Probabilistic Topic Models: Expectation-Maximization Algorithm

ChengXiang "Cheng" Zhai
Department of Computer Science
University of Illinois at Urbana-Champaign

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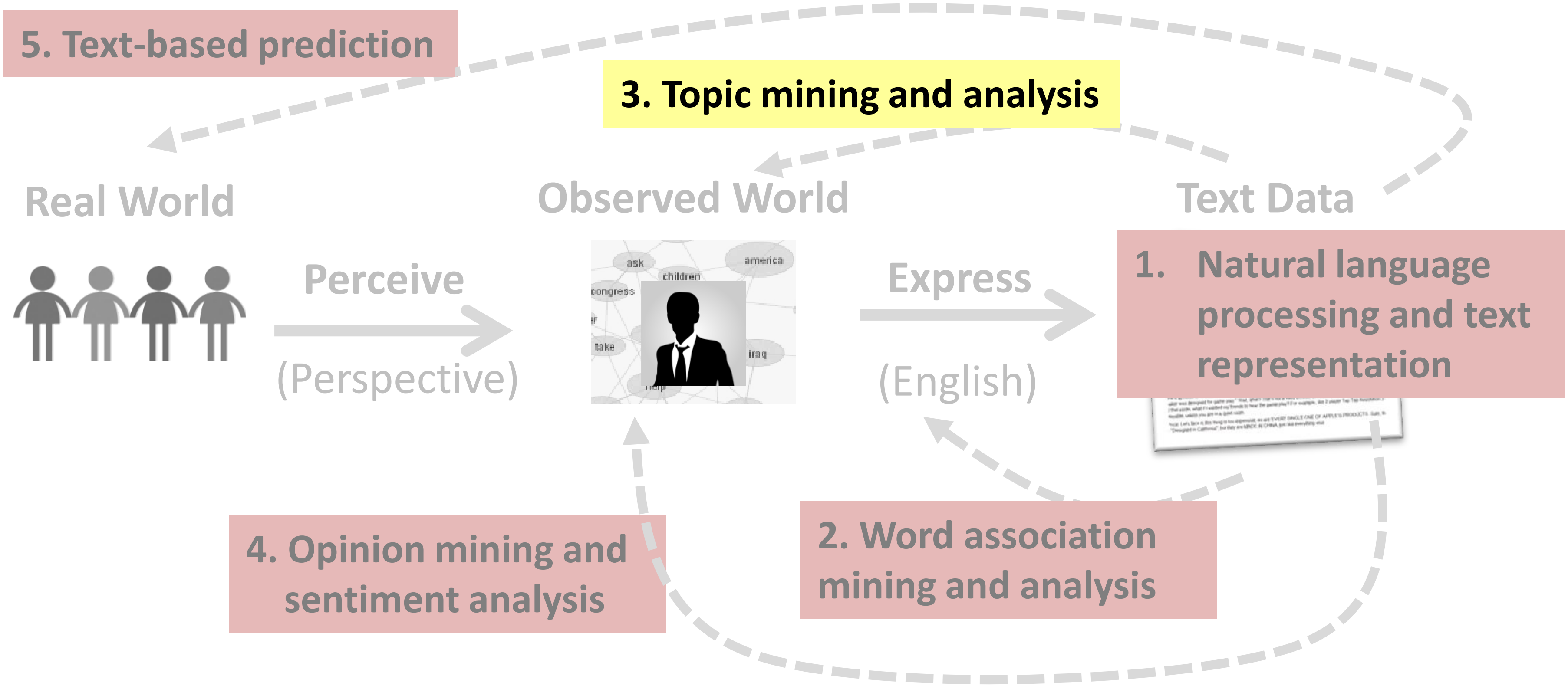


Estimation of One Topic: $P(w|\ \theta_d)$

How to set θ_d to maximize $p(d|\Lambda)$?
(all other parameters are known)

d

… text mining…
is… clustering…
we…. Text.. the

θ_d

text ?
mining ?
association ?
clustering ?
…
the ?

θ_B

the 0.03
a 0.02
is 0.015
we 0.01
food 0.003
…
text 0.000006

$p(\theta_d)+(\theta_B)=1$

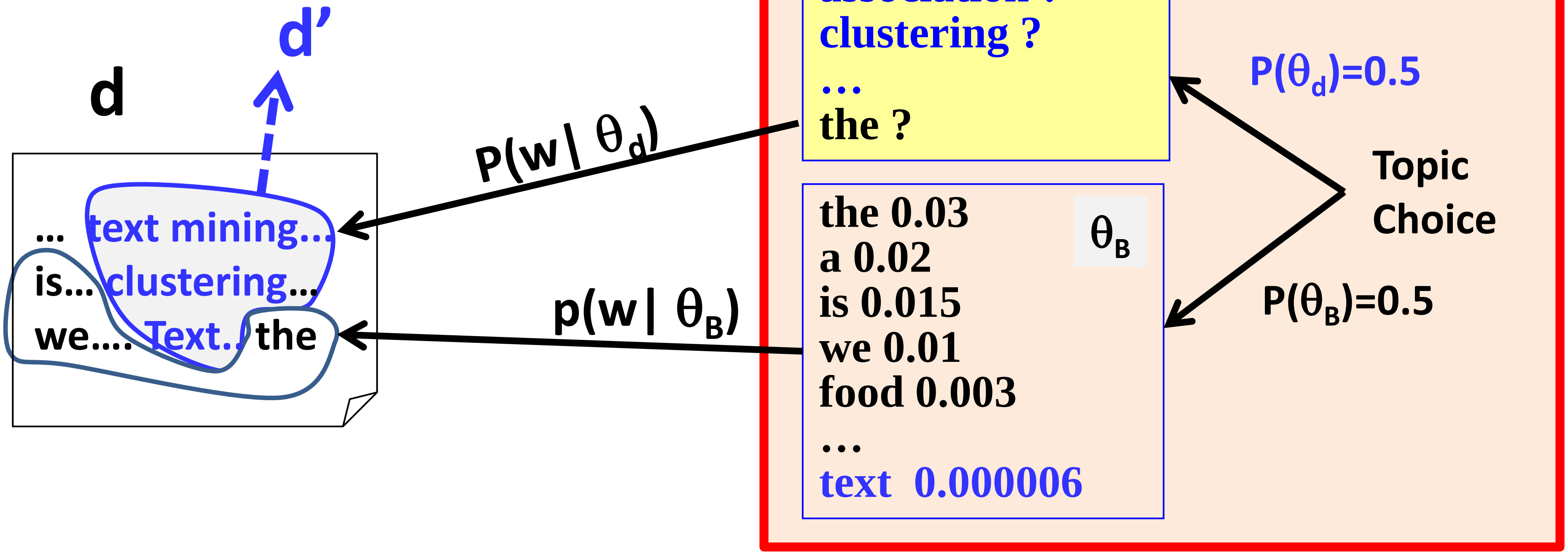
$P(\theta_d)=0.5$

Topic Choice

$P(\theta_B)=0.5$

If we know which word is from which distribution…

$$p(w_i \mid \theta_d) = \frac{c(w_i, d')}{\sum_{w' \in V} c(w', d')}$$



Given all the parameters, infer the distribution a word is from…

Is "text" more likely from θ_d or θ_B ?

From θ_d (Z=0)? → from θ_d.

$p(\theta_d)p(\text{"text"}|\theta_d)$

From θ_B (Z=1)? → from θ_B

$p(\theta_B)p(\text{"text"}|\theta_B)$

$P(w|\theta_d)$

text 0.04
mining 0.035
association 0.03
clustering 0.005
…
the 0.000001

θ_d

$p(w|\theta_B)$

the 0.03
a 0.02
is 0.015
we 0.01
food 0.003
…
text 0.000006

θ_B

$p(\theta_d)+p(\theta_B)=1$

$P(\theta_d)=0.5$

Topic Choice

$P(\theta_B)=0.5$

$$p(z=0|w=\text{"text"}) = \frac{p(\theta_d)p(\text{"text"}|\theta_d)}{p(\theta_d)p(\text{"text"}|\theta_d)+p(\theta_B)p(\text{"text"}|\theta_B)}$$

text|θ_d > text|θ_B.
但前面有 prior $P(\theta_d)$ 和 $P(\theta_B)$. ∴要综合以上两者

The Expectation-Maximization (EM) Algorithm

Hidden Variable: $z \in \{0, 1\}$

	z
the	1
paper	1
presents	1
a	1
text	0
mining	0
algorithm	0
for	1
clustering	0
...	...

① Initialize $p(w|\theta_d)$ with random values.
② Then iteratively improve it using E-step & M-step.
③ Stop when likelihood doesn't change.

$$p^{(n)}(z=0\,|\,w) = \frac{p(\theta_d)p^{(n)}(w\,|\,\theta_d)}{p(\theta_d)p^{(n)}(w\,|\,\theta_d) + p(\theta_B)p(w\,|\,\theta_B)}$$ E-step

How likely w is from θ_d 利用E-step推出的z.

$$p^{(n+1)}(w\,|\,\theta_d) = \frac{c(w,d)p^{(n)}(z=0\,|\,w)}{\sum_{w'\in V} c(w',d)p^{(n)}(z=0\,|\,w')}$$ M-step

获得进一步的预测

将相同组(z={0,1})分别组.

EM Computation in Action

E-step $p^{(n)}(z=0\mid w)=\dfrac{p(\theta_d)p^{(n)}(w\mid\theta_d)}{p(\theta_d)p^{(n)}(w\mid\theta_d)+p(\theta_B)p(w\mid\theta_B)}$

M-step $p^{(n+1)}(w\mid\theta_d)=\dfrac{c(w,d)p^{(n)}(z=0\mid w)}{\sum_{w'\in V}c(w',d)p^{(n)}(z=0\mid w')}$

Assume $p(\theta_d)=p(\theta_B)=0.5$ and $p(w|\theta_B)$ is known

Word	#	$p(w\|\theta_B)$	Iteration 1		Iteration 2		Iteration 3	
			$P(w\|\theta)$	$p(z=0\|w)$	$P(w\|\theta)$	$P(z=0\|w)$	$P(w\|\theta)$	$P(z=0\|w)$
The	4	0.5	**0.25**	0.33	**0.20**	0.29	**0.18**	0.26
Paper	2	0.3	**0.25**	0.45	**0.14**	0.32	**0.10**	0.25
Text	4	0.1	**0.25**	0.71	**0.44**	0.81	**0.50**	0.93
Mining	2	0.1	**0.25**	0.71	**0.22**	0.69	**0.22**	0.69
Log-Likelihood			-16.96		-16.13		-16.02	

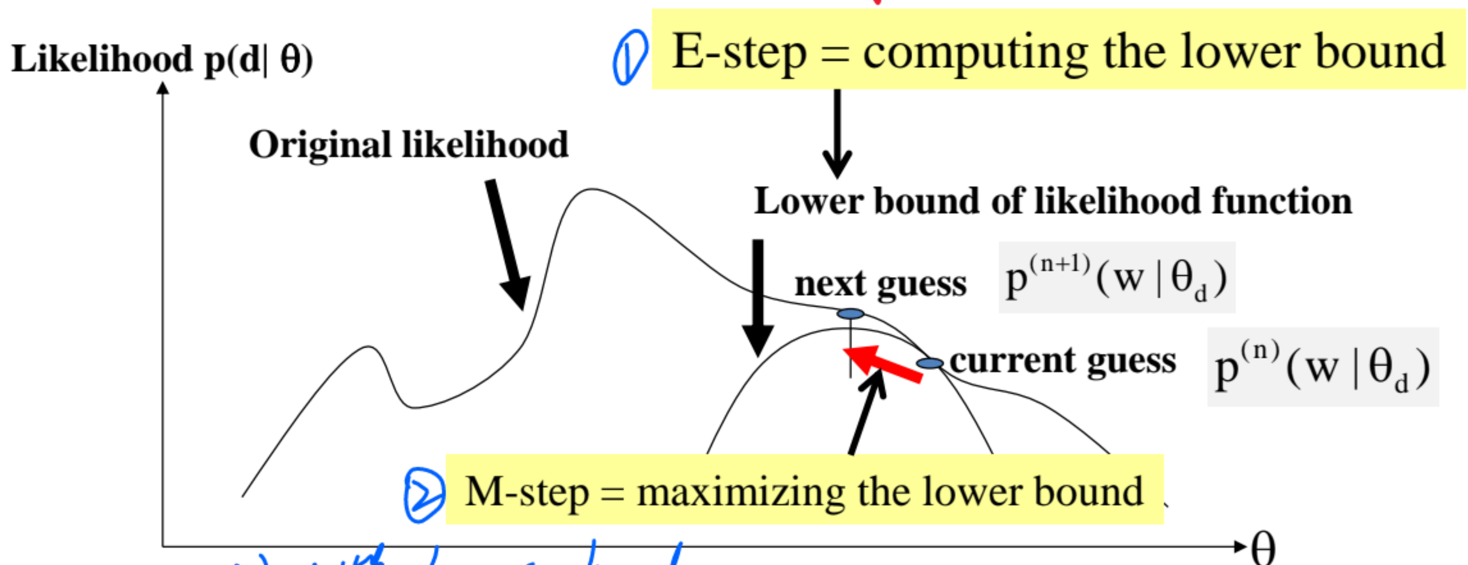
Likelihood increasing

值取值范围是 (0,1) ∴ log < 0.

"By products": Are they also useful?

EM As Hill-Climbing ➔ Converge to Local Maximum

类似于 梯度上升.



①: 计算 lower bound.

②: 找到 lower bound 最大值

③ 进一步提升.

Summary

- Expectation-Maximization (EM) algorithm
  - General algorithm for computing ML estimate of mixture models
  - Hill-climbing, so can only converge to a local maximum (depending on initial points) 类似梯度下降去理解.
- E-step: “augment” data by predicting values of useful hidden variables 通过预测有用的隐藏变量来简化计算.
- M-step: exploit the “augmented data” to improve estimate of parameters (“improve” is guaranteed in terms of likelihood)
- “Data augmentation” is probabilistic ➔ Split counts of events probabilistically