

天体物理辐射机制笔记

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第一章 辐射场理论

1.1 有源场: 电磁势—求解辐射场的经典方法

令 $\partial^a A_a(t, \mathbf{r}) = 0$, 得

$$\partial^a \partial_a A_b(t, \mathbf{r}) = -4\pi J_b(t, \mathbf{r}). \quad (1.1)$$

有解

$$A^a(t, \mathbf{r}) = \frac{1}{c} \int \frac{J^a(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}, \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV'. \quad (1.2)$$

对单粒子, $J^a(t, \mathbf{r}) = qU^a(t)\delta(\mathbf{r} - \mathbf{r}_q(t))$, 所以

$$A^a(t, \mathbf{r}) = \frac{q}{c} \int \frac{U^a(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})\delta(\mathbf{r}' - \mathbf{r}_q(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}))}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (1.3)$$

$$= \frac{q}{c} \int \frac{\int U^a(t')\delta(\mathbf{r}' - \mathbf{r}_q(t'))\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})) dt'}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (1.4)$$

$$= \frac{q}{c} \iint \frac{U^a(t')\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}))}{|\mathbf{r} - \mathbf{r}'|} \delta(\mathbf{r}' - \mathbf{r}_q(t')) dV' dt' \quad (1.5)$$

$$= \frac{q}{c} \int \frac{U^a(t')\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c}))}{|\mathbf{r} - \mathbf{r}_q(t')|} dt', \quad (1.6)$$

上式是对 t' 进行的积分. 把积分变量变成 $\tilde{t}' = t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c})$. 注意到积分中 t 和 \mathbf{r} 被当成常量, 所以 $d\tilde{t}' = dt' + \frac{\dot{R}(t')}{c} dt' = (1 + \frac{\dot{R}(t')}{c}) dt'$, 其中 $R(t') = |\mathbf{r} - \mathbf{r}_q(t')|$, 所以 $\dot{R}(t') = -\frac{(\mathbf{r} - \mathbf{r}_q(t'))}{|\mathbf{r} - \mathbf{r}_q(t')|} \cdot \dot{\mathbf{r}}_q(t') = -\hat{\mathbf{n}}(t') \cdot \mathbf{u}_q(t')$.

定义 $K(t') = 1 + \frac{\dot{R}(t')}{c} = 1 - \frac{\hat{\mathbf{n}}(t') \cdot \mathbf{u}_q(t')}{c} = 1 - \frac{\mathbf{u}_r(t')}{c}$, 则 $d\tilde{t}' = K(t') dt'$, 所以

$$A^a(t, \mathbf{r}) = \frac{q}{c} \int \frac{U^a(t')}{K(t') |\mathbf{r} - \mathbf{r}_q(t')|} \delta(\tilde{t}') d\tilde{t}', \quad (1.7)$$

其中 t' 是 \tilde{t} 和 t, \mathbf{r} 的函数. 令 $\tilde{t}' = t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c}) = 0$, 可得 $R(t') = |\mathbf{r} - \mathbf{r}_q(t')| = c(t - t')$, 所以

$$A^a(t, \mathbf{r}) = \left[\frac{qU^a(t')}{cK(t')R(t')} \right]_{R(t')=c(t-t')}. \quad (1.8)$$

t 时刻 \mathbf{r} 处接收到 t' 时刻 $\mathbf{r}_q(t')$ 处产生的辐射, 恰有 $R(t') = c(t - t')$. 给定 t 和 \mathbf{r} , 满足 $R(t') = |\mathbf{r} - \mathbf{r}_q(t')| = c(t - t')$ 的 t' 存在且唯一, 否则世界线不类时.

$$\mathbf{E} = q \left[\frac{(\hat{\mathbf{n}}(t') - \boldsymbol{\beta}(t'))(1 - \beta(t')^2)}{K(t')^3 R(t')^2} \right]_{R(t')=c(t-t')} \quad (1.9)$$

$$+ \frac{q}{c} \left[\frac{\hat{\mathbf{n}}(t') \times ((\hat{\mathbf{n}}(t') - \boldsymbol{\beta}(t')) \times \dot{\boldsymbol{\beta}}(t'))}{K(t')^3 R(t')} \right]_{R(t')=c(t-t')}, \quad (1.10)$$

$$\mathbf{B} = [\hat{\mathbf{n}} \times \mathbf{E}]_{R(t')=c(t-t')}. \quad (1.11)$$

$$\frac{dP(t)}{d\Omega} = \frac{c}{4\pi} R(t')^2 |\mathbf{E}(t')|^2. \quad (1.12)$$

$dP(t)$ 为因 t' 时刻电荷的辐射, 观者在 t 时刻测得的单位时间通过 $R(t')^2 d\Omega$ 的辐射. 求电荷处单位时间 $d\Omega$ 方向的能量 $dP(t')$. 能量守恒 $dP(t')dt' = dP(t)dt$, $K(t') = \frac{dt}{dt'}$,

$$\frac{dP(t')}{d\Omega} = \frac{c}{4\pi} K(t') R(t')^2 |\mathbf{E}(t')|^2. \quad (1.13)$$

非相对论, $K \simeq 1$, $\hat{\mathbf{n}} - \boldsymbol{\beta} \simeq \hat{\mathbf{n}}$,

$$\mathbf{E} = \frac{q}{cR} [\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \dot{\boldsymbol{\beta}})], \quad (1.14)$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \dot{\boldsymbol{\beta}}^2 \sin^2 \Theta_{\boldsymbol{\beta}, \hat{\mathbf{n}}}. \quad (1.15)$$

1.2 辐射谱: 电磁场能量在不同频率的分布

$$E(t) = \int E(\omega) e^{-i\omega t} d\omega, \quad (1.16)$$

$$E(\omega) = \frac{1}{2\pi} \int E(t) e^{i\omega t} dt. \quad (1.17)$$

对 $\frac{dP(t)}{d\Omega}$ 作 Fourier 分析 (警告: 不是 $\frac{dP(t')}{d\Omega}$, 因为观者测得频谱). 假定 $R \simeq \text{const}$, 由 Parseval 定理,

$$\frac{dW}{d\Omega d\omega} = cR^2 |\mathbf{E}(\omega)|^2. \quad (1.18)$$

$$\mathbf{E}(\omega) = \frac{1}{2\pi} \int \mathbf{E}(t') e^{i\omega t} dt, \quad (1.19)$$

由 $t' = t - \frac{R(t')}{c}$, $\frac{dt}{dt'} = K(t')$,

$$\mathbf{E}(\omega) = \frac{1}{2\pi} \int K(t') \mathbf{E}(t') e^{i\omega \left[t' + \frac{R(t')}{c}\right]} dt', \quad (1.20)$$

$R(t') = |\mathbf{r} - \mathbf{r}_q(t')| \simeq |\mathbf{r}| - \frac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{r}_q(t') = |\mathbf{r}| - \hat{\mathbf{n}} \cdot \mathbf{r}_q(t')$, 得

$$|\mathbf{E}(\omega)|^2 = \left| \frac{1}{2\pi} \int K(t') \mathbf{E}(t') e^{i\omega \left[t' - \frac{\hat{\mathbf{n}} \cdot \mathbf{r}_q(t')}{c}\right]} dt' \right|^2. \quad (1.21)$$

非相对论,

$$\frac{dW}{d\Omega d\omega} = \frac{\omega^4 q^2 |\mathbf{r}_q| (\omega)^2 \sin^2 \Theta_{\hat{\mathbf{r}}_q, \hat{\mathbf{n}}}}{c^3}. \quad (1.22)$$

第二章 相对论性带电粒子辐射

$$\frac{dP(t')}{d\Omega} = \frac{c}{4\pi} K(t') R(t')^2 |\mathbf{E}(t')|^2, \quad (2.1)$$

$$\mathbf{E} = \frac{q}{c} \frac{\hat{\mathbf{n}}(t') \times ((\hat{\mathbf{n}}(t') - \boldsymbol{\beta}(t')) \times \dot{\boldsymbol{\beta}}(t'))}{K(t')^3 R(t')}, \quad (2.2)$$

$$K(t') = 1 - \hat{\mathbf{n}}(t') \cdot \boldsymbol{\beta}(t'). \quad (2.3)$$

若 $\hat{\mathbf{n}} // \boldsymbol{\beta}$, 则 $K \rightarrow 0$, $\frac{dP}{d\Omega} \rightarrow \infty$, 速度方向极大.

$K = 2K_{\min} \rightarrow \theta \simeq \frac{1}{\gamma} := \sqrt{1 - \boldsymbol{\beta}^2}$ ¹.

粒子静系中 $\theta = \frac{\pi}{2}$ 出射的光子², 观者看来 $\theta \simeq \frac{1}{\gamma}$.

$\boldsymbol{\beta} // \dot{\boldsymbol{\beta}}$,

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \frac{\dot{\boldsymbol{\beta}}^2 \sin^2 \theta}{(1 - |\boldsymbol{\beta}| \cos \theta)^5}, \quad (2.4)$$

$$\theta_{\max} = \arccos \frac{\sqrt{15|\boldsymbol{\beta}|^2 + 1} - 1}{3|\boldsymbol{\beta}|}$$
³.

$\boldsymbol{\beta} \perp \dot{\boldsymbol{\beta}}$,

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \dot{\boldsymbol{\beta}}^2 \left[\frac{\gamma^2 (1 - |\boldsymbol{\beta}| \cos \theta)^2 - \sin^2 \theta \cos^2 \phi}{\gamma^2 (1 - |\boldsymbol{\beta}| \cos \theta)^5} \right], \quad (2.5)$$

其中 ϕ 坐标系为: 以 $\dot{\boldsymbol{\beta}}$ 方向为 x 轴正向, 以 $\boldsymbol{\beta}$ 方向为 z 轴正向.

¹ 此处 θ 是观者系中粒子运动方向和观者方向的夹角.

² 此处 θ 是粒子静系中光子出射方向和粒子运动方向的夹角.

³ 我算的, 不知对不对.