



Null Invariance Measures

Interestingness Measures & Null-Invariance

- ❑ **Null invariance:** Value does not change with the # of null-transactions
- ❑ A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant
$\chi^2(A, B)$	$\sum_{i,j=0,1} \frac{(e(a_i b_j) - o(a_i b_j))^2}{e(a_i b_j)}$	$[0, \infty]$	No
$Lift(A, B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$AllConf(A, B)$	$\frac{s(A \cup B)}{\max\{s(A), s(B)\}}$	$[0, 1]$	Yes
 $Jaccard(A, B)$	$\frac{s(A \cup B)}{s(A) + s(B) - s(A \cup B)}$	$[0, 1]$	Yes
 $Cosine(A, B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A, B)$	$\frac{1}{2} \left(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)} \right)$	$[0, 1]$	Yes
$MaxConf(A, B)$	$\max\left\{ \frac{s(A)}{s(A \cup B)}, \frac{s(B)}{s(A \cup B)} \right\}$	$[0, 1]$	Yes

χ^2 and lift are not null-invariant

Jaccard, cosine, AllConf, MaxConf, and Kulczynski are null-invariant measures

Null Invariance: An Important Property

□ Why is null invariance crucial for the analysis of massive transaction data?

□ Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	milk	\neg milk	Σ_{row}
coffee	mc	$\neg mc$	c
\neg coffee	$m\neg c$	$\neg m\neg c$	$\neg c$
Σ_{col}	m	$\neg m$	Σ

null transaction

□ Lift and χ^2 are not null-invariant: not good to evaluate data that contain too many or too few null transactions!

□ Many measures are not null-invariant!

Null-transactions w.r.t. m and c

→ $m\neg c$ 的值太大或太小，都不适合 χ^2 和 lift

Data set	mc	$\neg mc$	$m\neg c$	$\neg m\neg c$	χ^2	Lift
D_1	10,000	1,000	1,000	100,000	90557	9.26
D_2	10,000	1,000	1,000	100	0	1
D_3	100	1,000	1,000	100,000	670	8.44
D_4	1,000	1,000	1,000	100,000	24740	25.75
D_5	1,000	100	10,000	100,000	8173	9.18
D_6	1,000	10	100,000	100,000	965	1.97