

# 谱理论与量子力学

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# 第一章 自伴算符

**定义 1.1.** 对算符  $\hat{A} : D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$ , 若  $\overline{D_{\hat{A}}} = \mathcal{H}$ , 则称  $\hat{A}$  为稠定算符.

**定理 1.1.** 若  $\hat{A} : D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$  是线性算符,  $|\alpha\rangle \in \mathcal{H}$  则当且仅当  $\hat{A}$  为稠定算符, 满足  $\forall |\beta\rangle \in D_{\hat{A}}$ ,

$$\langle \alpha | (\hat{A}|\beta\rangle) = \langle \gamma | \beta \rangle$$

的  $|\gamma\rangle \in \mathcal{H}$  是唯一的.

**定义 1.2.** 对稠定线性算符  $\hat{A} : D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$ , 若  $\hat{A}^\dagger : D_{\hat{A}^\dagger} \subset \mathcal{H} \rightarrow \mathcal{H}$ ,  $|\alpha\rangle \mapsto \hat{A}^\dagger|\alpha\rangle$  满足  $\forall |\beta\rangle \in \mathcal{H}$ ,

$$\langle \alpha | (\hat{A}|\beta\rangle) = (\langle \alpha | \hat{A}^\dagger) |\beta\rangle$$

则称  $\hat{A}^\dagger$  为  $\hat{A}$  的伴随算符. 若  $\hat{A}^\dagger = \hat{A}$ , 则称  $\hat{A}$  为自伴算符.

**定义 1.3.** 对稠定线性算符  $\hat{A} : D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$ , 若  $\forall |\alpha\rangle, |\beta\rangle \in \mathcal{H}$ ,

$$\langle \alpha | (\hat{A}|\beta\rangle) = (\langle \alpha | \hat{A}) |\beta\rangle$$

则称  $\hat{A}$  为 Hermitean 算符.

**定义 1.4.** 对算符  $\hat{A} : D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$ ,  $\hat{B} : D_{\hat{B}} \subset \mathcal{H} \rightarrow \mathcal{H}$ ,

1. 若  $D_{\hat{A}} = D_{\hat{B}}$ , 且  $\forall |\varphi\rangle \in D_{\hat{A}}$ ,  $\hat{A}|\varphi\rangle = \hat{B}|\varphi\rangle$ , 则称  $\hat{A}$  与  $\hat{B}$  相等, 并记作  $\hat{A} = \hat{B}$ ;
2. 若  $D_{\hat{A}} \subset D_{\hat{B}}$ , 且  $\forall |\varphi\rangle \in D_{\hat{A}}$ ,  $\hat{A}|\varphi\rangle = \hat{B}|\varphi\rangle$ , 则称  $\hat{A}$  是  $\hat{B}$  的限制,  $\hat{B}$  是  $\hat{A}$  的延拓, 并记作  $\hat{A} \subset \hat{B}$ .

**定义 1.5.** 若  $\hat{A} : D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$  为稠定线性算符, 则当且仅当  $\hat{A} \subset \hat{A}^\dagger$ ,  $\hat{A}$  为 Hermitean 算符.

**定义 1.6.** 对线性算符  $\hat{A}: D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$ , 若  $\exists M \in \mathbb{R}$ , 使得  $\forall |\varphi\rangle \in D_{\hat{A}}$ ,

$$\|\hat{A}|\varphi\rangle\| \leq M\| |\varphi\rangle \|,$$

则称  $\hat{A}$  为有界算符, 否则称  $\hat{A}$  为无界算符.

**定义 1.7.** 若  $\hat{A}: D_{\hat{A}} \subset \mathcal{H} \rightarrow \mathcal{H}$  为有界 Hermitean 算符, 则  $\hat{A}$  为自伴算符.