LISA

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1 Monochromatic Sources

$$\Gamma_{ij} = \frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}\left[\partial_i A_\alpha(t)\partial_j A_\alpha(t) + A_\alpha^2(t)\partial_i\chi_\alpha(t)\partial_j\chi_\alpha(t)\right]\,\mathrm{d}t.$$

$$A_\alpha(t) = \left[A_+^2F_{+,\alpha}^2(t) + A_\times^2F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$\chi_\alpha(t) = 2\pi f_0 t + \varphi_0 + \varphi_{\mathrm{p},\alpha}(t) + \varphi_{\mathrm{D}}(t).$$

$$\varphi_{\mathrm{p},\alpha}(t) = \arctan\left(-\frac{A_\times F_{\times,\alpha}(t)}{A_+F_{+,\alpha}(t)}\right).$$

$$\varphi_{\mathrm{D}}(t) = 2\pi f_0 c^{-1}R\sin\bar\theta_{\mathrm{S}}\cos\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right].$$

$$A_+ = 2\left\{1 + \left[\cos\bar\theta_L\cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos(\bar\phi_L - \bar\phi_{\mathrm{S}})\right]^2\right\}.$$

$$A_\times = -4e^{\ln\mathcal{A}}\left[\cos\bar\theta_L\cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos(\bar\phi_L - \bar\phi_{\mathrm{S}})\right].$$

$$F_{+,\mathrm{I}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\cos 2\phi_{\mathrm{S}}\cos 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}}\sin 2\phi_{\mathrm{S}}\sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{I}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\cos 2\phi_{\mathrm{S}}\sin 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}}\sin 2\phi_{\mathrm{S}}\cos 2\psi_{\mathrm{S}}.$$

$$F_{+,\mathrm{II}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\sin 2\phi_{\mathrm{S}}\cos 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}}\cos 2\phi_{\mathrm{S}}\sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{II}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\sin 2\phi_{\mathrm{S}}\sin 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}}\cos 2\phi_{\mathrm{S}}\cos 2\psi_{\mathrm{S}}.$$

$$\cos\theta_{\mathrm{S}} = \frac{1}{2}\cos\bar\theta_{\mathrm{S}} - \frac{\sqrt{3}}{2}\sin\bar\theta_{\mathrm{S}}\cos\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right].$$

$$\phi_{\mathrm{S}} = \alpha_0 + 2\pi t/T + \arctan\left\{\frac{\sqrt{3}\cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_{\mathrm{S}}\cos\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right]}{2\sin\bar\theta_{\mathrm{S}}\sin\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right]}\right\}.$$

$$\psi_{\mathrm{S}} = \arctan\left\{\frac{\frac{1}{2}\cos\bar\theta_L - \frac{\sqrt{3}}{2}\sin\bar\theta_L\cos\left[\bar\phi(t) - \bar\phi_L\right] - \left[\cos\bar\theta_L\cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos(\bar\phi_L - \bar\phi_{\mathrm{S}})\right]\cos\theta_{\mathrm{S}}}{\frac{1}{2}\sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\sin(\bar\phi_L - \bar\phi_{\mathrm{S}}) - \frac{\sqrt{3}}{2}(\cos\bar\theta_L\sin\bar\theta_{\mathrm{S}}\sin\bar\phi_{\mathrm{S}} - \cos\bar\theta_{\mathrm{S}}\sin\bar\theta_L\sin\bar\phi_L)\cos\bar\phi(t) - \frac{\sqrt{3}}{2}(\cos\bar\theta_{\mathrm{S}}\sin\bar\theta_L\cos\bar\phi_L - \cos\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos\bar\phi_S)\sin\bar\phi(t)}\right\}$$

$$\bar\phi(t) = \bar\phi_0 + 2\pi t/T.$$

$$\alpha_0 = 0, \quad \bar\phi_0 = 0.$$

$$\Gamma_{1j} = \frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}A_\alpha(t)\partial_j A_\alpha(t)\,\mathrm{d}t.$$

$$\Gamma_{2j} = \frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}A_\alpha^2(t)\partial_j\chi_\alpha(t)\,\mathrm{d}t.$$

$$\Gamma_{3j} = \frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}A_\alpha^2(t)2\pi t\partial_j\chi_\alpha(t)\,\mathrm{d}t.$$

$$\ln\mathcal{A} := 0, \quad \varphi_0 := 0.$$

$$\frac{\Gamma_{ij}}{\Gamma_{ij,\ln\mathcal{A}=0}} = \frac{(S/N)^2}{(S/N)^2_{\ln\mathcal{A}=0}} = \frac{10^2}{(S/N)^2_{\ln\mathcal{A}=0}}.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}e^{2\pi ift_1}h_\alpha(t)\,\mathrm{d}t_1\right]\left[\int_{-\infty}^{\infty}e^{-2\pi ift_2}h_\alpha(t)\,\mathrm{d}t_2\right]\,\mathrm{d}f.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}\tilde h_\alpha^*(f)\tilde h_\alpha(f)\,\mathrm{d}f.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}h_\alpha^2(t)\,\mathrm{d}t.$$

2 SMBH Mergers

$$\Gamma_{ij} = 4 \sum_{\alpha=\mathrm{I,II}} \int_0^{f_{\max}} \frac{\partial_i \tilde{h}_\alpha^*(f) \partial_j \tilde{h}_\alpha(f)}{S_{\mathrm{n}}(f)} \, \mathrm{d}f.$$

$$\tilde{h}_\alpha(f) = \frac{\sqrt{3}}{2} \Lambda_\alpha(t) \mathcal{A} f^{-7/6} e^{i[\Psi(f) - \varphi_{\mathrm{p},\alpha}(t) + \varphi_{\mathrm{D}}(t)]} \tilde{H}(f).$$

$$\Lambda_\alpha(t) = A_\alpha(t)/A(t).$$

$$A_\alpha(t) = \left[A_+^2 F_{+,\alpha}^2(t) + A_\times^2 F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$A(t) = 2\frac{M_1 M_2 (1+z)}{r D_{\mathrm{L}}}.$$

$$\mathcal{A} = (5/96)^{1/2} \pi^{-2/3} [\mathcal{M}(1+z)]^{5/6} D_{\mathrm{L}}^{-1}.$$

$$\Psi(f) = 2\pi f t_{\mathrm{c}} - \phi_{\mathrm{c}} - \frac{\pi}{4} + \frac{3}{4}(8\pi f)^{-8/3}[\mathcal{M}(1+z)]^{-8/3} \left\{1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right)[\pi M(1+z) f]^{2/3} - 4\,(4\pi - \beta)\,[\pi M(1+z) f]\right\}.$$

$$\varphi_{\mathrm{p},\alpha}(t) = \arctan\left(-\frac{A_\times F_{\times,\alpha}(t)}{A_+ F_{+,\alpha}(t)}\right).$$

$$\varphi_{\mathrm{D}}(t) = 2\pi f c^{-1} R \sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right].$$

$$A_+ = 2\frac{M_1 M_2}{r D}\left\{1 + \left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right]^2\right\}.$$

$$A_\times = -4\frac{M_1 M_2}{r D}\left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right].$$

$$F_{+,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$F_{+,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$\cos\theta_{\mathrm{S}} = \frac{1}{2}\cos\bar{\theta}_{\mathrm{S}} - \frac{\sqrt{3}}{2}\sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right].$$

$$\phi_{\mathrm{S}} = \alpha_0 + 2\pi t/T + \arctan\left\{\frac{\sqrt{3}\cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right]}{2\sin\bar{\theta}_{\mathrm{S}} \sin\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right]}\right\}.$$

$$\psi_{\mathrm{S}} = \arctan\left\{\frac{\frac{1}{2}\cos\bar{\theta}_L - \frac{\sqrt{3}}{2}\sin\bar{\theta}_L \cos\left[\bar{\phi}(t) - \bar{\phi}_L\right] - \left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right]\cos\theta_{\mathrm{S}}}{\frac{1}{2}\sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \sin(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}}) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \sin\bar{\phi}_{\mathrm{S}} - \cos\bar{\theta}_{\mathrm{S}} \sin\bar{\theta}_L \sin\bar{\phi}_L)\cos\bar{\phi}(t) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_{\mathrm{S}} \sin\bar{\theta}_L \cos\bar{\phi}_L - \cos\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos\bar{\phi}_S)\sin\bar{\phi}(t)}\right\}$$

$$\bar{\phi}(t) = \bar{\phi}_0 + 2\pi t/T.$$

$$t(f) = t_{\mathrm{c}} - 5(8\pi f)^{-8/3}[\mathcal{M}(1+z)]^{-5/3}\left\{1 + \frac{4}{3}\left(\frac{743}{336} + \frac{11}{4}\eta\right)[\pi M(1+z) f]^{2/3} - \frac{8}{5}\,(4\pi - \beta)\,[\pi M(1+z) f]\right\}.$$

$$\alpha_0 = 0, \quad \bar{\phi}_0 = 0.$$

$$(S/N)^2 = 4 \sum_{\alpha=\mathrm{I,II}} \int_0^{f_{\max}} \frac{\tilde{h}_\alpha^*(f) \tilde{h}_\alpha(f)}{S_{\mathrm{n}}(f)} \, \mathrm{d}f.$$