

天体物理辐射机制笔记

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第一章 辐射场理论

1.1 有源场: 电磁势—求解辐射场的经典方法

令 $\partial^a A_a(t, \mathbf{r}) = 0$, 得

$$\partial^a \partial_a A_b(t, \mathbf{r}) = -4\pi J_b(t, \mathbf{r}). \quad (1.1)$$

有解

$$A^a(t, \mathbf{r}) = \frac{1}{c} \int \frac{J^a(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}, \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV'. \quad (1.2)$$

对单粒子, $J^a(t, \mathbf{r}) = qU^a(t)\delta(\mathbf{r} - \mathbf{r}_q(t))$, 所以

$$A^a(t, \mathbf{r}) = \frac{q}{c} \int \frac{U^a(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})\delta(\mathbf{r}' - \mathbf{r}_q(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}))}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (1.3)$$

$$= \frac{q}{c} \int \frac{\int U^a(t')\delta(\mathbf{r}' - \mathbf{r}_q(t'))\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})) dt'}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (1.4)$$

$$= \frac{q}{c} \iint \frac{U^a(t')\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}))}{|\mathbf{r} - \mathbf{r}'|} \delta(\mathbf{r}' - \mathbf{r}_q(t')) dV' dt' \quad (1.5)$$

$$= \frac{q}{c} \int \frac{U^a(t')\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c}))}{|\mathbf{r} - \mathbf{r}_q(t')|} dt', \quad (1.6)$$

上式是对 t' 进行的积分. 把积分变量变成 $\tilde{t}' = t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c})$. 注意到积分中 t 和 \mathbf{r} 被当成常量, 所以 $d\tilde{t}' = dt' + \frac{\dot{R}(t')}{c} dt' = (1 + \frac{\dot{R}(t')}{c}) dt'$, 其中 $R(t') = |\mathbf{r} - \mathbf{r}_q(t')|$, 所以 $\dot{R}(t') = -\frac{(\mathbf{r} - \mathbf{r}_q(t'))}{|\mathbf{r} - \mathbf{r}_q(t')|} \cdot \dot{\mathbf{r}}_q(t') = -\hat{\mathbf{n}}(t') \cdot \mathbf{u}_q(t')$.

定义 $K(t') = 1 + \frac{\dot{R}(t')}{c} = 1 - \frac{\hat{\mathbf{n}}(t') \cdot \mathbf{u}_q(t')}{c} = 1 - \frac{\mathbf{u}_r(t')}{c}$, 则 $d\tilde{t}' = K(t') dt'$, 所以

$$A^a(t, \mathbf{r}) = \frac{q}{c} \int \frac{U^a(t')}{K(t') |\mathbf{r} - \mathbf{r}_q(t')|} \delta(\tilde{t}') d\tilde{t}', \quad (1.7)$$

其中 t' 是 \tilde{t} 和 t, \mathbf{r} 的函数. 令 $\tilde{t}' = t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c}) = 0$, 可得 $R(t') = |\mathbf{r} - \mathbf{r}_q(t')| = c(t - t')$, 所以

$$A^a(t, \mathbf{r}) = \left[\frac{qU^a(t')}{cK(t')R(t')} \right]_{R(t')=c(t-t')}. \quad (1.8)$$

t 时刻 \mathbf{r} 处接收到 t' 时刻 $\mathbf{r}_q(t')$ 处产生的辐射, 恰有 $R(t') = c(t - t')$. 给定 t 和 \mathbf{r} , 满足 $R(t') = |\mathbf{r} - \mathbf{r}_q(t')| = c(t - t')$ 的 t' 存在且唯一, 否则世界线不类时.

$$\mathbf{E} = q \left[\frac{(\hat{\mathbf{n}}(t') - \boldsymbol{\beta}(t'))(1 - \boldsymbol{\beta}(t')^2)}{K(t')^3 R(t')^2} \right]_{R(t')=c(t-t')} \quad (1.9)$$

$$+ \frac{q}{c} \left[\frac{\hat{\mathbf{n}}(t') \times ((\hat{\mathbf{n}}(t') - \boldsymbol{\beta}(t')) \times \dot{\boldsymbol{\beta}}(t'))}{K(t')^3 R(t')} \right]_{R(t')=c(t-t')} \quad (1.10)$$

$$\mathbf{B} = [\hat{\mathbf{n}} \times \mathbf{E}]_{R(t')=c(t-t')} \quad (1.11)$$