LISA

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1 Monochromatic Sources

$$\Gamma_{ij} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} \left[\partial_i A_\alpha(t) \partial_j A_\alpha(t) + A_\alpha^2(t) \partial_i \chi_\alpha(t) \partial_j \chi_\alpha(t)\right] \mathrm{d}t.$$

$$A_\alpha(t) = \left[A_+^2 F_{+,\alpha}^2(t) + A_\times^2 F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$\chi_\alpha(t) = 2\pi f_0 t + \varphi_0 + \varphi_{\mathrm{p},\alpha}(t) + \varphi_{\mathrm{D}}(t).$$

$$\varphi_{\mathrm{p},\alpha}(t) = \arctan\left(-\frac{A_\times F_{\times,\alpha}(t)}{A_+ F_{+,\alpha}(t)}\right).$$

$$\varphi_{\mathrm{D}}(t) = 2\pi f_0 c^{-1} R \sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right].$$

$$A_+ = 2\left\{1 + \left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right]^2\right\}.$$

$$A_\times = -4e^{\ln\mathcal{A}} \left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right].$$

$$F_{+,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{I}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}}) \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$F_{+,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}} + \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}}.$$

$$F_{\times,\mathrm{II}}(\theta_{\mathrm{S}}, \phi_{\mathrm{S}}, \psi_{\mathrm{S}}) = \frac{1}{2}(1+\cos^2\theta_{\mathrm{S}}) \sin 2\phi_{\mathrm{S}} \sin 2\psi_{\mathrm{S}} - \cos\theta_{\mathrm{S}} \cos 2\phi_{\mathrm{S}} \cos 2\psi_{\mathrm{S}}.$$

$$\cos\theta_{\mathrm{S}} = \frac{1}{2}\cos\bar{\theta}_{\mathrm{S}} - \frac{\sqrt{3}}{2} \sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right].$$

$$\phi_{\mathrm{S}} = \alpha_0 + 2\pi t/T + \arctan\left\{\frac{\sqrt{3}\cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_{\mathrm{S}} \cos\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right]}{2\sin\bar{\theta}_{\mathrm{S}} \sin\left[\bar{\phi}(t) - \bar{\phi}_{\mathrm{S}}\right]}\right\}.$$

$$\psi_{\mathrm{S}} = \arctan\left\{\frac{\frac{1}{2}\cos\bar{\theta}_L - \frac{\sqrt{3}}{2}\sin\bar{\theta}_L \cos\left[\bar{\phi}(t) - \bar{\phi}_L\right] - \left[\cos\bar{\theta}_L \cos\bar{\theta}_{\mathrm{S}} + \sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}})\right]\cos\theta_{\mathrm{S}}}{\frac{1}{2}\sin\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \sin(\bar{\phi}_L - \bar{\phi}_{\mathrm{S}}) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \sin\bar{\phi}_{\mathrm{S}} - \cos\bar{\theta}_{\mathrm{S}} \sin\bar{\theta}_L \sin\bar{\phi}_L)\cos\bar{\phi}(t) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_{\mathrm{S}} \sin\bar{\theta}_L \cos\bar{\phi}_L - \cos\bar{\theta}_L \sin\bar{\theta}_{\mathrm{S}} \cos\bar{\phi}_S)\sin\bar{\phi}(t)}\right\}$$

$$\bar{\phi}(t) = \bar{\phi}_0 + 2\pi t/T.$$

$$\alpha_0 = 0, \quad \bar{\phi}_0 = 0.$$

$$\Gamma_{1j} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} A_\alpha(t) \partial_j A_\alpha(t) \,\mathrm{d}t.$$

$$\Gamma_{2j} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) \partial_j \chi_\alpha(t) \,\mathrm{d}t.$$

$$\Gamma_{3j} = \frac{3}{4} S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) 2\pi t \partial_j \chi_\alpha(t) \,\mathrm{d}t.$$

$$\ln\mathcal{A} := 0, \quad \varphi_0 := 0.$$

$$\frac{\Gamma_{ij}}{\Gamma_{ij,\ln\mathcal{A}=0}} = \frac{(S/N)^2}{(S/N)^2_{\ln\mathcal{A}=0}} = \frac{10^2}{(S/N)^2_{\ln\mathcal{A}=0}}.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} e^{2\pi i f t_1} h_\alpha(t) \,\mathrm{d}t_1\right] \left[\int_{-\infty}^{\infty} e^{-2\pi i f t_2} h_\alpha(t) \,\mathrm{d}t_2\right] \mathrm{d}f.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} \tilde{h}_\alpha^*(f) \tilde{h}_\alpha(f) \,\mathrm{d}f.$$

$$(S/N)^2 = 2S_{\mathrm{n}}(f_0)^{-1} \sum_{\alpha=\mathrm{I,II}} \int_{-\infty}^{\infty} h_\alpha^2(t) \,\mathrm{d}t.$$

2 SMBH Mergers

$$\Gamma_{ij} = 4\sum_{\alpha=\mathrm{I,II}}\int_0^{f_{\max}} \frac{\partial_i \tilde{h}^*_\alpha(f)\partial_j \tilde{h}_\alpha(f)}{S_\mathrm{n}(f)}\,\mathrm{d}f.$$

$$\tilde{h}_\alpha(f) = \frac{\sqrt{3}}{2}\Lambda_\alpha(t)\mathcal{A}f^{-7/6}e^{i[\Psi(f)-\varphi_{\mathrm{p},\alpha}(t)+\varphi_\mathrm{D}(t)]}.$$

$$\Lambda_\alpha(t) = A_\alpha(t)/A(t).$$

$$A_\alpha(t) = \left[A_+^2 F_{+,\alpha}^2(t) + A_\times^2 F_{\times,\alpha}^2(t)\right]^{1/2}.$$

$$A(t) = 2\frac{M_1 M_2(1+z)}{rD_\mathrm{L}}.$$

$$\mathcal{A} = (5/96)^{1/2}\pi^{-2/3}[\mathcal{M}(1+z)]^{5/6}D_\mathrm{L}^{-1}.$$

$$\Psi(f) = 2\pi f t_\mathrm{c} - \phi_\mathrm{c} - \frac{\pi}{4} + \frac{3}{4}(8\pi f)^{-8/3}[\mathcal{M}(1+z)]^{-8/3}\left\{1 + \frac{20}{9}\left(\frac{743}{336}+\frac{11}{4}\eta\right)[\pi M(1+z)f]^{2/3} - 4\left(4\pi-\beta\right)[\pi M(1+z)f]\right\}.$$

$$\varphi_{\mathrm{p},\alpha}(t) = \arctan\left(-\frac{A_\times F_{\times,\alpha}(t)}{A_+ F_{+,\alpha}(t)}\right).$$

$$\varphi_\mathrm{D}(t) = 2\pi f c^{-1} R\sin\bar{\theta}_\mathrm{S}\cos\left[\bar{\phi}(t)-\bar{\phi}_\mathrm{S}\right].$$

$$A_+ = 2\frac{M_1 M_2}{rD}\left\{1 + \left[\cos\bar{\theta}_L\cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\cos(\bar{\phi}_L-\bar{\phi}_\mathrm{S})\right]^2\right\}.$$

$$A_\times = -4\frac{M_1 M_2}{rD}\left[\cos\bar{\theta}_L\cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\cos(\bar{\phi}_L-\bar{\phi}_\mathrm{S})\right].$$

$$F_{+,\mathrm{I}}(\theta_\mathrm{S},\phi_\mathrm{S},\psi_\mathrm{S}) = \frac{1}{2}(1+\cos^2\theta_\mathrm{S})\cos 2\phi_\mathrm{S}\cos 2\psi_\mathrm{S} - \cos\theta_\mathrm{S}\sin 2\phi_\mathrm{S}\sin 2\psi_\mathrm{S}.$$

$$F_{\times,\mathrm{I}}(\theta_\mathrm{S},\phi_\mathrm{S},\psi_\mathrm{S}) = \frac{1}{2}(1+\cos^2\theta_\mathrm{S})\cos 2\phi_\mathrm{S}\sin 2\psi_\mathrm{S} + \cos\theta_\mathrm{S}\sin 2\phi_\mathrm{S}\cos 2\psi_\mathrm{S}.$$

$$F_{+,\mathrm{II}}(\theta_\mathrm{S},\phi_\mathrm{S},\psi_\mathrm{S}) = \frac{1}{2}(1+\cos^2\theta_\mathrm{S})\sin 2\phi_\mathrm{S}\cos 2\psi_\mathrm{S} + \cos\theta_\mathrm{S}\cos 2\phi_\mathrm{S}\sin 2\psi_\mathrm{S}.$$

$$F_{\times,\mathrm{II}}(\theta_\mathrm{S},\phi_\mathrm{S},\psi_\mathrm{S}) = \frac{1}{2}(1+\cos^2\theta_\mathrm{S})\sin 2\phi_\mathrm{S}\sin 2\psi_\mathrm{S} - \cos\theta_\mathrm{S}\cos 2\phi_\mathrm{S}\cos 2\psi_\mathrm{S}.$$

$$\cos\theta_\mathrm{S} = \frac{1}{2}\cos\bar{\theta}_\mathrm{S} - \frac{\sqrt{3}}{2}\sin\bar{\theta}_\mathrm{S}\cos\left[\bar{\phi}(t)-\bar{\phi}_\mathrm{S}\right].$$

$$\phi_\mathrm{S} = \alpha_0 + 2\pi t/T + \arctan\left\{\frac{\sqrt{3}\cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_\mathrm{S}\cos\left[\bar{\phi}(t)-\bar{\phi}_\mathrm{S}\right]}{2\sin\bar{\theta}_\mathrm{S}\sin\left[\bar{\phi}(t)-\bar{\phi}_\mathrm{S}\right]}\right\}.$$

$$\psi_\mathrm{S} = \arctan\left\{\frac{\frac{1}{2}\cos\bar{\theta}_L - \frac{\sqrt{3}}{2}\sin\bar{\theta}_L\cos\left[\bar{\phi}(t)-\bar{\phi}_L\right] - \left[\cos\bar{\theta}_L\cos\bar{\theta}_\mathrm{S} + \sin\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\cos(\bar{\phi}_L-\bar{\phi}_\mathrm{S})\right]\cos\theta_\mathrm{S}}{\frac{1}{2}\sin\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\sin(\bar{\phi}_L-\bar{\phi}_\mathrm{S}) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\sin\bar{\phi}_\mathrm{S} - \cos\bar{\theta}_\mathrm{S}\sin\bar{\theta}_L\sin\bar{\phi}_L)\cos\bar{\phi}(t) - \frac{\sqrt{3}}{2}(\cos\bar{\theta}_\mathrm{S}\sin\bar{\theta}_L\cos\bar{\phi}_L - \cos\bar{\theta}_L\sin\bar{\theta}_\mathrm{S}\cos\bar{\phi}_S)\sin\bar{\phi}(t)}\right\}$$

$$\bar{\phi}(t) = \bar{\phi}_0 + 2\pi t/T.$$

$$t(f) = t_\mathrm{c} - 5(8\pi f)^{-8/3}[\mathcal{M}(1+z)]^{-5/3}\left\{1 + \frac{4}{3}\left(\frac{743}{336}+\frac{11}{4}\eta\right)[\pi M(1+z)f]^{2/3} - \frac{8}{5}\left(4\pi-\beta\right)[\pi M(1+z)f]\right\}.$$

$$\alpha_0 = 0, \quad \bar{\phi}_0 = 0.$$

$$(S/N)^2 = 4\sum_{\alpha=\mathrm{I,II}}\int_0^{f_{\max}} \frac{\tilde{h}^*_\alpha(f)\tilde{h}_\alpha(f)}{S_\mathrm{n}(f)}\,\mathrm{d}f.$$