1 Memory

$$
\begin{aligned}
h_{l0}|_{t_1}^{t_2} &= \ldots \Re\left[\frac{4}{D}\int \Psi_2^{\circ}|_{t_1}^{t_2}\ldots\, d\theta d\phi - D\cdots\times\left(\int_{t_1}^{t_2}\dot{h}\dot{\bar{h}}\,dt - (\dot{h}\bar{h})|_{t_1}^{t_2}\right)\right]\\
&= \ldots \Re\left[\frac{4}{D}\int_{t_1}^{t_2}\left(\int \frac{\partial}{\partial t}\Psi_2^{\circ}\ldots\, d\theta d\phi\right)dt - D\cdots\times\left(\int_{t_1}^{t_2}\dot{h}\dot{\bar{h}}\,dt - \int_{t_1}^{t_2}(\ddot{h}\bar{h}+\dot{h}\dot{\bar{h}})\,dt\right)\right]\\
&= \ldots \Re\left[\frac{4}{D}\int_{t_1}^{t_2}\left(\int \frac{\partial}{\partial t}\Psi_2^{\circ}\ldots\, d\theta d\phi\right)dt + D\cdots\times\left(\int_{t_1}^{t_2}\ddot{h}\bar{h}\,dt\right)\right]
\end{aligned}
$$

$$
\dot{h}_{l0} = \ldots \Re\left[\frac{4}{D}\int \frac{\partial}{\partial t}\Psi_2^{\circ}\ldots\, d\theta d\phi + D\cdots\times\ddot{h}\bar{h}\right]
$$

$$
\Psi_2^{\circ} = -\frac{M}{\gamma^4}(1 - v_x\sin\theta\cos\phi - v_y\sin\theta\sin\phi - v_z\cos\theta)^{-3}
$$

$$
\Psi_2^{\circ} = -\frac{M(1-v_x^2-v_y^2-v_z^2)^2}{(1 - v_x\sin\theta\cos\phi - v_y\sin\theta\sin\phi - v_z\cos\theta)^3}
$$

$$
\Psi_2^{\circ} = -\frac{M(\frac{M^2-p_x^2-p_y^2-p_z^2}{M^2})^2}{(\frac{M - p_x\sin\theta\cos\phi - p_y\sin\theta\sin\phi - p_z\cos\theta}{M})^3}
$$

$$
\Psi_2^{\circ} = -\frac{(M^2-p_x^2-p_y^2-p_z^2)^2}{(M - p_x\sin\theta\cos\phi - p_y\sin\theta\sin\phi - p_z\cos\theta)^3}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}\Psi_2^{\circ} = &-\frac{2(M^2-p_x^2-p_y^2-p_z^2)(2M\frac{\partial M}{\partial t} - 2p_x\frac{\partial p_x}{\partial t} - 2p_y\frac{\partial p_y}{\partial t} - 2p_z\frac{\partial p_z}{\partial t})}{(M - p_x\sin\theta\cos\phi - p_y\sin\theta\sin\phi - p_z\cos\theta)^3}\\
&-\frac{-3(M^2-p_x^2-p_y^2-p_z^2)^2(\frac{\partial M}{\partial t} - \frac{\partial p_x}{\partial t}\sin\theta\cos\phi - \frac{\partial p_y}{\partial t}\sin\theta\sin\phi - \frac{\partial p_z}{\partial t}\cos\theta)}{(M - p_x\sin\theta\cos\phi - p_y\sin\theta\sin\phi - p_z\cos\theta)^4}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}\Psi_2^{\circ} = &-\frac{4(M\frac{\partial M}{\partial t} - p_x\frac{\partial p_x}{\partial t} - p_y\frac{\partial p_y}{\partial t} - p_z\frac{\partial p_z}{\partial t})}{M}\\
&+3(\frac{\partial M}{\partial t} - \frac{\partial p_x}{\partial t}\sin\theta\cos\phi - \frac{\partial p_y}{\partial t}\sin\theta\sin\phi - \frac{\partial p_z}{\partial t}\cos\theta)
\end{aligned}
$$

$$
\frac{\partial}{\partial t}\Psi_2^{\circ} = -\frac{\partial M}{\partial t} - 3(\frac{\partial p_x}{\partial t}\sin\theta\cos\phi + \frac{\partial p_y}{\partial t}\sin\theta\sin\phi + \frac{\partial p_z}{\partial t}\cos\theta)
$$

$$
\frac{\partial}{\partial t}\Psi_2^{\circ} = \frac{\partial E_{\mathrm{GW}}}{\partial t} - 3(\frac{\partial p_x}{\partial t}\sin\theta\cos\phi + \frac{\partial p_y}{\partial t}\sin\theta\sin\phi + \frac{\partial p_z}{\partial t}\cos\theta)
$$

$$
l = 2, \quad \vec{p} = 0
$$

$$
\frac{\partial}{\partial t}\Psi_2 = \frac{\partial E_{\mathrm{GW}}}{\partial t} = \frac{D^2}{16\pi}(\left|\dot{h}_{2-2}\right|^2 + \left|\dot{h}_{22}\right|^2) = \frac{D^2}{8\pi}\left|\dot{h}_{22}\right|^2
$$

$$
\begin{aligned}
h_{20}|_{t_1}^{t_2} &= -\frac{1}{\sqrt{24}}\Re\left[\frac{4}{D}\left(\int_{t_1}^{t_2}\frac{D^2}{8\pi}\left|\dot{h}_{22}\right|^2\,dt\right)\left(\int [{}^0Y_{20}]\sin\theta\,d\theta d\phi\right) + D\cdots\times\left(\int_{t_1}^{t_2}\ddot{h}\bar{h}\right)\right]\\
&= -\frac{1}{\sqrt{24}}\Re\left[D\cdots\times\left(\int_{t_1}^{t_2}\ddot{h}\bar{h}\right)\right]\\
&= -\frac{1}{\sqrt{24}}\Re\left[D\sum_{\substack{-2\le m'\le 2\\ m'\neq 0}}\sum_{\substack{-2\le m''\le 2\\ m''\neq 0}} G_{222m'-m''0}\left(\int_{t_1}^{t_2}\ddot{h}_{2m'}\bar{h}_{2m''}\,dt\right)\right]\\
&= -\frac{1}{\sqrt{24}}\Re\left[D[G_{222-220}\left(\int_{t_1}^{t_2}\ddot{h}_{2-2}\bar{h}_{2-2}\,dt\right) + G_{2222-20}\left(\int_{t_1}^{t_2}\ddot{h}_{22}\bar{h}_{22}\,dt\right)]\right]\\
&= \frac{2G_{2222-20}D}{\sqrt{24}}\left(\int_{t_1}^{t_2}\left|\dot{h}_{22}\right|^2\,dt\right)\\
&= \frac{16\pi G_{2222-20}}{\sqrt{24}D}\left(\int_{t_1}^{t_2}\frac{D^2}{8\pi}\left|\dot{h}_{22}\right|^2\,dt\right)\\
&= \frac{8\pi}{\sqrt{6}}G_{2222-20}\frac{E_{\mathrm{GW}}|_{t_1}^{t_2}}{D}
\end{aligned}
$$

$$
\begin{aligned}
G_{2222-20} &= \int [{}^{-2}Y_{22}][{}^{-2}\overline{Y}_{22}][{}^{0}\overline{Y}_{20}] \sin\theta \, d\theta d\phi \\
&= \int [\frac{1}{2}\sqrt{\frac{15}{48\pi}}(1+\cos\theta)^2 e^{2i\phi}][\frac{1}{2}\sqrt{\frac{15}{48\pi}}(1+\cos\theta)^2 e^{-2i\phi}][\frac{1}{2}\sqrt{\frac{5}{4\pi}}(3\cos^2\theta - 1)] \sin\theta \, d\theta d\phi \\
&= \frac{\sqrt{5}}{7\sqrt{\pi}}
\end{aligned}
$$

$$
\begin{aligned}
h_{20}|_{t_1}^{t_2} &= \frac{8}{7}\sqrt{\frac{5\pi}{6}}\frac{E_{\mathrm{GW}}|_{t_1}^{t_2}}{D} \\
\dot{h}_{20} &= \frac{8}{7}\sqrt{\frac{5\pi}{6}}\frac{L_{\mathrm{GW}}}{D} = \frac{32\pi}{7}\sqrt{\frac{5\pi}{6}}\bar{F}_{\mathrm{GW}} D
\end{aligned}
$$

2 Waveform

$$
\begin{aligned}
h_{22} &= \int h \left[{}^{-2}\overline{Y}_{22}\right] dS \\
&= \iint \left\{ \frac{2\nu M}{r} v^2 (1+\cos^2\theta)\cos[2(\varphi-\phi)] - i\frac{4\nu M}{r} v^2 \cos\theta \sin[2(\varphi-\phi)] \right\} \left[\frac{1}{2}\sqrt{\frac{15}{48\pi}}(1+\cos\theta)^2 e^{-2i\phi}\right] \sin\theta \, d\theta \, d\phi \\
&= \frac{1}{2}\sqrt{\frac{15}{48\pi}}\frac{4\nu M}{r} v^2 \left\{ \int \frac{1+\cos^2\theta}{2}(1+\cos\theta)^2 \sin\theta \, d\theta \int \cos[2(\varphi-\phi)] e^{-2i\phi} \, d\phi - i \int \cos\theta (1+\cos\theta)^2 \sin\theta \, d\theta \int \sin[2(\varphi-\phi)] e^{-2i\phi} \, d\phi \right\} \\
&= \frac{1}{2}\sqrt{\frac{15}{48\pi}}\frac{4\nu M}{r} v^2 \left\{ \left(\frac{28}{15}\right)\left(\pi e^{-2i\varphi}\right) - i\left(\frac{4}{3}\right)\left(i\pi e^{-2i\varphi}\right) \right\} \\
&= \frac{1}{2}\sqrt{\frac{15}{48\pi}}\frac{4\nu M}{r} v^2 \left\{ \frac{16}{5}\pi e^{-2i\varphi} \right\} \\
&= \frac{\sqrt{4\pi}}{5}\frac{4\nu M}{r} v^2 e^{-2i\varphi}
\end{aligned}
$$

$$
\begin{aligned}
h_{2-2} &= \int h \left[{}^{-2}\overline{Y}_{2-2}\right] dS \\
&= \iint \left\{ \frac{2\nu M}{r} v^2 (1+\cos^2\theta)\cos[2(\varphi-\phi)] - i\frac{4\nu M}{r} v^2 \cos\theta \sin[2(\varphi-\phi)] \right\} \left[\frac{1}{2}\sqrt{\frac{15}{48\pi}}(1-\cos\theta)^2 e^{2i\phi}\right] \sin\theta \, d\theta \, d\phi \\
&= \frac{1}{2}\sqrt{\frac{15}{48\pi}}\frac{4\nu M}{r} v^2 \left\{ \int \frac{1+\cos^2\theta}{2}(1-\cos\theta)^2 \sin\theta \, d\theta \int \cos[2(\varphi-\phi)] e^{2i\phi} \, d\phi - i \int \cos\theta (1-\cos\theta)^2 \sin\theta \, d\theta \int \sin[2(\varphi-\phi)] e^{2i\phi} \, d\phi \right\} \\
&= \frac{1}{2}\sqrt{\frac{15}{48\pi}}\frac{4\nu M}{r} v^2 \left\{ \left(\frac{28}{15}\right)\left(\pi e^{2i\varphi}\right) - i\left(-\frac{4}{3}\right)\left(-i\pi e^{2i\varphi}\right) \right\} \\
&= \frac{1}{2}\sqrt{\frac{15}{48\pi}}\frac{4\nu M}{r} v^2 \left\{ \frac{16}{5}\pi e^{2i\varphi} \right\} \\
&= \frac{\sqrt{4\pi}}{5}\frac{4\nu M}{r} v^2 e^{2i\varphi}
\end{aligned}
$$

$$
\begin{aligned}
\dot{h}_{22} &= \frac{\sqrt{4\pi}}{5}\frac{4\nu M}{r}(2v\frac{dv}{d\tau}e^{-2i\varphi} - v^2 e^{-2i\varphi} 2i\frac{d\varphi}{d\tau})\frac{d\tau}{dt} \\
\dot{h}_{2-2} &= \frac{\sqrt{4\pi}}{5}\frac{4\nu M}{r}(2v\frac{dv}{d\tau}e^{2i\varphi} + v^2 e^{2i\varphi} 2i\frac{d\varphi}{d\tau})\frac{d\tau}{dt}
\end{aligned}
$$

$\varphi = \frac{-1}{\nu\tau^5}\{1 + (\frac{3715}{8064} + \frac{55}{96}\nu)\tau^2 - \frac{3\pi}{4}\tau^3 + (\frac{9275495}{14450688} + \frac{284875}{258048}\nu + \frac{1855}{2048}\nu^2)\tau^4 + (-\frac{38645}{172032} + \frac{65}{2048}\nu)\pi\tau^5 \ln\tau + [\frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}(\gamma + \ln(2\tau)) + (-\frac{126510089885}{4161798144} + \frac{2255}{2048}\pi^2)\nu + \frac{154565}{1835008}\nu^2 - \frac{1179625}{1769472}\nu^3]\tau^6 + (\frac{188516689}{173408256} + \frac{488825}{516096}\nu - \frac{141769}{516096}\nu^2)\pi\tau^7\}$

$v^2 = \frac{\tau^2}{4}\{1 + (\frac{743}{4032} + \frac{11}{48}\nu)\tau^2 - \frac{\pi}{5}\tau^3 + (\frac{19583}{254016} + \frac{24401}{193536}\nu + \frac{31}{288}\nu^2)\tau^4 + (-\frac{11891}{53760} + \frac{109}{1920}\nu)\pi\tau^5 + [-\frac{10052469856691}{6008596070400} + \frac{\pi^2}{6} + \frac{107}{420}(\gamma + \ln 2\tau) + (\frac{3147553127}{780337152} - \frac{451}{3072}\pi^2)\nu - \frac{15211}{442368}\nu^2 + \frac{25565}{331776}\nu^3]\tau^6 + (-\frac{113868647}{433520640} - \frac{31821}{143360}\nu + \frac{294941}{3870720}\nu^2)\pi\tau^7\}$

$\tau = [\nu/(5M)]^{-1/8}(t_{\mathrm{c}} - t)^{-1/8}$

$\frac{d\tau}{dt} = (1/8)[\nu/(5M)]^{-1/8}(t_{\mathrm{c}} - t)^{-1/8-1}$

$\tau = [\nu/(5M)]^{-1/8}[-(t_? + t)]^{-1/8}$

$\frac{d\tau}{dt} = (1/8)[\nu/(5M)]^{-1/8}[-(t_? + t)]^{-9/8}$

$\tau = [\nu/(5M)]^{-1/8}[-t_? - t]^{-1/8}$

$\frac{d\tau}{dt} = (1/8)[\nu/(5M)]^{-1/8}[-t_? - t]^{-9/8}$

$$\tau = [\nu/(5M)]^{-1/8}[-t_?]^{-1/8}\frac{[-t_?-t]^{-1/8}}{[-t_?]^{-1/8}}$$
$$\frac{d\tau}{dt} = (1/8)[\nu/(5M)]^{-1/8}[-t_?]^{-9/8}\frac{[-t_?-t]^{-9/8}}{[-t_?]^{-9/8}}$$
$$\tau = [\nu/(5M)]^{-1/8}[-t_?]^{-1/8}[1+\frac{t}{t_?}]^{-1/8}$$
$$\frac{d\tau}{dt} = (1/8)[\nu/(5M)]^{-1/8}[-t_?]^{-9/8}[1+\frac{t}{t_?}]^{-9/8}$$