Syntagmatic Relation Discovery: Entropy

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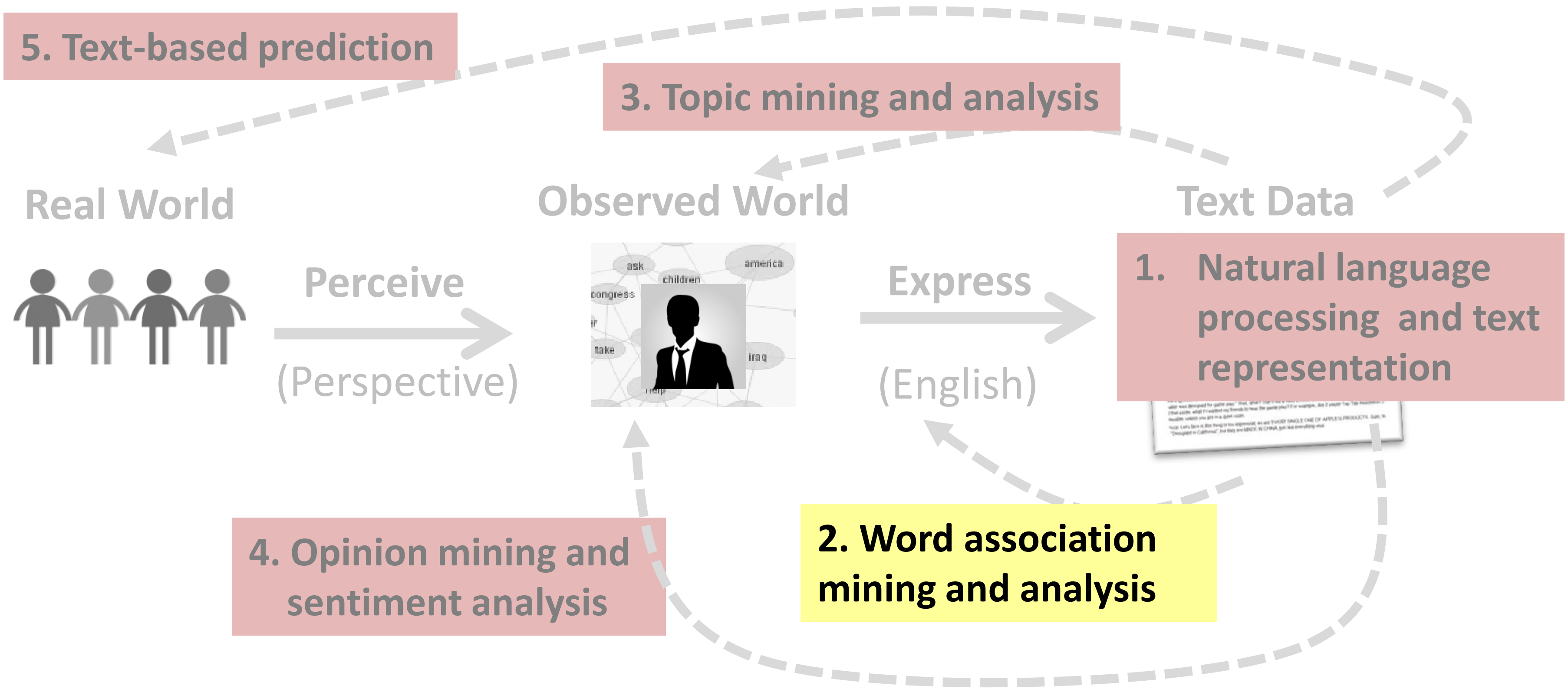


Syntagmatic Relation = Correlated Occurrences

Whenever "**eats**" occurs, what **other words** also tend to occur?

My cat **eats** fish on Saturday
His cat **eats** turkey on Tuesday
My dog **eats** meat on Sunday
His dog **eats** turkey on Tuesday
…

My __ **eats** __ on Saturday
His __ **eats** __ on Tuesday
My __ **eats** __ on Sunday
His __ **eats** __ on Tuesday
…

What words tend to occur to the **left** of **"eats"?**

What words are to the **right?**

Word Prediction: Intuition

Prediction Question: Is word **W** present (or absent) in this segment?

Text Segment (any unit, e.g., sentence, paragraph, document)

Are some words easier to predict than others?

1) W = "meat" medium

2) W="the" frequent

3) W="unicorn" rare

Word Prediction: Formal Definition

Binary Random Variable :
$X_w \in \{0, 1\}$

$$X_w = \begin{cases} 1 & \text{w is present 存在} \\ 0 & \text{w is absent 不存在.} \end{cases}$$

$$p(X_w = 1) + p(X_w = 0) = 1$$

Xw越随机，则预测越困难.

The more random X_w is, the more difficult the prediction would be.

How does one quantitatively measure the "randomness" of a random variable like Xw?

Entropy H(X) Measures Randomness of X

熵描述了随机程度.

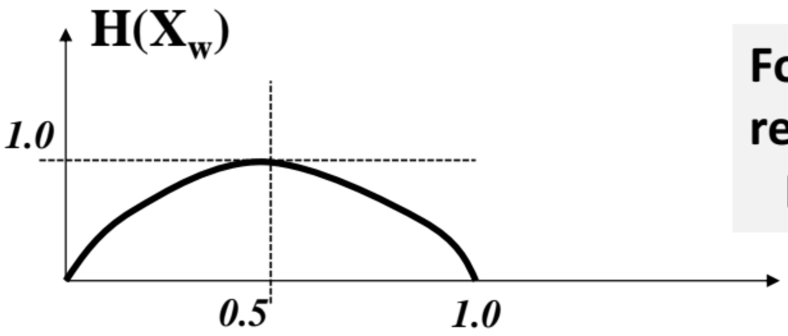
$$H(X_w) = \sum_{v\in\{0,1\}} -p(X_w = v)\log_2 p(X_w = v)$$

$$X_w = \begin{cases} 1 & \text{w is present} \\ 0 & \text{w is absent} \end{cases}$$

展开

$$= -p(X_w = 0)\log_2 p(X_w = 0) - p(X_w = 1)\log_2 p(X_w = 1)$$

Define $0\log_2 0 = 0$



P(Xw=1)

or equivalently P(Xw=0) (Why?)

For what X_w, does $H(X_w)$ reach maximum/minimum?

E.g., $P(X_w=1)=1$? $P(X_w=1)=0.5$?

Entropy H(X): Coin Tossing

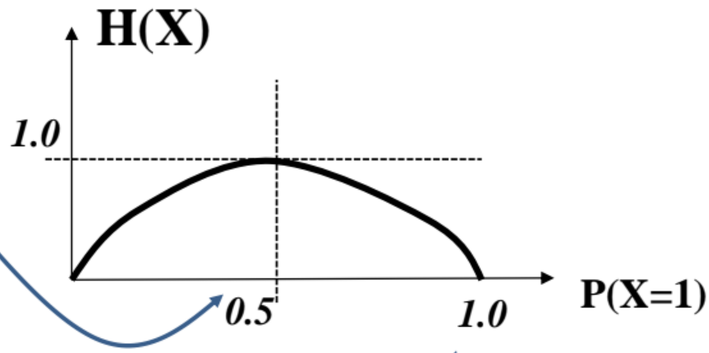
$$H(X_{coin}) = -p(X_{coin}=0)\log_2 p(X_{coin}=0) - p(X_{coin}=1)\log_2 p(X_{coin}=1)$$

X_{coin}: tossing a coin $\quad X_{coin} = \begin{cases} 1 & \text{Head} \\ 0 & \text{Tail} \end{cases}$

Fair coin: p(X=1)=p(X=0)=1/2

此时熵最大，随机程度最大，很难预测

$$H(X) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} = 1$$



Completely biased: p(X=1)=1

熵最小，随机程度最小，最容易预测。

$$H(X) = -0*\log_2 0 - 1*\log_2 1 = 0$$

Entropy for Word Prediction

Is word **W** present (or absent) in this segment?

1) W = “meat” 2) W = “the” 3) W = “unicorn”

Which is **high/low**? **H(X_{meat}), H(X_{the}), or H($X_{unicorn}$)?**

the 几乎很容易出现.

熵很高.

H(X_{the})≈0 ➔ no uncertainty since p(X_{the}=1) ≈ 1

High entropy words are harder to predict!