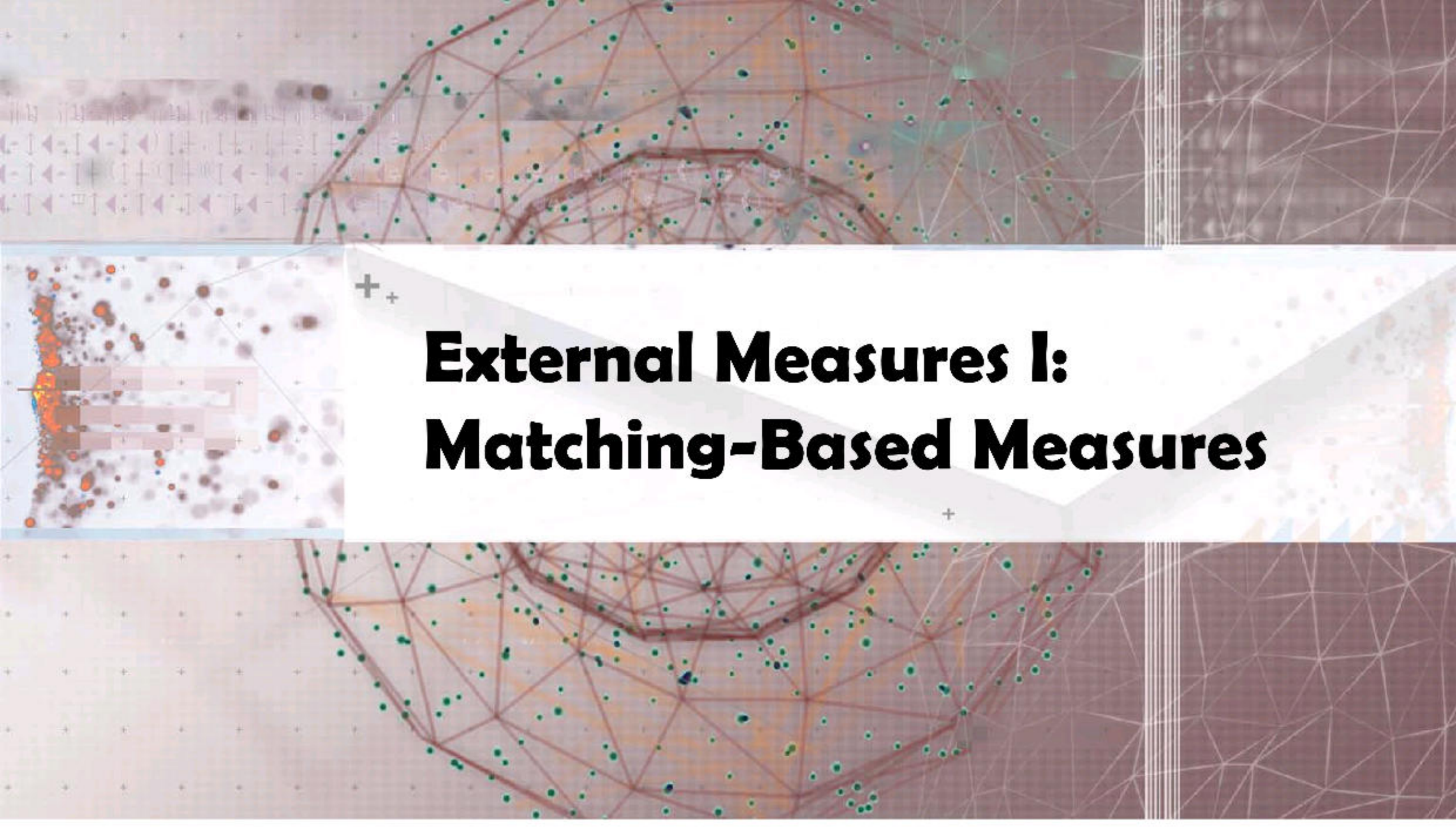


External Measures I: Matching-Based Measures

Matching-Based Measures (I): Purity vs. Maximum Matching

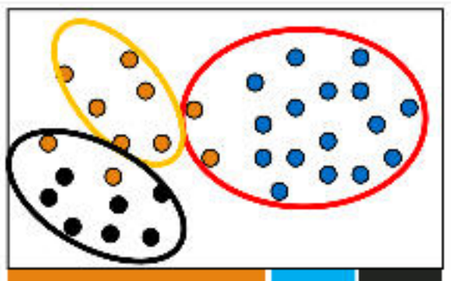
- **Purity**: Quantifies the extent that cluster C_i contains points only from one (ground truth) partition:

$$purity_i = \frac{1}{n_i} \max_{j=1}^{k} \{n_{ij}\}$$

  - Total purity of clustering C:

$$purity = \sum_{i=1}^{r} \frac{n_i}{n} purity_i = \frac{1}{n} \sum_{i=1}^{r} \max_{j=1}^{k} \{n_{ij}\}$$

  - Perfect clustering if purity = 1 and $r = k$ (the number of clusters obtained is the same as that in the ground truth)
  - Ex. 1 (green or orange): $purity_1$ = 30/50; $purity_2$ = 20/25; $purity_3$ = 25/25; *purity* = (30 + 20 + 25)/100 = 0.75
  - Two clusters may share the same majority partition



C\T	T_1	T_2	T_3	Sum
C_1	0	20	30	50
C_2	0	20	5	25
C_3	25	0	0	25
m_j	25	40	35	100

- **Maximum matching**: Only one cluster can match one partition
  - Match: Pairwise matching, weight $w(e_{ij}) = n_{ij}$ $\quad w(M) = \sum_{e \in M} w(e)$
  - Maximum weight matching: $match = \arg\max_{M} \{\frac{w(M)}{n}\}$
  - Ex2. (green) *match* = *purity* = 0.75; (orange) *match* = 0.65 > 0.6

C\T	T_1	T_2	T_3	Sum
C_1	0	30	20	50
C_2	0	20	5	25
C_3	25	0	0	25
m_j	25	50	25	100

Matching-Based Measures (II): F-Measure

- **Precision**: The fraction of points in C_i from the majority partition T_{j_i} (i.e., the same as purity), where j_i is the partition that contains the maximum # of points from C_i

$$prec_i = \frac{1}{n_i} \max_{j=1}^{k} \{n_{ij}\} = \frac{n_{ij_i}}{n_i}$$

  - Ex. For the green table
    - $prec_1$ = 30/50; $prec_2$ = 20/25; $prec_3$ = 25/25
- **Recall**: The fraction of point in partition T_{j_i} shared in common with cluster C_i, where $m_{j_i} = |T_{j_i}|$

$$recall_i = \frac{n_{ij_i}}{|T_{j_i}|} = \frac{n_{ij_i}}{m_{j_i}}$$

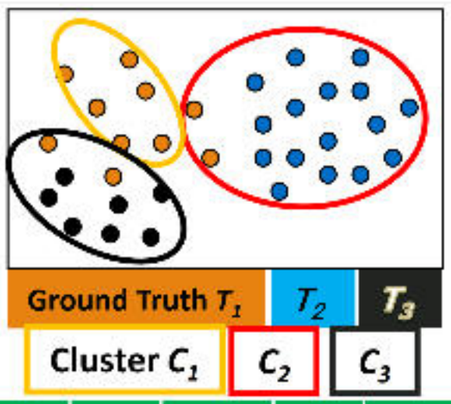
  - Ex. For the green table
    - $recall_1$ = 30/35; $recall_2$ = 20/40; $recall_3$ = 25/25
- **F-measure** for C_i: The harmonic means of $prec_i$ and $recall_i$:

$$F_i = \frac{2n_{ij_i}}{n_i + m_{j_i}}$$

- F-measure for clustering C: average of all clusters:

$$F = \frac{1}{r} \sum_{i=1}^{r} F_i$$

  - Ex. For the green table
    - F_1 = 60/85; F_2 = 40/65; F_3 = 1; F = 0.774



$C \backslash T$	T_1	T_2	T_3	Sum
C_1	0	20	30	50
C_2	0	20	5	25
C_3	25	0	0	25
m_j	25	40	35	100