1);
for i = 1 : num
    cvx_begin
        variable theta(p,p) semidefinite
```

```
        minimize(trace(S*theta) - log_det(theta) + (i-
1)*ones(1,p)*abs(theta)*ones(p,1))
    cvx_end
    theta_1_norm(i) = ones(1,p)*abs(theta)*ones(p,1);
end

a = 0 : num-1;
plot(a,theta_1_norm);
xlabel('${\alpha}$','Interpreter','latex','fontsize', 16)
ylabel('Quantity $||\Theta^*(\alpha)||$','Interpreter','latex','fontsize', 16)
```

Figure on optimal solution

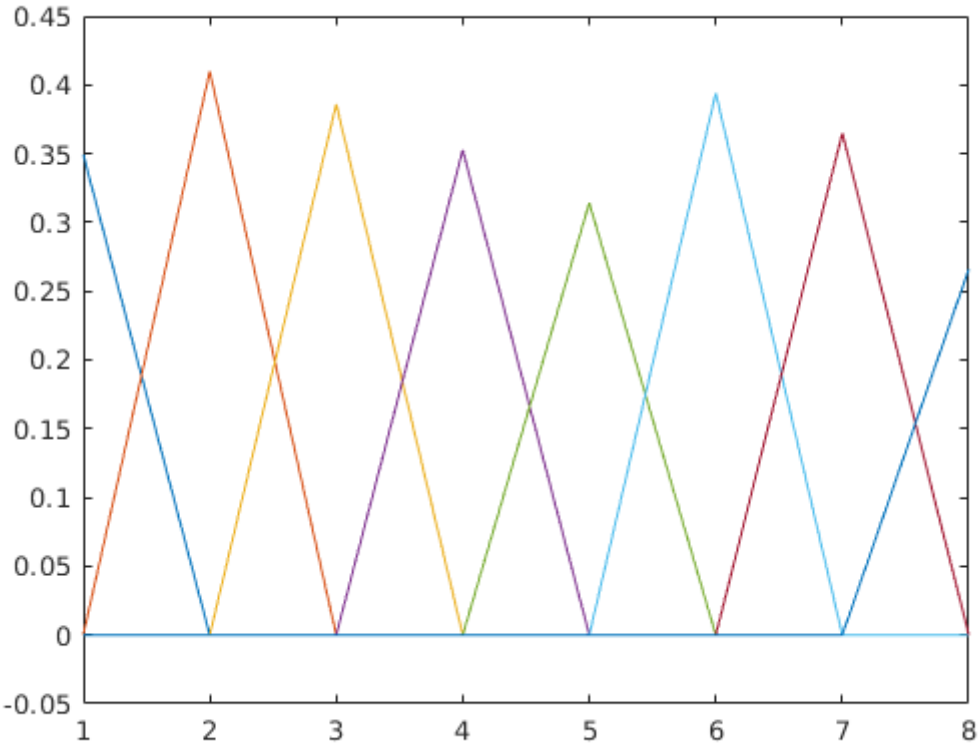
$\alpha = 0$

```
Theta(0) =

    1.9792    2.6422   -0.6617   -1.6734   -1.6157   -2.1220   -1.4403   -0.1402
    2.6422   10.4610    2.1443   -0.1575   -0.0112   -5.8660    0.4330   -1.1647
   -0.6617    2.1443   30.2081   27.2235   33.7761   14.8297   29.4359   -5.2033
   -1.6734   -0.1575   27.2235   28.8241   33.1012   16.1056   28.5817   -3.9418
   -1.6157   -0.0112   33.7761   33.1012   41.0997   18.3963   35.8448   -5.5727
   -2.1220   -5.8660   14.8297   16.1056   18.3963   15.2509   14.9636   -2.2237
   -1.4403    0.4330   29.4359   28.5817   35.8448   14.9636   33.3469   -5.4812
   -0.1402   -1.1647   -5.2033   -3.9418   -5.5727   -2.2237   -5.4812    1.9783
```



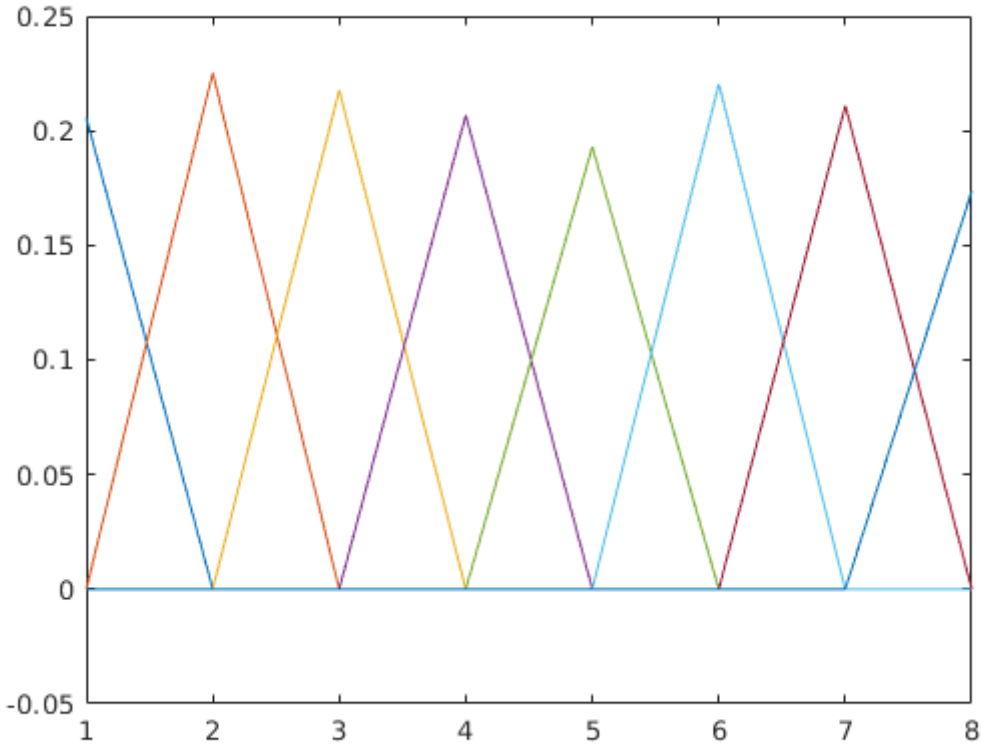
$\alpha = 2$

```
Theta(2) =

    0.3494    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000
    0.0000    0.4098    0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.0000    0.0000    0.3859   -0.0000    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000   -0.0000    0.3525    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.3142    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.0000    0.3941   -0.0000   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.3649   -0.0000
    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    0.2661
```



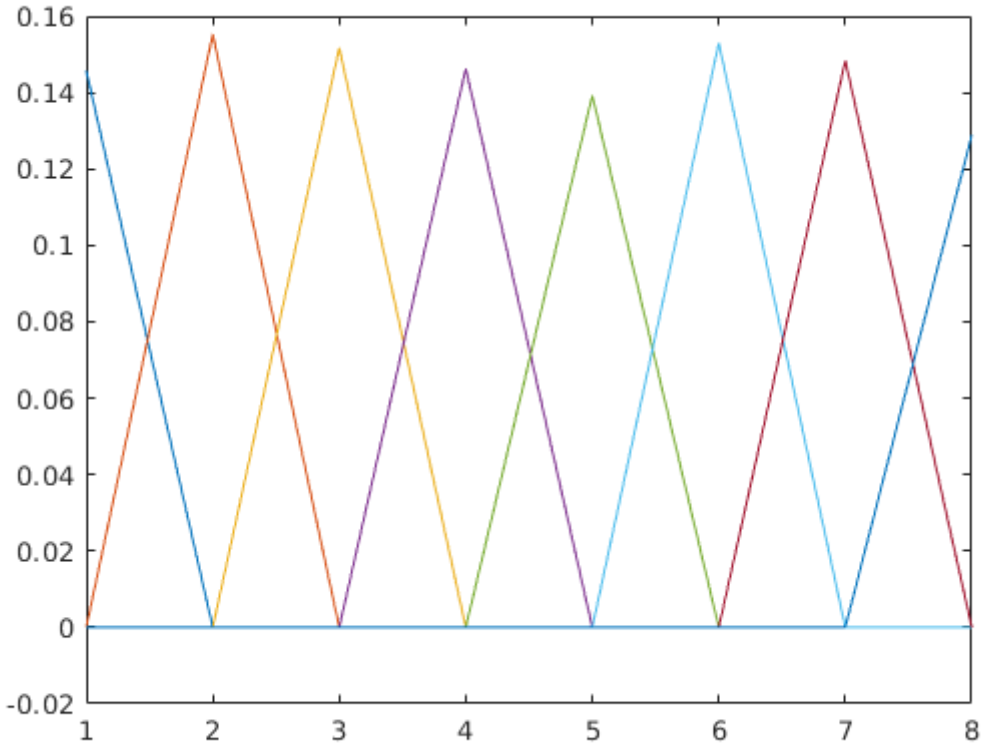
$\alpha = 4$

```
Theta(4) =

    0.2057    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000
    0.0000    0.2252    0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.0000    0.0000    0.2178   -0.0000    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000   -0.0000    0.2068    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.1929    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.0000    0.2204   -0.0000   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.2109   -0.0000
    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    0.1737
```



$\alpha = 6$

```
Theta(6) =

    0.1457    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000
    0.0000    0.1553    0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.0000    0.0000    0.1517   -0.0000    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000   -0.0000    0.1463    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.1392    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.0000    0.1530   -0.0000   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.1484   -0.0000
    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    0.1289
```



$\alpha = 8$

```
Theta(8) =

    0.1128    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000
    0.0000    0.1185    0.0000    0.0000   -0.0000   -0.0000   -0.0000   -0.0000
    0.0000    0.0000    0.1164   -0.0000    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000   -0.0000    0.1132    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.1089    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000    0.0000    0.1171   -0.0000   -0.0000
   -0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000    0.1144   -0.0000
    0.0000   -0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000    0.1025
```

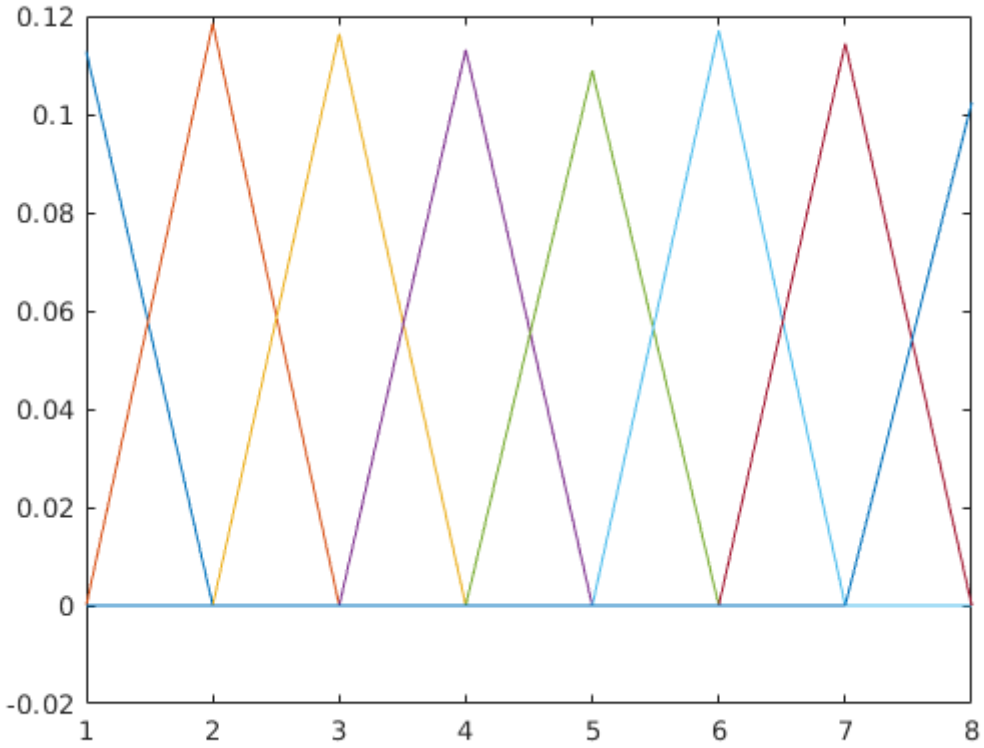


Figure on quanlity and α