

If we only take the lowest order effects of the motion of mass sources into account and neglect stresses, then we can define two vector fields,  $\vec{E}$  and  $\vec{B}$ , which are similar to the electric and magnetic field. We can define  $A_\mu = -\frac{1}{4}c_0\bar{h}_{0\mu} = (\varphi/c_0, \vec{A})$ , then  $A_\mu$ , which are similar to the electromagnetic 4-potential, will satisfy

$$\partial^\nu \partial_\nu A_\mu = -\frac{4\pi G_0}{c_0^2} J_\mu, \quad \partial^\mu A_\mu = 0, \quad (1)$$

where  $J_\mu = -c_0 T_{0\mu}/c_0^2 = (c_0\rho, \vec{j}) = \rho(c_0, \vec{v})$  is 4-momentum density. If we define  $\vec{E} = -\vec{\nabla}\varphi - \frac{\partial}{\partial t}\vec{A}$ ,  $\vec{B} = \vec{\nabla} \times \vec{A}$ , and  $\varepsilon_{G0} = \frac{1}{4\pi G_0}$ ,  $\mu_{G0} = \frac{4\pi G_0}{c_0^2}$ , then there will be

$$\begin{cases} \vec{\nabla} \cdot (\varepsilon_{G0}\vec{E}) = \rho, \\ \vec{\nabla} \cdot \vec{B} = 0, \\ \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}, \\ \vec{\nabla} \times (\mu_{G0}^{-1}\vec{B}) = \vec{j} + \frac{\partial}{\partial t}(\varepsilon_{G0}\vec{E}), \end{cases} \quad (2)$$

and the acceleration of mass particle  $\vec{a}$  will satisfy

$$\vec{a} = -\vec{E} - 4\vec{v} \times \vec{B}. \quad (3)$$

Since (??) are identical to Maxwell's equations, and (??) is identical to the Lorentz force equation except for an overall minus sign and a factor of 4 in the "magnetic force" term, we assume that the constants  $\varepsilon_{G0}$  and  $\mu_{G0}$  can simply vary as constants  $\varepsilon_0$  and  $\mu_0$  in electromagnetic theories, which means, (??) is still valid and (??) becomes

$$\begin{cases} \vec{\nabla} \cdot \vec{D} = \rho, \\ \vec{\nabla} \cdot \vec{B} = 0, \\ \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}, \\ \vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial}{\partial t}\vec{D}, \end{cases} \quad (4)$$

where  $\vec{D} = \varepsilon_G \vec{E}$ ,  $\vec{B} = \mu_G \vec{H}$ . Also, we define  $c = \frac{1}{\sqrt{\varepsilon_G \mu_G}}$ ,  $G = \frac{1}{4\pi \varepsilon_G}$ , together with  $\vec{E} = -\vec{\nabla}\varphi - \frac{\partial}{\partial t}\vec{A}$ ,  $\vec{B} = \vec{\nabla} \times \vec{A}$ , and  $\bar{h}_{0\mu} = -4A_\mu/c = -4(\varphi/c, \vec{A})/c$ .

We assume that the source of GW is located in a spherical coordinate origin and  $c = c(r)$ ,  $G = G(r)$ . We suppose that  $\bar{h}_{0\mu}(\rho, \vec{j}; c_0, G_0)$  is the solution of  $\bar{h}_{0\mu}$  when  $\rho$  and  $\vec{j}$  are assumed and  $c \equiv c_0$ ,  $G \equiv G_0$  everywhere, then if  $c \equiv c_s$ ,  $G \equiv G_s$ , where  $c_s$  and  $G_s$  are two constants, within a region  $r \leq R$ , then  $\bar{h}_{0\mu}(\rho, \vec{j}; c_s, G_s)$  can be a solution of  $\bar{h}_{0\mu}$  within the region  $r \leq R$ , and now we can solve the equations

$$\begin{cases} \vec{\nabla} \cdot \vec{D} = 0, \\ \vec{\nabla} \cdot \vec{B} = 0, \\ \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}, \\ \vec{\nabla} \times \vec{H} = +\frac{\partial}{\partial t}\vec{D}, \end{cases} \quad (5)$$

with the boundary conditions  $\vec{E}|_{r=R} = \vec{E}(\rho, \vec{j}; c_s, G_s)$ ,  $\vec{B}|_{r=R} = \vec{B}(\rho, \vec{j}; c_s, G_s)$ , where  $\vec{E}(\rho, \vec{j}; c_s, G_s)$  and  $\vec{B}(\rho, \vec{j}; c_s, G_s)$  can be derived from  $\bar{h}_{0\mu}(\rho, \vec{j}; c_s, G_s)$ .

Again, since (??) are identical to Maxwell's equations, we can use the Liénard–Wiechert potentials formula to calculate the  $\vec{E}$  and  $\vec{B}$  field produced by a moving particle, whose position is  $\vec{r}'$ , in vacuum, and the results are

$$\vec{E} = \frac{m}{4\pi\epsilon_{G0}} \left[ \frac{(\vec{n}' - \vec{\beta})}{(1 - \vec{\beta} \cdot \vec{n}')^3 \gamma^2 r'^2} + \frac{\vec{n}' \times \{(\vec{n}' - \vec{\beta}) \times \dot{\vec{\beta}}\}}{(1 - \vec{\beta} \cdot \vec{n}')^3 c_0 r'} \right]_{\text{ret}}, \vec{B} = \left[ \frac{\vec{n}' \times \vec{E}}{c_0} \right]_{\text{ret}}, \quad (6)$$

where  $r' = |\vec{r}'|$ ,  $\vec{n}' = \vec{r}'/r'$ ,  $\vec{\beta} = \dot{\vec{r}}'/c_0$ ,  $\gamma = 1/\sqrt{1 - |\vec{\beta}|^2}$ , and “ret” denotes that the fields are “retarded fields”. Both  $\vec{E}$  and  $\vec{B}$  in (??) can be divided into two parts: one, called “inherent field part”, is proportional to  $1/r'^2$  and the other one, called “radiation part”, is proportional to  $1/r'$ . We only take the lowest order effects of the motion of mass sources into account, therefore we can assume that  $|\vec{\beta}| \ll 1$ . The “inherent field part” of  $\vec{E}$  will be parallel to  $\vec{r}'$  and the “inherent field part” of  $\vec{B}$  will be almost zero vector. The “radiation part” of  $\vec{E}$  and  $\vec{B}$  will be perpendicular to  $\vec{r}'$ . Since ?? are linear equations, we can discuss two parts of  $\vec{E}$  and  $\vec{B}$  separately for a binary GW source. If we assume that the distance between two components of binary is much less than  $R$  mentioned above, then the “inherent field part” of  $\vec{E}$  will have only  $r$  component, and the “radiation part” of  $\vec{E}$  and  $\vec{B}$  will have no  $r$  component.

For the “inherent field part”, ?? will become  $\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) = 0$ , then  $D_r \propto 1/r^2$ , and  $E_r \propto \epsilon_G^{-1} (1/r^2)$ . As mentioned below, for the “radiation part”,  $E \propto c^{-1} \epsilon_G^{-1/2} (1/r)$ , therefore the “inherent field part” will vanish in the region far away from source.