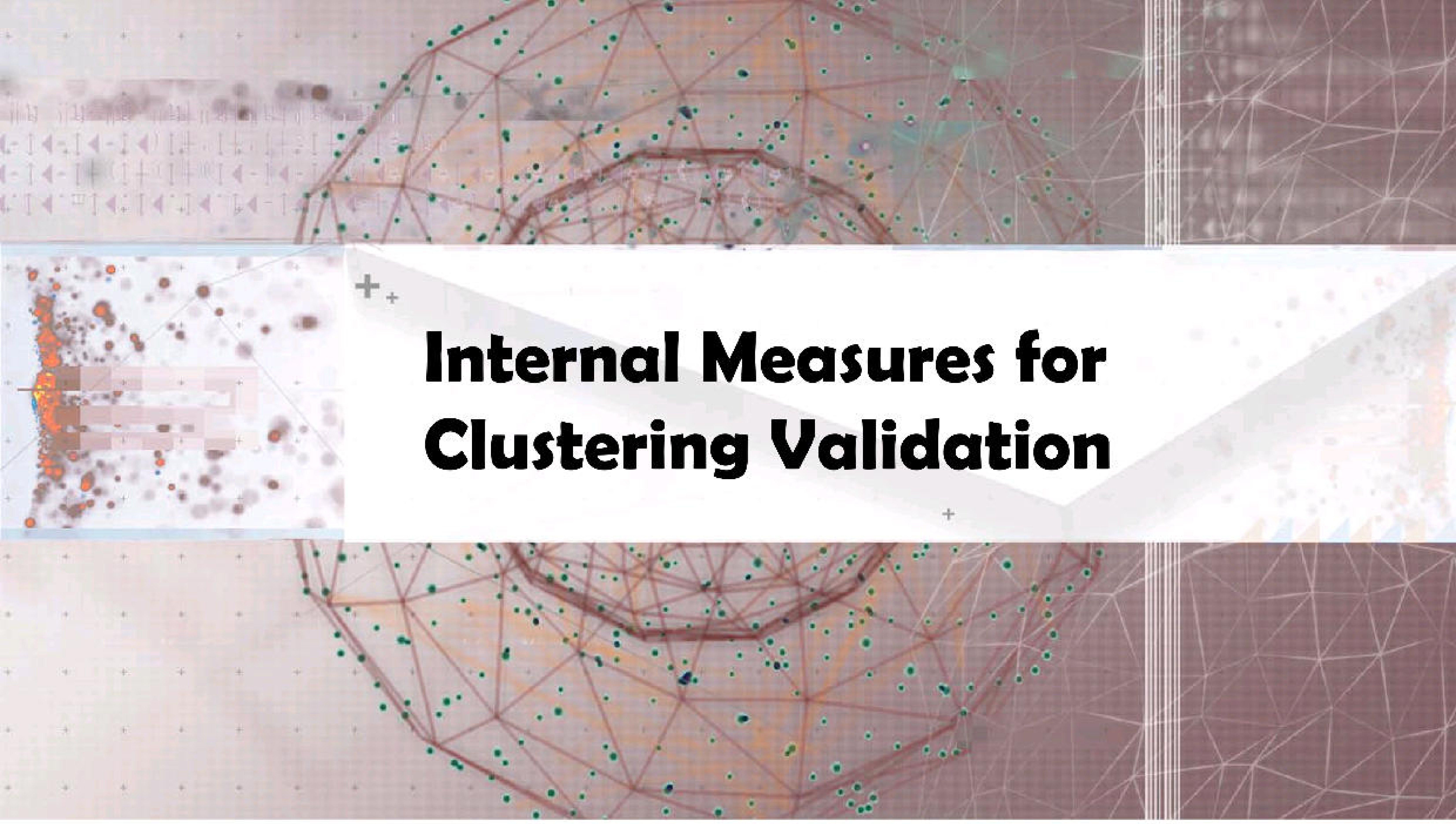
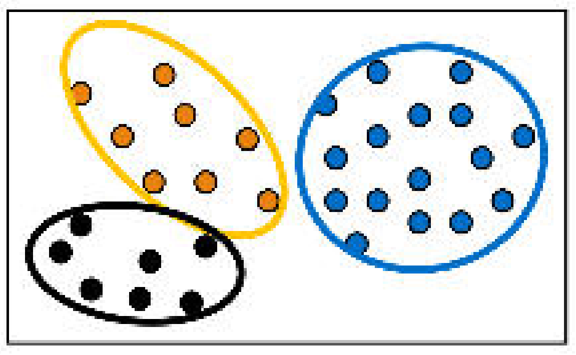


Internal Measures for Clustering Validation

Internal Measures (I): BetaCV Measure

- A trade-off in maximizing intra-cluster compactness and inter-cluster separation
- Given a clustering $C = \{C_1, \ldots, C_k\}$ with k clusters, cluster C_i containing $n_i = |C_i|$ points
  - Let $W(S, R)$ be sum of weights on all edges with one vertex in S and the other in R
  - The sum of all the intra-cluster weights over all clusters: $W_{in} = \frac{1}{2}\sum_{i=1}^{k} W(C_i, C_i)$
  - The sum of all the inter-cluster weights: $W_{out} = \frac{1}{2}\sum_{i=1}^{k} W(C_i, \overline{C_i}) = \sum_{i=1}^{k-1}\sum_{j>i} W(C_i, C_j)$
  - The number of distinct intra-cluster edges: $N_{in} = \sum_{i=1}^{k}\binom{n_i}{2}$
  - The number of distinct inter-cluster edges: $N_{out} = \sum_{i=1}^{k-1}\sum_{j=i+1}^{k} n_i n_j$



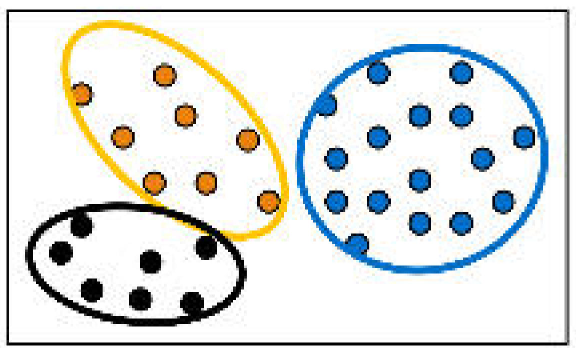
- **Beta-CV measure**: $BetaCV = \frac{W_{in} / N_{in}}{W_{out} / N_{out}}$
  - The ratio of the mean intra-cluster distance to the mean inter-cluster distance
  - The smaller, the better the clustering

Internal Measures (II): Normalized Cut and Modularity

- **Normalized cut:** $NC=\sum_{i=1}^{k}\frac{W(C_i,\overline{C_i})}{vol(C_i)}=\sum_{i=1}^{k}\frac{W(C_i,\overline{C_i})}{W(C_i,V)}=\sum_{i=1}^{k}\frac{W(C_i,\overline{C_i})}{W(C_i,C_i)+W(C_i,\overline{C_i})}=\sum_{i=1}^{k}\frac{1}{\frac{W(C_i,C_i)}{W(C_i,\overline{C_i})}+1}$

  where $vol(C_i) = W(C_i, V)$ is the volume of cluster C_i

  - The higher normalized cut value, the better the clustering



- **Modularity** (for graph clustering) $Q=\sum_{i=1}^{k}\left(\frac{W(C_i,C_i)}{W(V,V)}-\left(\frac{W(C_i,V)}{W(V,V)}\right)^2\right)$
  - Modularity Q is defined as

    where $W(V,V)=\sum_{i=1}^{k}W(C_i,V)=\sum_{i=1}^{k}W(C_i,C_i)+\sum_{i=1}^{k}W(C_i,\overline{C_i})=2(W_{in}+W_{out})$
  - Modularity measures the difference between the observed and expected fraction of weights on edges within the clusters.
  - The smaller the value, the better the clustering—the intra-cluster distances are lower than expected