引力波天文学笔记

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第一章　引力波

[1].

1.1 Linearized Gravity

流形 $\mathbb{R}^4$. 任意坐标系 $\{x^\mu\}$, $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}=\eta_{\mu\nu}+\gamma_{\mu\nu}s+\mathrm{O}(s^2)$, 得

$$R_{\mu\nu\lambda\sigma}=\partial_\sigma\partial_{[\mu}h_{\lambda]\nu}-\partial_\nu\partial_{[\mu}h_{\lambda]\sigma}+\mathrm{O}(s^2). \tag{1.1}$$

$\bar{h}_{\mu\nu}:=h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}\eta^{\lambda\sigma}h_{\lambda\sigma}=h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h$.

$$-\frac{1}{2}\partial^\lambda\partial_\lambda\bar{h}_{\mu\nu}+\partial^\lambda\partial_{(\mu}\bar{h}_{\nu)\lambda}-\frac{1}{2}\eta_{\mu\nu}\partial^\lambda\partial^\sigma\bar{h}_{\lambda\sigma}+\mathrm{O}(s^2)=8\pi T_{\mu\nu}. \tag{1.2}$$

存在 $\{x^\mu\}$, 使得 $\partial^\nu\bar{h}_{\mu\nu}+\mathrm{O}(s^2)=0$ (Lorentz gauge). 令 $\{x^\mu\}$ 满足 $\partial^\nu\bar{h}_{\mu\nu}+\mathrm{O}(s^2)=0$, 则

$$\partial^\lambda\partial_\lambda\bar{h}_{\mu\nu}+\mathrm{O}(s^2)=-16\pi T_{\mu\nu}. \tag{1.3}$$

略去 $\mathrm{O}(s^2)$ 条件: $h_{\mu\nu}$, $\partial_\lambda h_{\mu\nu}$…小.

1.2 Radiation Gauge

存在 $\{x^\mu\}$, 使得 $h+\mathrm{O}(s^2)=0$ (TT gauge [2]) 且 $h_{0\mu}+\mathrm{O}(s^2)=0$.

1.3 quadrupole Approximation

下略 $\mathrm{O}(s^2)$. 由(1.3)得

$$\bar{h}_{\mu\nu}(t,\vec{r})=4\int\frac{T_{\mu\nu}(t-|\vec{r}-\vec{r}'|,\vec{r}')}{|\vec{r}-\vec{r}'|}\,\mathrm{d}V'. \tag{1.4}$$

$$\hat{\bar{h}}_{\mu\nu}(\omega,\vec{r}) := \frac{1}{\sqrt{2\pi}}\int \bar{h}_{\mu\nu}(t,\vec{r})e^{i\omega t}\mathrm{d}t \tag{1.5}$$

$$= 4\int \frac{\hat{T}_{\mu\nu}(\omega,\vec{r}')}{|\vec{r}-\vec{r}'|}e^{i\omega|\vec{r}-\vec{r}'|}\,\mathrm{d}V'. \tag{1.6}$$

由 $\partial^\nu \bar{h}_{\mu\nu}=0$,

$$-i\omega\hat{\bar{h}}_{0\mu} = \sum_i \frac{\partial\hat{\bar{h}}_{i\mu}}{\partial x^i}. \tag{1.7}$$

$|\vec{r}|\gg|\vec{r}'|$ 且 $\omega \ll 1/|\vec{r}'|$,

$$\hat{\bar{h}}_{ij}(\omega,\vec{r}) = 4\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\int \hat{T}_{ij}(\omega,\vec{r}')\,\mathrm{d}V'. \tag{1.8}$$

$$\int \hat{T}_{ij}\,\mathrm{d}V' = -\frac{\omega^2}{2}\int \hat{T}_{00}\,x^i x^j\,\mathrm{d}V', \quad ? \tag{1.9}$$

$$q_{ij}(t) := \int \hat{T}_{00}\,x^i x^j\,\mathrm{d}V' \tag{1.10}$$

$$\hat{\bar{h}}_{ij}(\omega,\vec{r}) = -2\omega^2\frac{e^{i\omega|\vec{r}|}}{|\vec{r}|}\hat{q}_{ij}(\omega), \tag{1.11}$$

$$\bar{h}_{ij}(t,\vec{r}) = \frac{2}{|\vec{r}|}\frac{\mathrm{d}^2}{\mathrm{d}t^2}q_{ij}(t-|\vec{r}|). \tag{1.12}$$

参考文献

[1] Robert M. Wald. *General Relativity*. University of Chicago Pr., 1984.

[2] 王运永. 引力波探测. 科学出版社, 2020.