LISA

GasinAn

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1 Monochromatic Sources

$$\Gamma_{ij}=\frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}\left[\partial_i A_\alpha(t)\partial_j A_\alpha(t)+A_\alpha^2(t)\partial_i\chi_\alpha(t)\partial_j\chi_\alpha(t)\right]\,\mathrm{d}t.$$

$$A_\alpha(t)=\left[A_+^2F_{+,\alpha}^2(t)+A_\times^2F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$\chi_\alpha(t)=2\pi f_0t+\varphi_0+\varphi_{\mathrm{p},\alpha}(t)+\varphi_{\mathrm{D}}(t).$$

$$\varphi_{\mathrm{p},\alpha}(t)=\arctan\left(-\frac{A_\times F_{\times,\alpha}(t)}{A_+F_{+,\alpha}(t)}\right).$$

$$\varphi_{\mathrm{D}}(t)=2\pi f_0c^{-1}R\sin\bar\theta_{\mathrm{S}}\cos\left[\bar\phi(t)-\bar\phi_{\mathrm{S}}\right].$$

$$A_+=2\left\{1+\left[\cos\bar\theta_L\cos\bar\theta_{\mathrm{S}}+\sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos(\bar\phi_L-\bar\phi_{\mathrm{S}})\right]^2\right\}.$$

$$A_\times=-4e^{\ln\mathcal{A}}\left[\cos\bar\theta_L\cos\bar\theta_{\mathrm{S}}+\sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos(\bar\phi_L-\bar\phi_{\mathrm{S}})\right].$$

$$F_{+,\mathrm{I}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}})=\frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\cos2\phi_{\mathrm{S}}\cos2\psi_{\mathrm{S}}-\cos\theta_{\mathrm{S}}\sin2\phi_{\mathrm{S}}\sin2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{I}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}})=\frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\cos2\phi_{\mathrm{S}}\sin2\psi_{\mathrm{S}}+\cos\theta_{\mathrm{S}}\sin2\phi_{\mathrm{S}}\cos2\psi_{\mathrm{S}}.$$

$$F_{+,\mathrm{II}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}})=\frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\sin2\phi_{\mathrm{S}}\cos2\psi_{\mathrm{S}}+\cos\theta_{\mathrm{S}}\cos2\phi_{\mathrm{S}}\sin2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{II}}(\theta_{\mathrm{S}},\phi_{\mathrm{S}},\psi_{\mathrm{S}})=\frac{1}{2}(1+\cos^2\theta_{\mathrm{S}})\sin2\phi_{\mathrm{S}}\sin2\psi_{\mathrm{S}}-\cos\theta_{\mathrm{S}}\cos2\phi_{\mathrm{S}}\cos2\psi_{\mathrm{S}}.$$

$$\cos\theta_{\mathrm{S}}=\frac{1}{2}\cos\bar\theta_{\mathrm{S}}-\frac{\sqrt3}{2}\sin\bar\theta_{\mathrm{S}}\cos\left[\bar\phi(t)-\bar\phi_{\mathrm{S}}\right].$$

$$\phi_{\mathrm{S}}=\alpha_0+2\pi t/T+\arctan\left\{\frac{\sqrt3\cos\bar\theta_{\mathrm{S}}+\sin\bar\theta_{\mathrm{S}}\cos\left[\bar\phi(t)-\bar\phi_{\mathrm{S}}\right]}{2\sin\bar\theta_{\mathrm{S}}\sin\left[\bar\phi(t)-\bar\phi_{\mathrm{S}}\right]}\right\}.$$

$$\psi_{\mathrm{S}}=\arctan\left\{\frac{\frac12\cos\bar\theta_L-\frac{\sqrt3}{2}\sin\bar\theta_L\cos\left[\bar\phi(t)-\bar\phi_L\right]-\left[\cos\bar\theta_L\cos\bar\theta_{\mathrm{S}}+\sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos(\bar\phi_L-\bar\phi_{\mathrm{S}})\right]\cos\theta_{\mathrm{S}}}{\frac12\sin\bar\theta_L\sin\bar\theta_{\mathrm{S}}\sin(\bar\phi_L-\bar\phi_{\mathrm{S}})-\frac{\sqrt3}{2}(\cos\bar\theta_L\sin\bar\theta_{\mathrm{S}}\sin\bar\phi_{\mathrm{S}}-\cos\bar\theta_{\mathrm{S}}\sin\bar\theta_L\sin\bar\phi_L)\cos\bar\phi(t)-\frac{\sqrt3}{2}(\cos\bar\theta_{\mathrm{S}}\sin\bar\theta_L\cos\bar\phi_L-\cos\bar\theta_L\sin\bar\theta_{\mathrm{S}}\cos\bar\phi_S)\sin\bar\phi(t)}\right\}.$$

$$\bar\phi(t)=\bar\phi_0+2\pi t/T.$$

$$\alpha_0=0,\quad\bar\phi_0=0.$$

$$\Gamma_{1j}=\frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}A_\alpha(t)\partial_jA_\alpha(t)\,\mathrm{d}t.$$

$$\Gamma_{2j}=\frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}A_\alpha^2(t)\partial_j\chi_\alpha(t)\,\mathrm{d}t.$$

$$\Gamma_{3j}=\frac{3}{4}S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}A_\alpha^2(t)2\pi t\partial_j\chi_\alpha(t)\,\mathrm{d}t.$$

$$\ln\mathcal{A}:=0,\quad\varphi_0:=0.$$

$$\frac{\Gamma_{ij}}{\Gamma_{ij,\ln\mathcal{A}=0}}=\frac{(S/N)^2}{(S/N)^2_{\ln\mathcal{A}=0}}=\frac{10^2}{(S/N)^2_{\ln\mathcal{A}=0}}.$$

$$(S/N)^2=2S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}e^{2\pi ift_1}h_\alpha(t)\,\mathrm{d}t_1\right]\left[\int_{-\infty}^{\infty}e^{-2\pi ift_2}h_\alpha(t)\,\mathrm{d}t_2\right]\,\mathrm{d}f.$$

$$(S/N)^2=2S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}\tilde h_\alpha^*(f)\tilde h_\alpha(f)\,\mathrm{d}f.$$

$$(S/N)^2=2S_{\mathrm{n}}(f_0)^{-1}\sum_{\alpha=\mathrm{I,II}}\int_{-\infty}^{\infty}h_\alpha^2(t)\,\mathrm{d}t.$$

2 SMBH Mergers

$$\Gamma_{ij} = 4 \sum_{\alpha=\mathrm{I,II}} \int_0^{f_{\max}} \frac{\partial_i \tilde{h}_\alpha^*(f) \partial_j \tilde{h}_\alpha(f)}{S_{\mathrm{n}}(f)} \mathrm{d}f.$$

$$f_{\max} = [3^{3/2}\pi \frac{GM}{c^3}(1+z)]^{-1}$$

$$\tilde{h}_\alpha(f) = \frac{\sqrt{3}}{2} \Lambda_\alpha(t) \mathcal{A} f^{-7/6} e^{i[\Psi(f) - \varphi_{\mathrm{p},\alpha}(t) + \varphi_{\mathrm{D}}(t)]}.$$

$$\Lambda_\alpha(t) = \left[\Lambda_+^2 F_{+,\alpha}^2(t) + \Lambda_\times^2 F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$\mathcal{A} = (5/96)^{1/2} \pi^{-2/3} [\frac{G\mathcal{M}}{c^3}(1+z)]^{5/6} (e^{\ln \frac{D_{\mathrm{L}}}{c}})^{-1}.$$

$$\Psi(f) = 2\pi f t_{\mathrm{c}} - \phi_{\mathrm{c}} - \frac{\pi}{4} + \frac{3}{4}(8\pi f)^{-5/3} [\frac{G\mathcal{M}}{c^3}(1+z)]^{-5/3} \left\{1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right)\left[\pi \frac{GM}{c^3}(1+z) f\right]^{2/3} - 4(4\pi - \beta)\left[\pi \frac{GM}{c^3}(1+z) f\right]\right\}.$$

$$\varphi_{\mathrm{p},\alpha}(t) = \arctan\left(-\frac{\Lambda_\times F_{\times,\alpha}(t)}{\Lambda_+ F_{+,\alpha}(t)}\right).$$

$$\varphi_{\mathrm{D}}(t) = 2\pi f \frac{R}{c} \sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right].$$

$$\Lambda_+ = 1 + \left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right]^2.$$

$$\Lambda_\times = -2\left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right].$$

$$F_{+,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$F_{+,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$\cos\theta_{\mathrm{S}} = \frac{1}{2}\cos\bar{\theta}_{\mathrm{S}} - \frac{\sqrt{3}}{2}\sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right].$$

$$\phi_{\mathrm{S}} = \alpha_0 + 2\pi t/T + \arctan\left\{\frac{\sqrt{3}\cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right]}{2\sin\bar{\theta}_{\mathrm{S}} \sin\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right]}\right\}.$$

$$\psi_{\mathrm{S}} = \arctan\left\{\frac{\frac{1}{2}\cos\bar{\theta}_L - \frac{\sqrt{3}}{2}\sin\bar{\theta}_L \cos\left[\bar{\phi}(t) - \bar{\phi}_L\right] - \left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right] \cos\theta_{\mathrm{S}}}{\frac{1}{2}\sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \sin(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}}) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \sin\bar{\phi}_{\mathrm{S}} - \cos\bar{\theta}_{\mathrm{S}} \sin\bar{\theta}_L \sin\bar{\phi}_L)\cos\bar{\phi}(t) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_{\mathrm{S}} \sin\bar{\theta}_L \cos\bar{\phi}_L - \cos\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos\bar{\phi}_S)\sin\bar{\phi}(t)}\right\}.$$

$$\bar{\phi}(t) = \bar{\phi}_0 + 2\pi t/T.$$

$$t(f) = t_{\mathrm{c}} - 5(8\pi f)^{-8/3} [\frac{G\mathcal{M}}{c^3}(1+z)]^{-5/3} \left\{1 + \frac{4}{3}\left(\frac{743}{336} + \frac{11}{4}\eta\right)\left[\pi \frac{GM}{c^3}(1+z) f\right]^{2/3} - \frac{8}{5}(4\pi - \beta)\left[\pi \frac{GM}{c^3}(1+z) f\right]\right\}.$$

$$\alpha_0 = 0, \quad \bar{\phi}_0 = 0, \quad z = 1.$$

$$\beta = 0, \quad \ln\frac{D_{\mathrm{L}}}{c} = \ln H_0^{-1} z.$$

$$\phi_{\mathrm{c}} := 0, \quad t_{\mathrm{c}} := 0.$$

$$(S/N)^2 = 4 \sum_{\alpha=\mathrm{I,II}} \int_0^{f_{\max}} \frac{\tilde{h}_\alpha^*(f) \tilde{h}_\alpha(f)}{S_{\mathrm{n}}(f)} \mathrm{d}f.$$

$$\partial_1 h_\alpha(f) = -\frac{5}{4} i (8\pi f)^{-5/3} [\frac{G\mathcal{M}}{c^3}(1+z)]^{-5/3} \left\{1 + \frac{55}{6}\eta\left[\pi \frac{GM}{c^3}(1+z) f\right]^{2/3} + (8\pi - 2\beta)\left[\pi \frac{GM}{c^3}(1+z) f\right]\right\} h_\alpha(f).$$

$$\partial_2 h_\alpha(f) = \frac{3}{4} i (8\pi f)^{-5/3} [\frac{G\mathcal{M}}{c^3}(1+z)]^{-5/3} \left\{\left(-\frac{3715}{756} + \frac{55}{6}\eta\right)\left[\pi \frac{GM}{c^3}(1+z) f\right]^{2/3} + (24\pi - 6\beta)\left[\pi \frac{GM}{c^3}(1+z) f\right]\right\} h_\alpha(f).$$

$$\partial_3 h_\alpha(f) = 3i (8\pi f)^{-5/3} [\frac{G\mathcal{M}}{c^3}(1+z)]^{-5/3} \left[\pi \frac{GM}{c^3}(1+z) f\right] h_\alpha(f).$$

$$\partial_4 h_\alpha(f) = -i h_\alpha(f).$$

$$\partial_5 h_\alpha(f) = 2\pi i f h_\alpha(f).$$

$$\partial_6 h_\alpha(f) = -h_\alpha(f).$$