

$$I(\nu) = \frac{2kT\nu^2}{c^2},$$

$$L(\nu) = 4\pi R^2 \times \int_{-\pi}^{\pi} \int_0^{\frac{\pi}{2}} \cos \theta \sin \theta \, d\theta \, d\phi \times I(\nu) = 4\pi R^2 \times \pi \times I(\nu),$$

$$F(\nu) = \frac{L(\nu)}{4\pi r^2} \approx 60\mu\text{Jy}.$$

$$R_{\odot} \approx 6.957 \times 10^8 \text{m},$$

$$L_{\odot} = 4\pi R_{\odot}^2 \sigma T_{\odot}^4 \approx 3.9 \times 10^{26} \text{W}.$$

$$L_{\lambda} = L_{\odot} \frac{\pi R_{\text{M}}^2}{4\pi r_{\odot \text{M}}^2},$$

$$L_{\text{M}} = 4\pi R_{\text{M}}^2 \sigma T_{\text{M}}^4,$$

$$T_{\text{M}} = \sqrt[4]{\frac{L_{\odot}}{16\pi\sigma r_{\odot \text{M}}^2}} = \sqrt[4]{\frac{R_{\odot}^2}{4r_{\odot \text{M}}^2}} T_{\odot} \approx 230 \text{K}.$$

$$R_{\text{M}} = r_{\text{EM}} \theta_{\text{M}},$$

$$I_{\text{M}}(\nu) = \frac{2kT_{\text{M}}\nu^2}{c^2},$$

$$L_{\text{M}}(\nu) = 4\pi R_{\text{M}}^2 \times \int_{-\pi}^{\pi} \int_0^{\frac{\pi}{2}} \cos \theta \sin \theta \, d\theta \, d\phi \times I_{\text{M}}(\nu) = 4\pi R_{\text{M}}^2 \times \pi \times I_{\text{M}}(\nu),$$

$$F_{\text{M}}(\nu) = \frac{L_{\text{M}}(\nu)}{4\pi r_{\text{EM}}^2} = 4\theta_{\text{M}}^2 I_{\text{M}}(\nu) \approx 4.8 \times 10^{-25} \text{W}/(\text{m}^2 \cdot \text{Hz}).$$

$$T_{\text{M}} = \sqrt[4]{\frac{(1-\alpha)L_{\odot}}{16\pi\sigma r_{\odot \text{M}}^2}} = \sqrt[4]{\frac{(1-\alpha)R_{\odot}^2}{4r_{\odot \text{M}}^2}} T_{\odot} \approx 220 \text{K}.$$

$$F_{\text{M}}(\nu) = \frac{L_{\text{M}}(\nu)}{4\pi r_{\text{EM}}^2} = 4\theta_{\text{M}}^2 I_{\text{M}}(\nu) \approx 4.7 \times 10^{-25} \text{W}/(\text{m}^2 \cdot \text{Hz}).$$

$$P = \frac{2}{3} \frac{q^2 \ddot{x}^2}{c^3} = \frac{2}{3} \frac{q^2 A^2 \omega^4 \cos^2(\omega t)}{c^3},$$

$$\bar{P} = \frac{2q^2 A^2 \omega^4}{3c^3} \frac{\int_{\omega t=-2\pi}^{\omega t=2\pi} \cos^2(\omega t) \, dt}{\int_{\omega t=-2\pi}^{\omega t=2\pi} dt} = \frac{2q^2 A^2 \omega^4}{3c^3} \frac{\int_{-2\pi}^{2\pi} \cos^2 \theta \, d\theta}{\int_{-2\pi}^{2\pi} d\theta} = \frac{q^2 A^2 \omega^4}{3c^3}.$$

$$E_{\perp} = \Re \left[\frac{-i\pi \sin \theta}{c} \frac{I_0 l}{\lambda} \frac{e^{-i\omega t}}{r} \right],$$

$$S = \frac{c}{4\pi} E_{\perp}^2 \propto \frac{\sin^2 \theta}{r^2},$$

$$P(\Omega) d\Omega = S r^2 d\Omega \propto \sin^2 \theta d\Omega,$$

$$\iint \sin^2 \theta \sin \theta \, d\theta \, d\phi = \int \frac{4}{3} \, d\phi = \frac{8\pi}{3},$$

$$G(\theta, \phi) = \frac{3}{8\pi} \sin^2 \theta.$$

$$A_e(\theta, \phi) = \frac{\lambda^2}{4\pi} G(\theta, \phi) = \frac{3\lambda^2}{32\pi^2} \sin^2 \theta = \frac{3c^2}{32\pi^2 \nu^2} \sin^2 \theta,$$

$$P(\nu) = A_e \frac{S}{2} \approx 5.1 \times 10^{-26} \text{W/Hz}.$$

$$T_A = \frac{P(\nu)}{k} \approx 3.7 \times 10^{-3} \text{K}.$$

$$R_{\text{ff}} \approx \frac{2D^2}{\lambda} \approx 2 \times 10^6 \text{m}.$$

$$f(l) = g \int_{-\frac{D}{2\lambda}}^{\frac{D}{2\lambda}} e^{-i2\pi l u} du = g \left[\frac{e^{-i2\pi l u}}{-i2\pi l} \right]_{-\frac{D}{2\lambda}}^{\frac{D}{2\lambda}} = g \frac{\sin(\pi l \frac{D}{\lambda})}{\pi l}.$$

$$\lambda_D \approx \sqrt{\frac{kT}{4\pi n_e e^2}},$$

$$b_{\text{max}} \approx \frac{v}{2\pi\nu},$$

$$\frac{1}{2} m_e v^2 \approx \frac{3}{2} kT,$$

$$b_{\text{max}} \approx \frac{\sqrt{\frac{3kT}{m_e}}}{2\pi\nu} \approx 1 \times 10^{-4} \text{m}.$$

$$n_e \approx \frac{kT}{4\pi e^2 \lambda_D^2} \approx 4 \times 10^{15} \text{m}^{-3}.$$

$$T \approx 10^4 \text{K},$$

$$\nu = 100 \text{MHz} = 0.1 \text{GHz},$$

$$\text{EM} = \int_{R_\odot}^{r_\oplus} \left[N_{e\oplus} \left(\frac{r_\oplus}{r} \right)^2 \right]^2 dr = \left[-\frac{1}{3} N_{e\oplus}^2 r_\oplus^4 r^{-3} \right]_{R_\odot}^{r_\oplus} \approx \frac{N_{e\oplus}^2 r_\oplus^4}{R_\odot^3} \approx 5000 \text{pc/cm}^6,$$

$$\tau_\nu \approx 3.28 \times 10^7 \left(\frac{T}{10^4 \text{K}} \right)^{-1.35} \left(\frac{\nu}{\text{GHz}} \right)^{-2.1} \left(\frac{\text{EM}}{\text{pc/cm}^6} \right) \approx 2 \times 10^{13}.$$

$$\frac{d}{dt}(\gamma m_e \vec{v}) = -\frac{e}{c}(\vec{v} \times \vec{B}),$$

$$\frac{d}{dt}\gamma = 0,$$

$$\frac{d}{dt}\vec{v} = -\frac{e}{\gamma m_e c}(\vec{v} \times \vec{B}),$$

$$\omega^2 r = \frac{e}{\gamma m_e c} v B = \frac{e}{\gamma m_e c} \omega r B,$$

$$\omega = \frac{eB}{\gamma m_e c} = \frac{\omega_G}{\gamma},$$

$$\sin \alpha \approx 1,$$

$$\nu_c = \frac{3}{2} \gamma^2 \nu_G \sin \alpha = \frac{3}{2} \gamma^2 \frac{eB}{2\pi m_e c},$$

$$\gamma^2 = \frac{4\pi m_e c \nu_c}{3eB},$$

$$E = \gamma m_e c^2,$$

$$P = \frac{2e^2}{3c^3} \gamma^2 \frac{e^2 B^2}{m_e^2 c^2} v^2 \sin^2 \alpha = \frac{2e^2}{3c^3} \gamma^2 \frac{e^2 B^2}{m_e^2} \left(1 - \frac{1}{\gamma^2} \right) \approx \frac{2e^2}{3c^3} \gamma^2 \frac{e^2 B^2}{m_e^2},$$

$$\tau \approx \frac{E}{p} \approx \frac{3m_e^3 c^5}{2\gamma e^4 B^2} \approx 9 \times 10^7 \text{yr}.$$