LISA

GasinAn

2022 年 1 月 19 日

1 Monochromatic Sources

$$\Gamma_{ij} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} \left[\partial_i A_\alpha(t) \partial_j A_\alpha(t) + A_\alpha^2(t) \partial_i \chi_\alpha(t) \partial_j \chi_\alpha(t)\right] \mathrm{d}t.$$

$$A_\alpha(t) = \left[A_+^2 F_{+,\alpha}^2(t) + A_\times^2 F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$\chi_\alpha(t) = 2\pi f_0 t + \varphi_0 + \varphi_{\mathrm{p},\alpha}(t) + \varphi_{\mathrm{D}}(t).$$

$$\varphi_{\mathrm{p},\alpha}(t) = \arctan\left(-\frac{A_\times F_{\times,\alpha}(t)}{A_+ F_{+,\alpha}(t)}\right).$$

$$\varphi_{\mathrm{D}}(t) = 2\pi f_0 c^{-1} R \sin\bar\theta_{\mathrm{S}} \cos\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right].$$

$$A_+ = 2\left\{1 + \left[\cos\bar\theta_L \cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_L \sin\bar\theta_{\mathrm{S}} \cos(\bar\phi_L - \bar\phi_{\mathrm{S}})\right]^2\right\}.$$

$$A_\times = -4e^{\ln\mathcal{A}}\left[\cos\bar\theta_L \cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_L \sin\bar\theta_{\mathrm{S}} \cos(\bar\phi_L - \bar\phi_{\mathrm{S}})\right].$$

$$F_{+,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$F_{+,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1 + \cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$\cos\theta_{\mathrm{S}} = \frac{1}{2}\cos\bar\theta_{\mathrm{S}} - \frac{\sqrt{3}}{2}\sin\bar\theta_{\mathrm{S}} \cos\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right].$$

$$\phi_{\mathrm{S}} = \alpha_0 + 2\pi t/T + \arctan\left\{\frac{\sqrt{3}\cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_{\mathrm{S}} \cos\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right]}{2\sin\bar\theta_{\mathrm{S}} \sin\left[\bar\phi(t) - \bar\phi_{\mathrm{S}}\right]}\right\}.$$

$$\psi_{\mathrm{S}} = \arctan\left\{\frac{\frac{1}{2}\cos\bar\theta_L - \frac{\sqrt{3}}{2}\sin\bar\theta_L \cos\left[\bar\phi(t) - \bar\phi_L\right] - \left[\cos\bar\theta_L \cos\bar\theta_{\mathrm{S}} + \sin\bar\theta_L \sin\bar\theta_{\mathrm{S}} \cos(\bar\phi_L - \bar\phi_{\mathrm{S}})\right]\cos\theta_{\mathrm{S}}}{\frac{1}{2}\sin\bar\theta_L \sin\bar\theta_{\mathrm{S}} \sin(\bar\phi_L - \bar\phi_{\mathrm{S}}) - \frac{\sqrt{3}}{2}(\cos\bar\theta_L \sin\bar\theta_{\mathrm{S}} \sin\bar\phi_{\mathrm{S}} - \cos\bar\theta_{\mathrm{S}} \sin\bar\theta_L \sin\bar\phi_L)\cos\bar\phi(t) - \frac{\sqrt{3}}{2}(\cos\bar\theta_{\mathrm{S}} \sin\bar\theta_L \cos\bar\phi_L - \cos\bar\theta_L \sin\bar\theta_{\mathrm{S}} \cos\bar\phi_S)\sin\bar\phi(t)}\right\}.$$

$$\bar\phi(t) = \bar\phi_0 + 2\pi t/T.$$

$$\alpha_0 = 0, \quad \bar\phi_0 = 0.$$

$$\Gamma_{1j} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} A_\alpha(t) \partial_j A_\alpha(t) \,\mathrm{d}t.$$

$$\Gamma_{2j} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) \partial_j \chi_\alpha(t) \,\mathrm{d}t.$$

$$\Gamma_{3j} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) 2\pi t \partial_j \chi_\alpha(t) \,\mathrm{d}t.$$

$$\ln\mathcal{A} := 0, \quad \varphi_0 := 0.$$

$$\frac{\Gamma_{ij}}{\Gamma_{ij,\ln\mathcal{A}=0}} = \frac{(S/N)^2}{(S/N)^2_{\ln\mathcal{A}=0}} = \frac{10^2}{(S/N)^2_{\ln\mathcal{A}=0}}.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} e^{2\pi i f t_1} h_\alpha(t) \,\mathrm{d}t_1\right] \left[\int_{-\infty}^{\infty} e^{-2\pi i f t_2} h_\alpha(t) \,\mathrm{d}t_2\right] \mathrm{d}f.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} \tilde h_\alpha^*(f) \tilde h_\alpha(f) \,\mathrm{d}f.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} h_\alpha^2(t) \,\mathrm{d}t.$$

2 SMBH Mergers

$$\Gamma_{ij} = 4 \sum_{\alpha=\mathrm{I,II}} \int_0^{f_{\max}} \frac{\partial_i \tilde{h}_\alpha^*(f) \partial_j \tilde{h}_\alpha(f)}{S_\mathrm{n}(f)} \,\mathrm{d}f.$$

$$\tilde{h}_\alpha(f) = \frac{\sqrt{3}}{2} \Lambda_\alpha(t) \mathcal{A} f^{-7/6} e^{i[\Psi(f) - \varphi_{\mathrm{p},\alpha}(t) + \varphi_\mathrm{D}(t)]}.$$

$$\Lambda_\alpha(t) = \left[\Lambda_+^2 F_{+,\alpha}^2(t) + \Lambda_\times^2 F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$\mathcal{A} = (5/96)^{1/2} \pi^{-2/3} [\mathcal{M}(1+z)]^{5/6} (e^{\ln D_\mathrm{L}})^{-1}.$$

$$\Psi(f) = 2\pi f t_\mathrm{c} - \phi_\mathrm{c} - \frac{\pi}{4} + \frac{3}{4}(8\pi f)^{-8/3}[\mathcal{M}(1+z)]^{-8/3} \left\{1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right)[\pi M(1+z)f]^{2/3} - 4\,(4\pi - \beta)\,[\pi M(1+z)f]\right\}.$$

$$\varphi_{\mathrm{p},\alpha}(t) = \arctan\left(-\frac{\Lambda_\times F_{\times,\alpha}(t)}{\Lambda_+ F_{+,\alpha}(t)}\right).$$

$$\varphi_\mathrm{D}(t) = 2\pi f c^{-1} R \sin\bar{\theta}_\mathrm{S} \cos\left[\bar{\phi}(t) - \bar{\phi}_\mathrm{S}\right].$$

$$\Lambda_+ = 1 + \left[\cos\bar{\theta}_L \cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_L \sin\bar{\theta}_\mathrm{S} \cos(\bar{\phi}_L - \bar{\phi}_\mathrm{S})\right]^2.$$

$$\Lambda_\times = -2\left[\cos\bar{\theta}_L \cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_L \sin\bar{\theta}_\mathrm{S} \cos(\bar{\phi}_L - \bar{\phi}_\mathrm{S})\right].$$

$$F_{+,\mathrm{I}}(\theta_\mathrm{S}, \phi_\mathrm{S}, \psi_\mathrm{S}) = \frac{1}{2}(1 + \cos^2\theta_\mathrm{S}) \cos 2\phi_\mathrm{S} \cos 2\psi_\mathrm{S} - \cos\theta_\mathrm{S} \sin 2\phi_\mathrm{S} \sin 2\psi_\mathrm{S}.$$

$$F_{\times,\mathrm{I}}(\theta_\mathrm{S}, \phi_\mathrm{S}, \psi_\mathrm{S}) = \frac{1}{2}(1 + \cos^2\theta_\mathrm{S}) \cos 2\phi_\mathrm{S} \sin 2\psi_\mathrm{S} + \cos\theta_\mathrm{S} \sin 2\phi_\mathrm{S} \cos 2\psi_\mathrm{S}.$$

$$F_{+,\mathrm{II}}(\theta_\mathrm{S}, \phi_\mathrm{S}, \psi_\mathrm{S}) = \frac{1}{2}(1 + \cos^2\theta_\mathrm{S}) \sin 2\phi_\mathrm{S} \cos 2\psi_\mathrm{S} + \cos\theta_\mathrm{S} \cos 2\phi_\mathrm{S} \sin 2\psi_\mathrm{S}.$$

$$F_{\times,\mathrm{II}}(\theta_\mathrm{S}, \phi_\mathrm{S}, \psi_\mathrm{S}) = \frac{1}{2}(1 + \cos^2\theta_\mathrm{S}) \sin 2\phi_\mathrm{S} \sin 2\psi_\mathrm{S} - \cos\theta_\mathrm{S} \cos 2\phi_\mathrm{S} \cos 2\psi_\mathrm{S}.$$

$$\cos\theta_\mathrm{S} = \frac{1}{2}\cos\bar{\theta}_\mathrm{S} - \frac{\sqrt{3}}{2}\sin\bar{\theta}_\mathrm{S} \cos\left[\bar{\phi}(t) - \bar{\phi}_\mathrm{S}\right].$$

$$\phi_\mathrm{S} = \alpha_0 + 2\pi t/T + \arctan\left\{\frac{\sqrt{3}\cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_\mathrm{S}\cos\left[\bar{\phi}(t) - \bar{\phi}_\mathrm{S}\right]}{2\sin\bar{\theta}_\mathrm{S}\sin\left[\bar{\phi}(t) - \bar{\phi}_\mathrm{S}\right]}\right\}.$$

$$\psi_\mathrm{S} = \arctan\left\{\frac{\frac{1}{2}\cos\bar{\theta}_L - \frac{\sqrt{3}}{2}\sin\bar{\theta}_L\cos\left[\bar{\phi}(t) - \bar{\phi}_L\right] - \left[\cos\bar{\theta}_L\cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\cos(\bar{\phi}_L - \bar{\phi}_\mathrm{S})\right]\cos\theta_\mathrm{S}}{\frac{1}{2}\sin\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\sin(\bar{\phi}_L - \bar{\phi}_\mathrm{S}) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\sin\bar{\phi}_\mathrm{S} - \cos\bar{\theta}_\mathrm{S}\sin\bar{\theta}_L\sin\bar{\phi}_L)\cos\bar{\phi}(t) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_\mathrm{S}\sin\bar{\theta}_L\cos\bar{\phi}_L - \cos\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\cos\bar{\phi}_S)\sin\bar{\phi}(t)}\right\}.$$

$$\bar{\phi}(t) = \bar{\phi}_0 + 2\pi t/T.$$

$$t(f) = t_\mathrm{c} - 5(8\pi f)^{-8/3}[\mathcal{M}(1+z)]^{-5/3}\left\{1 + \frac{4}{3}\left(\frac{743}{336} + \frac{11}{4}\eta\right)[\pi M(1+z)f]^{2/3} - \frac{8}{5}\,(4\pi - \beta)\,[\pi M(1+z)f]\right\}.$$

$$\alpha_0 = 0, \quad \bar{\phi}_0 = 0, \quad z = 1.$$

$$\beta = 0, \quad \ln D_\mathrm{L} = \ln H_0^{-1} z.$$

$$\phi_\mathrm{c} := 0, \quad t_\mathrm{c} := 0.$$

$$(S/N)^2 = 4 \sum_{\alpha=\mathrm{I,II}} \int_0^{f_{\max}} \frac{\tilde{h}_\alpha^*(f)\tilde{h}_\alpha(f)}{S_\mathrm{n}(f)} \,\mathrm{d}f.$$

$$\partial_1 h_\alpha(f) = -\frac{5}{4} i [8\pi(1+z)f]^{-5/3} \mathcal{M}^{-2/3} \left\{1 + \frac{55}{6}\eta\,[\pi M(1+z)f] + (8\pi - 2\beta)\,[\pi M(1+z)f]^{3/2}\right\} h_\alpha(f).$$

$$\partial_2 h_\alpha(f) = \frac{3}{4} i [8\pi(1+z)f]^{-5/3} \mathcal{M}^{-5/3} \mu \left\{\left(-\frac{3715}{756} + \frac{55}{6}\eta\right)[\pi M(1+z)f] + (24\pi - 6\beta)\,[\pi M(1+z)f]^{3/2}\right\} h_\alpha(f).$$

$$\partial_3 h_\alpha(f) = 3i[8\pi(1+z)f]^{-5/3}\mathcal{M}^{-5/3}\,[\pi M(1+z)f]\,h_\alpha(f).$$

$$\partial_4 h_\alpha(f) = -i h_\alpha(f).$$

$$\partial_5 h_\alpha(f) = 2\pi i f h_\alpha(f).$$

$$\partial_6 h_\alpha(f) = -h_\alpha(f).$$