

# 天体物理辐射机制笔记

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# 第一章 辐射场理论

## 1.1 有源场: 电磁势—求解辐射场的经典方法

令  $\partial^a A_a(t, \mathbf{r}) = 0$ , 得

$$\partial^a \partial_a A_b(t, \mathbf{r}) = -4\pi J_b(t, \mathbf{r}). \quad (1.1)$$

有解

$$A^a(t, \mathbf{r}) = \frac{1}{c} \int \frac{J^a(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}, \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV'. \quad (1.2)$$

对单粒子,  $J^a(t, \mathbf{r}) = qU^a(t)\delta(\mathbf{r} - \mathbf{r}_q(t))$ , 所以

$$A^a(t, \mathbf{r}) = \frac{q}{c} \int \frac{U^a(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})\delta(\mathbf{r}' - \mathbf{r}_q(t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}))}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (1.3)$$

$$= \frac{q}{c} \int \frac{\int U^a(t')\delta(\mathbf{r}' - \mathbf{r}_q(t'))\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}'|}{c})) dt'}{|\mathbf{r} - \mathbf{r}'|} dV' \quad (1.4)$$

$$= \frac{q}{c} \iint \frac{U^a(t')\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}'|}{c}))}{|\mathbf{r} - \mathbf{r}'|} \delta(\mathbf{r}' - \mathbf{r}_q(t')) dV' dt' \quad (1.5)$$

$$= \frac{q}{c} \int \frac{U^a(t')\delta(t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c}))}{|\mathbf{r} - \mathbf{r}_q(t')|} dt', \quad (1.6)$$

上式是对  $t'$  进行的积分. 把积分变量变成  $\tilde{t}' = t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c})$ . 注意到积分中  $t$  和  $\mathbf{r}$  被当成常量, 所以  $d\tilde{t}' = dt' + \frac{\dot{R}(t')}{c} dt' = (1 + \frac{\dot{R}(t')}{c}) dt'$ , 其中  $R(t') = |\mathbf{r} - \mathbf{r}_q(t')|$ , 所以  $\dot{R}(t') = -\frac{(\mathbf{r} - \mathbf{r}_q(t'))}{|\mathbf{r} - \mathbf{r}_q(t')|} \cdot \dot{\mathbf{r}}_q(t') = -\hat{\mathbf{n}}(t') \cdot \mathbf{u}_q(t')$ .

定义  $K(t') = 1 + \frac{\dot{R}(t')}{c} = 1 - \frac{\hat{\mathbf{n}}(t') \cdot \mathbf{u}_q(t')}{c} = 1 - \frac{\mathbf{u}_r(t')}{c}$ , 则  $d\tilde{t}' = K(t') dt'$ , 所以

$$A^a(t, \mathbf{r}) = \frac{q}{c} \int \frac{U^a(t')}{K(t') |\mathbf{r} - \mathbf{r}_q(t')|} \delta(\tilde{t}') d\tilde{t}', \quad (1.7)$$

其中  $t'$  是  $\tilde{t}'$  和  $t, \mathbf{r}$  的函数. 令  $\tilde{t}' = t' - (t - \frac{|\mathbf{r} - \mathbf{r}_q(t')|}{c}) = 0$ , 可得  $R(t') = |\mathbf{r} - \mathbf{r}_q(t')| = c(t - t')$ , 所以

$$A^a(t, \mathbf{r}) = \left[ \frac{qU^a(t')}{cK(t')R(t')} \right]_{R(t')=c(t-t')}. \quad (1.8)$$

$t$  时刻  $\mathbf{r}$  处接收到  $t'$  时刻  $\mathbf{r}_q(t')$  处产生的辐射, 恰有  $R(t') = c(t - t')$ . 给定  $t$  和  $\mathbf{r}$ , 满足  $R(t') = |\mathbf{r} - \mathbf{r}_q(t')| = c(t - t')$  的  $t'$  存在且唯一, 否则世界线不类时.

$$\mathbf{E} = q \left[ \frac{(\hat{\mathbf{n}}(t') - \boldsymbol{\beta}(t'))(1 - \boldsymbol{\beta}(t')^2)}{K(t')^3 R(t')^2} \right]_{R(t')=c(t-t')} \quad (1.9)$$

$$+ \frac{q}{c} \left[ \frac{\hat{\mathbf{n}}(t') \times ((\hat{\mathbf{n}}(t') - \boldsymbol{\beta}(t')) \times \dot{\boldsymbol{\beta}}(t'))}{K(t')^3 R(t')} \right]_{R(t')=c(t-t')}, \quad (1.10)$$

$$\mathbf{B} = [\hat{\mathbf{n}} \times \mathbf{E}]_{R(t')=c(t-t')}. \quad (1.11)$$

$$\frac{dP(t)}{d\Omega} = \frac{c}{4\pi} R(t')^2 |\mathbf{E}(t')|^2. \quad (1.12)$$

$dP(t)$  为因  $t'$  时刻电荷的辐射, 观者在  $t$  时刻测得的单位时间通过  $R(t')^2 d\Omega$  的辐射. 求电荷处单位时间  $d\Omega$  方向的能量  $dP(t')$ . 能量守恒  $dP(t')dt' = dP(t)dt$ ,  $K(t') = \frac{dt}{dt'}$ ,

$$\frac{dP(t')}{d\Omega} = \frac{c}{4\pi} K(t') R(t')^2 |\mathbf{E}(t')|^2. \quad (1.13)$$

非相对论,  $K \simeq 1$ ,  $\hat{\mathbf{n}} - \boldsymbol{\beta} \simeq \hat{\mathbf{n}}$ ,

$$\mathbf{E} = \frac{q}{cR} [\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \dot{\boldsymbol{\beta}})], \quad (1.14)$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \dot{\boldsymbol{\beta}}^2 \sin^2 \Theta_{\boldsymbol{\beta}, \hat{\mathbf{n}}}. \quad (1.15)$$