

# 引力波天文学笔记

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2022 年 6 月 17 日

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Printed in China

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# 第一章 引力波

[3].

## 1.1 Linearized Gravity

流形  $\mathbb{R}^4$ . 任意坐标系  $\{x^\mu\}$ ,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}s + O(s^2)$ , 得

$$R_{\mu\nu\lambda\sigma} = \partial_\sigma \partial_{[\mu} h_{\lambda]\nu} - \partial_\nu \partial_{[\mu} h_{\lambda]\sigma} + O(s^2). \quad (1.1)$$

$$\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\sigma}h_{\lambda\sigma} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h.$$

$$-\frac{1}{2}\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} + \partial^\lambda \partial_{(\mu} \bar{h}_{\nu)\lambda} - \frac{1}{2}\eta_{\mu\nu} \partial^\lambda \partial^\sigma \bar{h}_{\lambda\sigma} + O(s^2) = 8\pi T_{\mu\nu}. \quad (1.2)$$

存在  $\{x^\mu\}$ , 使得  $\partial^\nu \bar{h}_{\mu\nu} + O(s^2) = 0$  (Lorentz gauge). 令  $\{x^\mu\}$  满足  $\partial^\nu \bar{h}_{\mu\nu} + O(s^2) = 0$ , 则

$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} + O(s^2) = -16\pi T_{\mu\nu}. \quad (1.3)$$

略去  $O(s^2)$  条件:  $h_{\mu\nu}, \partial_\lambda h_{\mu\nu} \dots$  小.

## 1.2 Radiation Gauge

存在  $\{x^\mu\}$ , 使得  $h + O(s^2) = 0$  (TT gauge [4]) 且  $h_{0\mu} + O(s^2) = 0$ .

## 1.3 Quadrupole Approximation

下略  $O(s^2)$ . 由(1.3)得

$$\bar{h}_{\mu\nu}(t, \vec{r}) = 4 \int \frac{T_{\mu\nu}(t - |\vec{r} - \vec{r}'|, \vec{r}')}{|\vec{r} - \vec{r}'|} dV'. \quad (1.4)$$

$$\hat{h}_{\mu\nu}(\omega, \vec{r}) := \frac{1}{\sqrt{2\pi}} \int \bar{h}_{\mu\nu}(t, \vec{r}) e^{i\omega t} dt \quad (1.5)$$

$$= 4 \int \frac{\hat{T}_{\mu\nu}(\omega, \vec{r}')}{|\vec{r} - \vec{r}'|} e^{i\omega|\vec{r} - \vec{r}'|} dV'. \quad (1.6)$$

由  $\partial^\nu \bar{h}_{\mu\nu} = 0$ ,

$$-i\omega \hat{h}_{0\mu} = \sum_i \frac{\partial \hat{h}_{i\mu}}{\partial x^i}. \quad (1.7)$$

$|\vec{r}| \gg |\vec{r}'|$  且  $\omega \ll 1/|\vec{r}'|$ ,

$$\hat{h}_{ij}(\omega, \vec{r}) = 4 \frac{e^{i\omega|\vec{r}|}}{|\vec{r}|} \int \hat{T}_{ij}(\omega, \vec{r}') dV'. \quad (1.8)$$

$$\int \hat{T}_{ij} dV' = -\frac{\omega^2}{2} \int \hat{T}_{00} x'^i x'^j dV' \quad (1.9)$$

$$q_{ij}(t) := \int T_{00} x'^i x'^j dV' \quad (1.10)$$

$$\hat{h}_{ij}(\omega, \vec{r}) = -2\omega^2 \frac{e^{i\omega|\vec{r}|}}{|\vec{r}|} \hat{q}_{ij}(\omega), \quad (1.11)$$

$$\bar{h}_{ij}(t, \vec{r}) = \frac{2}{|\vec{r}|} \frac{d^2}{dt^2} q_{ij}(t - |\vec{r}|). \quad (1.12)$$

## 1.4 + Mode and $\times$ Mode

寻新标架  $(e'^1)_a = (e^+)_a$ ,  $(e'^2)_a = (e^\times)_a$ ,  $(e'^3)_a = (e^r)_a$ ,  $\bar{h}_{ij}(e^i)_a (e^j)_b = \bar{h}'_{ij}(e'^i)_a (e'^j)_b$ , 取  $x, y$  分量后去迹,  $h_+ = \frac{1}{2}(\bar{h}'_{11} - \bar{h}'_{22})$ ,  $h_\times = \bar{h}'_{12} = \bar{h}'_{21}$ ? [2]

[1],  $\vec{n} := \frac{\vec{r}}{|\vec{r}|}$ ,

$$h_{ij}^{\text{TT}} = \frac{2}{|\vec{r}|} \mathcal{P}_{ijkm} \frac{d^2}{dt^2} Q^{km}(t - |\vec{r}|), \quad (1.13)$$

$$\mathcal{P}_{ijkm} := (\delta_{ik} - \vec{n}_i \vec{n}_k) (\delta_{jm} - \vec{n}_j \vec{n}_m) - \frac{1}{2} (\delta_{ij} - \vec{n}_i \vec{n}_j) (\delta_{km} - \vec{n}_k \vec{n}_m), \quad (1.14)$$

$$Q^{km}(t) := \int T_{00} \left( x'^k x'^m - \frac{1}{3} \delta^{km} \sum_n x'^n x'^n \right) dV' \quad (1.15)$$

## 参考文献

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