引力波天文学笔记

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目录

第一章　Basic

1.1　Linearized Gravity

[25, 11, 16]. 流形 $\mathbb{R}^4$. 任意坐标系 $\{x^\mu\}$, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} s + \mathrm{O}(s^2)$. 设 $g^{\mu\nu} = ?^{\mu\nu} + ??^{\mu\nu} s + \mathrm{O}(s^2)$, 则 $\delta^\mu{}_\lambda = ?^{\mu\nu}\eta_{\nu\lambda} + ?^{\mu\nu}\gamma_{\nu\lambda} s + ??^{\mu\nu}\eta_{\nu\lambda} s + \mathrm{O}(s^2)$, 所以 $?^{\mu\nu} = \eta^{\mu\nu}$, $??^{\mu\nu} = ??^{\mu\sigma}\delta_\sigma{}^\nu = ??^{\mu\sigma}\eta_{\sigma\lambda}\eta^{\lambda\nu} = -?^{\mu\sigma}\gamma_{\sigma\lambda}\eta^{\lambda\nu} = -\eta^{\mu\sigma}\gamma_{\sigma\lambda}\eta^{\lambda\nu} = -\gamma^{\mu\nu}$, 所以 $g^{\mu\nu} = \eta^{\mu\nu} - \gamma^{\mu\nu} s + \mathrm{O}(s^2) = \eta^{\mu\nu} - h^{\mu\nu} + \mathrm{O}(s^2)$.

$$R_{\mu\lambda\nu\sigma} = \partial_\sigma \partial_{[\mu} h_{\lambda]\nu} - \partial_\nu \partial_{[\mu} h_{\lambda]\sigma} + \mathrm{O}(s^2). \tag{1.1}$$

$\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\sigma}h_{\lambda\sigma} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$.

$$-\frac{1}{2}\partial^\lambda\partial_\lambda \bar{h}_{\mu\nu} + \partial^\lambda \partial_{(\mu}\bar{h}_{\nu)\lambda} - \frac{1}{2}\eta_{\mu\nu}\partial^\lambda\partial^\sigma \bar{h}_{\lambda\sigma} + \mathrm{O}(s^2) = 8\pi T_{\mu\nu}. \tag{1.2}$$

存在 $\{x^\mu\}$, 使得 $\partial^\nu \bar{h}_{\mu\nu} + \mathrm{O}(s^2) = 0$ (Lorentz gauge). [证: 设 $x'^\mu = x^\mu - \xi^\mu = x^\mu - \zeta^\mu s - \mathrm{O}(s^2)$, 则 $\frac{\partial ?}{\partial x'^\mu} = \frac{\partial ?}{\partial x^\lambda}\frac{\partial x^\lambda}{\partial x'^\mu} = \frac{\partial ?}{\partial x^\lambda}(\delta^\lambda{}_\mu + \frac{\partial \xi^\lambda}{\partial x'^\mu}) = \frac{\partial ?}{\partial x^\nu} + \mathrm{O}(s^2)$, $g'_{\mu\nu} = g_{\lambda\sigma}\frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} = g_{\lambda\sigma}(\delta^\lambda{}_\mu + \frac{\partial \xi^\lambda}{\partial x'^\mu})(\delta^\sigma{}_\nu + \frac{\partial \xi^\sigma}{\partial x'^\nu}) = g_{\mu\nu} + g_{\mu\sigma}\frac{\partial \xi^\sigma}{\partial x'^\nu} + g_{\lambda\nu}\frac{\partial \xi^\lambda}{\partial x'^\mu} = g_{\mu\nu} + (\eta_{\mu\sigma} + \mathrm{O}(s))(\frac{\partial \xi^\sigma}{\partial x^\nu} + \mathrm{O}(s^2)) + (\eta_{\lambda\nu} + \mathrm{O}(s))(\frac{\partial \xi^\lambda}{\partial x^\mu} + \mathrm{O}(s^2)) = g_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \mathrm{O}(s^2)$, 所以 $h'_{\mu\nu} = g'_{\mu\nu} - \eta_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu + \mathrm{O}(s^2) = h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu + \mathrm{O}(s^2)$, 因此存在 ξ^μ, 使得 $\partial'^\nu \bar{h}'_{\mu\nu} + \mathrm{O}(s^2) = 0$.] 令 $\{x^\mu\}$ 满足 $\partial^\nu \bar{h}_{\mu\nu} + \mathrm{O}(s^2) = 0$, 则

$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} + \mathrm{O}(s^2) = -16\pi T_{\mu\nu}. \tag{1.3}$$

略去 $\mathrm{O}(s^2)$ 条件: $h_{\mu\nu}$, $\partial_\lambda h_{\mu\nu}$...小. 下略 $\mathrm{O}(s^2)$.

Lorentz gauge 等价于协和坐标条件.

1.2 Radiation Gauge

[25]. 存在 $\{x^\mu\}$, 使得 "无源处" $h=0 \Rightarrow \bar{h}_{\mu\nu}=h_{\mu\nu}$ (TT gauge [26, 11]) 且 $h_{0\mu}=0$. [16], 解 $\partial^\lambda\partial_\lambda\bar{h}_{ij}=0$ 得 $h_{ij}=A_{ij}(\vec{k})e^{ik^\mu x_\mu}$ (A_{ij} 称为 polarization tensor). $h_{(ij)}=0$, $h=0$, $\partial^j h_{ij}=0 \Rightarrow A_{(ij)}=0$, $A=0$, $k^jA_{ij}=0$. 令 $\vec{e}_z \parallel \vec{k}$,

$$h_{xy}=\begin{bmatrix}+h_+ & h_\times \\ h_\times & -h_+\end{bmatrix}e^{i\omega(t-z)}. \tag{1.4}$$

[16]. Lorentz gauge $\to$ radiation gauge, $P_{ij}:=\delta_{ij}-n_in_j$, $\Lambda_{ijkl}=P_{ik}P_{jl}-\frac{1}{2}P_{ij}P_{kl}$, $h^{\mathrm{r}}_{ij}=\Lambda_{ijkl}h^{\mathrm{L}}_{kl}=\Lambda_{ijkl}\bar{h}^{\mathrm{L}}_{kl}$. [22]. Step 1: 坐标系空间旋转, 使 $\vec{e}_z \parallel \vec{n}$. Step 2: 取 x, y 分量 h_{xy}. Step 3: 去迹. [$h_+=\frac{1}{2}(h_{xx}-h_{yy})$, $h_\times=h_{xy}=h_{yx}$.]

$x'^\mu=x^\mu-\xi^\mu$, $h'_{\mu\nu}=h_{\mu\nu}+\partial_\mu\xi_\nu+\partial_\nu\xi_\mu$, $h_{\mu\nu}=h'_{\mu\nu}-\partial'_\mu\xi_\nu-\partial'_\nu\xi_\mu$.

辩曰: 令

$$\xi=\frac{1}{2}\begin{bmatrix}+h_+ & h_\times \\ h_\times & -h_+\end{bmatrix}\begin{bmatrix}x \\ y\end{bmatrix}e^{i\omega(t-z)}, \tag{1.5}$$

则式 (1.4) 中 h_{xy} 为 0, 岂非无波欤? 对曰: 诚如是, 然 h_{01}, h_{02}, h_{31}, h_{32} 非为 0, 故不可谓无波也.

另可考 [13].

1.3 Fourier Transformation

[16].

$$h_{ij}=\frac{1}{(2\pi)^3}\int \mathrm{d}^3\vec{k}\left[\mathcal{A}_{ij}(\vec{k})e^{+ik_\mu x^\mu}+\mathcal{A}^*_{ij}(\vec{k})e^{-ik_\mu x^\mu}\right] \tag{1.6}$$

$\mathrm{d}^2\vec{n}:=\sin\theta\mathrm{d}\theta\mathrm{d}\phi$,

$$\begin{aligned}
h_{ij}&=\int_0^\infty \mathrm{d}f\, f^2\int \mathrm{d}^2\vec{n}\left[\mathcal{A}_{ij}(f,\vec{n})e^{-2\pi if(t-\vec{n}\cdot\vec{x})}+\text{c.c.}\right] &(1.7)\\
&=\int_0^\infty \mathrm{d}f\left[e^{-2\pi ift}f^2\int \mathrm{d}^2\vec{n}\,\mathcal{A}_{ij}(f,\vec{n})e^{+2\pi if\vec{n}\cdot\vec{x}}+\text{c.c.}\right] &(1.8)\\
&:=\int_0^\infty \mathrm{d}f\left[\tilde{h}_{ij}(f,\vec{x})e^{-2\pi ift}+\tilde{h}^*_{ij}(f,\vec{x})e^{+2\pi ift}\right] &(1.9)\\
&:=\int \mathrm{d}f\,\tilde{h}_{ij}(f,\vec{x})e^{-2\pi ift}. &(1.10)
\end{aligned}$$

When we observe on Earth a GW emitted by a single astrophysical source, and the linear dimensions of the detector are much smaller than wavelength of the GW, choosing the origin of the coordinate system centered on the detector, $\tilde{h}_{ij}(f,\vec{x}) \approx \tilde{h}_{ij}(f) := \tilde{h}_{ij}(f,\vec{x}=\vec{0})$,

$$h_{ij} = \int \mathrm{d}f\, \tilde{h}_{ij}(f) e^{-2\pi i f t}. \tag{1.11}$$

The dependence on $\vec{x}$ must be kept in some cases (see [16]).

1.4 TT frame

TT gauge $\Rightarrow$ TT frame. [16], free test body $x^\mu(\tau)$, $\frac{\mathrm{d}x^i}{\mathrm{d}\tau}|_{\tau=0} = 0 \Rightarrow \frac{\mathrm{d}x^i}{\mathrm{d}\tau} \equiv 0$ and $\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \equiv 1$. $\frac{\mathrm{d}x^i}{\mathrm{d}t}|_{t=0} = 0$ 可得相似结论.

设一测试体在 $(0,0,0)$, 另一测试体在 $(\Delta x^1, \Delta x^2, \Delta x^3)$, 定义 $(\Delta x)^2 = \delta_{ij}\Delta x^i \Delta x^j$, 则 $(\Delta s)^2 = g_{ij}\Delta x^i \Delta x^j = (\Delta x)^2(1 + h_{ij}\frac{\Delta x^i}{\Delta x}\frac{\Delta x^j}{\Delta x})$, $\Delta s \approx \Delta x(1 + \frac{1}{2}h_{ij}\frac{\Delta x^i}{\Delta x}\frac{\Delta x^j}{\Delta x})$, $\Delta\ddot{s} \approx \frac{1}{2}\ddot{h}_{ij}\frac{\Delta x^i}{\Delta x}\frac{\Delta x^j}{\Delta x}\Delta x$. 定义 $n^i = \frac{\Delta x^i}{\Delta x}$, 则 $\Delta\ddot{s} \approx n^i(\frac{1}{2}\ddot{h}_{ij}\Delta x^j)$. 定义 $\Delta s^i = \Delta s\, n^i$, 则 $\Delta s = \Delta s\, n^i n_i = \Delta s^i n_i = n^i \Delta s_i$, 则 $\Delta s_i \approx \frac{1}{2}\ddot{h}_{ij}\Delta x^j \approx \frac{1}{2}\ddot{h}_{ij}\Delta s^j$.

1.5 Proper detector frame

设一基准测试体, 取其固有坐标系, 另一测试体世界线 $x^i(t)$. [13], Let us now imagine that for times $t \le 0$ there were no waves ($h^{\mathrm{TT}}_{ij} = 0$) in the vicinity of the two observers and that the observers were at rest with respect to each other before the wave has come, so $x^i(t) = x^i_0 = \text{const.}$, $\mathrm{d}x^i/\mathrm{d}t = 0$, for $t \le 0$. At $t = 0$ some wave arrives. Then according to geodesic deviation equation,

$$\frac{\mathrm{d}^2 x^i}{\mathrm{d}t^2} = \frac{1}{2}\frac{\partial^2 h^{\mathrm{TT}}_{ij}}{\partial t^2} x^j_0, \tag{1.12}$$

$$x^i = (\delta_{ij} + \frac{1}{2}h^{\mathrm{TT}}_{ij})x^j_0. \tag{1.13}$$

考虑对观测器材有影响的其他效应, 见 [16], [18].

第二章　宇宙学效应

2.1 Luminosity Distance

[16],

$$\int_{t_{\text{emis}}}^{t_{\text{obs}}} \frac{c\,\mathrm{d}t}{a(t)} = \int_{t_{\text{emis}}+\Delta t_{\text{emis}}}^{t_{\text{obs}}+\Delta t_{\text{obs}}} \frac{c\,\mathrm{d}t}{a(t)} = \int_0^r \frac{\mathrm{d}r}{(1-kr^2)^{1/2}}, \tag{2.1}$$

故 $\Delta t_{\text{obs}} = \Delta t_{\text{emis}}(1+z)$. 另由 $E = \hbar\omega$, 有 $\Delta E_{\text{obs}} = \Delta E_{\text{emis}}(1+z)$, 故 $d_{\text{L}} = a(t_{\text{obs}})r(1+z)$.

不用光子论, 见 [8] 7.4 节.

2.2 Propagation

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)], \tag{2.2}$$

$\mathrm{d}t := a\mathrm{d}\eta$, $(1/\sqrt{1-kr^2})\mathrm{d}r := \mathrm{d}\psi$,

$$\mathrm{d}s^2 = a^2[-\mathrm{d}\eta^2 + \mathrm{d}\psi^2 + r(\psi)^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)]. \tag{2.3}$$

[16], $\Box\Phi(\eta,\psi) := 0$,

$$-\partial_\eta(a^2r^2\partial_\eta\Phi) + \partial_\psi(a^2r^2\partial_\psi\Phi) = 0, \tag{2.4}$$

$$-ar\frac{\partial^2\Phi}{\partial\eta^2} + ar\frac{\partial^2\Phi}{\partial\psi^2} + 2a\frac{\partial\Phi}{\partial\psi}\frac{\mathrm{d}r}{\mathrm{d}\psi} - 2r\frac{\partial\Phi}{\partial\eta}\frac{\mathrm{d}a}{\mathrm{d}\eta} = 0, \tag{2.5}$$

$\Phi := g/ar$,

$$-ag\frac{\mathrm{d}^2r}{\mathrm{d}\psi^2} - ar\frac{\partial^2g}{\partial\eta^2} + ar\frac{\partial^2g}{\partial\psi^2} + gr\frac{\mathrm{d}^2a}{\mathrm{d}\eta^2} = 0. \tag{2.6}$$

$\mathrm{d}^2r/\mathrm{d}\psi^2 = kr$,

$$\partial_\psi^2 g - kg + (\partial_\eta^2 a/a)g - \partial_\eta^2 g = 0, \tag{2.7}$$

$\partial_\eta^2 a/a \sim \eta^2$, 故 $k=0$ 时, 若 $\omega^2 \gg 1/\eta^2$,

$$g \simeq e^{i\omega(\eta-\psi)} = e^{i\omega(\eta-r)}. \tag{2.8}$$

$g := N(\eta)\Psi(\psi)$,

$$N\partial_\psi^2\Psi - kN\Psi - \Psi\partial_\eta^2 N \simeq 0, \tag{2.9}$$

$$\Psi^{-1}\partial_\psi^2\Psi - k \simeq N^{-1}\partial_\eta^2 N :\simeq C, \tag{2.10}$$

$$\Psi \simeq e^{\sqrt{k-C}\psi}, \tag{2.11}$$

$$N \simeq e^{\sqrt{-C}\eta}, \tag{2.12}$$

$$g \simeq e^{i(\omega\eta - \sqrt{\omega^2-k}\psi)}. \tag{2.13}$$

第三章　能量

3.1　Isaacson's Method

[25],

$$G^{[1]}_{ab}(h^{[1]}_{cd}) + G^{[1]}_{ab}(h^{[2]}_{cd}) + G^{[2]}_{ab}(h^{[1]}_{cd}) = 8\pi T_{ab}, \tag{3.1}$$

$$G^{[1]}_{ab}(h^{[1]}_{cd} + h^{[2]}_{cd}) = 8\pi(T_{ab} + t_{ab}) := 8\pi(T_{ab} - \frac{G^{[2]}_{ab}(h^{[1]}_{cd})}{8\pi}), \tag{3.2}$$

Thus, in the 2nd order, $h^{[2]}_{ab}$ causes the same correction to g_{ab} as would be produced by ordinary matter with effect stress-energy tensor t_{ab}. If not $T_{ab} \gg t_{ab}$, derivations in chapter 一 are not valid.

[16, 12], $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$. $R_{\mu\nu} = R^{(0)}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} \dots$,

$$R^{(0)}_{\mu\nu} + [R^{(2)}_{\mu\nu}]^{\text{low}} = 8\pi(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})^{\text{low}}, \tag{3.3}$$

$$R^{(1)}_{\mu\nu} + [R^{(2)}_{\mu\nu}]^{\text{high}} = 8\pi(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})^{\text{high}}, \tag{3.4}$$

(3.3) $\Rightarrow$

$$\begin{aligned} R^{(0)}_{\mu\nu} &= 8\pi\langle(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})^{\text{low}}\rangle - \langle[R^{(2)}_{\mu\nu}]^{\text{low}}\rangle & (3.5)\\ &= 8\pi\langle(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu})\rangle - \langle[R^{(2)}_{\mu\nu}]\rangle & (3.6)\\ &:= 8\pi(T^{(0)}_{\mu\nu} - \frac{1}{2}T^{(0)}g^{(0)}_{\mu\nu}) + 8\pi(t_{\mu\nu} - \frac{1}{2}tg^{(0)}_{\mu\nu}), & (3.7)\end{aligned}$$

$\Rightarrow$

$$G^{(0)}_{\mu\nu} = 8\pi(T^{(0)}_{\mu\nu} + t_{\mu\nu}). \tag{3.8}$$

We are interested in the energy and momentum carried by the GWs at large distances from the source (e.g. at the position of the detector), where we can approximate the background space-time as flat. In TT gauge,

$$t_{\mu\nu} = \frac{1}{32\pi} \langle \partial_\mu h^{\alpha\beta} \partial_\nu h_{\alpha\beta} \rangle. \tag{3.9}$$

第四章　多极矩

注意, 以下内容需要 $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. 更多内容见 [16] 第 3 章.

4.1 Quadrupole Approximation

[25]. 由 (1.3) 得

$$\bar{h}_{\mu\nu}(t, \vec{r}) = 4\int \frac{T_{\mu\nu}(t - |\vec{r} - \vec{r}'|, \vec{r}')}{|\vec{r} - \vec{r}'|}\,\mathrm{d}V'. \tag{4.1}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega, \vec{r}) := \frac{1}{\sqrt{2\pi}}\int \bar{h}_{\mu\nu}(t, \vec{r})e^{i\omega t}\mathrm{d}t \tag{4.2}$$

$$= 4\int \frac{\hat{T}_{\mu\nu}(\omega, \vec{r}')}{|\vec{r} - \vec{r}'|}e^{i\omega|\vec{r} - \vec{r}'|}\,\mathrm{d}V'. \tag{4.3}$$

由 $\partial^\nu \bar{h}_{\mu\nu} = 0$,

$$-i\omega\hat{\bar{h}}_{0\mu} = \sum_i \frac{\partial\hat{\bar{h}}_{i\mu}}{\partial x^i}. \tag{4.4}$$

$|\vec{r}| \gg |\vec{r}'|$ 且 $\omega \ll 1/|\vec{r}'|$,

$$\hat{\bar{h}}_{ij}(\omega, \vec{r}) = 4\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{ij}(\omega, \vec{r}')\,\mathrm{d}V'. \tag{4.5}$$

$$\int \hat{T}_{ij}\,\mathrm{d}V' = \int \sum_k (\hat{T}_{kj}\frac{\partial x'^i}{\partial x'^k})\,\mathrm{d}V' \tag{4.6}$$

$$= \sum_k \left[\int \frac{\partial}{\partial x'^k}(\hat{T}_{kj}x'^i)\,\mathrm{d}V' - \int \frac{\partial\hat{T}_{kj}}{\partial x'^k}x'^i\,\mathrm{d}V'\right] \tag{4.7}$$

$$= \sum_k \int \partial'_k\,(\hat{T}_{kj}x'^i)\,\mathrm{d}V' - \sum_k \int \frac{\partial\hat{T}_{kj}}{\partial x'^k}x'^i\,\mathrm{d}V' \tag{4.8}$$

$$= \int \hat{T}_{kj} x'^i \, \mathrm{d}S' - \sum_k \int \frac{\partial \hat{T}_{kj}}{\partial x'^k} x'^i \, \mathrm{d}V' \tag{4.9}$$

$$= -\sum_k \int \frac{\partial \hat{T}_{kj}}{\partial x'^k} x'^i \, \mathrm{d}V' \tag{4.10}$$

$$= -\int (\sum_k \partial'_k \hat{T}_{kj}) x'^i \, \mathrm{d}V' \tag{4.11}$$

$$= -\int (\partial_0 \hat{T}_{0j}) x'^i \, \mathrm{d}V' \tag{4.12}$$

$$= -i\omega \int \hat{T}_{0j} x'^i \, \mathrm{d}V' \tag{4.13}$$

$$= \int \hat{T}_{(ij)} \, \mathrm{d}V' \tag{4.14}$$

$$= -i\omega \int \hat{T}_{0(j} x'^{i)} \, \mathrm{d}V' \tag{4.15}$$

$$= -\frac{i\omega}{2} \int (\hat{T}_{0j} x'^i + \hat{T}_{0i} x'^j) \, \mathrm{d}V', \tag{4.16}$$

$$-\frac{i\omega}{2} \int (\hat{T}_{0j} x'^i + \hat{T}_{0i} x'^j) \, \mathrm{d}V' = -\frac{i\omega}{2} \int \sum_k (\hat{T}_{0k} x'^i \frac{\partial x'^j}{\partial x'^k} + \hat{T}_{0k} \frac{\partial x'^i}{\partial x'^k} x'^j) \, \mathrm{d}V' \tag{4.17}$$

$$= -\frac{i\omega}{2} \sum_k \left[\int \frac{\partial}{\partial x'^k} (\hat{T}_{0k} x'^i x'^j) \, \mathrm{d}V' - \int \frac{\partial \hat{T}_{0k}}{\partial x'^k} x'^i x'^j \, \mathrm{d}V' \right] \tag{4.18}$$

$$= -\frac{i\omega}{2} \sum_k \int \partial'_k \, (\hat{T}_{0k} x'^i x'^j) \, \mathrm{d}V' + \frac{i\omega}{2} \sum_k \int \frac{\partial \hat{T}_{0k}}{\partial x'^k} x'^i x'^j \, \mathrm{d}V' \tag{4.19}$$

$$= -\frac{i\omega}{2} \sum_k \int \hat{T}_{0k} x'^i x'^j \, \mathrm{d}S' + \frac{i\omega}{2} \sum_k \int \frac{\partial \hat{T}_{0k}}{\partial x'^k} x'^i x'^j \, \mathrm{d}V' \tag{4.20}$$

$$= \frac{i\omega}{2} \sum_k \int \frac{\partial \hat{T}_{0k}}{\partial x'^k} x'^i x'^j \, \mathrm{d}V' \tag{4.21}$$

$$= \frac{i\omega}{2} \int (\sum_k \partial'_k \hat{T}_{0k}) x'^i x'^j \, \mathrm{d}V' \tag{4.22}$$

$$= \frac{i\omega}{2} \int (\partial_0 \hat{T}_{00}) x'^i x'^j \, \mathrm{d}V' \tag{4.23}$$

$$= -\frac{\omega^2}{2}\int \hat{T}_{00}\, x'^i x'^j \,\mathrm{d}V'. \tag{4.24}$$

$$q_{ij}(t) := \int T_{00}\, x'^i x'^j \,\mathrm{d}V', \tag{4.25}$$

$$\hat{\bar{h}}_{ij}(\omega,\vec{r}) = -2\omega^2 \frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\hat{q}_{ij}(\omega), \tag{4.26}$$

$$\bar{h}_{ij}(t,\vec{r}) = \frac{2}{|\vec{r}|}\frac{\mathrm{d}^2}{\mathrm{d}t^2}q_{ij}(t-|\vec{r}|). \tag{4.27}$$

4.2 电磁—引力对比

$$A_\mu(t,\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J_\mu(t-|\vec{r}-\vec{r}'|,\vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{d}V' \tag{4.28}$$

$$\bar{h}_{\mu\nu}(t,\vec{r}) = 4G\int \frac{T_{\mu\nu}(t-|\vec{r}-\vec{r}'|,\vec{r}')}{|\vec{r}-\vec{r}'|}\,\mathrm{d}V' \tag{4.29}$$

$$A_\mu(t,\vec{r}) = \frac{1}{\sqrt{2\pi}}\int \hat{A}_\mu(\omega,\vec{r})e^{-i\omega t}\mathrm{d}t \tag{4.30}$$

$$\bar{h}_{\mu\nu}(t,\vec{r}) = \frac{1}{\sqrt{2\pi}}\int \hat{\bar{h}}_{\mu\nu}(\omega,\vec{r})e^{-i\omega t}\mathrm{d}t \tag{4.31}$$

$$\hat{A}_\mu(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\hat{J}_\mu(\omega,\vec{r}')}{|\vec{r}-\vec{r}'|}e^{i\omega|\vec{r}-\vec{r}'|}\,\mathrm{d}V' \tag{4.32}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) = 4G\int \frac{\hat{T}_{\mu\nu}(\omega,\vec{r}')}{|\vec{r}-\vec{r}'|}e^{i\omega|\vec{r}-\vec{r}'|}\,\mathrm{d}V' \tag{4.33}$$

$$\hat{A}_\mu(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{J}_\mu(\omega,\vec{r}')e^{-i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}')}\,\mathrm{d}V' \tag{4.34}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{\mu\nu}(\omega,\vec{r}')e^{-i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}')}\,\mathrm{d}V' \tag{4.35}$$

$$\hat{A}_\mu(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{J}_\mu(\omega,\vec{r}')\left[1-i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}')-\dots\right]\,\mathrm{d}V' \tag{4.36}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{\mu\nu}(\omega,\vec{r}')\left[1-i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}')-\dots\right]\,\mathrm{d}V' \tag{4.37}$$

4.2.1 电偶极—引力对比

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{J}_i \,\mathrm{d}V' \tag{4.38}$$

$$\hat{\bar{h}}_{ij} = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{ij} \,\mathrm{d}V' \tag{4.39}$$

$$\int \hat{J}_i \,\mathrm{d}V' = -i\omega \int \hat{J}_0 x'^i \,\mathrm{d}V' \tag{4.40}$$

$$\int \hat{T}_{ij} \,\mathrm{d}V' = -\frac{\omega^2}{2}\int \hat{T}_{00}\, x'^i x'^j \,\mathrm{d}V' \tag{4.41}$$

$$\hat{p}_i = \int \hat{J}_0 x'^i \,\mathrm{d}V' \tag{4.42}$$

$$\hat{q}_{ij} = \int \hat{T}_{00}\, x'^i x'^j \,\mathrm{d}V' \tag{4.43}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega\hat{p}_i) \tag{4.44}$$

$$\hat{\bar{h}}_{ij} = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-\frac{\omega^2}{2}\hat{q}_{ij}) \tag{4.45}$$

$$A_i = \frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|}\frac{\mathrm{d}}{\mathrm{d}t}p_i(t-|\vec{r}|) \tag{4.46}$$

$$\bar{h}_{ij} = 4G\frac{1}{|\vec{r}|}\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}q_{ij}(t-|\vec{r}|) \tag{4.47}$$

4.2.2 电四极—引力对比

$$\hat{A}_i(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega)\int \hat{J}_i(\omega,\vec{r}')(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}') \,\mathrm{d}V' \tag{4.48}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega)\int \hat{J}'_i n^j x'_j \,\mathrm{d}V' \tag{4.49}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega)\int n^j x'_j \hat{J}'_i \,\mathrm{d}V' \tag{4.50}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega)n^j\left[\int x'_{(j}\hat{J}'_{i)}\,\mathrm{d}V'\right] \tag{4.51}$$

$$\begin{aligned}
\int x'_{(j}\hat{J}'_{i)}\,\mathrm{d}V' &= \frac{1}{2}\int(\hat{J}'_j x'_i + \hat{J}'_i x'_j)\,\mathrm{d}V' &(4.52)\\
&= \frac{1}{2}\int\sum_k(\hat{J}'_k x'^i\frac{\partial x'^j}{\partial x'^k} + \hat{J}'_k\frac{\partial x'^i}{\partial x'^k}x'^j)\,\mathrm{d}V' &(4.53)\\
&= \frac{1}{2}\sum_k\left[\int\frac{\partial}{\partial x'^k}(\hat{J}'_k x'^i x'^j)\,\mathrm{d}V' - \int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V'\right] &(4.54)\\
&= \frac{1}{2}\sum_k\int\partial'_k\,(\hat{J}'_k x'^i x'^j)\,\mathrm{d}V' - \frac{1}{2}\sum_k\int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V' &(4.55)\\
&= \frac{1}{2}\sum_k\int\hat{J}'_k x'^i x'^j\,\mathrm{d}S' - \frac{1}{2}\sum_k\int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V' &(4.56)\\
&= -\frac{1}{2}\sum_k\int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V' &(4.57)\\
&= -\frac{1}{2}\int(\sum_k\partial'_k\hat{J}'_k)x'^i x'^j\,\mathrm{d}V' &(4.58)\\
&= -\frac{1}{2}\int(\partial_0\hat{J}'_0)x'^i x'^j\,\mathrm{d}V' &(4.59)\\
&= -\frac{i\omega}{2}\int\hat{J}'_0 x'^i x'^j\,\mathrm{d}V' &(4.60)
\end{aligned}$$

$$\hat{D}_{ij} = \int\hat{J}'_0\,x'^i x'^j\,\mathrm{d}V' \tag{4.61}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-\frac{\omega^2}{2}n^j\hat{D}_{ij}) \tag{4.62}$$

$$A_i = \frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|}n^j\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}D_{ij}(t-|\vec{r}|) \tag{4.63}$$

第五章　频段

5.1　波源

[26, 22, 4, 10, 21, 23, 7, 20]

5.2　探测器

[26, 17]

(i) 超高频段 ($>$ 1 THz): 检测方法包括 THz 共振器, 光学共振器以及尚未发明的巧妙方法. 引力波, 暗能量和膨胀.

(ii) 甚高频段 (100 kHz–1 THz): 微波共振器/波导探测器, 光学干涉仪和 Gaussian 光束探测器对该频段敏感.

(iii) 高频段 (10 Hz–100 kHz): 低温共振器和激光干涉地面探测器对该频段最敏感.

(iv) 中频段 (0.1 Hz–10 Hz): 空间干涉仪探测器 (臂长 1000–100000 km).

(v) 低频段 (100 nHz–0.1 Hz): 激光干涉空间探测器对该频段最敏感.

(vi) 极低频段 (300 pHz–100 nHz): 脉冲星计时观测对该频段最敏感.

(vii) 超低频段 (10 fHz–300 pHz): 类星体自行的天文观测对该频段最敏感.

(viii) 极低 (Hubble) 频段 (1 aHz–10 fHz): 宇宙微波背景实验对该频段最敏感.

(ix) 超越 Hubble 频率带 ($<$ 1Hz): 暴涨宇宙学模型给出了该频带内引力波的强度. 可以通过验证暴涨宇宙学模型间接验证这些引力波的存在.

第六章　PN

[6, 16].

第七章　双星系统

7.1　基本公式

$$\mathcal{M} := \mu^{3/5} M^{2/5} \tag{7.1}$$

$$h_+ = \frac{4\mathcal{M}}{D}[\pi \mathcal{M} F(t)]^{2/3} \frac{1+\cos^2\iota}{2} \cos\Phi(t) \tag{7.2}$$

$$h_\times = \frac{4\mathcal{M}}{D}[\pi \mathcal{M} F(t)]^{2/3} \cos\iota \sin\Phi(t) \tag{7.3}$$

$$h = F_+ h_+ + F_\times h_\times \tag{7.4}$$

7.2　Post-Newtonian Approximation

7.3　Stationary Phase Approximation

[19], if $\zeta(t)$ varies slowly near $t = t_0$ where the phase has a stationary point: $\phi'(t_0) = 0$,

$$\begin{aligned}
\int \zeta(t) e^{i\phi(t;f)}\,\mathrm{d}t &= \int \zeta(t) e^{i[\phi(t_0)+\phi'(t_0)(t-t_0)+\frac{1}{2}\phi''(t_0)(t-t_0)^2+\dots]}\,\mathrm{d}t && (7.5)\\
&\simeq e^{i\phi(t_0)} \int \zeta(t) e^{i[\frac{1}{2}\phi''(t_0)(t-t_0)^2]}\,\mathrm{d}t && (7.6)\\
&\simeq e^{i\phi(t_0)} \int \zeta(t_0) e^{\frac{-\sqrt{-i\phi''(t_0)}^2(t-t_0)^2}{2}}\,\mathrm{d}t && (7.7)\\
&= \frac{\sqrt{2\pi}}{\sqrt{-i\phi''(t_0)}} \zeta(t_0) e^{i\phi(t_0)}. && (7.8)
\end{aligned}$$

$$h = \frac{\mathcal{M}}{D}[\pi \mathcal{M} F(t)]^{2/3} Q \cos\Phi(t) \tag{7.9}$$

$$= \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}[e^{i\Phi(t)} + e^{-i\Phi(t)}] \tag{7.10}$$

$$\tilde{h}(f) = \int h(t)e^{i2\pi ft}\,\mathrm{d}t \tag{7.11}$$

$$= \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}[e^{i\Phi(t)} + e^{-i\Phi(t)}]e^{i2\pi ft}\,\mathrm{d}t \tag{7.12}$$

$$= \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}\{e^{i[2\pi ft+\Phi(t)]} + e^{i[2\pi ft-\Phi(t)]}\}\,\mathrm{d}t \tag{7.13}$$

$$\simeq \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}e^{i[2\pi ft-\Phi(t)]}\,\mathrm{d}t \tag{7.14}$$

$$= \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F]^{2/3}Q\frac{1}{2}e^{i[2\pi ft(F)-\Phi(F)]}\frac{\mathrm{d}t}{\mathrm{d}F}\,\mathrm{d}F \tag{7.15}$$

$$\simeq \frac{\sqrt{2\pi}}{\sqrt{-i[2\pi ft(F) - \Phi(F)]''_{F=f}}} \tag{7.16}$$

$$\left[\frac{\mathcal{M}}{D}(\pi\mathcal{M}F)^{2/3}Q\frac{1}{2}\frac{\mathrm{d}t}{\mathrm{d}F}\right]_{F=f} e^{i[2\pi ft(f)-\Phi(f)]} \tag{7.17}$$

$$\simeq \frac{\sqrt{2\pi}}{\sqrt{-i\left\{2\pi f\left[-\frac{5}{256}\mathcal{M}(\pi\mathcal{M}F)^{-8/3}\right] - \left[\frac{1}{16}(\pi\mathcal{M}F)^{-5/3}\right]\right\}''_{F=f}}} \tag{7.18}$$

$$\left\{\frac{\mathcal{M}}{D}(\pi\mathcal{M}F)^{2/3}Q\frac{1}{2}\left[\frac{5\pi\mathcal{M}^2}{96}(\pi\mathcal{M}F)^{-11/3}\right]\right\}_{F=f} e^{i[2\pi ft(f)-\Phi(f)]} \tag{7.19}$$

$$= \frac{\sqrt{30}}{48\pi^{2/3}}\frac{\mathcal{M}^{5/6}Q}{D}f^{-7/6}e^{i[2\pi ft(f)-\Phi(f)-\frac{\pi}{4}]} \quad (\texttt{pnspa.py}) \tag{7.20}$$

另可考 [5]. 其中 $\frac{\mathrm{d}\Phi}{\mathrm{d}t} = 2\pi F$.

第八章 Perturbation Theory

[11]

第九章　波形

LALSimIMR.h. lalsuite-extra.

第十章　干涉仪

10.1　类光测地线

$h_{ab} = h\cos[\omega_{\mathrm{GW}}(x^0 - x^3)][(\mathrm{d}x^1)_a(\mathrm{d}x^1)_b - (\mathrm{d}x^2)_a(\mathrm{d}x^2)_b]$, 不妨设 $x^3 = 0$. 设切矢 $T^a = (\partial/\partial\beta)^a$, $T^\mu = \mathrm{d}x^\mu/\mathrm{d}\beta$,

$$\frac{\mathrm{d}T^\mu}{\mathrm{d}\beta} + \Gamma^\mu{}_{\nu\sigma}T^\nu T^\sigma = 0, \tag{10.1}$$

$$g_{\mu\nu}T^\mu T^\nu = 0. \tag{10.2}$$

$$\Gamma^\mu{}_{\nu\sigma} = \frac{1}{2}\eta^{\mu\lambda}(\partial_\nu h_{\sigma\lambda} + \partial_\sigma h_{\nu\lambda} - \partial_\lambda h_{\nu\sigma}) \tag{10.3}$$

$$\Gamma^0{}_{\nu\sigma} = \frac{1}{2}\partial_0 h_{\nu\sigma} \tag{10.4}$$

$$\Gamma^1{}_{\nu\sigma} = \frac{1}{2}(\partial_\nu h_{\sigma 1} + \partial_\sigma h_{\mu 1}) \tag{10.5}$$

$$\frac{\mathrm{d}T^0}{\mathrm{d}\beta} + \frac{1}{2}\partial_0 h_{11}T^1T^1 = 0, \tag{10.6}$$

$$\frac{\mathrm{d}T^1}{\mathrm{d}\beta} + \partial_0 h_{11}T^0T^1 = 0, \tag{10.7}$$

$$-T^0T^0 + (1 + h_{11})T^1T^1 = 0. \tag{10.8}$$

猜测: $x^0 \approx x^1 \approx \beta$, $T^0 \approx T^1 \approx 1$,

$$\frac{\mathrm{d}T^0}{\mathrm{d}\beta} \approx -\frac{1}{2}\partial_0 h_{11}, \tag{10.9}$$

$$\frac{\mathrm{d}T^1}{\mathrm{d}\beta} \approx -\partial_0 h_{11}, \tag{10.10}$$

$\partial/\partial x^0 \approx \partial/\partial\beta$,

$$T^0 \approx 1 - \frac{1}{2}h_{11}, \tag{10.11}$$

$$T^1 \approx 1 - h_{11}. \tag{10.12}$$

验证: $(\partial/\partial x^1)^a$ 是 Killing 的, 沿测地线 $g_{ab}(\partial/\partial x^1)^a(\partial/\partial\beta)^b = g_{11}(\mathrm{d}x^1)_b(\partial/\partial\beta)^b = g_{11}(\mathrm{d}x^1/\mathrm{d}\beta) = g_{11}T^1 = 0$ 守恒. 于是

$$T^a = (1 - \frac{1}{2}h_{11})(\partial/\partial x^0)^a + (1 - h_{11})(\partial/\partial x^1)^a \tag{10.13}$$

$$T_a = -(1 - \frac{1}{2}h_{11})(\mathrm{d}x^0)_a + (\mathrm{d}x^1)_a, \tag{10.14}$$

如此可过渡到 10.2 节.

$E^a = C^a \cos\theta$, $\nabla_a\theta = \partial_a\theta = (\mathrm{d}\theta)_a = \frac{\partial\theta}{\partial x^0}(\mathrm{d}x^0)_a + \frac{\partial\theta}{\partial 1^0}(\mathrm{d}x^1)_a = \omega T_a$, $\theta = \int T_0\,\mathrm{d}x^0 + \int T_1\,\mathrm{d}x^1 + \theta_0$.

辩曰: 引力波至时, 臂长有伸缩, 然光波长无伸缩欤? 对曰: 光波长伸缩同! 以 "坐标距离" 考之, 臂长与光波长不变, 而波速变为 $1 \mp \frac{1}{2}h_{11}$ 倍, 往返时间变为 $1 \pm \frac{1}{2}h_{11}$ 倍, 知有图样. 以 "物理距离" 考之, 臂长与光波长变为 $1 \mp \frac{1}{2}h_{11}$ 倍, 波速不变, 而往返时间变为 $1 \pm \frac{1}{2}h_{11}$ 倍, 亦知有图样. LSC FAQ 亦云: "引力波确实会拉伸和挤压臂中光的波长. 但是干涉图案并不是由于臂的长度和光的波长之间的差异而产生的. 相反, 它是由光波的 "波峰和波谷" 从一只臂到达的时间与光在另一只臂传播到达的时间不同引起的. 因此, 激光的作用与其说是尺子, 不如说是秒表. "

10.2 干涉图样 (TT frame)

[16]. 设入射电场 $\vec{E}_0 e^{-i\omega_\mathrm{L}t + i\vec{k}_\mathrm{L}\cdot\vec{x}}$. 设 splitter 在 $(0,0)$, reflector x 在 $(L_x, 0)$, reflector y 在 $(0, L_y)$.

无 GW 时, 有 $\vec{E}_\mathrm{in} = \vec{E}_0 e^{-i\omega_\mathrm{L}t}$. $\vec{E}_\mathrm{out} = \vec{E}_{\mathrm{form\ x}} + \vec{E}_{\mathrm{form\ y}}$, t 时的 $\vec{E}_{\mathrm{form\ x}}$ 在 $t - \frac{2L_x}{c}$ 时入 splitter, t 时的 $\vec{E}_{\mathrm{form\ y}}$ 在 $t - \frac{2L_y}{c}$ 时入 splitter, 考虑反相, $\vec{E}_{\mathrm{form\ x}} = -\frac{1}{2}\vec{E}_0 e^{-i\omega_\mathrm{L}t + 2ik_\mathrm{L}L_x}$, $\vec{E}_{\mathrm{form\ y}} = +\frac{1}{2}\vec{E}_0 e^{-i\omega_\mathrm{L}t + 2ik_\mathrm{L}L_y}$, $\vec{E}_\mathrm{out} = \vec{E}_0 \sin(\phi_0) e^{-i\omega_\mathrm{L}(t - \frac{2L}{c}) - i\frac{\pi}{2}}$, where $\phi_0 = k_\mathrm{L}(L_y - L_x)$ and $L = (L_x + L_y)/2$.

有 GW 时, 设 GW 只有 +mode 且方向为 z_+, $h_+ = h_0\cos[\omega_\mathrm{gw}(t - z/c)]$,

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + (1 + h_+)\mathrm{d}x^2 + (1 - h_+)\mathrm{d}y^2 + \mathrm{d}z^2. \tag{10.15}$$

$h_+(t) := h_+|_{z=0}$. 光 $\mathrm{d}s^2 = 0$, 保留一阶项, x 方向光轨迹

$$\mathrm{d}x = \pm c\mathrm{d}t[1 - \frac{1}{2}h_+(t)], \tag{10.16}$$

y 方向光轨迹

$$\mathrm{d}y = \pm c\mathrm{d}t[1 + \frac{1}{2}h_+(t)], \tag{10.17}$$

$+$ 号是 splitter 到 reflector, $-$ 号是 reflector 到 splitter.

设 photon $t = t_0$ 到 splitter, $t = t_1$ 到 x reflector, $t = t_2$ 到 splitter, 则

$$t_2 - t_0 = \frac{2L_x}{c} + \frac{1}{2}\int_{t_0}^{t_2} \mathrm{d}t' h_+(t') \tag{10.18}$$

$$\approx \frac{2L_x}{c} + \frac{1}{2}\int_{t_0}^{t_0+\frac{2L_x}{c}} \mathrm{d}t' h_+(t') \tag{10.19}$$

$$= \frac{2L_x}{c} + \frac{L_x}{c}h_+(t_0 + \frac{L_x}{c})\,\mathrm{sinc}(\omega_{\mathrm{gw}}\frac{L_x}{c}). \tag{10.20}$$

特殊情况: $\omega_{\mathrm{gw}}\frac{L_x}{c} \ll 1$, $t_2 - t_0 \approx \frac{2L_x}{c} + \frac{L_x}{c}h_+(t_1)$. $\omega_{\mathrm{gw}}\frac{L_x}{c} \gg 1$, $t_2 - t_0 \approx \frac{2L_x}{c}$.

y 方向, x 改成 y, $+h_+$ 改成 $-h_+$.

$\vec{E}_{\mathrm{in}} = \vec{E}_0 e^{-i\omega_{\mathrm{L}}t}$, t 时的 $\vec{E}_{\text{form x}}$ 在 $t - \frac{2L_x}{c} - \frac{L_x}{c}h_+(t - \frac{L_x}{c})\,\mathrm{sinc}(\omega_{\mathrm{gw}}\frac{L_x}{c})$ 时入 splitter, t 时的 $\vec{E}_{\text{form y}}$ 在 $t - \frac{2L_y}{c} + \frac{L_y}{c}h_+(t - \frac{L_y}{c})\,\mathrm{sinc}(\omega_{\mathrm{gw}}\frac{L_y}{c})$ 时入 splitter, $\vec{E}_{\text{form x}} = -\frac{1}{2}\vec{E}_0 e^{-i\omega_{\mathrm{L}}(t-\frac{2L}{c})+i\phi_0+i\Delta\phi(t)}$, $\vec{E}_{\text{form y}} = +\frac{1}{2}\vec{E}_0 e^{-i\omega_{\mathrm{L}}(t-\frac{2L}{c})-i\phi_0-i\Delta\phi(t)}$, where $\phi_0 = k_{\mathrm{L}}(L_y - L_x)$, $\Delta\phi(t) = h_+(t - \frac{L}{c})k_{\mathrm{L}}L\,\mathrm{sinc}(\omega_{\mathrm{gw}}\frac{L}{c})$, and $L = (L_x + L_y)/2$. 特殊情况: $\omega_{\mathrm{gw}}\frac{L_x}{c} \ll 1$, $\Delta\phi(t) \approx h_+(t - \frac{L}{c})k_{\mathrm{L}}L$. $\omega_{\mathrm{gw}}\frac{L_x}{c} \gg 1$, $\Delta\phi(t) \approx 0$. Finally, $\vec{E}_{\mathrm{out}} = \vec{E}_0 \sin[\phi_0 + \Delta\phi(t)]e^{-i\omega_{\mathrm{L}}(t-\frac{2L}{c})-i\frac{\pi}{2}}$.

辩曰: 反射镜运动, 有 Doppler 效应, 何以应对? 对曰: 需有 long wave approximation $\omega_{\mathrm{gw}}\frac{L}{c} \ll 1$, 此情形下单纯传播引入相移 $\Delta\phi_{\mathrm{p}} \propto h$, Doppler 效应引入相移 $\Delta\phi_{\mathrm{d}} \propto \frac{\frac{\mathrm{d}h}{\mathrm{d}t}L}{c} \sim \frac{\omega_{\mathrm{gw}}hL}{c} \ll \Delta\phi_{\mathrm{p}}$ 可忽略. 详细论证: reflector 接收, 相移 $2\pi[(1 \pm h)L/\lambda'_{\mathrm{L}}]$, splitter 再接收, 相移 $2\pi[(1 \pm h)L/\lambda''_{\mathrm{L}}]$, 其中 $\lambda''_{\mathrm{L}}/\lambda'_{\mathrm{L}} = \lambda'_{\mathrm{L}}/\lambda_{\mathrm{L}} = 1 \pm (\mathrm{d}h/\mathrm{d}t)L/c$, $(1/\lambda''_{\mathrm{L}})/(1/\lambda'_{\mathrm{L}}) = (1/\lambda'_{\mathrm{L}})/(1/\lambda_{\mathrm{L}}) = 1 \mp (\mathrm{d}h/\mathrm{d}t)L/c$, 总相移 $2\pi[2(1 \pm h)(1 \mp (\mathrm{d}h/\mathrm{d}t)L/c)L/\lambda_{\mathrm{L}}] \approx 2\pi[2(1 \pm h \mp (\mathrm{d}h/\mathrm{d}t)L/c)L/\lambda_{\mathrm{L}}]$. 若 $\omega_{\mathrm{gw}}L/c \ll 1$ $(v/c \ll h)$, 则 $h \gg (\mathrm{d}h/\mathrm{d}t)L/c \approx h(\omega_{\mathrm{gw}}L/c)$, 总相移 $2\pi[2(1 \pm h)L/\lambda_{\mathrm{L}}]$. 注: $(\mathrm{d}h/\mathrm{d}t)|_{t_2} - (\mathrm{d}h/\mathrm{d}t)|_{t_1} \approx (\mathrm{d}^2h/\mathrm{d}t^2)(L/c) \approx (\mathrm{d}h/\mathrm{d}t)(\omega_{\mathrm{gw}}L/c) \ll (\mathrm{d}h/\mathrm{d}t)$.

第十一章　数据分析

[1] (翻译见 `others/data_analysis/`). [16], [14, 13], [9], [24].

11.1　主要步骤

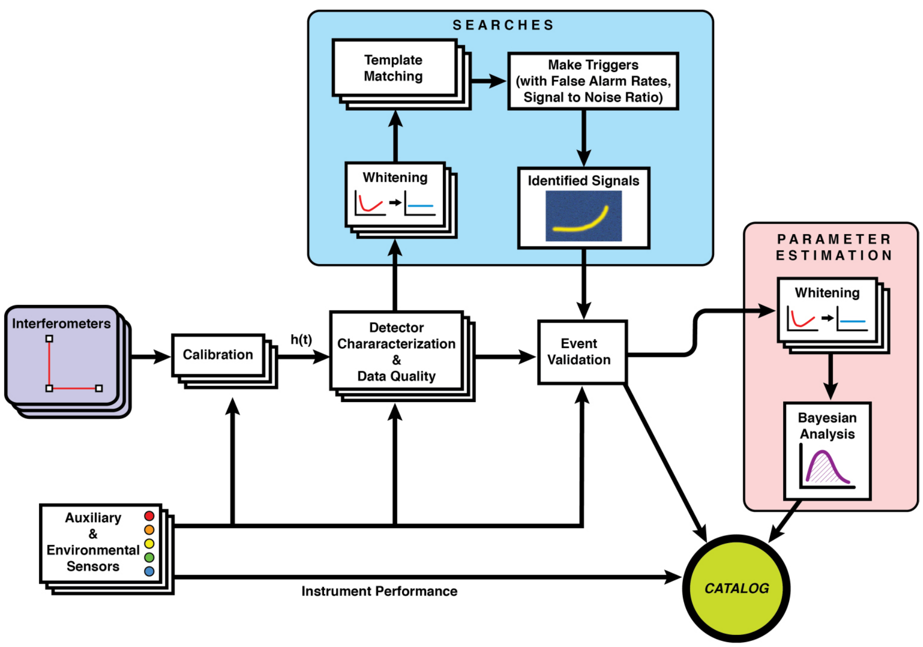

图 11.1:　一个简化的示意图, 总结 LIGO-Virgo 数据处理的主要步骤, 从数据输出到瞬态事件表中报告的结果.

11.2 定义

简单认为

$$S_t = h(t) + N_t \tag{11.1}$$

详细讨论见 [16].

内积

$$\langle p \mid q \rangle := 4\mathrm{Re}\int_0^\infty \frac{\tilde{p}^*(f)\tilde{q}(f)}{S_N(f)}\,\mathrm{d}f. \tag{11.2}$$

11.3 PSD

$$R(\tau) := \mathrm{E}(N_t N_{t+\tau}), \tag{11.3}$$

$$\frac{1}{2}S_N(f) := S_N^{\text{双边}}(f) := \tilde{R}(f) := \int R(\tau)e^{i2\pi f\tau}\,\mathrm{d}\tau. \tag{11.4}$$

若 $\tilde{N}_f$ 存在, 则

$$\mathrm{E}(N_f^* N_{f'}) = \delta(f-f')S_N^{\text{双边}}(f). \tag{11.5}$$

11.4 Matched filtering

$$\mathcal{S}(K) := \int S_t K(t)\,\mathrm{d}t \tag{11.6}$$

$$\mathcal{N}(K) := \int N_t K(t)\,\mathrm{d}t \tag{11.7}$$

$$\frac{\mathrm{S}}{\mathrm{N}}(K) := \frac{\mathrm{E}(\mathcal{S})}{\sqrt{\mathrm{D}(\mathcal{N})}} \tag{11.8}$$

$$= \frac{\mathrm{E}(\int S_t K(t)\,\mathrm{d}t)}{\sqrt{\mathrm{D}(\int N_t K(t)\,\mathrm{d}t)}} \tag{11.9}$$

$$= \frac{\int h(t)K(t)\,\mathrm{d}t}{\sqrt{\int \mathrm{E}(N_{t_1}N_{t_2})K(t_1)K(t_2)\,\mathrm{d}t_1\mathrm{d}t_2}} \tag{11.10}$$

$$= \frac{\int h(t)K(t)\,\mathrm{d}t}{\sqrt{\int R(t_2-t_1)K(t_1)K(t_2)\,\mathrm{d}t_1\mathrm{d}t_2}} \tag{11.11}$$

$$= \frac{\int \tilde{h}(f)\tilde{K}^*(f)\,\mathrm{d}f}{\sqrt{\int \frac{1}{2}S_N(f)\tilde{K}(f)\tilde{K}^*(f)\,\mathrm{d}f}} \tag{11.12}$$

$$= \frac{\langle \frac{1}{2}S_N\tilde{K} \mid h\rangle}{\langle \frac{1}{2}S_N\tilde{K} \mid \frac{1}{2}S_N\tilde{K}\rangle^{1/2}} \tag{11.13}$$

$$\max\,[\frac{\mathrm{S}}{\mathrm{N}}(K)] = \langle h \mid h\rangle^{1/2} \tag{11.14}$$

11.5 Event validation

$$\rho := \frac{\mathcal{S}}{\mathrm{N}}(K) \tag{11.15}$$

When $h = 0$,

$$\rho = \frac{\mathcal{N}}{\mathrm{N}}(K) \tag{11.16}$$

Since N_t is Gaussian, $\mathcal{N}(K) = \int N_t K(t)\,\mathrm{d}t$ is Gaussian,

$$p(\rho|0) = \frac{1}{\sqrt{2\pi}}e^{-\rho^2/2}\,\mathrm{d}\rho, \tag{11.17}$$

$$p_{\mathrm{FA}} = 2\,\mathrm{erfc}\,\rho_{\mathrm{t}}/\sqrt{2}. \tag{11.18}$$

定理 1. 事件 $A_1, \ldots, A_n$, η 是 $A_1, \ldots, A_n$ 中事件的发生数, 则 $\mathrm{E}(\eta) = \sum_{i=1}^n P(A_i)$.

证明. $\eta = \sum_{i=1}^n \mathrm{I}_{A_i}$, $\mathrm{E}(\eta) = \int \eta\,\mathrm{d}P = \sum_{i=1}^n \int \mathrm{I}_{A_i}\,\mathrm{d}P = \sum_{i=1}^n P(A_i)$. □

11.6 Parameter estimation

$$p(\mu \mid d) \propto p(\mu)\exp\left[-\frac{1}{2}\sum_{m,n} C^{-1}_{mn}(d_m - h_m)(d_n - h_n)\right], \tag{11.19}$$

$$p(\mu \mid d) \propto p(\mu)\exp\left[-\frac{1}{2}\langle d - h \mid d - h\rangle\right]. \tag{11.20}$$

11.7 Sensitivity

Fisher information matrix

$$\Gamma_{mn} := \mathrm{E}(\partial_m \ln p \, \partial_n \ln p), \tag{11.21}$$

$p = p(s|\vec{\theta}) = \Lambda(\vec{\theta}|s)$,

$$\Gamma_{mn} = \langle \partial_m h \mid \partial_n h \rangle. \tag{11.22}$$

第十二章　电磁引力

[15].

12.1　时空张量转化为空间张量

$$h_{ab} := g_{ab} + Z_a Z_b. \tag{12.1}$$

$$h_a{}^b = \delta_a{}^b + Z_a Z^b. \tag{12.2}$$

$$Z^a h_{ab} = 0. \tag{12.3}$$

$$V_{\langle a\rangle} := h_a{}^b V_b. \tag{12.4}$$

$$Z^a V_{\langle a\rangle} = 0. \tag{12.5}$$

$$T_{\langle ab\rangle} := h_{(a}{}^c h_{b)}{}^d T_{cd} - \frac{1}{3} h_{cd} T^{cd} h_{ab}. \tag{12.6}$$

$$Z^a (h_a{}^c h_b{}^d T_{cd}) = 0. \tag{12.7}$$

$$Z^a (h_b{}^c h_a{}^d T_{cd}) = 0. \tag{12.8}$$

$$Z^a (h_{(a}{}^c h_{b)}{}^d T_{cd}) = 0. \tag{12.9}$$

$$Z^a (h_{cd} T^{cd} h_{ab}) = 0. \tag{12.10}$$

$$Z^a T_{\langle ab\rangle} = 0. \tag{12.11}$$

$$T_{(\langle ab\rangle)} = T_{\langle ab\rangle}. \tag{12.12}$$

$$h^{ab} T_{\langle ab\rangle} = 0. \tag{12.13}$$

$$\varepsilon_{abc} := \varepsilon_{abcd} Z^d. \tag{12.14}$$

$$\varepsilon_{0123} := -\sqrt{|g|}. \tag{12.15}$$

$$T_a := \frac{1}{2}\varepsilon_{abc} T^{[bc]}. \tag{12.16}$$

$$[U, V]_a := \varepsilon_{abc} U^b V^c. \tag{12.17}$$

$$[S, T]_a := \varepsilon_{abc} g_{de} S^{bd} T^{ce}. \tag{12.18}$$

$$\mathrm{D}_t T^{a\dots}{}_{b\dots} := Z^c \nabla_c T^{a\dots}{}_{b\dots}. \tag{12.19}$$

$${}^3\nabla_a T^{b\dots}{}_{c\dots} := h_a{}^p h^b{}_q \dots h_c{}^r \dots \nabla_p T^{q\dots}{}_{r\dots}. \tag{12.20}$$

$$(\operatorname{div} V) := {}^3\nabla^a V_a. \tag{12.21}$$

$$(\operatorname{curl} V)_a := \varepsilon_{bca} {}^3\nabla^b V^c. \tag{12.22}$$

$$(\operatorname{div} T)_a := {}^3\nabla^b T_{ab}. \tag{12.23}$$

$$(\operatorname{curl} T)_{ab} := \varepsilon_{cd(a} {}^3\nabla^c g_{b)e} T^{ed}. \tag{12.24}$$

12.2 电磁空间矢量

$${}^*F_{ab} := \frac{1}{2}\varepsilon_{abcd} F^{cd} \tag{12.25}$$

$$E_a := F_{ab} Z^b = E_{\langle a\rangle}. \tag{12.26}$$

$$B_a := {}^*F_{ab} Z^b = B_{\langle a\rangle}. \tag{12.27}$$

$$\rho = -Z^a J_a. \tag{12.28}$$

$$j_a = h_a{}^b J_b. \tag{12.29}$$

$$\nabla_{[a} F_{bc]} = 0. \tag{12.30}$$

$$\nabla^a F_{ab} = \mu J_b. \tag{12.31}$$

$$(\operatorname{div} E) = \mu\rho - \dots. \tag{12.32}$$

$$(\operatorname{div} B) = + \dots. \tag{12.33}$$

$$(\operatorname{curl} E)_a + \dots = -\mathrm{D}_t B_{\langle a\rangle} - \dots. \tag{12.34}$$

$$(\operatorname{curl} B)_a + \dots = \mu j_a + \mathrm{D}_t E_{\langle a\rangle} + \dots. \tag{12.35}$$

12.3　引力空间张量

$$^{*}C_{abcd} := \frac{1}{2}\varepsilon_{abef}C^{ef}{}_{cd}. \tag{12.36}$$

$$E_{ab} := C_{acbd}Z^c Z^d = E_{\langle ab\rangle}. \tag{12.37}$$

$$B_{ab} := {}^{*}C_{acbd}Z^c Z^d = B_{\langle ab\rangle}. \tag{12.38}$$

$$(\operatorname{div} E)_a = \kappa\frac{1}{3}{}^3\nabla_a\rho - \dots. \tag{12.39}$$

$$(\operatorname{div} B)_a = \kappa(\rho + p)\omega_a + \dots. \tag{12.40}$$

$$(\operatorname{curl} E)_{ab} + \dots = -\mathrm{D}_t B_{\langle ab\rangle} - \dots. \tag{12.41}$$

$$(\operatorname{curl} B)_{ab} + \dots = \kappa\frac{1}{2}(\rho + p)\sigma_{ab} + \mathrm{D}_t E_{\langle ab\rangle} + \dots. \tag{12.42}$$

第十三章　Varying G

13.1　Modification of Amplitude

$$\partial^c \partial_c \bar{h}_{ab} = -16\pi \frac{G_0}{c_0^4} T_{ab}, \quad \partial^a \bar{h}_{ab} = 0 \tag{13.1}$$

$$\Gamma^c{}_{ab} = \frac{1}{2}\eta^{cd}(2\partial_{(a}h_{b)d} - \partial_d h_{ab}) \tag{13.2}$$

$$U^a \partial_a U^c + \Gamma^c{}_{ab} U^a U^b = 0 \tag{13.3}$$

$$U^a \partial_a U^c = -\frac{1}{2}\eta^{cd}(2\partial_{(a}h_{b)d} - \partial_d h_{ab}) U^a U^b \tag{13.4}$$

$$T_{ab} = c_0^2(2U_{(a}J_{b)} + U^c J_c U_a U_b) \tag{13.5}$$

$$J_b c_0^2 = -U^a T_{ab} \tag{13.6}$$

$$A_b = -\frac{1}{4} U^a \bar{h}_{ab} \tag{13.7}$$

$$A_0 = -\frac{1}{4} c_0 \bar{h}_{00} = -\frac{1}{2} c_0 (\bar{h}_{00} - \frac{1}{2}\eta_{00}\eta^{00}\bar{h}_{00}) = -\frac{1}{2} c_0 h_{00} \tag{13.8}$$

$$A_i = -\frac{1}{4} c_0 \bar{h}_{0i} = -\frac{1}{4} c_0 h_{0i} \tag{13.9}$$

$$U^\mu \partial_\mu U^i = -\frac{1}{2}\eta^{i\sigma}(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}) U^\mu U^\nu \tag{13.10}$$

$$\begin{aligned} -\frac{1}{2}\eta^{i\sigma}(\partial_0 h_{0\sigma} + \partial_0 h_{0\sigma} - \partial_\sigma h_{00}) U^0 U^0 &= \frac{1}{2} c_0^2 \eta^{i\sigma} \partial_\sigma h_{00} && (13.11) \\ &= \frac{1}{2} c_0^2 \partial^i h_{00} && (13.12) \end{aligned}$$

$$= -c_0 \partial^i A_0 \tag{13.13}$$

$$= -E^i \tag{13.14}$$

$$-\frac{1}{2}\eta^{i\sigma}(\partial_0 h_{j\sigma} + \partial_j h_{0\sigma} - \partial_\sigma h_{0j})U^0 U^j = -\frac{1}{2}c_0\eta^{i\sigma}(\partial_j h_{0\sigma} - \partial_\sigma h_{0j})v^j \tag{13.15}$$

$$= -\frac{1}{2}c_0\eta^{ik}(\partial_j h_{0k} - \partial_k h_{0j})v^j \tag{13.16}$$

$$= 2\eta^{ik}(\partial_j A_k - \partial_k A_j)v^j \tag{13.17}$$

$$= -2\eta^{ik}(\partial_k A_j - \partial_j A_k)v^j \tag{13.18}$$

$$= -2(\partial^i A_j - \partial_j A^i)v^j \tag{13.19}$$

$$= -2\varepsilon^i{}_{jk} v^j B^k \tag{13.20}$$

$$-\frac{1}{2}\eta^{i\sigma}(\partial_j h_{k\sigma} + \partial_k h_{j\sigma} - \partial_\sigma h_{jk})U^j U^k = 0 \tag{13.21}$$

$$a^i = -E^i - 4\varepsilon^i{}_{jk} v^j B^k \tag{13.22}$$

$$\partial^i(\frac{1}{4\pi G_0}E_i) = \rho \tag{13.23}$$

$$\partial^i B_i = 0 \tag{13.24}$$

$$\varepsilon^i{}_{jk}\partial^j E^k = -\partial_t B^i \tag{13.25}$$

$$\varepsilon^i{}_{jk}\partial^j(\frac{c_0^2}{4\pi G_0}B^k) = j^i + \partial_t(\frac{1}{4\pi G_0}E^i) \tag{13.26}$$

$$\varepsilon_{\mathrm{G}0} := \frac{1}{4\pi G_0}, \quad \mu_{\mathrm{G}0} := \frac{4\pi G_0}{c_0^2} \tag{13.27}$$

$$\begin{cases} \vec{\nabla}\cdot(\varepsilon_{\mathrm{G}0}\vec{E}) = \rho \\ \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B} \\ \vec{\nabla}\times(\mu_{\mathrm{G}0}^{-1}\vec{B}) = \vec{j} + \frac{\partial}{\partial t}(\varepsilon_{\mathrm{G}0}\vec{E}) \end{cases} \tag{13.28}$$

$$\vec{a} = -\vec{E} - 4\vec{v}\times\vec{B} \tag{13.29}$$

$$\varepsilon_{\mathrm{G}} = \frac{1}{4\pi G}, \quad \mu_{\mathrm{G}} = \frac{4\pi G}{c^2} \tag{13.30}$$

$$x^\mu = (ct, x, y, z) \tag{13.31}$$

$$\begin{cases} \vec{\nabla} \cdot (\varepsilon_{\mathrm{G}} \vec{E}) = \rho \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \\ \vec{\nabla} \times (\mu_{\mathrm{G}}^{-1} \vec{B}) = \vec{j} + \frac{\partial}{\partial t} (\varepsilon_{\mathrm{G}} \vec{E}) \end{cases} \tag{13.32}$$

$$\vec{a} = -\vec{E} - 4\vec{v} \times \vec{B} \tag{13.33}$$

$$A_\mu = -\frac{1}{4} c \bar{h}_{0\mu} \tag{13.34}$$

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = \varepsilon_{\mathrm{G}}^{-1} \rho \\ \vec{\nabla} \times \vec{B} = \mu_{\mathrm{G}} \vec{j} + \varepsilon_{\mathrm{G}} \mu_{\mathrm{G}} \frac{\partial}{\partial t} \vec{E} \end{cases} \tag{13.35}$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \varphi + \vec{\nabla} \cdot \vec{A} = 0 \tag{13.36}$$

$$\begin{cases} -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \varphi + \vec{\nabla}^2 \varphi = \varepsilon_{\mathrm{G}}^{-1} \rho \\ -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{A} + \vec{\nabla}^2 \vec{A} = \mu_{\mathrm{G}} \vec{j} \end{cases} \tag{13.37}$$

$$\begin{cases} -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} c^{-1} \varphi + \vec{\nabla}^2 c^{-1} \varphi = \mu_{\mathrm{G}} c \rho \\ -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{A} + \vec{\nabla}^2 \vec{A} = \mu_{\mathrm{G}} \vec{j} \end{cases} \tag{13.38}$$

$$\begin{cases} \vec{\nabla} \cdot (\varepsilon_{\mathrm{G}} \vec{E}) = 0 \\ \vec{\nabla} \cdot (\mu_{\mathrm{G}} \vec{H}) = 0 \\ \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} (\mu_{\mathrm{G}} \vec{H}) \\ \vec{\nabla} \times \vec{H} = +\frac{\partial}{\partial t} (\varepsilon_{\mathrm{G}} \vec{E}) \end{cases} \tag{13.39}$$

$$E_r = 0, \quad H_r = 0 \tag{13.40}$$

$$\begin{cases} \frac{\varepsilon_{\mathrm{G}}}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta E_\theta) + \frac{\varepsilon_{\mathrm{G}}}{r \sin\theta} \frac{\partial}{\partial \phi} (E_\phi) = 0 \\ \frac{\mu_{\mathrm{G}}}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta H_\theta) + \frac{\mu_{\mathrm{G}}}{r \sin\theta} \frac{\partial}{\partial \phi} (H_\phi) = 0 \\ \frac{1}{r \sin\theta} [\frac{\partial}{\partial \theta} (\sin\theta E_\phi) - \frac{\partial}{\partial \phi} (E_\theta)] \vec{e}_r - \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \vec{e}_\theta + \frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) \vec{e}_\phi = -\mu_{\mathrm{G}} \frac{\partial}{\partial t} (H_\theta \vec{e}_\theta + H_\phi \vec{e}_\phi) \\ \frac{1}{r \sin\theta} [\frac{\partial}{\partial \theta} (\sin\theta H_\phi) - \frac{\partial}{\partial \phi} (H_\theta)] \vec{e}_r - \frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) \vec{e}_\theta + \frac{1}{r} \frac{\partial}{\partial r} (r H_\theta) \vec{e}_\phi = +\varepsilon_{\mathrm{G}} \frac{\partial}{\partial t} (E_\theta \vec{e}_\theta + E_\phi \vec{e}_\phi) \end{cases} \tag{13.41}$$

$$\vec{E} = E_\theta \vec{e}_\theta, \quad \vec{H} = H_\phi \vec{e}_\phi \tag{13.42}$$

$$\begin{cases} \frac{\varepsilon_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) = 0 \\ \frac{\mu_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\phi}(H_\phi) = 0 \\ -\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(E_\theta)\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)\vec{e}_\phi = -\mu_{\rm G}\frac{\partial}{\partial t}(H_\phi)\vec{e}_\phi \\ +\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta H_\phi)\vec{e}_r - \frac{1}{r}\frac{\partial}{\partial r}(rH_\phi)\vec{e}_\theta = +\varepsilon_{\rm G}\frac{\partial}{\partial t}(E_\theta)\vec{e}_\theta \end{cases} \tag{13.43}$$

$$\begin{cases} \frac{\partial}{\partial r}(rE_\theta) + \mu_{\rm G}\frac{\partial}{\partial t}(rH_\phi) = 0 \\ \frac{\partial}{\partial r}(rH_\phi) + \varepsilon_{\rm G}\frac{\partial}{\partial t}(rE_\theta) = 0 \end{cases} \tag{13.44}$$

$$\begin{cases} \mu_{\rm G}\frac{\partial}{\partial r}\mu_{\rm G}^{-1}\frac{\partial}{\partial r}(rE_\theta) - \varepsilon_{\rm G}\mu_{\rm G}\frac{\partial}{\partial t}\frac{\partial}{\partial t}(rE_\theta) = 0 \\ \varepsilon_{\rm G}\frac{\partial}{\partial r}\varepsilon_{\rm G}^{-1}\frac{\partial}{\partial r}(rH_\phi) - \varepsilon_{\rm G}\mu_{\rm G}\frac{\partial}{\partial t}\frac{\partial}{\partial t}(rH_\phi) = 0 \end{cases} \tag{13.45}$$

$$\begin{cases} \mu_{\rm G}\frac{\partial}{\partial r}\mu_{\rm G}^{-1}\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rE_\theta) = 0 \\ \varepsilon_{\rm G}\frac{\partial}{\partial r}\varepsilon_{\rm G}^{-1}\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rH_\phi) = 0 \end{cases} \tag{13.46}$$

$$\begin{cases} \frac{\partial}{\partial r}\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial}{\partial r}(\ln\mu_{\rm G})\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rE_\theta) = 0 \\ \frac{\partial}{\partial r}\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial}{\partial r}(\ln\varepsilon_{\rm G})\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rH_\phi) = 0 \end{cases} \tag{13.47}$$

$$\frac{\partial^2}{\partial r^2}f(r,t) - p(r)\frac{\partial}{\partial r}f(r,t) - \frac{\partial^2}{\partial(ct)^2}f(r,t) = 0 \tag{13.48}$$

$$f(r,t) = f(r)e^{-ikct} \tag{13.49}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}f(r) - p(r)\frac{\mathrm{d}}{\mathrm{d}r}f(r) + k^2 f(r) = 0 \tag{13.50}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}f(r) - p\frac{\mathrm{d}}{\mathrm{d}r}f(r) + k^2 f(r) = 0 \tag{13.51}$$

$$f(r) = e^{(p/2)r}[C_+e^{i\sqrt{k^2-(p/2)^2}r} + C_-e^{-i\sqrt{k^2-(p/2)^2}r}] \tag{13.52}$$

$$f(r,t) = e^{(p/2)r}[C_+e^{i(+\sqrt{k^2-(p/2)^2}r-kct)} + C_-e^{i(-\sqrt{k^2-(p/2)^2}r-kct)}] \tag{13.53}$$

$$f(r,t) = e^{(p/2)r}[C_+e^{i(+\sqrt{(\omega/c)^2-(p/2)^2}r-\omega t)} + C_-e^{i(-\sqrt{(\omega/c)^2-(p/2)^2}r-\omega t)}] \tag{13.54}$$

$$f(r,t) = e^{\int(p/2)\mathrm{d}r}[C_+e^{i(+\int\sqrt{(\omega/c)^2-(p/2)^2}\mathrm{d}r-\omega t)} + C_-e^{i(-\int\sqrt{(\omega/c)^2-(p/2)^2}\mathrm{d}r-\omega t)}] \tag{13.55}$$

$$\begin{cases} r_2\left|E_\theta\right|_{r=r_2} = r_1\left|E_\theta\right|_{r=r_1} e^{\int_{r_1}^{r_2}\frac{1}{2}\frac{\partial}{\partial r}(\ln\mu_{\rm G})\,\mathrm{d}r} \\ r_2\left|H_\phi\right|_{r=r_2} = r_1\left|H_\phi\right|_{r=r_1} e^{\int_{r_1}^{r_2}\frac{1}{2}\frac{\partial}{\partial r}(\ln\varepsilon_{\rm G})\,\mathrm{d}r} \end{cases} \tag{13.56}$$

$$
\begin{cases} E_2 = \sqrt{\frac{\mu_{\mathrm{G}\,2}}{\mu_{\mathrm{G}\,1}}} \frac{r_1}{r_2} E_1 \\ H_2 = \sqrt{\frac{\varepsilon_{\mathrm{G}\,2}}{\varepsilon_{\mathrm{G}\,1}}} \frac{r_1}{r_2} H_1 \end{cases} \tag{13.57}
$$

$$
\begin{cases} E_2/c_2 = \sqrt{\frac{\mu_{\mathrm{G}\,2}}{\mu_{\mathrm{G}\,1}}} \frac{c_1}{c_2} \frac{r_1}{r_2} E_1/c_1 \\ B_2 = \sqrt{\frac{\mu_{\mathrm{G}\,2}}{\mu_{\mathrm{G}\,1}}} \frac{c_1}{c_2} \frac{r_1}{r_2} B_1 \end{cases} \tag{13.58}
$$

$$
\begin{cases} (\omega/c_2)c_2(\bar{h}_{00})_2 = \sqrt{\frac{\mu_{\mathrm{G}\,2}}{\mu_{\mathrm{G}\,1}}} \frac{c_1}{c_2} \frac{r_1}{r_2} (\omega/c_1)c_1(\bar{h}_{00})_1 \\ (\omega/c_2)c_2(\bar{h}_{0i})_2 = \sqrt{\frac{\mu_{\mathrm{G}\,2}}{\mu_{\mathrm{G}\,1}}} \frac{c_1}{c_2} \frac{r_1}{r_2} (\omega/c_1)c_1(\bar{h}_{0i})_1 \end{cases} \tag{13.59}
$$

$$
h_2 = \sqrt{\frac{c_1^4/G_1}{c_2^4/G_2}} \frac{r_1}{r_2} h_1 \tag{13.60}
$$

双星系统引力辐射本为

$$
h = \frac{\mathcal{M}[\pi \mathcal{M} F(t)]^{2/3}}{r} Q(\theta, \phi, \psi, \iota) \cos[\int 2\pi F(t)\,\mathrm{d}t] \tag{13.61}
$$

设双星系统常量 c^*, G^*, 一观者临近双星系统且与双星系统相对静止, 其与双星系统距离为 r, 测得强度 h_r, 频率 F_r, 则[1]

$$
h_r = \frac{\mathcal{M}[\pi \mathcal{M} F_r(t)]^{2/3}}{r/c^*} Q(\theta, \phi, \psi, \iota) \tag{13.62}
$$

设地球观者与双星系统距离为 d, 双星系统红移为 z, 测得强度 h_d, 频率 $F_d = F_r/(1+z)$, 则

$$
h_d = \sqrt{\frac{c^{*4}/G^*}{c^4/G}} \frac{r}{d} h_r \tag{13.63}
$$

$$
= \sqrt{\frac{c^{*4}/G^*}{c^4/G}} \frac{\mathcal{M}[\pi \mathcal{M} F_r(t)]^{2/3}}{d/c^*} Q(\theta, \phi, \psi, \iota) \tag{13.64}
$$

所以地球观者测得

$$
h = \sqrt{\frac{c^{*4}/G^*}{c^4/G}} \frac{\mathcal{M}[\pi \mathcal{M} F_r(t)]^{2/3}}{d/c^*} Q(\theta, \phi, \psi, \iota) \cos[\int 2\pi \frac{F_r(t)}{1+z}\,\mathrm{d}t] \tag{13.65}
$$

记 $F_{\mathrm{obs}}(t) = F_r(t)/(1+z)$, $\mathcal{M}_{\mathrm{obs}} = \mathcal{M}(1+z)$, 光度距离 $d_{\mathrm{L}} = d(1+z)$, 则

$$
h = \sqrt{\frac{c^{*4}/G^*}{c^4/G}} \frac{\mathcal{M}[\pi \mathcal{M} F_r(t)]^{2/3}}{d(1+z)/c^*} Q(\theta, \phi, \psi, \iota) \cos[\int 2\pi F_{\mathrm{obs}}(t)\,\mathrm{d}t] \tag{13.66}
$$

[1] $\mathcal{M}$ 和 c^*, G^* 简并, 所以可以笼统地仍记作 $\mathcal{M}$.

$$= \sqrt{\frac{c^{*4}/G^*}{c^4/G}} \frac{\mathcal{M}_{\mathrm{obs}}[\pi \mathcal{M}_{\mathrm{obs}} F_{\mathrm{obs}}(t)]^{2/3}}{d_{\mathrm{L}}/c^*} Q(\theta, \phi, \psi, \iota) \cos[\int 2\pi F_{\mathrm{obs}}(t)\,\mathrm{d}t] \tag{13.67}$$

$$= \sqrt{\frac{c^{*6}/G^*}{c^6/G}} \frac{\mathcal{M}_{\mathrm{obs}}[\pi \mathcal{M}_{\mathrm{obs}} F_{\mathrm{obs}}(t)]^{2/3}}{d_{\mathrm{L}}/c} Q(\theta, \phi, \psi, \iota) \cos[\int 2\pi F_{\mathrm{obs}}(t)\,\mathrm{d}t] \tag{13.68}$$

用引力波测距测得 $d_{\mathrm{L,G}}$, 则

$$d_{\mathrm{L,G}} = d_{\mathrm{L}} \sqrt{\frac{c^6/G}{c^{*6}/G^*}} \tag{13.69}$$

[19]

$$h(t) = \frac{\mathcal{M}[\pi \mathcal{M} F(t)]^{2/3}}{\xi\, d_{\mathrm{L}}} Q(\mathrm{angles}) \cos\Phi(t) \tag{13.70}$$

$$\tilde{h}(f) = \frac{\sqrt{30}}{48\pi^{2/3}} \frac{\mathcal{M}^{5/6} Q}{\xi\, d_{\mathrm{L}}} f^{-7/6} e^{i[2\pi f t(f) - \Phi(f) - \frac{\pi}{4}]} \tag{13.71}$$

问题转化为估计 ξ

$$p(\mu) \propto p^{(0)}(\mu) \exp[-\frac{1}{2}\Gamma_{ab}(\mu^a - \hat{\mu}^a)(\mu^b - \hat{\mu}^b)] \tag{13.72}$$

$$p^{(0)}(\mu) \propto \exp[-\frac{1}{2}\Gamma^{(0)}_{ab}(\mu^a - \bar{\mu}^a)(\mu^b - \bar{\mu}^b)] \tag{13.73}$$

设待估参数为 $\mu = (\ln\xi, \ln(d_{\mathrm{L}}/d_{\mathrm{L}0}), \ln Q, \ldots)$, $\ldots$ 为其他参数 (如 $\mathcal{M}$), 则 $\tilde{h}_{,\ln\xi} = \tilde{h}_{,\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} = -\tilde{h}_{,\ln Q} = -\tilde{h}$, $\tilde{h}$ 对其他参数求偏导皆为纯虚数, 则由 $\Gamma_{ab} = \langle h_{,a} \mid h_{,b}\rangle$ 和 $\mathrm{SNR} := \rho = \sqrt{\langle h \mid h\rangle}$ 得

$$\Gamma_{ab} = \begin{bmatrix} \rho^2 & \rho^2 & -\rho^2 & 0 & \ldots \\ \rho^2 & \rho^2 & -\rho^2 & 0 & \ldots \\ -\rho^2 & -\rho^2 & \rho^2 & 0 & \ldots \\ 0 & 0 & 0 & ? & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{13.74}$$

又设

$$\Gamma^{(0)}_{ab} = \begin{bmatrix} 0 & 0 & 0 & 0 & \ldots \\ 0 & 1/\sigma^2_{\ln d_{\mathrm{L}}} & 0 & 0 & \ldots \\ 0 & 0 & 1/\sigma^2_{\ln Q} & 0 & \ldots \\ 0 & 0 & 0 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{13.75}$$

则由 $\Sigma_{ab} = (\Gamma_{ab}^{(0)} + \Gamma_{ab})^{-1}$ 得

$$\Sigma_{ab} = \begin{bmatrix} \begin{bmatrix} \rho^2 & \rho^2 & -\rho^2 \\ \rho^2 & \rho^2 + 1/\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & -\rho^2 \\ -\rho^2 & -\rho^2 & \rho^2 + 1/\sigma^2_{\ln Q} \end{bmatrix}^{-1} & 0 \\ 0 & [?]^{-1} \end{bmatrix} \tag{13.76}$$

而

$$\begin{bmatrix} \rho^2 & \rho^2 & -\rho^2 \\ \rho^2 & \rho^2 + 1/\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & -\rho^2 \\ -\rho^2 & -\rho^2 & \rho^2 + 1/\sigma^2_{\ln Q} \end{bmatrix}^{-1} \tag{13.77}$$

$$= \begin{bmatrix} 1/\rho^2 + \sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} + \sigma^2_{\ln Q} & -\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & \sigma^2_{\ln Q} \\ -\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & \sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & 0 \\ \sigma^2_{\ln Q} & 0 & \sigma^2_{\ln Q} \end{bmatrix} \tag{13.78}$$

13.2 Modification of Phase

$$\frac{d^2}{dz^2}H(z) + 2p(z)\frac{d}{dz}H(z) + \left[\omega^2 + q(z)\right]H(z) = 0. \tag{13.79}$$

$$H = Ae^{i\Phi}. \tag{13.80}$$

$k = \frac{d\Phi}{dz}$,

$$\frac{d^2A}{dz^2} + 2p\frac{dA}{dz} + \left[\omega^2\left(1 - \frac{k^2}{\omega^2}\right) + q\right]A = 0, \tag{13.81}$$

$$2\frac{dA}{dz}k + A\frac{dk}{dz} + 2pAk = 0, \tag{13.82}$$

$$2\frac{1}{A}\frac{dA}{dz} + \frac{1}{k}\frac{dk}{dz} + 2p = 0, \tag{13.83}$$

$$A \propto e^{-\int p\,dz}k^{-1/2}. \tag{13.84}$$

$\Gamma = e^{\int p\,dz}$ and $K = (k/\omega)^{-1/2}$,

$$\frac{d^2K}{dz^2} - \left(\frac{1}{\Gamma}\frac{d^2\Gamma}{dz^2} - q\right)K + \omega^2 K(1 - K^{-4}) = 0, \tag{13.85}$$

$\Xi = \frac{1}{\Gamma}\frac{d^2\Gamma}{dz^2} - q = p^2 + \frac{dp}{dz} - q$ and make $\omega = 1$,

$$\frac{d^2K}{dz^2} + K[(1 - \Xi) - K^{-4}] = 0. \tag{13.86}$$

$\Xi = \text{const}$,

$$K = (1-\Xi)^{-1/4} = 1 + \frac{1}{4}\Xi + \frac{5}{32}\Xi^2 + O(\Xi^3), \tag{13.87}$$

$$k = (1-\Xi)^{1/2} = 1 - \frac{1}{2}\Xi - \frac{1}{8}\Xi^2 + O(\Xi^3), \tag{13.88}$$

$\Xi \neq \text{const}$, $\Xi(z) = \kappa^2\tilde{\Xi}(\tilde{z})$, where $\tilde{z} = \kappa z$.

$$K^3\frac{d^2K}{d\tilde{z}^2}\kappa^2 - K^4\tilde{\Xi}(\tilde{z})\kappa^2 + K^4 - 1 = 0. \tag{13.89}$$

$$K = \sum_{n=0}^{\infty} K_n(\tilde{z})\kappa^{2n}, \tag{13.90}$$

$$K_0^4 - 1 = 0, \tag{13.91}$$

$$K_0^3K_0'' - K_0^4\tilde{\Xi} + 4K_0^3K_1 = 0, \tag{13.92}$$

$$(K_0^3K_1'' + 3K_0^2K_1K_0'') - 4K_0^3K_1\tilde{\Xi} + (4K_0^3K_2 + 6K_0^2K_1^2) = 0. \tag{13.93}$$

$$K_0 = 1, \tag{13.94}$$

$$K_1 = \frac{1}{4}\tilde{\Xi}, \tag{13.95}$$

$$K_2 = \frac{5}{32}\tilde{\Xi}^2 - \frac{1}{16}\frac{d^2\tilde{\Xi}}{d\tilde{z}^2}. \tag{13.96}$$

$p = -\frac{1}{2}\frac{d\ln G}{dz} = -\frac{1}{2}\frac{1}{G}\frac{dG}{dz}$, $\Gamma = C_\Gamma G^{-1/2}$, $\frac{1}{\Gamma}\frac{d^2\Gamma}{dz^2} = G^{1/2}\frac{d^2G^{-1/2}}{dz^2} = p^2 + \frac{dp}{dz} = -\frac{1}{2}\frac{1}{G}\frac{d^2G}{dz^2} + \frac{3}{4}\left(\frac{1}{G}\frac{dG}{dz}\right)^2$, $q = 2G\frac{d^2G^{-1}}{dz^2} = -2\frac{1}{G}\frac{d^2G}{dz^2} + 4\left(\frac{1}{G}\frac{dG}{dz}\right)^2$, $\frac{1}{\Gamma}\frac{d^2\Gamma}{dz^2} - q = \frac{3}{2}\frac{1}{G}\frac{d^2G}{dz^2} - \frac{13}{4}\left(\frac{1}{G}\frac{dG}{dz}\right)^2 = aG^b\frac{d^2G^{-b}}{dz^2} = a(-b)\frac{1}{G}\frac{d^2G}{dz^2} + a(-b)(-b-1)\left(\frac{1}{G}\frac{dG}{dz}\right)^2$, $a(-b) = \frac{3}{2}$, $a(-b)(-b-1) = -\frac{13}{4}$, $(-b-1) = -\frac{13}{6}$, $b = \frac{7}{6}$, $a = -\frac{9}{7}$.

$$h(z,t) = h(z)e^{-i\omega t} \tag{13.97}$$

$$h(z) = \Gamma^{-1}(z)K(z)(C_+e^{+i\omega\int K^{-2}(z)\,\mathrm{d}z} + C_-e^{-i\omega\int K^{-2}(z)\,\mathrm{d}z}) \tag{13.98}$$

$$h(z,t) = \int_{-\infty}^{+\infty}\tilde{h}(z;f)e^{-i2\pi ft}\,\mathrm{d}f \tag{13.99}$$

$$\tilde{h}(z;f) = \Gamma^{-1}(z)K(z;f)[C_+(f)e^{+i2\pi f\int K^{-2}(z;f)\,\mathrm{d}z} + C_-(f)e^{-i2\pi f\int K^{-2}(z;f)\,\mathrm{d}z}] \tag{13.100}$$

$$\tilde{h}(z;f) = \Gamma^{-1}(z)K(z;f)[C_+(f)e^{+i2\pi f\int_0^z K^{-2}(z';f)\,\mathrm{d}z'} + C_-(f)e^{-i2\pi f\int_0^z K^{-2}(z';f)\,\mathrm{d}z'}] \tag{13.101}$$

$$\tilde{h}_0(z;f) = \Gamma^{-1}(0)K(0;f)[C_+(f)e^{+i2\pi f\int_0^z \mathrm{d}z'} + C_-(f)e^{-i2\pi f\int_0^z \mathrm{d}z'}] \quad (13.102)$$

$$\tilde{h}(z;f) = \frac{\Gamma^{-1}(z)K(z;f)}{\Gamma^{-1}(0)K(0;f)}[C_+(f)e^{+i2\pi f\int_0^z K^{-2}(z';f)\,\mathrm{d}z'} + C_-(f)e^{-i2\pi f\int_0^z K^{-2}(z';f)\,\mathrm{d}z'}] \quad (13.103)$$

$$\tilde{h}_0(z;f) = [C_+(f)e^{+i2\pi f\int_0^z \mathrm{d}z'} + C_-(f)e^{-i2\pi f\int_0^z \mathrm{d}z'}] \quad (13.104)$$

$$\tilde{h}_0(z;f) = C_+(f)e^{+i2\pi fz} + C_-(f)e^{-i2\pi fz} \quad (13.105)$$

$$\partial_z\tilde{h}_0(z;f) = C_+(f)(i2\pi f)e^{+i2\pi fz} - C_-(f)(i2\pi f)e^{-i2\pi fz} \quad (13.106)$$

$$C_+(f)e^{+i2\pi fz} = \frac{1}{2}[\tilde{h}_0(z;f) + \partial_z\tilde{h}_0(z;f)(i2\pi f)^{-1}] \quad (13.107)$$

$$C_-(f)e^{-i2\pi fz} = \frac{1}{2}[\tilde{h}_0(z;f) - \partial_z\tilde{h}_0(z;f)(i2\pi f)^{-1}] \quad (13.108)$$

$$\tilde{h}(z;f) = \frac{\Gamma^{-1}(z)K(z;f)}{\Gamma^{-1}(0)K(0;f)}C_+(f)e^{+i2\pi f\int_0^z K^{-2}(z';f)\,\mathrm{d}z'} \quad (13.109)$$

$$\tilde{h}_0(z;f) = C_+(f)e^{+i2\pi f\int_0^z \mathrm{d}z'} \quad (13.110)$$

$$K(z) = 1 + \frac{1}{4\omega^2}\Xi(z) \quad (13.111)$$

$$\tilde{h}(z;f) = \frac{\Gamma^{-1}(z)[1+\frac{\Xi(z)}{4(2\pi f)^2}]}{\Gamma^{-1}(0)[1+\frac{\Xi(0)}{4(2\pi f)^2}]}e^{+i(2\pi f)\int_0^z -\frac{\Xi(z')}{2(2\pi f)^2}\,\mathrm{d}z'}\tilde{h}_0(z;f) \quad (13.112)$$

$$\tilde{h}(z;f) = \frac{\Gamma^{-1}(z)}{\Gamma^{-1}(0)}[1+\frac{\Xi(z)-\Xi(0)}{4}(2\pi f)^{-2}]e^{-i(2\pi f)\int_0^z \frac{\Xi(z')}{2}(2\pi f)^{-2}\,\mathrm{d}z'}\tilde{h}_0(z;f) \quad (13.113)$$

$$\tilde{h}(f) = \gamma(1+\xi f^{-2})e^{i\Omega f^{-1}}\tilde{h}_0(f) \quad (13.114)$$

$$h(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{h}(z;\omega)e^{-i\omega t}\,\mathrm{d}\omega \quad (13.115)$$

$$\tilde{h}(z;\omega) = \Gamma^{-1}(z)K(z;\omega)[C_+(\omega)e^{+i\omega\int K^{-2}(z;\omega)\,\mathrm{d}z} + C_-(\omega)e^{-i\omega\int K^{-2}(z;\omega)\,\mathrm{d}z}] \quad (13.116)$$

$$h(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{h}(t;k)e^{+ikz}\,\mathrm{d}k \quad (13.117)$$

$$\hat{h}(t;k) = \Gamma^{-1}(t)K(t;k)[C_+(k)e^{+ik\int K^{-2}(t;k)\,\mathrm{d}t} + C_-(k)e^{-ik\int K^{-2}(t;k)\,\mathrm{d}t}] \quad (13.118)$$

13.3 Relationship to LVK

[3], $\varphi_2 \to \varphi_2(1+\delta\hat{\varphi}_2)$,

$$\tilde{\Omega} = \frac{3\varphi_2\delta\hat{\varphi}_2}{128\pi\eta} \tag{13.119}$$

$$\delta\hat{\varphi}_2 = \frac{128\pi\eta\tilde{\Omega}}{3\varphi_2} \tag{13.120}$$

13.4 Hierarchical combination

[2, 24], For a given beyond-GR parameter x, this distribution $p(x \mid d)$ is the expectation for x after marginalizing over the hyperparameters μ and σ,

$$p(x \mid d) = \int p(x \mid \mu, \sigma) p(\mu, \sigma \mid d)\, \mathrm{d}\mu \mathrm{d}\sigma, \tag{13.121}$$

where d represents the data for *all* detected events, and $p(x \mid \mu, \sigma) \sim \mathcal{N}(\mu, \sigma)$ by construction.

$p(\mu, \sigma \mid d)$ 算法见 `mid hp.pymid` .

附录 A　Bayesian 统计

[(Shujia Wang, 贝叶斯统计及应用), (kausiujik, 统计小站: 贝叶斯分析)]

A.1　Bayesian 公式

$$p(\vec{\theta} \mid \vec{x}) = \frac{\mathcal{L}(\vec{\theta} \mid \vec{x})\pi(\vec{\theta})}{\mathcal{Z}}. \tag{A.1}$$

$p(\vec{\theta} \mid \vec{x})$ 是 posterior, $\pi(\vec{\theta})$ 是 prior, $\mathcal{L}(\vec{\theta} \mid \vec{x}) = p(\vec{x} \mid \vec{\theta})$ 是 likelihood, $\mathcal{Z} = \int \mathcal{L}(\vec{\theta} \mid \vec{x})\pi(\vec{\theta})\,\mathrm{d}\vec{\theta}$ 是 evidence.

A.2　点估计

$\hat{\theta} = \mathrm{E}[p(\vec{\theta} \mid \vec{x})]$ 使 $\mathrm{E}[p(\vec{\theta} - \hat{\theta} \mid \vec{x})] = \mathrm{D}[p(\vec{\theta} \mid \vec{x})]$ 最小.

A.3　区间估计

若存在 $C \in \{\theta\}$, 使得 $\int_C \partial(\theta \mid \vec{x}) \geq 1 - \alpha$, 则称 $[\hat{\theta}_{\mathrm{L}}, \hat{\theta}_{\mathrm{U}}]$ 为 θ 的可信水平为 $1 - \alpha$ 的 Bayesian 可信集. 警告: Bayesian 可信区间不是经典置信区间.

若存在 $C \in \{\theta\}$, 使得 $\int_C \partial(\theta \mid \vec{x}) = 1 - \alpha$, 且任意 $\theta_1 \in C$, $\theta_2 \notin C$, $\pi(\theta_1 \mid \vec{x}) \geq \pi(\theta_2 \mid \vec{x})$, 则称 C 为 θ 的可信水平为 $1 - \alpha$ 的 Bayesian HPD 可信集. 作法: 在 $\theta - p$ 图中, 划一平行于 θ 轴的横线, 与 $p(\theta)$ 曲线的交点向下划垂直于 θ 轴的竖线, 获得可信集端点.

Bayesian HPD 可信区间使 Bayesian 可信区间最短.

若 Bayesian HPD 可信集不是可信区间, 则不用 Bayesian HPD 可信集, 而用 $\alpha/2$ 和 $1-\alpha/2$ 分位数获得等尾可信区间, 并检查 prior.

A.4 Prior

A.4.1 共轭 Prior

A.4.2 无信息 Prior

均匀 Prior

Jeffreys Prior

Fisher 信息矩阵 $\mathcal{I}_{ij}(\vec{\theta}) := \mathrm{E}[\frac{\partial \ln p(\vec{x}|\vec{\theta})}{\partial \theta_i} \cdot \frac{\partial \ln p(\vec{x}|\vec{\theta})}{\partial \theta_j}]$.
Jeffreys prior $\pi_{\mathrm{J}}(\vec{\theta}) :\propto \det(\mathcal{I}_{ij}(\vec{\theta}))^{1/2}$,

A.4.3 有信息 Prior

A.5 Hierarchical Prior

$\pi(\vec{\theta} \mid \vec{\lambda})$ 中未定且需定的 parameter 称为 hyperparameter. 为难定的 hyperparameter 给的 prior 称为 hyperprior. prior 和 hyperprior 组成 hierarchical prior.

A.6 算法

A.6.1 MCMC

用 Monte Carlo 法模拟根据 posterior 构造的 Markov chain 来估计 posterior 的方法.

构造一个 Markov chain $\vec{\theta}^{(1)}, \dots, \vec{\theta}^{(n)}, \dots$, 使得 $\lim_{n\to\infty} p(\vec{\theta}^{(n)}) = p(\vec{\theta} \mid \vec{x})$, 用 Monte Carlo 法模拟 $\vec{\theta}^{(1)}, \dots, \vec{\theta}^{(n)}$, 去除 $\vec{\theta}^{(1)}, \dots, \vec{\theta}^{(m)}$, 用 $\vec{\theta}^{(m+1)}, \dots, \vec{\theta}^{(n)}$ 估计 $p(\vec{\theta} \mid \vec{x})$.

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