

# LISA

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## 1 Monochromatic Sources

$$\begin{aligned}
 \Gamma_{ij} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} [\partial_i A_\alpha(t) \partial_j A_\alpha(t) + A_\alpha^2(t) \partial_i \chi_\alpha(t) \partial_j \chi_\alpha(t)] \, dt. \\
 A_\alpha(t) &= [A_+^2 F_{+,\alpha}^2(t) + A_\times^2 F_{\times,\alpha}^2(t)]^{1/2}. \\
 \chi_\alpha(t) &= 2\pi f_0 t + \varphi_0 + \varphi_{\text{p},\alpha}(t) + \varphi_{\text{D}}(t). \\
 \varphi_{\text{p},\alpha}(t) &= \arctan \left( -\frac{A_\times F_{\times,\alpha}(t)}{A_+ F_{+,\alpha}(t)} \right). \\
 \varphi_{\text{D}}(t) &= 2\pi f_0 c^{-1} R \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]. \\
 A_+ &= 2 \left\{ 1 + [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})]^2 \right\}. \\
 A_\times &= -4e^{\ln A} [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})]. \\
 F_{+,\text{I}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \cos 2\phi_{\text{S}} \cos 2\psi_{\text{S}} - \cos \theta_{\text{S}} \sin 2\phi_{\text{S}} \sin 2\psi_{\text{S}}. \\
 F_{\times,\text{I}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \cos 2\phi_{\text{S}} \sin 2\psi_{\text{S}} + \cos \theta_{\text{S}} \sin 2\phi_{\text{S}} \cos 2\psi_{\text{S}}. \\
 F_{+,\text{II}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \sin 2\phi_{\text{S}} \cos 2\psi_{\text{S}} + \cos \theta_{\text{S}} \cos 2\phi_{\text{S}} \sin 2\psi_{\text{S}}. \\
 F_{\times,\text{II}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \sin 2\phi_{\text{S}} \sin 2\psi_{\text{S}} - \cos \theta_{\text{S}} \cos 2\phi_{\text{S}} \cos 2\psi_{\text{S}}. \\
 \cos \theta_{\text{S}} &= \frac{1}{2} \cos \bar{\theta}_{\text{S}} - \frac{\sqrt{3}}{2} \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]. \\
 \phi_{\text{S}} &= \alpha_0 + 2\pi t/T + \arctan \left\{ \frac{\sqrt{3} \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]}{2 \sin \bar{\theta}_{\text{S}} \sin [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]} \right\}. \\
 \psi_{\text{S}} &= \arctan \left\{ \frac{\frac{1}{2} \cos \bar{\theta}_L - \frac{\sqrt{3}}{2} \sin \bar{\theta}_L \cos [\bar{\phi}(t) - \bar{\phi}_L] - [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})] \cos \theta_{\text{S}}}{\frac{1}{2} \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \sin(\bar{\phi}_L - \bar{\phi}_{\text{S}}) - \frac{\sqrt{3}}{2} (\cos \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \sin \bar{\phi}_{\text{S}} - \cos \bar{\theta}_{\text{S}} \sin \bar{\theta}_L \sin \bar{\phi}_L) \cos \bar{\phi}(t) - \frac{\sqrt{3}}{2} (\cos \bar{\theta}_{\text{S}} \sin \bar{\theta}_L \cos \bar{\phi}_L - \cos \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos \bar{\phi}_{\text{S}}) \sin \bar{\phi}(t)} \right\}. \\
 \bar{\phi}(t) &= \bar{\phi}_0 + 2\pi t/T. \\
 \alpha_0 &= 0, \quad \bar{\phi}_0 = 0. \\
 \Gamma_{1j} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} A_\alpha(t) \partial_j A_\alpha(t) \, dt. \\
 \Gamma_{2j} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) \partial_j \chi_\alpha(t) \, dt. \\
 \Gamma_{3j} &= \frac{3}{4} S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} A_\alpha^2(t) 2\pi t \partial_j \chi_\alpha(t) \, dt. \\
 \ln \mathcal{A} &:= 0, \quad \varphi_0 := 0. \\
 \frac{\Gamma_{ij}}{\Gamma_{ij, \ln \mathcal{A}=0}} &= \frac{(S/N)^2}{(S/N)_{\ln \mathcal{A}=0}^2} = \frac{10^2}{(S/N)_{\ln \mathcal{A}=0}^2}. \\
 (S/N)^2 &= 2S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} e^{2\pi i f t_1} h_\alpha(t) \, dt_1 \right] \left[ \int_{-\infty}^{\infty} e^{-2\pi i f t_2} h_\alpha(t) \, dt_2 \right] \, df. \\
 (S/N)^2 &= 2S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} \tilde{h}_\alpha^*(f) \tilde{h}_\alpha(f) \, df. \\
 (S/N)^2 &= 2S_n(f_0)^{-1} \sum_{\alpha=\text{I,II}} \int_{-\infty}^{\infty} h_\alpha^2(t) \, dt.
 \end{aligned}$$

## 2 SMBH Mergers

$$\begin{aligned}
\Gamma_{ij} &= 4 \sum_{\alpha=\text{I,II}} \int_0^{f_{\max}} \frac{\partial_i \tilde{h}_\alpha^*(f) \partial_j \tilde{h}_\alpha(f)}{S_n(f)} df. \\
f_{\max} &= [3^{3/2} \pi \frac{GM}{c^3} (1+z)]^{-1} \\
\tilde{h}_\alpha(f) &= \frac{\sqrt{3}}{2} \Lambda_\alpha(t) \mathcal{A} f^{-7/6} e^{i[\Psi(f) - \varphi_{\text{p},\alpha}(t) + \varphi_{\text{D}}(t)]}. \\
\Lambda_\alpha(t) &= [\Lambda_+^2 F_{+,\alpha}^2(t) + \Lambda_\times^2 F_{\times,\alpha}^2(t)]^{1/2}. \\
\mathcal{A} &= (5/96)^{1/2} \pi^{-2/3} [\frac{GM}{c^3} (1+z)]^{5/6} (e^{\ln \frac{D_L}{c}})^{-1}. \\
\Psi(f) &= 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4} (8\pi f)^{-5/3} [\frac{GM}{c^3} (1+z)]^{-5/3} \left\{ 1 + \frac{20}{9} \left( \frac{743}{336} + \frac{11}{4} \eta \right) \left[ \pi \frac{GM}{c^3} (1+z) f \right]^{2/3} - 4(4\pi - \beta) \left[ \pi \frac{GM}{c^3} (1+z) f \right] \right\}. \\
\varphi_{\text{p},\alpha}(t) &= \arctan \left( -\frac{\Lambda_\times F_{\times,\alpha}(t)}{\Lambda_+ F_{+,\alpha}(t)} \right). \\
\varphi_{\text{D}}(t) &= 2\pi f \frac{R}{c} \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]. \\
\Lambda_+ &= 1 + [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})]^2. \\
\Lambda_\times &= -2 [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})]. \\
F_{+,\text{I}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \cos 2\phi_{\text{S}} \cos 2\psi_{\text{S}} - \cos \theta_{\text{S}} \sin 2\phi_{\text{S}} \sin 2\psi_{\text{S}}. \\
F_{\times,\text{I}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \cos 2\phi_{\text{S}} \sin 2\psi_{\text{S}} + \cos \theta_{\text{S}} \sin 2\phi_{\text{S}} \cos 2\psi_{\text{S}}. \\
F_{+,\text{II}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \sin 2\phi_{\text{S}} \cos 2\psi_{\text{S}} + \cos \theta_{\text{S}} \cos 2\phi_{\text{S}} \sin 2\psi_{\text{S}}. \\
F_{\times,\text{II}}(\theta_{\text{S}}, \phi_{\text{S}}, \psi_{\text{S}}) &= \frac{1}{2} (1 + \cos^2 \theta_{\text{S}}) \sin 2\phi_{\text{S}} \sin 2\psi_{\text{S}} - \cos \theta_{\text{S}} \cos 2\phi_{\text{S}} \cos 2\psi_{\text{S}}. \\
\cos \theta_{\text{S}} &= \frac{1}{2} \cos \bar{\theta}_{\text{S}} - \frac{\sqrt{3}}{2} \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]. \\
\phi_{\text{S}} &= \alpha_0 + 2\pi t/T + \arctan \left\{ \frac{\sqrt{3} \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_{\text{S}} \cos [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]}{2 \sin \bar{\theta}_{\text{S}} \sin [\bar{\phi}(t) - \bar{\phi}_{\text{S}}]} \right\}. \\
\psi_{\text{S}} &= \arctan \left\{ \frac{\frac{1}{2} \cos \bar{\theta}_L - \frac{\sqrt{3}}{2} \sin \bar{\theta}_L \cos [\bar{\phi}(t) - \bar{\phi}_L] - [\cos \bar{\theta}_L \cos \bar{\theta}_{\text{S}} + \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos(\bar{\phi}_L - \bar{\phi}_{\text{S}})] \cos \theta_{\text{S}}}{\frac{1}{2} \sin \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \sin(\bar{\phi}_L - \bar{\phi}_{\text{S}}) - \frac{\sqrt{3}}{2} (\cos \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \sin \bar{\phi}_{\text{S}} - \cos \bar{\theta}_{\text{S}} \sin \bar{\theta}_L \sin \bar{\phi}_L) \cos \bar{\phi}(t) - \frac{\sqrt{3}}{2} (\cos \bar{\theta}_{\text{S}} \sin \bar{\theta}_L \cos \bar{\phi}_L - \cos \bar{\theta}_L \sin \bar{\theta}_{\text{S}} \cos \bar{\phi}_{\text{S}}) \sin \bar{\phi}(t)} \right\}. \\
\bar{\phi}(t) &= \bar{\phi}_0 + 2\pi t/T. \\
t(f) &= t_c - 5(8\pi f)^{-8/3} [\frac{GM}{c^3} (1+z)]^{-5/3} \left\{ 1 + \frac{4}{3} \left( \frac{743}{336} + \frac{11}{4} \eta \right) \left[ \pi \frac{GM}{c^3} (1+z) f \right]^{2/3} - \frac{8}{5} (4\pi - \beta) \left[ \pi \frac{GM}{c^3} (1+z) f \right] \right\}. \\
\alpha_0 &= 0, \quad \bar{\phi}_0 = 0, \quad z = 1. \\
\beta &= 0, \quad \ln \frac{D_L}{c} = \ln H_0^{-1} z. \\
\phi_c &:= 0, \quad t_c := 0. \\
(S/N)^2 &= 4 \sum_{\alpha=\text{I,II}} \int_0^{f_{\max}} \frac{\tilde{h}_\alpha^*(f) \tilde{h}_\alpha(f)}{S_n(f)} df. \\
\partial_1 h_\alpha(f) &= -\frac{5}{4} i [8\pi(1+z)f]^{-5/3} \frac{GM}{c^3}^{-2/3} \left\{ 1 + \frac{55}{6} \eta \left[ \pi \frac{GM}{c^3} (1+z) f \right]^{2/3} + (8\pi - 2\beta) \left[ \pi \frac{GM}{c^3} (1+z) f \right] \right\} h_\alpha(f). \\
\partial_2 h_\alpha(f) &= \frac{3}{4} i [8\pi(1+z)f]^{-5/3} \frac{GM}{c^3}^{-5/3} \mu \left\{ \left( -\frac{3715}{756} + \frac{55}{6} \eta \right) \left[ \pi \frac{GM}{c^3} (1+z) f \right]^{2/3} + (24\pi - 6\beta) \left[ \pi \frac{GM}{c^3} (1+z) f \right] \right\} h_\alpha(f). \\
\partial_3 h_\alpha(f) &= 3i [8\pi(1+z)f]^{-5/3} \frac{GM}{c^3}^{-5/3} \left[ \pi \frac{GM}{c^3} (1+z) f \right] h_\alpha(f). \\
\partial_4 h_\alpha(f) &= -i h_\alpha(f). \\
\partial_5 h_\alpha(f) &= 2\pi i f h_\alpha(f). \\
\partial_6 h_\alpha(f) &= -h_\alpha(f).
\end{aligned}$$