引力波天文学笔记

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目录

第一章　引力波

1.1　Linearized Gravity

[?]. 流形 $\mathbb{R}^4$. 任意坐标系 $\{x^\mu\}$, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} s + \mathrm{O}(s^2)$, 得

$$R_{\mu\nu\lambda\sigma} = \partial_\sigma \partial_{[\mu} h_{\lambda]\nu} - \partial_\nu \partial_{[\mu} h_{\lambda]\sigma} + \mathrm{O}(s^2). \tag{1.1}$$

$\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\sigma}h_{\lambda\sigma} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$.

$$-\frac{1}{2}\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} + \partial^\lambda \partial_{(\mu} \bar{h}_{\nu)\lambda} - \frac{1}{2}\eta_{\mu\nu}\partial^\lambda \partial^\sigma \bar{h}_{\lambda\sigma} + \mathrm{O}(s^2) = 8\pi T_{\mu\nu}. \tag{1.2}$$

存在 $\{x^\mu\}$, 使得 $\partial^\nu \bar{h}_{\mu\nu} + \mathrm{O}(s^2) = 0$ (Lorentz gauge). 令 $\{x^\mu\}$ 满足 $\partial^\nu \bar{h}_{\mu\nu} + \mathrm{O}(s^2) = 0$, 则

$$\partial^\lambda \partial_\lambda \bar{h}_{\mu\nu} + \mathrm{O}(s^2) = -16\pi T_{\mu\nu}. \tag{1.3}$$

略去 $\mathrm{O}(s^2)$ 条件: $h_{\mu\nu}$, $\partial_\lambda h_{\mu\nu}$...小.

1.2　Radiation Gauge

[?]. 存在 $\{x^\mu\}$, 使得 “无源处” $h + \mathrm{O}(s^2) = 0$ (TT gauge [?]) 且 $h_{0\mu} + \mathrm{O}(s^2) = 0$.

1.3　Quadrupole Approximation

[?]. 下略 $\mathrm{O}(s^2)$. 由(1.3)得

$$\bar{h}_{\mu\nu}(t, \vec{r}) = 4 \int \frac{T_{\mu\nu}(t - |\vec{r} - \vec{r}'|, \vec{r}')}{|\vec{r} - \vec{r}'|} \mathrm{d}V'. \tag{1.4}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) := \frac{1}{\sqrt{2\pi}}\int \bar{h}_{\mu\nu}(t,\vec{r})e^{i\omega t}\mathrm{d}t \tag{1.5}$$

$$= 4\int \frac{\hat{T}_{\mu\nu}(\omega,\vec{r}')}{|\vec{r}-\vec{r}'|}e^{i\omega|\vec{r}-\vec{r}'|}\,\mathrm{d}V'. \tag{1.6}$$

由 $\partial^\nu \bar{h}_{\mu\nu} = 0$,

$$-i\omega\hat{\bar{h}}_{0\mu} = \sum_i \frac{\partial\hat{\bar{h}}_{i\mu}}{\partial x^i}. \tag{1.7}$$

$|\vec{r}| \gg |\vec{r}'|$ 且 $\omega \ll 1/|\vec{r}'|$,

$$\hat{\bar{h}}_{ij}(\omega,\vec{r}) = 4\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{ij}(\omega,\vec{r}')\,\mathrm{d}V'. \tag{1.8}$$

$$\int \hat{T}_{ij}\,\mathrm{d}V' = \int \sum_k (\hat{T}_{kj}\frac{\partial x'^i}{\partial x'^k})\,\mathrm{d}V' \tag{1.9}$$

$$= \sum_k \left[\int \frac{\partial}{\partial x'^k}(\hat{T}_{kj}x'^i)\,\mathrm{d}V' - \int \frac{\partial \hat{T}_{kj}}{\partial x'^k}x'^i\,\mathrm{d}V'\right] \tag{1.10}$$

$$= \sum_k \int \partial'_k\,(\hat{T}_{kj}x'^i)\,\mathrm{d}V' - \sum_k \int \frac{\partial \hat{T}_{kj}}{\partial x'^k}x'^i\,\mathrm{d}V' \tag{1.11}$$

$$= \int \hat{T}_{kj}x'^i\,\mathrm{d}S' - \sum_k \int \frac{\partial \hat{T}_{kj}}{\partial x'^k}x'^i\,\mathrm{d}V' \tag{1.12}$$

$$= -\sum_k \int \frac{\partial \hat{T}_{kj}}{\partial x'^k}x'^i\,\mathrm{d}V' \tag{1.13}$$

$$= -\int (\sum_k \partial'_k\hat{T}_{kj})x'^i\,\mathrm{d}V' \tag{1.14}$$

$$= -\int (\partial_0\hat{T}_{0j})x'^i\,\mathrm{d}V' \tag{1.15}$$

$$= -i\omega\int \hat{T}_{0j}x'^i\,\mathrm{d}V' \tag{1.16}$$

$$= \int \hat{T}_{(ij)}\,\mathrm{d}V' \tag{1.17}$$

$$= -i\omega\int \hat{T}_{0(j}x'^{i)}\,\mathrm{d}V' \tag{1.18}$$

$$= -\frac{i\omega}{2}\int (\hat{T}_{0j}x'^i + \hat{T}_{0i}x'^j)\,\mathrm{d}V', \tag{1.19}$$

$$\tag{1.20}$$

$$-\frac{i\omega}{2}\int(\hat{T}_{0j}x'^i+\hat{T}_{0i}x'^j)\,\mathrm{d}V' = -\frac{i\omega}{2}\int\sum_k(\hat{T}_{0k}x'^i\frac{\partial x'^j}{\partial x'^k}+\hat{T}_{0k}\frac{\partial x'^i}{\partial x'^k}x'^j)\,\mathrm{d}V' \tag{1.21}$$

$$= -\frac{i\omega}{2}\sum_k\left[\int\frac{\partial}{\partial x'^k}(\hat{T}_{0k}x'^ix'^j)\,\mathrm{d}V'-\int\frac{\partial\hat{T}_{0k}}{\partial x'^k}x'^ix'^j\,\mathrm{d}V'\right] \tag{1.22}$$

$$= -\frac{i\omega}{2}\sum_k\int\partial'_k\,(\hat{T}_{0k}x'^ix'^j)\,\mathrm{d}V'+\frac{i\omega}{2}\sum_k\int\frac{\partial\hat{T}_{0k}}{\partial x'^k}x'^ix'^j\,\mathrm{d}V' \tag{1.23}$$

$$= -\frac{i\omega}{2}\sum_k\int\hat{T}_{0k}x'^ix'^j\,\mathrm{d}S'+\frac{i\omega}{2}\sum_k\int\frac{\partial\hat{T}_{0k}}{\partial x'^k}x'^ix'^j\,\mathrm{d}V' \tag{1.24}$$

$$= \frac{i\omega}{2}\sum_k\int\frac{\partial\hat{T}_{0k}}{\partial x'^k}x'^ix'^j\,\mathrm{d}V' \tag{1.25}$$

$$= \frac{i\omega}{2}\int(\sum_k\partial'_k\hat{T}_{0k})x'^ix'^j\,\mathrm{d}V' \tag{1.26}$$

$$= \frac{i\omega}{2}\int(\partial_0\hat{T}_{00})x'^ix'^j\,\mathrm{d}V' \tag{1.27}$$

$$= -\frac{\omega^2}{2}\int\hat{T}_{00}\,x'^ix'^j\,\mathrm{d}V'. \tag{1.28}$$

$$q_{ij}(t) := \int T_{00}\,x'^ix'^j\,\mathrm{d}V', \tag{1.29}$$

$$\hat{\bar{h}}_{ij}(\omega,\vec{r}) = -2\omega^2\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\hat{q}_{ij}(\omega), \tag{1.30}$$

$$\bar{h}_{ij}(t,\vec{r}) = \frac{2}{|\vec{r}|}\frac{\mathrm{d}^2}{\mathrm{d}t^2}q_{ij}(t-|\vec{r}|). \tag{1.31}$$

1.4 $+$ Mode and $\times$ Mode

寻新标架 $(e'^1)_a=(e^+)_a$, $(e'^2)_a=(e^\times)_a$, $(e'^3)_a=(e^r)_a$, $\bar{h}_{ij}(e^i)_a(e^j)_b=\bar{h}'_{ij}(e'^i)_a(e'^j)_b$, 取 x, y 分量后去迹, $h_+=\frac{1}{2}(\bar{h}'_{11}-\bar{h}'_{22})$, $h_\times=\bar{h}'_{12}=\bar{h}'_{21}$? **[?]** **[?]**, $\vec{n}:=\frac{\vec{r}}{|\vec{r}|}$,

$$h_{ij}^{\mathrm{TT}} = \frac{2}{|\vec{r}|}\mathcal{P}_{ijkm}\frac{\mathrm{d}^2}{\mathrm{d}t^2}Q^{km}(t-|\vec{r}|), \tag{1.32}$$

$$\mathcal{P}_{ijkm} := (\delta_{ik} - \vec{n}_i\vec{n}_k)(\delta_{jm} - \vec{n}_j\vec{n}_m) - \frac{1}{2}(\delta_{ij} - \vec{n}_i\vec{n}_j)(\delta_{km} - \vec{n}_k\vec{n}_m), \tag{1.33}$$

$$Q^{km}(t) := \int T_{00}\left(x'^k x'^m - \frac{1}{3}\delta^{km}\sum_n x'^n x'^n\right)\mathrm{d}V' \tag{1.34}$$

1.5 电磁—引力对比

$$A_\mu(t,\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J_\mu(t - |\vec{r}-\vec{r}'|, \vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{d}V' \tag{1.35}$$

$$\bar{h}_{\mu\nu}(t,\vec{r}) = 4G\int \frac{T_{\mu\nu}(t - |\vec{r}-\vec{r}'|, \vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{d}V' \tag{1.36}$$

$$A_\mu(t,\vec{r}) = \frac{1}{\sqrt{2\pi}}\int \hat{A}_\mu(\omega,\vec{r})e^{-i\omega t}\mathrm{d}t \tag{1.37}$$

$$\bar{h}_{\mu\nu}(t,\vec{r}) = \frac{1}{\sqrt{2\pi}}\int \hat{\bar{h}}_{\mu\nu}(\omega,\vec{r})e^{-i\omega t}\mathrm{d}t \tag{1.38}$$

$$\hat{A}_\mu(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\hat{J}_\mu(\omega,\vec{r}')}{|\vec{r}-\vec{r}'|}e^{i\omega|\vec{r}-\vec{r}'|}\mathrm{d}V' \tag{1.39}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) = 4G\int \frac{\hat{T}_{\mu\nu}(\omega,\vec{r}')}{|\vec{r}-\vec{r}'|}e^{i\omega|\vec{r}-\vec{r}'|}\mathrm{d}V' \tag{1.40}$$

$$\hat{A}_\mu(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{J}_\mu(\omega,\vec{r}')e^{-i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}')}\mathrm{d}V' \tag{1.41}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{\mu\nu}(\omega,\vec{r}')e^{-i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}')}\mathrm{d}V' \tag{1.42}$$

$$\hat{A}_\mu(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{J}_\mu(\omega,\vec{r}')\left[1 - i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}') - \dots\right]\mathrm{d}V' \tag{1.43}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{\mu\nu}(\omega,\vec{r}')\left[1 - i\omega(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}') - \dots\right]\mathrm{d}V' \tag{1.44}$$

1.5.1 电偶极—引力对比

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{J}_i \,\mathrm{d}V' \tag{1.45}$$

$$\hat{\bar{h}}_{ij} = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{ij}\,\mathrm{d}V' \tag{1.46}$$

$$\int \hat{J}_i \,\mathrm{d}V' = -i\omega\int \hat{J}_0 x'^i \,\mathrm{d}V' \tag{1.47}$$

$$\int \hat{T}_{ij}\,\mathrm{d}V' = -\frac{\omega^2}{2}\int \hat{T}_{00}\, x'^i x'^j \,\mathrm{d}V' \tag{1.48}$$

$$\hat{p}_i = \int \hat{J}_0 x'^i \,\mathrm{d}V' \tag{1.49}$$

$$\hat{q}_{ij} = \int \hat{T}_{00}\, x'^i x'^j \,\mathrm{d}V' \tag{1.50}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega\hat{p}_i) \tag{1.51}$$

$$\hat{\bar{h}}_{ij} = 4G\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-\frac{\omega^2}{2}\hat{q}_{ij}) \tag{1.52}$$

$$A_i = \frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|}\frac{\mathrm{d}}{\mathrm{d}t}p_i(t-|\vec{r}|) \tag{1.53}$$

$$\bar{h}_{ij} = 4G\frac{1}{|\vec{r}|}\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}q_{ij}(t-|\vec{r}|) \tag{1.54}$$

1.5.2 电四极—引力对比

$$\hat{A}_i(\omega,\vec{r}) = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\ \ (-i\omega)\int \hat{J}_i(\omega,\vec{r}')(\frac{\vec{r}}{|\vec{r}|}\cdot\vec{r}')\,\mathrm{d}V' \tag{1.55}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega)\int \hat{J}_i' n^j x_j' \,\mathrm{d}V' \tag{1.56}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega)\int n^j x_j' \hat{J}_i' \,\mathrm{d}V' \tag{1.57}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-i\omega)n^j\left[\int x'_{(j}\hat{J}'_{i)}\,\mathrm{d}V'\right] \tag{1.58}$$

$$\begin{aligned}
\int x'_{(j}\hat{J}'_{i)}\,\mathrm{d}V' &= \frac{1}{2}\int(\hat{J}'_j x'_i + \hat{J}'_i x'_j)\,\mathrm{d}V' && (1.59)\\
&= \frac{1}{2}\int\sum_k(\hat{J}'_k x'^i\frac{\partial x'^j}{\partial x'^k} + \hat{J}'_k\frac{\partial x'^i}{\partial x'^k}x'^j)\,\mathrm{d}V' && (1.60)\\
&= \frac{1}{2}\sum_k\left[\int\frac{\partial}{\partial x'^k}(\hat{J}'_k x'^i x'^j)\,\mathrm{d}V' - \int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V'\right] && (1.61)\\
&= \frac{1}{2}\sum_k\int\partial'_k\,(\hat{J}'_k x'^i x'^j)\,\mathrm{d}V' - \frac{1}{2}\sum_k\int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V' && (1.62)\\
&= \frac{1}{2}\sum_k\int\hat{J}'_k x'^i x'^j\,\mathrm{d}S' - \frac{1}{2}\sum_k\int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V' && (1.63)\\
&= -\frac{1}{2}\sum_k\int\frac{\partial\hat{J}'_k}{\partial x'^k}x'^i x'^j\,\mathrm{d}V' && (1.64)\\
&= -\frac{1}{2}\int(\sum_k\partial'_k\hat{J}'_k)x'^i x'^j\,\mathrm{d}V' && (1.65)\\
&= -\frac{1}{2}\int(\partial_0\hat{J}'_0)x'^i x'^j\,\mathrm{d}V' && (1.66)\\
&= -\frac{i\omega}{2}\int\hat{J}'_0 x'^i x'^j\,\mathrm{d}V' && (1.67)
\end{aligned}$$

$$\hat{D}_{ij} = \int\hat{J}'_0\,x'^i x'^j\,\mathrm{d}V' \tag{1.68}$$

$$\hat{A}_i = \frac{\mu_0}{4\pi}\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}(-\frac{\omega^2}{2}n^j\hat{D}_{ij}) \tag{1.69}$$

$$A_i = \frac{\mu_0}{4\pi}\frac{1}{|\vec{r}|}n^j\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}D_{ij}(t-|\vec{r}|) \tag{1.70}$$

1.6 Varying G

$$\partial^c\partial_c\bar{h}_{ab} = -16\pi\frac{G_0}{c_0^4}T_{ab}, \quad \partial^a\bar{h}_{ab} = 0 \tag{1.71}$$

$$\Gamma^c{}_{ab} = \frac{1}{2}\eta^{cd}(2\partial_{(a}h_{b)d} - \partial_d h_{ab}) \tag{1.72}$$

$$U^a\partial_a U^c + \Gamma^c{}_{ab}U^a U^b = 0 \tag{1.73}$$

$$U^a\partial_a U^c = -\frac{1}{2}\eta^{cd}(2\partial_{(a}h_{b)d} - \partial_d h_{ab})U^a U^b \tag{1.74}$$

$$T_{ab} = c_0^2(2U_{(a}J_{b)} + U^c J_c U_a U_b) \tag{1.75}$$

$$J_b c_0^2 = -U^a T_{ab} \tag{1.76}$$

$$A_b = -\frac{1}{4}U^a\bar{h}_{ab} \tag{1.77}$$

$$A_0 = -\frac{1}{4}c_0\bar{h}_{00} = -\frac{1}{2}c_0(\bar{h}_{00} - \frac{1}{2}\eta_{00}\eta^{00}\bar{h}_{00}) = -\frac{1}{2}c_0 h_{00} \tag{1.78}$$

$$A_i = -\frac{1}{4}c_0\bar{h}_{0i} = -\frac{1}{4}c_0 h_{0i} \tag{1.79}$$

$$U^\mu\partial_\mu U^i = -\frac{1}{2}\eta^{i\sigma}(\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu})U^\mu U^\nu \tag{1.80}$$

$$\begin{aligned}
-\frac{1}{2}\eta^{i\sigma}(\partial_0 h_{0\sigma} + \partial_0 h_{0\sigma} - \partial_\sigma h_{00})U^0 U^0 &= \frac{1}{2}c_0^2\eta^{i\sigma}\partial_\sigma h_{00} && (1.81)\\
&= \frac{1}{2}c_0^2\partial^i h_{00} && (1.82)\\
&= -c_0\partial^i A_0 && (1.83)\\
&= -E^i && (1.84)
\end{aligned}$$

$$\begin{aligned}
-\frac{1}{2}\eta^{i\sigma}(\partial_0 h_{j\sigma} + \partial_j h_{0\sigma} - \partial_\sigma h_{0j})U^0 U^j &= -\frac{1}{2}c_0\eta^{i\sigma}(\partial_j h_{0\sigma} - \partial_\sigma h_{0j})v^j && (1.85)\\
&= -\frac{1}{2}c_0\eta^{ik}(\partial_j h_{0k} - \partial_k h_{0j})v^j && (1.86)\\
&= 2\eta^{ik}(\partial_j A_k - \partial_k A_j)v^j && (1.87)\\
&= -2\eta^{ik}(\partial_k A_j - \partial_j A_k)v^j && (1.88)\\
&= -2(\partial^i A_j - \partial_j A^i)v^j && (1.89)\\
&= -2\varepsilon^i{}_{jk}v^j B^k && (1.90)
\end{aligned}$$

$$-\frac{1}{2}\eta^{i\sigma}(\partial_j h_{k\sigma} + \partial_k h_{j\sigma} - \partial_\sigma h_{jk})U^j U^k = 0 \tag{1.91}$$

$$a^i = -E^i - 4\varepsilon^i{}_{jk} v^j B^k \tag{1.92}$$

$$\partial^i (\frac{1}{4\pi G_0} E_i) = \rho \tag{1.93}$$

$$\partial^i B_i = 0 \tag{1.94}$$

$$\varepsilon^i{}_{jk} \partial^j E^k = -\partial_t B^i \tag{1.95}$$

$$\varepsilon^i{}_{jk} \partial^j (\frac{c_0^2}{4\pi G_0} B^k) = j^i + \partial_t (\frac{1}{4\pi G_0} E^i) \tag{1.96}$$

$$\varepsilon_{\mathrm{G}0} := \frac{1}{4\pi G_0}, \quad \mu_{\mathrm{G}0} := \frac{4\pi G_0}{c_0^2} \tag{1.97}$$

$$\begin{cases} \vec{\nabla} \cdot (\varepsilon_{\mathrm{G}0} \vec{E}) = \rho \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \\ \vec{\nabla} \times (\mu_{\mathrm{G}0}^{-1} \vec{B}) = \vec{j} + \frac{\partial}{\partial t} (\varepsilon_{\mathrm{G}0} \vec{E}) \end{cases} \tag{1.98}$$

$$\vec{a} = -\vec{E} - 4\vec{v} \times \vec{B} \tag{1.99}$$

$$\varepsilon_{\mathrm{G}} = \frac{1}{4\pi G}, \quad \mu_{\mathrm{G}} = \frac{4\pi G}{c^2} \tag{1.100}$$

$$x^\mu = (ct, x, y, z) \tag{1.101}$$

$$\begin{cases} \vec{\nabla} \cdot (\varepsilon_{\mathrm{G}} \vec{E}) = \rho \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \\ \vec{\nabla} \times (\mu_{\mathrm{G}}^{-1} \vec{B}) = \vec{j} + \frac{\partial}{\partial t} (\varepsilon_{\mathrm{G}} \vec{E}) \end{cases} \tag{1.102}$$

$$\vec{a} = -\vec{E} - 4\vec{v} \times \vec{B} \tag{1.103}$$

$$A_\mu = -\frac{1}{4} c \bar{h}_{0\mu} \tag{1.104}$$

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = \varepsilon_{\mathrm{G}}^{-1} \rho \\ \vec{\nabla} \times \vec{B} = \mu_{\mathrm{G}} \vec{j} + \varepsilon_{\mathrm{G}} \mu_{\mathrm{G}} \frac{\partial}{\partial t} \vec{E} \end{cases} \tag{1.105}$$

$$\frac{1}{c^2} \frac{\partial}{\partial t} \varphi + \vec{\nabla} \cdot \vec{A} = 0 \tag{1.106}$$

$$\begin{cases} -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varphi + \vec{\nabla}^2\varphi = \varepsilon_{\rm G}^{-1}\rho \\ -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} + \vec{\nabla}^2\vec{A} = \mu_{\rm G}\vec{j} \end{cases} \tag{1.107}$$

$$\begin{cases} -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}c^{-1}\varphi + \vec{\nabla}^2 c^{-1}\varphi = \mu_{\rm G}c\rho \\ -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} + \vec{\nabla}^2\vec{A} = \mu_{\rm G}\vec{j} \end{cases} \tag{1.108}$$

$$\begin{cases} \vec{\nabla}\cdot(\varepsilon_{\rm G}\vec{E}) = 0 \\ \vec{\nabla}\cdot(\mu_{\rm G}\vec{H}) = 0 \\ \vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}(\mu_{\rm G}\vec{H}) \\ \vec{\nabla}\times\vec{H} = +\frac{\partial}{\partial t}(\varepsilon_{\rm G}\vec{E}) \end{cases} \tag{1.109}$$

$$E_r = 0, \quad H_r = 0 \tag{1.110}$$

$$\begin{cases} \frac{\varepsilon_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) + \frac{\varepsilon_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\phi}(E_\phi) = 0 \\ \frac{\mu_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta H_\theta) + \frac{\mu_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\phi}(H_\phi) = 0 \\ \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta E_\phi) - \frac{\partial}{\partial\phi}(E_\theta)]\vec{e}_r - \frac{1}{r}\frac{\partial}{\partial r}(rE_\phi)\vec{e}_\theta + \frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)\vec{e}_\phi = -\mu_{\rm G}\frac{\partial}{\partial t}(H_\theta\vec{e}_\theta + H_\phi\vec{e}_\phi) \\ \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta H_\phi) - \frac{\partial}{\partial\phi}(H_\theta)]\vec{e}_r - \frac{1}{r}\frac{\partial}{\partial r}(rH_\phi)\vec{e}_\theta + \frac{1}{r}\frac{\partial}{\partial r}(rH_\theta)\vec{e}_\phi = +\varepsilon_{\rm G}\frac{\partial}{\partial t}(E_\theta\vec{e}_\theta + E_\phi\vec{e}_\phi) \end{cases} \tag{1.111}$$

$$\vec{E} = E_\theta\vec{e}_\theta, \quad \vec{H} = H_\phi\vec{e}_\phi \tag{1.112}$$

$$\begin{cases} \frac{\varepsilon_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) = 0 \\ \frac{\mu_{\rm G}}{r\sin\theta}\frac{\partial}{\partial\phi}(H_\phi) = 0 \\ -\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(E_\theta)\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)\vec{e}_\phi = -\mu_{\rm G}\frac{\partial}{\partial t}(H_\phi)\vec{e}_\phi \\ +\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta H_\phi)\vec{e}_r - \frac{1}{r}\frac{\partial}{\partial r}(rH_\phi)\vec{e}_\theta = +\varepsilon_{\rm G}\frac{\partial}{\partial t}(E_\theta)\vec{e}_\theta \end{cases} \tag{1.113}$$

$$\begin{cases} \frac{\partial}{\partial r}(rE_\theta) + \mu_{\rm G}\frac{\partial}{\partial t}(rH_\phi) = 0 \\ \frac{\partial}{\partial r}(rH_\phi) + \varepsilon_{\rm G}\frac{\partial}{\partial t}(rE_\theta) = 0 \end{cases} \tag{1.114}$$

$$\begin{cases} \mu_{\rm G}\frac{\partial}{\partial r}\mu_{\rm G}^{-1}\frac{\partial}{\partial r}(rE_\theta) - \varepsilon_{\rm G}\mu_{\rm G}\frac{\partial}{\partial t}\frac{\partial}{\partial t}(rE_\theta) = 0 \\ \varepsilon_{\rm G}\frac{\partial}{\partial r}\varepsilon_{\rm G}^{-1}\frac{\partial}{\partial r}(rH_\phi) - \varepsilon_{\rm G}\mu_{\rm G}\frac{\partial}{\partial t}\frac{\partial}{\partial t}(rH_\phi) = 0 \end{cases} \tag{1.115}$$

$$\begin{cases} \mu_{\rm G}\frac{\partial}{\partial r}\mu_{\rm G}^{-1}\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rE_\theta) = 0 \\ \varepsilon_{\rm G}\frac{\partial}{\partial r}\varepsilon_{\rm G}^{-1}\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rH_\phi) = 0 \end{cases} \tag{1.116}$$

$$\begin{cases} \frac{\partial}{\partial r}\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial}{\partial r}(\ln \mu_{\mathrm{G}})\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rE_\theta) = 0 \\ \frac{\partial}{\partial r}\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial}{\partial r}(\ln \varepsilon_{\mathrm{G}})\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial}{\partial(ct)}\frac{\partial}{\partial(ct)}(rH_\phi) = 0 \end{cases} \tag{1.117}$$

$$\frac{\partial^2}{\partial r^2}f(r,t) - p(r)\frac{\partial}{\partial r}f(r,t) - \frac{\partial^2}{\partial(ct)^2}f(r,t) = 0 \tag{1.118}$$

$$f(r,t) = f(r)e^{-ikct} \tag{1.119}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}f(r) - p(r)\frac{\mathrm{d}}{\mathrm{d}r}f(r) + k^2 f(r) = 0 \tag{1.120}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}f(r) - p\frac{\mathrm{d}}{\mathrm{d}r}f(r) + k^2 f(r) = 0 \tag{1.121}$$

$$f(r) = e^{(p/2)r}[C_+ e^{i\sqrt{k-(p/2)^2}r} + C_- e^{-i\sqrt{k-(p/2)^2}r}] \tag{1.122}$$

$$f(r,t) = e^{(p/2)r}[C_+ e^{i(+\sqrt{k-(p/2)^2}r-kct)} + C_- e^{i(-\sqrt{k-(p/2)^2}r-kct)}] \tag{1.123}$$

$$f(r,t) = e^{(p/2)r}[C_+ e^{i(+\sqrt{k-(p/2)^2}r-kct)} + C_- e^{i(-\sqrt{k-(p/2)^2}r-kct)}] \tag{1.124}$$

$$f(r,t) = e^{(p/2)r}[C_+ e^{i(+\sqrt{(\omega/c)-(p/2)^2}r-\omega t)} + C_- e^{i(-\sqrt{(\omega/c)-(p/2)^2}r-\omega t)}] \tag{1.125}$$

$$f(r,t) = e^{\int(p/2)\mathrm{d}r}[C_+ e^{i(+\int\sqrt{(\omega/c)-(p/2)^2}\mathrm{d}r-\omega t)} + C_- e^{i(-\int\sqrt{(\omega/c)-(p/2)^2}\mathrm{d}r-\omega t)}] \tag{1.126}$$

$$\begin{cases} r_2\,|E_\theta|_{r=r_2} = r_1\,|E_\theta|_{r=r_1}\,e^{\int_{r_1}^{r_2}\frac{1}{2}\frac{\partial}{\partial r}(\ln\mu_{\mathrm{G}})\,\mathrm{d}r} \\ r_2\,|H_\phi|_{r=r_2} = r_1\,|H_\phi|_{r=r_1}\,e^{\int_{r_1}^{r_2}\frac{1}{2}\frac{\partial}{\partial r}(\ln\varepsilon_{\mathrm{G}})\,\mathrm{d}r} \end{cases} \tag{1.127}$$

$$\begin{cases} E_2 = \sqrt{\frac{\mu_{\mathrm{G}2}}{\mu_{\mathrm{G}1}}}\frac{r_1}{r_2}E_1 \\ H_2 = \sqrt{\frac{\varepsilon_{\mathrm{G}2}}{\varepsilon_{\mathrm{G}1}}}\frac{r_1}{r_2}H_1 \end{cases} \tag{1.128}$$

$$\begin{cases} E_2/c_2 = \sqrt{\frac{\mu_{\mathrm{G}2}}{\mu_{\mathrm{G}1}}}\frac{c_1}{c_2}\frac{r_1}{r_2}E_1/c_1 \\ B_2 = \sqrt{\frac{\mu_{\mathrm{G}2}}{\mu_{\mathrm{G}1}}}\frac{c_1}{c_2}\frac{r_1}{r_2}B_1 \end{cases} \tag{1.129}$$

$$\begin{cases} (\omega/c_2)c_2(\bar{h}_{00})_2 = \sqrt{\frac{\mu_{\mathrm{G}2}}{\mu_{\mathrm{G}1}}}\frac{c_1}{c_2}\frac{r_1}{r_2}(\omega/c_1)c_1(\bar{h}_{00})_1 \\ (\omega/c_2)c_2(\bar{h}_{0i})_2 = \sqrt{\frac{\mu_{\mathrm{G}2}}{\mu_{\mathrm{G}1}}}\frac{c_1}{c_2}\frac{r_1}{r_2}(\omega/c_1)c_1(\bar{h}_{0i})_1 \end{cases} \tag{1.130}$$

$$h_2 = \sqrt{\frac{c_1^5/G_1}{c_2^5/G_2}}\frac{r_1}{r_2}h_1 \tag{1.131}$$

双星系统引力辐射本为

$$h = \frac{\mathcal{M}[\pi\mathcal{M}F(t)]^{2/3}}{r} Q(\theta,\phi,\psi,\iota)\cos[\int 2\pi F(t)\,\mathrm{d}t] \tag{1.132}$$

设双星系统常量 c^*, G^*, 一观者临近双星系统且与双星系统相对静止, 其与双星系统距离为 r, 测得强度 h_r, 频率 F_r, 则[1]

$$h_r = \frac{\mathcal{M}[\pi\mathcal{M}F_r(t)]^{2/3}}{r/c^*} Q(\theta,\phi,\psi,\iota) \tag{1.133}$$

设地球观者与双星系统距离为 d, 双星系统红移为 z, 测得强度 h_d, 频率 $F_d = F_r/(1+z)$, 则

$$h_d = \sqrt{\frac{{c^*}^5/G^*}{c^5/G}}\frac{r}{d}h_r \tag{1.134}$$

$$= \sqrt{\frac{{c^*}^5/G^*}{c^5/G}}\frac{\mathcal{M}[\pi\mathcal{M}F_r(t)]^{2/3}}{d/c^*} Q(\theta,\phi,\psi,\iota) \tag{1.135}$$

所以地球观者测得

$$h = \sqrt{\frac{{c^*}^5/G^*}{c^5/G}}\frac{\mathcal{M}[\pi\mathcal{M}F_r(t)]^{2/3}}{d/c^*} Q(\theta,\phi,\psi,\iota)\cos[\int 2\pi\frac{F_r(t)}{1+z}\,\mathrm{d}t] \tag{1.136}$$

记 $F_{\mathrm{obs}}(t) = F_r(t)/(1+z)$, $\mathcal{M}_{\mathrm{obs}} = \mathcal{M}(1+z)$, 光度距离 $d_{\mathrm{L}} = d(1+z)$, 则

$$h = \sqrt{\frac{{c^*}^5/G^*}{c^5/G}}\frac{\mathcal{M}[\pi\mathcal{M}F_r(t)]^{2/3}}{d(1+z)/c^*} Q(\theta,\phi,\psi,\iota)\cos[\int 2\pi F_{\mathrm{obs}}(t)\,\mathrm{d}t] \tag{1.137}$$

$$= \sqrt{\frac{{c^*}^5/G^*}{c^5/G}}\frac{\mathcal{M}_{\mathrm{obs}}[\pi\mathcal{M}_{\mathrm{obs}}F_{\mathrm{obs}}(t)]^{2/3}}{d_{\mathrm{L}}/c^*} Q(\theta,\phi,\psi,\iota)\cos[\int 2\pi F_{\mathrm{obs}}(t)\,\mathrm{d}t] \tag{1.138}$$

$$= \sqrt{\frac{{c^*}^7/G^*}{c^7/G}}\frac{\mathcal{M}_{\mathrm{obs}}[\pi\mathcal{M}_{\mathrm{obs}}F_{\mathrm{obs}}(t)]^{2/3}}{d_{\mathrm{L}}/c} Q(\theta,\phi,\psi,\iota)\cos[\int 2\pi F_{\mathrm{obs}}(t)\,\mathrm{d}t] \tag{1.139}$$

用引力波测距测得 $d_{\mathrm{L,G}}$, 则

$$d_{\mathrm{L,G}} = d_{\mathrm{L}}\sqrt{\frac{c^7/G}{{c^*}^7/G^*}} \tag{1.140}$$

[1] $\mathcal{M}$ 和 c^*, G^* 简并, 所以可以笼统地仍记作 $\mathcal{M}$.

[?]

$$h(t)=\frac{\mathcal{M}[\pi\mathcal{M}F(t)]^{2/3}}{\xi\, d_{\mathrm{L}}}Q(\mathrm{angles})\cos\Phi(t) \tag{1.141}$$

$$\tilde{h}(f)=\frac{\sqrt{30}}{48\pi^{2/3}}\frac{\mathcal{M}^{5/6}Q}{\xi\, d_{\mathrm{L}}}f^{-7/6}e^{i[2\pi f t(f)-\Phi(f)-\frac{\pi}{4}]} \tag{1.142}$$

问题转化为估计 ξ

$$p(\mu)\propto p^{(0)}(\mu)\exp[-\frac{1}{2}\Gamma_{ab}(\mu^a-\hat{\mu}^a)(\mu^b-\hat{\mu}^b)] \tag{1.143}$$

$$p^{(0)}(\mu)\propto \exp[-\frac{1}{2}\Gamma^{(0)}_{ab}(\mu^a-\bar{\mu}^a)(\mu^b-\bar{\mu}^b)] \tag{1.144}$$

设待估参数为 $\mu=(\ln\xi,\ln(d_{\mathrm{L}}/d_{\mathrm{L}0}),\ln Q,\dots)$, $\dots$ 为其他参数 (如 $\mathcal{M}$), 则 $\tilde{h}_{,\ln\xi}=\tilde{h}_{,\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})}=-\tilde{h}_{,\ln Q}=-\tilde{h}$, $\tilde{h}$ 对其他参数求偏导皆为纯虚数, 则由 $\Gamma_{ab}=\langle h_{,a}|h_{,b}\rangle$ 和 $\mathrm{SNR}:=\rho=\sqrt{\langle h|h\rangle}$ 得

$$\Gamma_{ab}=\begin{bmatrix}\rho^2 & \rho^2 & -\rho^2 & 0 & \dots\\ \rho^2 & \rho^2 & -\rho^2 & 0 & \dots\\ -\rho^2 & -\rho^2 & \rho^2 & 0 & \dots\\ 0 & 0 & 0 & ? & \dots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{bmatrix} \tag{1.145}$$

又设

$$\Gamma^{(0)}_{ab}=\begin{bmatrix}0 & 0 & 0 & 0 & \dots\\ 0 & 1/\sigma^2_{\ln d_{\mathrm{L}}} & 0 & 0 & \dots\\ 0 & 0 & 1/\sigma^2_{\ln Q} & 0 & \dots\\ 0 & 0 & 0 & 0 & \dots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{bmatrix} \tag{1.146}$$

则由 $\Sigma_{ab}=(\Gamma^{(0)}_{ab}+\Gamma_{ab})^{-1}$ 得

$$\Sigma_{ab}=\begin{bmatrix}\begin{bmatrix}\rho^2 & \rho^2 & -\rho^2\\ \rho^2 & \rho^2+1/\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & -\rho^2\\ -\rho^2 & -\rho^2 & \rho^2+1/\sigma^2_{\ln Q}\end{bmatrix}^{-1} & 0\\ 0 & [?]^{-1}\end{bmatrix} \tag{1.147}$$

而

$$\begin{bmatrix} \rho^2 & \rho^2 & -\rho^2 \\ \rho^2 & \rho^2 + 1/\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & -\rho^2 \\ -\rho^2 & -\rho^2 & \rho^2 + 1/\sigma^2_{\ln Q} \end{bmatrix}^{-1} \tag{1.148}$$

$$= \begin{bmatrix} 1/\rho^2 + \sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} + \sigma^2_{\ln Q} & -\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & \sigma^2_{\ln Q} \\ -\sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & \sigma^2_{\ln(d_{\mathrm{L}}/d_{\mathrm{L}0})} & 0 \\ \sigma^2_{\ln Q} & 0 & \sigma^2_{\ln Q} \end{bmatrix} \tag{1.149}$$

第二章　能量

第三章　双星系统

3.1　基本公式

$$\mathcal{M} := \mu^{3/5} M^{2/5} \tag{3.1}$$

$$h_+ = \frac{4\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}\frac{1+\cos^2\iota}{2}\cos\Phi(t) \tag{3.2}$$

$$h_\times = \frac{4\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}\cos\iota\sin\Phi(t) \tag{3.3}$$

$$h = F_+h_+ + F_\times h_\times \tag{3.4}$$

3.2　Post-Newtonian Approximation

3.3　Stationary Phase Approximation

[**?**], if $\zeta(t)$ varies slowly near $t = t_0$ where the phase has a stationary point: $\phi'(t_0) = 0$,

$$\begin{aligned}
\int \zeta(t)e^{i\phi(t;f)}\,\mathrm{d}t &= \int \zeta(t)e^{i[\phi(t_0)+\phi'(t_0)(t-t_0)+\frac{1}{2}\phi''(t_0)(t-t_0)^2+\dots]}\,\mathrm{d}t && (3.5)\\
&\simeq e^{i\phi(t_0)}\int \zeta(t)e^{i[\frac{1}{2}\phi''(t_0)(t-t_0)^2]}\,\mathrm{d}t && (3.6)\\
&\simeq e^{i\phi(t_0)}\int \zeta(t_0)e^{\frac{-\sqrt{-i\phi''(t_0)}^2(t-t_0)^2}{2}}\,\mathrm{d}t && (3.7)\\
&= \frac{\sqrt{2\pi}}{\sqrt{-i\phi''(t_0)}}\zeta(t_0)e^{i\phi(t_0)}. && (3.8)
\end{aligned}$$

$$
\begin{aligned}
h &= \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\cos\Phi(t) && (3.9)\\
&= \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}[e^{i\Phi(t)}+e^{-i\Phi(t)}] && (3.10)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{h}(f) &= \int h(t)e^{i2\pi ft}\,\mathrm{d}t && (3.11)\\
&= \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}[e^{i\Phi(t)}+e^{-i\Phi(t)}]e^{i2\pi ft}\,\mathrm{d}t && (3.12)\\
&= \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}\{e^{i[2\pi ft+\Phi(t)]}+e^{i[2\pi ft-\Phi(t)]}\}\,\mathrm{d}t && (3.13)\\
&\simeq \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F(t)]^{2/3}Q\frac{1}{2}e^{i[2\pi ft-\Phi(t)]}\,\mathrm{d}t && (3.14)\\
&= \int \frac{\mathcal{M}}{D}[\pi\mathcal{M}F]^{2/3}Q\frac{1}{2}e^{i[2\pi ft(F)-\Phi(F)]}\frac{\mathrm{d}t}{\mathrm{d}F}\,\mathrm{d}F && (3.15)\\
&\simeq \frac{\sqrt{2\pi}}{\sqrt{-i[2\pi ft(F)-\Phi(F)]''_{F=f}}} && (3.16)\\
&\left[\frac{\mathcal{M}}{D}(\pi\mathcal{M}F)^{2/3}Q\frac{1}{2}\frac{\mathrm{d}t}{\mathrm{d}F}\right]_{F=f}e^{i[2\pi ft(f)-\Phi(f)]} && (3.17)\\
&\simeq \frac{\sqrt{2\pi}}{\sqrt{-i\left\{2\pi f\left[-\frac{5}{256}\mathcal{M}(\pi\mathcal{M}F)^{-8/3}\right]-\left[\frac{1}{16}(\pi\mathcal{M}F)^{-5/3}\right]\right\}''_{F=f}}} && (3.18)\\
&\left\{\frac{\mathcal{M}}{D}(\pi\mathcal{M}F)^{2/3}Q\frac{1}{2}\left[\frac{5\pi\mathcal{M}^2}{96}(\pi\mathcal{M}F)^{-11/3}\right]\right\}_{F=f}e^{i[2\pi ft(f)-\Phi(f)]} && (3.19)\\
&= \frac{\sqrt{30}}{48\pi^{2/3}}\frac{\mathcal{M}^{5/6}Q}{D}f^{-7/6}e^{i[2\pi ft(f)-\Phi(f)-\frac{\pi}{4}]}\quad(\texttt{pnspa.py}) && (3.20)
\end{aligned}
$$

或 [?], $h(t)=2A(t)\cos\phi(t)$, $\mathrm{d}\ln A/\mathrm{d}t \ll \mathrm{d}\phi/\mathrm{d}t$ 且 $|\mathrm{d}^2\phi/\mathrm{d}t^2| \ll (\mathrm{d}\phi/\mathrm{d}t)^2$.

第四章　宇宙学效应

$$\frac{\mathrm{d}\eta}{\mathrm{d}(ct)} = \frac{1}{a} \tag{4.1}$$

$$\mathrm{d}s^2 = -\mathrm{d}(ct)^2 + a^2[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)] \tag{4.2}$$

$$\mathrm{d}s^2 = a^2[-\mathrm{d}\eta^2 + \frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)] \tag{4.3}$$

第五章　电磁引力

[?].

5.1　时空张量转化为空间张量

$$h_{ab} := g_{ab} + Z_a Z_b. \tag{5.1}$$

$$h_a{}^b = \delta_a{}^b + Z_a Z^b. \tag{5.2}$$

$$Z^a h_{ab} = 0. \tag{5.3}$$

$$V_{\langle a\rangle} := h_a{}^b V_b. \tag{5.4}$$

$$Z^a V_{\langle a\rangle} = 0. \tag{5.5}$$

$$T_{\langle ab\rangle} := h_{(a}{}^c h_{b)}{}^d T_{cd} - \frac{1}{3} h_{cd} T^{cd} h_{ab}. \tag{5.6}$$

$$Z^a (h_a{}^c h_b{}^d T_{cd}) = 0. \tag{5.7}$$

$$Z^a (h_b{}^c h_a{}^d T_{cd}) = 0. \tag{5.8}$$

$$Z^a (h_{(a}{}^c h_{b)}{}^d T_{cd}) = 0. \tag{5.9}$$

$$Z^a (h_{cd} T^{cd} h_{ab}) = 0. \tag{5.10}$$

$$Z^a T_{\langle ab\rangle} = 0. \tag{5.11}$$

$$T_{(\langle ab\rangle)} = T_{\langle ab\rangle}. \tag{5.12}$$

$$h^{ab} T_{\langle ab\rangle} = 0. \tag{5.13}$$

$$\varepsilon_{abc} := \varepsilon_{abcd} Z^d. \tag{5.14}$$

$$\varepsilon_{0123} := -\sqrt{|g|}. \tag{5.15}$$

$$T_a := \frac{1}{2}\varepsilon_{abc} T^{[bc]}. \tag{5.16}$$

$$[U, V]_a := \varepsilon_{abc} U^b V^c. \tag{5.17}$$

$$[S, T]_a := \varepsilon_{abc} g_{de} S^{bd} T^{ce}. \tag{5.18}$$

$$\mathrm{D}_t T^{a\dots}{}_{b\dots} := Z^c \nabla_c T^{a\dots}{}_{b\dots}. \tag{5.19}$$

$${}^3\nabla_a T^{b\dots}{}_{c\dots} := h_a{}^p h^b{}_q \dots h_c{}^r \dots \nabla_p T^{q\dots}{}_{r\dots}. \tag{5.20}$$

$$(\operatorname{div} V) := {}^3\nabla^a V_a. \tag{5.21}$$

$$(\operatorname{curl} V)_a := \varepsilon_{bca} {}^3\nabla^b V^c. \tag{5.22}$$

$$(\operatorname{div} T)_a := {}^3\nabla^b T_{ab}. \tag{5.23}$$

$$(\operatorname{curl} T)_{ab} := \varepsilon_{cd(a} {}^3\nabla^c g_{b)e} T^{ed}. \tag{5.24}$$

5.2 电磁空间矢量

$${}^*F_{ab} := \frac{1}{2}\varepsilon_{abcd} F^{cd} \tag{5.25}$$

$$E_a := F_{ab} Z^b = E_{\langle a \rangle}. \tag{5.26}$$

$$B_a := {}^*F_{ab} Z^b = B_{\langle a \rangle}. \tag{5.27}$$

$$\rho = -Z^a J_a. \tag{5.28}$$

$$j_a = h_a{}^b J_b. \tag{5.29}$$

$$\nabla_{[a} F_{bc]} = 0. \tag{5.30}$$

$$\nabla^a F_{ab} = \mu J_b. \tag{5.31}$$

$$(\operatorname{div} E) = \mu\rho - \dots. \tag{5.32}$$

$$(\operatorname{div} B) = + \dots. \tag{5.33}$$

$$(\operatorname{curl} E)_a + \dots = -\mathrm{D}_t B_{\langle a\rangle} - \dots. \tag{5.34}$$

$$(\operatorname{curl} B)_a + \dots = \mu j_a + \mathrm{D}_t E_{\langle a\rangle} + \dots. \tag{5.35}$$

5.3 引力空间张量

$$^{*}C_{abcd} := \frac{1}{2}\varepsilon_{abef}C^{ef}{}_{cd}. \tag{5.36}$$

$$E_{ab} := C_{acbd}Z^c Z^d = E_{\langle ab\rangle}. \tag{5.37}$$

$$B_{ab} := {}^{*}C_{acbd}Z^c Z^d = B_{\langle ab\rangle}. \tag{5.38}$$

$$(\operatorname{div} E)_a = \kappa\frac{1}{3}{}^3\nabla_a\rho - \dots. \tag{5.39}$$

$$(\operatorname{div} B)_a = \kappa(\rho + p)\omega_a + \dots. \tag{5.40}$$

$$(\operatorname{curl} E)_{ab} + \dots = -\mathrm{D}_t B_{\langle ab\rangle} - \dots. \tag{5.41}$$

$$(\operatorname{curl} B)_{ab} + \dots = \kappa\frac{1}{2}(\rho + p)\sigma_{ab} + \mathrm{D}_t E_{\langle ab\rangle} + \dots. \tag{5.42}$$