Text Categorization: Discriminative Classifiers

Part 1

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Overview

- What is text categorization?
- Why text categorization?
- How to do text categorization?
  - Generative probabilistic models
  - **Discriminative approaches**
- How to evaluate categorization results?

Anatomy of Naïve Bayes Classifier

Two categories: θ_1 and θ_2

$$\text{score}(d) = \log \frac{p(\theta_1 | d)}{p(\theta_2 | d)} = \log \frac{p(\theta_1) \prod_{w \in V} p(w | \theta_1)^{c(w,d)}}{p(\theta_2) \prod_{w \in V} p(w | \theta_2)^{c(w,d)}}$$

$$= \boxed{\log \frac{p(\theta_1)}{p(\theta_2)}} + \sum_{w \in V} \underline{c(w,d)} \boxed{\log \frac{p(w | \theta_1)}{p(w | \theta_2)}}$$

Category bias (β_0) doesn't depend on d!

Sum over all words (features $\{x_i\}$)

Feature value: x_i=c(w,d)

Weight on each word (feature) β_i

Generalize

$$d = (x_1, x_2, \ldots, x_M), \quad x_i \in \Re$$

$$\text{score}(d) = \beta_0 + \sum_{i=1}^{M} x_i \beta_i \qquad \beta_i \in \Re$$

= Logistic Regression!

Discriminative Classifier 1: Logistic Regression

Binary Response Variable: Y $\in$ {0,1}

Predictors: $X = (x_1, x_2, \ldots, x_M), \;\; x_i \in \Re$

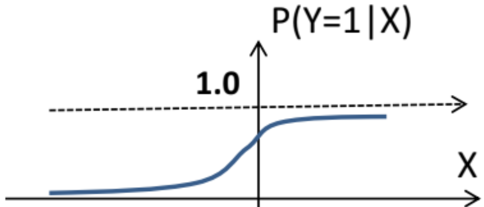
$$Y = \begin{cases} 1 & \text{category}(d) = \theta_1 \\ 0 & \text{category}(d) = \theta_2 \end{cases}$$

Modeling p(Y|X) directly

Allow many other features than words!

$$\log \frac{p(\theta_1 \mid d)}{p(\theta_2 \mid d)} = \log \frac{p(Y=1 \mid X)}{p(Y=0 \mid X)} = \log \frac{p(Y=1 \mid X)}{1 - p(Y=1 \mid X)} = \beta_0 + \sum_{i=1}^{M} x_i \beta_i \qquad \beta_i \in \Re$$

$$p(Y=1 \mid X) = \frac{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i}}{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i} + 1}$$



$\ln \frac{x}{1-x} = y,$

$\frac{x}{1-x} = e^y. \Rightarrow 1 + \frac{x}{1-x} = e^y + 1 \Rightarrow \frac{1}{1-x} = e^y + 1 \Rightarrow 1 - x = \frac{1}{e^y+1} \quad x = 1 - \frac{1}{e^y+1} = \frac{e^y}{e^y+1}$

Estimation of Parameters

- Training Data: T={(Xi, Yi)}, i=1,2, …, |T|
- Parameters: $\vec{\beta} = (\beta_0, \beta_1, \ldots, \beta_M)$
- Conditional likelihood: $p(T \mid \vec{\beta}) = \prod_{i=1}^{|T|} p(Y = Y_i \mid X = X_i, \vec{\beta})$

Y_i =1

$$p(Y = 1 \mid X) = \frac{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i}}{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i} + 1}$$

Y_i =0

$$p(Y = 0 \mid X) = \frac{1}{e^{\beta_0 + \sum_{i=1}^{M} x_i \beta_i} + 1}$$

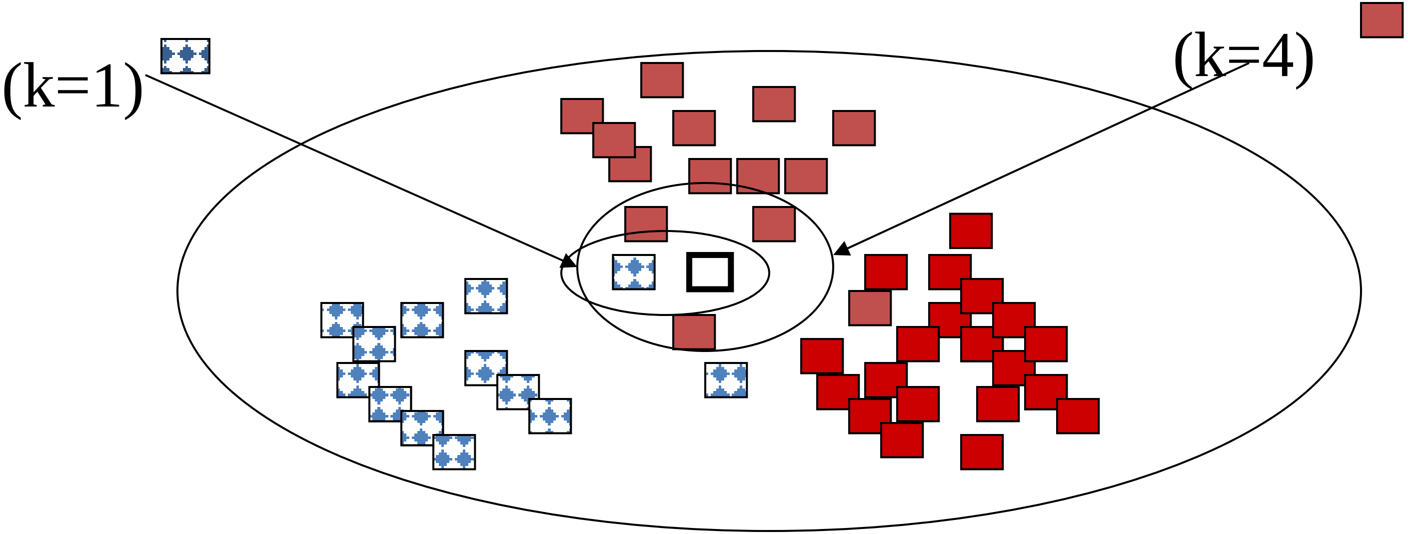
- Maximum Likelihood estimate $\vec{\beta}^* = \arg\max_{\vec{\beta}} p(T \mid \vec{\beta})$

Can be computed in many ways (e.g., Newton’s method)

Discriminative Classifier 2: K-Nearest Neighbors (K-NN)

- Find k examples in the training set that are most similar to the text object to be classified ("neighbor" documents)
- Assign the category that is most common in these neighbor text objects (neighbors vote for the category)
- Can be improved by considering the distance of a neighbor (a closer neighbor has more influence)
- Can be regarded as a way to directly estimate the conditional probability of label given data instance, i.e., p(Y|X)
- Need a similarity function to measure similarity of two text objects

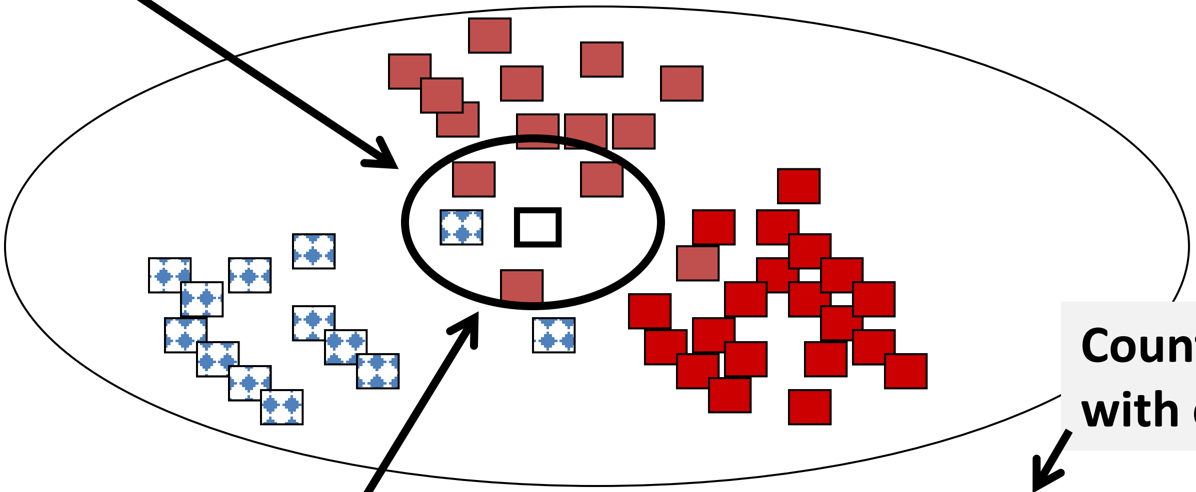
Illustration of K-NN Classifier



K-NN as an Estimate of p(Y|X)

Assume $p(\theta_i|d)$ is <u>locally smooth</u>, i.e., the same for all the d's in this region R

$p(\theta_i|d) = p(\theta_i|R)$



Count of d's in R with category θ_i

Estimate $p(\theta_i|R)$ based on the known categories in the region

$$p(\theta_i \mid R) = \frac{c(\theta_i, R)}{|R|}$$

Total # of docs in R