

LISA

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1 Monochromatic Sources

$$\begin{aligned}
\Gamma_{ij} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} [\partial_i A_\alpha(t) \partial_j A_\alpha(t) + A_\alpha^2(t) \partial_i \chi_\alpha(t) \partial_j \chi_\alpha(t)] \, dt. \\
A_\alpha(t) &= [A_+^2 F_{+, \alpha}^2(t) + A_\times^2 F_{\times, \alpha}^2(t)]^{1/2}. \\
\chi_\alpha(t) &= 2\pi f_0 t + \varphi_0 + \varphi_{\text{p}, \alpha}(t) + \varphi_{\text{D}}(t). \\
\varphi_{\text{p}, \alpha}(t) &= \arctan \left(-\frac{A_\times F_{\times, \alpha}(t)}{A_+ F_{+, \alpha}(t)} \right). \\
\varphi_{\text{D}}(t) &= 2\pi f_0 c^{-1} R \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]. \\
A_+ &= 2e^{\ln \mathcal{A}} \left\{ 1 + [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})]^2 \right\}. \\
A_\times &= -4e^{\ln \mathcal{A}} [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})]. \\
F_{+, \text{I}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \cos 2\phi_{\text{S}} \cos 2\psi_{\text{S}} - \cos \theta_{\text{S}} \sin 2\phi_{\text{S}} \sin 2\psi_{\text{S}}. \\
F_{\times, \text{I}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \cos 2\phi_{\text{S}} \sin 2\psi_{\text{S}} - \cos \theta_{\text{S}} \sin 2\phi_{\text{S}} \cos 2\psi_{\text{S}}. \\
F_{+, \text{II}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \sin 2\phi_{\text{S}} \cos 2\psi_{\text{S}} + \cos \theta_{\text{S}} \cos 2\phi_{\text{S}} \sin 2\psi_{\text{S}}. \\
F_{\times, \text{II}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \sin 2\phi_{\text{S}} \sin 2\psi_{\text{S}} + \cos \theta_{\text{S}} \cos 2\phi_{\text{S}} \cos 2\psi_{\text{S}}. \\
\cos \theta_{\text{S}} &= \frac{1}{2} \cos \bar{\theta}_{\text{S}} - \frac{\sqrt{3}}{2} \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]. \\
\phi_{\text{S}} &= \alpha_0 + 2\pi t/T + \arctan \left\{ \frac{\sqrt{3} \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]}{2 \sin \bar{\theta}_{\text{S}} \sin [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]} \right\}. \\
\psi_{\text{S}} &= \arctan \left\{ \frac{\frac{1}{2} \cos \bar{\theta}_L - \frac{\sqrt{3}}{2} \sin \bar{\theta}_L \cos [\bar{\phi}(t) - \bar{\phi}_L] - [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})] \cos \theta_{\text{S}}}{\frac{1}{2} \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \sin(\bar{\phi}_L - \bar{\phi}_{\text{S}}) - \frac{\sqrt{3}}{2} (\cos \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \sin \bar{\phi}_{\text{S}} - \cos \bar{\theta}_{\text{S}} \sin \bar{\theta}_L \sin \bar{\phi}_L) \cos \bar{\phi}(t) - \frac{\sqrt{3}}{2} (\cos \bar{\theta}_{\text{S}} \sin \bar{\theta}_L \cos \bar{\phi}_L - \cos \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos \bar{\phi}_L) \sin \bar{\phi}(t)} \right\} \\
\bar{\phi}(t) &= \bar{\phi}_0 + 2\pi t/T. \\
\alpha_0 &= 0, \quad \bar{\phi}_0 = 0. \\
\Gamma_{1j} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} A_\alpha(t) \partial_j A_\alpha(t) \, dt. \\
\Gamma_{2j} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) \partial_j \chi_\alpha(t) \, dt. \\
\Gamma_{3j} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) 2\pi t \partial_j \chi_\alpha(t) \, dt. \\
\ln \mathcal{A} &:= 0, \quad \varphi_0 := 0. \\
\frac{\Gamma_{ij}}{\Gamma_{ij, \ln \mathcal{A}=0}} &= \frac{(S/N)^2}{(S/N)_{\ln \mathcal{A}=0}^2} = \frac{10^2}{(S/N)_{\ln \mathcal{A}=0}^2}. \\
(S/N)^2 &= 2S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} e^{2\pi i f t_1} h_\alpha(t) \, dt_1 \right] \left[\int_{-\infty}^{\infty} e^{-2\pi i f t_2} h_\alpha(t) \, dt_2 \right] \, df. \\
(S/N)^2 &= 2S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} \tilde{h}_\alpha(f) \tilde{h}_\alpha^*(f) \, df. \\
(S/N)^2 &= 2S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} h_\alpha^2(t) \, dt.
\end{aligned}$$