An Approach to Test the Open-Loop Parameters of Feedback Amplifiers

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***Abstract*—A novel method for determining open-loop parameters, such as unity-gain frequency and phase margin, in feedback amplifiers is given. It is based on relations between open-loop and closed-loop configurations and is very useful for evaluating the above parameters in a unity-gain amplifier from a simple measure of its closed-loop 3-dB cutoff frequency and the respective phase for a given feedback factor, β. The approach is validated by simulations of an ideal amplifier and of a real OTA. Moreover, measures carried out for a μA-741 operational amplifier show a percentage error between real and calculated values lower than 2%.**

***Index Terms*—Close-loop systems, feedback amplifiers, frequency measurement, open-loop systems, phase measurement.**

I. Introduction

The commonly used parameters, which describe the stability and the bandwidth of a feedback amplifier, are the phase margin and the unity-gain frequency in open-loop configuration [1]–[4].

Unfortunately, their measurement can present difficulties in many cases, especially when the amplifier under test is a low-voltage, high-gain amplifier. Indeed, due to the high gain, the input signal must be kept low in order to get the output signal level less than the power supply. At the same time, the input signal must be higher than the equivalent input noise level, which grows with the decreasing of bias currents. Obviously, if the noise level multiplied by the amplifier gain is greater than the power supply value, we cannot have a direct measure of the amplifier open-loop parameters easily.

Several methods have been proposed to evaluate the phase margin and the unity-gain frequency of an amplifier both in the frequency and in the time domain [5]. In particular, [6] suggests achieving the desired parameters measuring the overshoot in the time domain. In [7], [8] strategies to measure the open-loop gain of an amplifier directly in the frequency domain are given. The approach in [9] is based on the implementation of an oscillator as test stage. Finally, [10] proposes to measure the phase of the amplifier in unity gain configuration and does not require external components.

This paper presents a novel and accurate method for the determination of the phase margin and the unity-gain frequency of an amplifier. Considering the achieved accuracy, the method is simpler with respect to previous approaches and has also the advantage to allow an easy measurement of such low-voltage IC amplifiers which cannot be closed in unity-gain configuration [11], [12]. The approach is based on the measure of the 3-dB cutoff frequency and its respective phase value for a given feedback factor. Moreover, the same measure gives us information on the same parameter values of the amplifier in unity-gain feedback configuration.

Manuscript received June 29, 2000; revised March 6, 2201. This paper was recommended by Associate Editor N. M. K. Rao.

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Publisher Item Identifier S 1057-7122(02)00284-2.

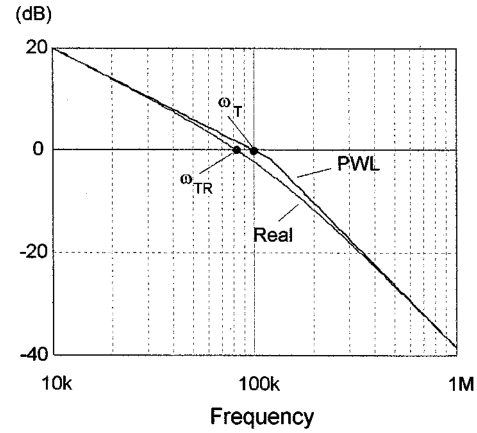

Fig. 1. Frequency gain magnitude: Real and PWL approximation.

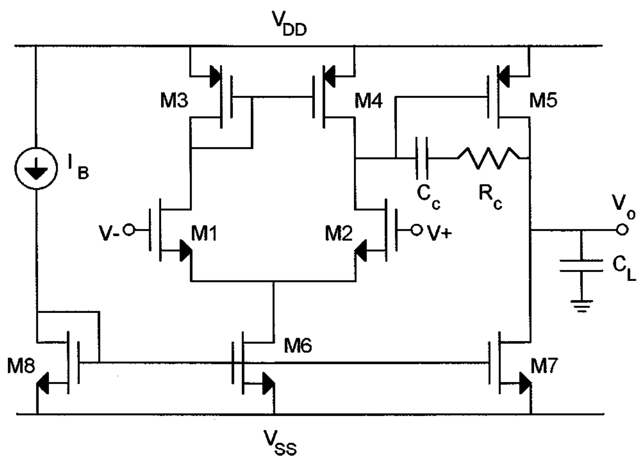

Fig. 2. Schematic of the OTA.

II. Parameters Extraction

The frequency behavior of an open-loop amplifier is approximately defined by two real poles,[1] ω_1 and ω_2, and a dc gain, a_0

$$a(s) = \frac{a_0}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}. \tag{1}$$

Assuming a feedback factor, β, the closed-loop transfer function, which has usually complex conjugate poles, is

$$A_{CL}(s) = \frac{A_{CL}(0)}{1+\frac{s}{\omega_0 Q}+\frac{s^2}{\omega_0^2}} \tag{2}$$

where ω_0 and Q which are the *pole frequency* and the *pole-Q factor*, respectively, are given by

$$\omega_0 = \sqrt{\omega_1 \omega_2 (1+\beta a_0)} \tag{3}$$

$$Q = \frac{\omega_0}{\omega_1+\omega_2} \tag{4}$$

and $\beta a_0 \gg 1$. Note that the term $A_{CL}(0)$ is often reported to be equal to $a_0/(1+\beta a_0)$ and, in general, it is set by a ratio of passive elements (resistors or capacitors). Actually, although it depends not only on the

[1]Every amplifier with more than one pole or zeros close to the transition frequency can be well approximated in a form similar to that given in (1) since its behavior mainly depends on its phase margin and its transition frequency [14], [15].

TABLE I
SIMULATED VALUES

ω_{GBW}=2π·1MHz	$\omega_3/2\pi$	φ_3	$\omega_{TR}/2\pi$	φ_{mR}
φ_m=60°	1.41 MHz	-95.74°	890 kHz	62.80°
φ_m=50°	1.33 MHz	-110.13°	824 kHz	55.00°
φ_m=45°	1.27 MHz	-115.80°	787 kHz	51.96°

TABLE II
ESTIMATED VALUES

ω_{GBW}=2π·1MHz	$\omega_T/2\pi$ eq. (14)	φ_m eq. (15)	$\omega_{TR}/2\pi$ eq. (19)	φ_{mR} eq. (21)
φ_m=60°	1.00 MHz	60.04°	892 kHz	62.84°
φ_m=50°	1.00 MHz	49.86°	823 kHz	55.28°
φ_m=45°	997 kHz	45.10°	785 kHz	51.96°

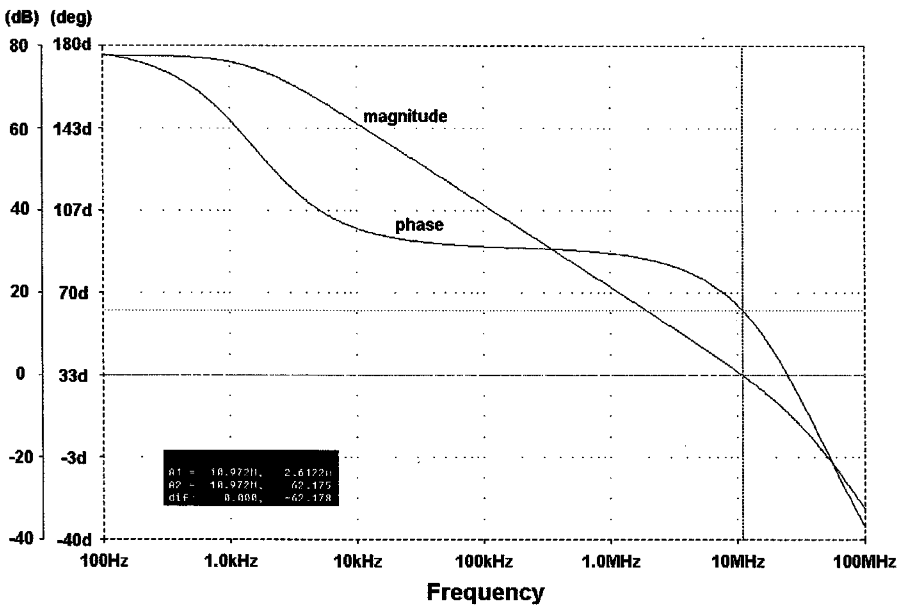

Fig. 3. OTA open-loop magnitude and phase ($\beta = 1$).

feedback network but also on the input terminal where the signal is applied (i.e., inverting or noninverting), its value has no influence in the following, as it will be simplified.

In stable feedback amplifiers the second pole, ω_2, is at frequency much higher than that of the first pole, ω_1, and higher than the unity-gain frequency, ω_T, (i.e., $\omega_1 \ll \omega_2, \omega_2 \geq \omega_T$). Hence, the unity-gain frequency is almost equal to the gain-bandwidth product, $\omega_{GBW} = \beta a_0 \omega_1$, and defining the separation factor, K, as

$$K = \frac{\omega_2}{\omega_T} \cong \frac{\omega_2}{\omega_{GBW}} = \frac{\omega_2}{\beta a_0 \omega_1} = tg\left(\varphi_m\right) \tag{5}$$

relationship (2) can be expressed versus the phase margin, φ_m, and ω_T [13]

$$A_{CL}(s) = \frac{A_{CL}(0)}{1 + \dfrac{s}{\omega_T} + \dfrac{s^2}{K\omega_T^2}}. \tag{6}$$

Let us define ω_3 the frequency where the gain of the closed-loop amplifier is reduced of 3 dB with respect to its dc value, and φ_3 its corresponding phase. Thus

$$|A_{CL}(j\omega_3)| = \frac{A_{CL}(0)}{\sqrt{2}} \tag{7}$$

$$\tan(\varphi_3) = \frac{\mathrm{Im}\left[A_{CL}(j\omega_3)\right]}{\mathrm{Re}\left[A_{CL}(j\omega_3)\right]}. \tag{8}$$

Evaluating (6) for $s = j\omega$ and substituting it in (7) and (8) we get

$$\left[1 - \frac{1}{K}\left(\frac{\omega_3}{\omega_T}\right)^2\right]^2 + \left(\frac{\omega_3}{\omega_T}\right)^2 = 2 \tag{9}$$

$$\tan(\varphi_3) = -\frac{\dfrac{\omega_3}{\omega_T}}{1 - \dfrac{1}{K}\left(\dfrac{\omega_3}{\omega_T}\right)^2}. \tag{10}$$

Solving (9) with respect $1/K$

$$\frac{1}{K} = \frac{1 - \sqrt{2 - \left(\dfrac{\omega_3}{\omega_T}\right)^2}}{\left(\dfrac{\omega_3}{\omega_T}\right)^2} \tag{11}$$

and substituting the value in (10), the 3-dB phase becomes

$$\tan(\varphi_3) = -\frac{\dfrac{\omega_3}{\omega_T}}{\sqrt{2 - \left(\dfrac{\omega_3}{\omega_T}\right)^2}}. \tag{12}$$

Solving relationship (12) for the ratio ω_3/ω_T, we get

$$\left(\frac{\omega_3}{\omega_T}\right)^2 = \frac{2tg^2(\varphi_3)}{1 + tg^2(\varphi_3)} = 2\sin^2(\varphi_3). \tag{13}$$

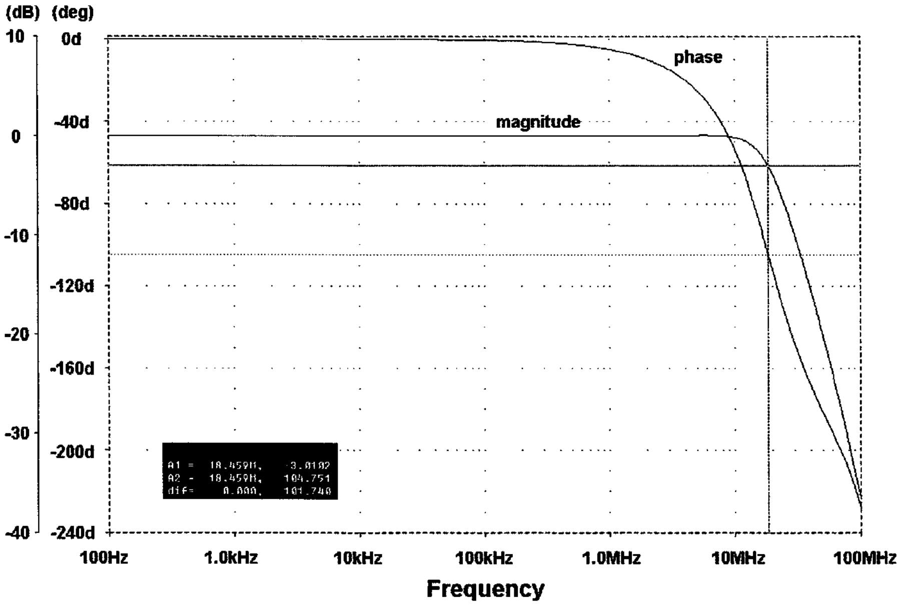

Fig. 4. OTA closed-loop magnitude and phase ($\beta = 1$).

Hence, the unity-gain frequency, ω_T, can be expressed as a function of ω_3 and φ_3

$$\omega_T = \frac{\omega_3}{\sqrt{2}\,|\sin(\varphi_3)|}. \tag{14}$$

Finally, substituting (13) in (11) we can have the phase margin, φ_m, as a function of ω_3 and φ_3

$$\varphi_m = \arctan(K) = \arctan\left[\frac{2\sin^2(\varphi_3)}{1-\sqrt{2}\cos(\varphi_3)}\right]. \tag{15}$$

III. An Accurate Procedure

Although the simplified approach, which gives ω_T and φ_m versus measured ω_3 and φ_3, holds when the amplifier under test is well compensated, it has a low accuracy when the second pole, ω_2, is close to the transition frequency, ω_T.

This fact arises because a frequency gain whose magnitude is represented by an asymptotic piecewise-linear (PWL) function in logarithmic axes is assumed (the same approximation is used to graph Bode plots). In fact, under this approximation, at second pole frequency, the slope of the magnitude is assumed to change abruptly from 20 dB/dec to 40 dB/dec. Actually, as shown in Fig. 1, the slope change is smooth and the real unity-gain frequency, ω_{TR}, is less than that given by (14) since the second pole reduces the actual gain with respect to that one calculated by assuming a PWL approximation.

The relationship between ω_T and φ_m (parameters evaluated when a PWL function is assumed for the magnitude) and ω_{TR} (the real transition frequency), can be found by considering that close to the transition frequency, the PWL approximation of the frequency gain magnitude, $|\beta a_{\mathrm{PWL}}(j\omega)|$, can be written as

$$|\beta a_{\mathrm{PWL}}\,(j\omega)| = \frac{\beta a_0}{\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}} \cong \frac{\beta a_0}{\frac{\omega}{\omega_1}}, \quad \text{for} \quad \omega < \omega_T \tag{16}$$

and that the real frequency gain magnitude, $|\beta a_R(j\omega)|$, can be obtained by multiplying (16) by the attenuation determined by the second pole, that is

$$|\beta a_R\,(j\omega)| = \frac{\frac{\beta a_0 \omega_1}{\omega}}{\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}} \tag{17}$$

Evaluating (17) for $\omega = \omega_{TR}$ and considering that $|\beta a_R(j\omega_{TR})| = 1$, $\omega_2 = K\omega_T$ and $\omega_1\beta a_0 = \omega_T$, we get

$$\sqrt{1+\left(\frac{\omega_{TR}}{\omega_T}\right)^2\frac{1}{\tan^2(\varphi_{\mathrm{m}})}} = \frac{\omega_T}{\omega_{TR}}. \tag{18}$$

Solving (18) with respect to ω_{TR}/ω_T, results

$$\omega_{TR} = \omega_T\sqrt{\frac{K\left(\sqrt{K^2+4}-K\right)}{2}} \tag{19}$$

where, according to (15), K depends only on φ_3, and ω_T, given by (14), depends only on both φ_3 and ω_3.

Once the real transition frequency is determined, the real phase margin, φ_{mR}, can be expressed as the arctangent of the ratio between the second pole and the real unity-gain frequency. Hence

$$\varphi_{mR} = \arctan\left(\frac{\omega_2}{\omega_{\mathrm{TR}}}\right) = \arctan\left(\frac{\mathrm{K}\omega_{\mathrm{T}}}{\omega_{\mathrm{TR}}}\right) \tag{20}$$

and substituting (19) in (20) we get

$$\varphi_{mR} = \arctan\left(\sqrt{\frac{2\mathrm{K}}{\sqrt{\mathrm{K}^2+4}-\mathrm{K}}}\right). \tag{21}$$

Moreover, defining ω_{T1} and K_1 equal to ω_{TR} and K_R for $\beta = 1$, that is the unity-gain configuration, it can be simply shown that

$$K_1 = \beta\tan(\varphi_{\mathrm{mR}}) \tag{22a}$$

$$\omega_{T1} = \frac{\omega_{TR}}{\beta}. \tag{22b}$$

Relationships (22) give both the transition frequency and the phase margin of the pure open-loop amplifier without a feedback network.

At this point, an indirect measurement of both the transition frequency and the phase margin can be done by means of a direct measurement of the cutoff frequency and its respective phase for the amplifier in closed loop gain.

In fact, a first approximation can be obtained by using (14) and (15) for the transition frequency and the phase margin, respectively. It is apparent that these estimated values are as precise as the second pole of the open loop amplifier is far from the transition frequency.

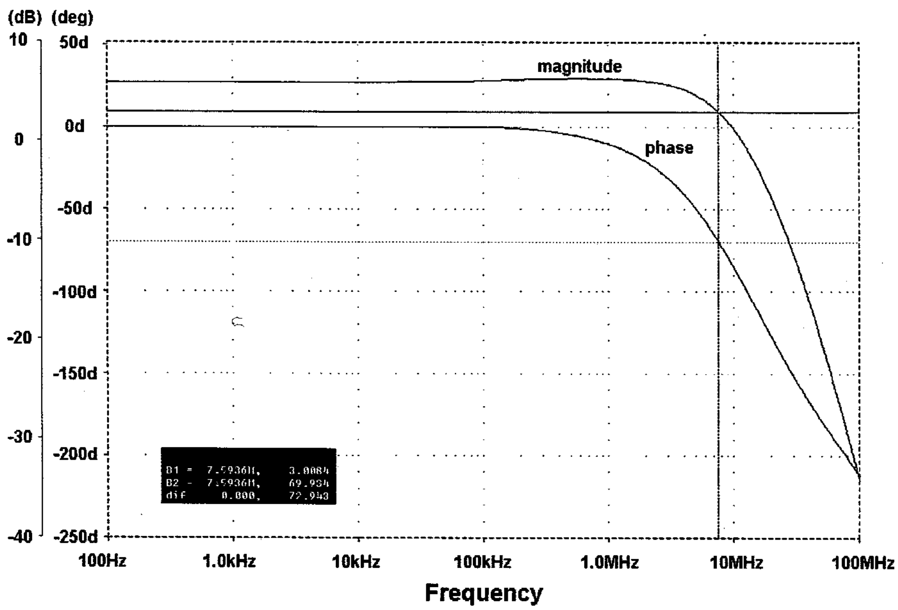

Fig. 5. OTA closed-loop magnitude and phase ($\beta = 1/2$).

TABLE III
ERRORS BETWEEN SIMULATED AND ESTIMATED VALUES

ω_{GBW}=2π·1MHz	$\omega_T/2\pi$ eq. (14)	φ_m eq. (15)	$\omega_{TR}/2\pi$ eq. (19)	φ_{mR} eq. (21)
φ_m=60°	12.4 %	-4.4 %	0.22 %	0.06 %
φ_m=50°	21.4 %	-9.3 %	-0.12 %	0.51 %
φ_m=45°	27.1 %	-13.2 %	-0.25 %	0.00 %

TABLE IV
COMPONENT OF THE OTA

Component	Value
M1, M2	100/2
M3, M4	50/2
M5	400/2
M6	50/2
M7	200/2
M8	50/2
C_c	5 pF
R_c	720 Ω
I_B	50 μA
V_{DD}=-V_{SS}	2.5 V

A more realistic result can be found by means of (19) and (21) and, as the Appendix will show, this yield to a very precise result.

Finally, if a measure of open-loop parameters in unity-gain configuration is requested, (22) let us to determine them even if the closed-loop amplifier was not tested with a unitary feedback factor β.

IV. VALIDATION RESULTS

The proposed procedure has been extensively validated. In particular, in order to give a theoretical validation, the method is applied to the ideal case of a pure two poles amplifier and simulated results with estimated ones are compared, and the accuracy is analytically evaluated in the Appendix. It is again validated by means of simulations performed on an amplifier designed at transistor level. Finally, the approach is applied to a real μA-741 and measured values are compared with estimated ones.

A. Simulation Results

An ideal two poles amplifier with a 90-dB open loop gain and a 1-MHz gain-bandwidth product was considered, and its second pole was set to give a phase margin equal to 60, 50, and 45 degrees. Spice simulation results, estimated values and related errors are summarized in Tables I–III, respectively. It is apparent that as expected the error given by (14) and (15) increases when the second pole is closer to the unity-gain frequency. However, relationship (19) and (21) give good accuracy regardless of the second pole position.

Then the approach has been applied to the two-stage CMOS OTA in Fig. 2, usually used in switched capacitor circuit [3], [4]. The amplifier is designed in a 1.2-μm standard process with the transistor aspect ratios in Table IV. Setting a unitary feedback factor, open-loop and closed-loop gains are plotted in Figs. 3 and 4, respectively. In particular, when loaded with a 5-pF capacitor, the OTA has a unity-gain frequency and a phase margin equal to 10.97 MHz and 62.17°, respectively. Moreover, in unitary closed-loop configuration it has 3-dB frequency and a phase equal to 18.46 MHz and $-104.75°$, respectively. By using this two values into relationship (14) and (15) we get estimated values of unity-gain frequency and a phase margin equal to 13.5 MHz and 53.99°, respectively, which means errors equal to 23.1% and -13.2%, respectively. Of course a better agreement is found by using relationships (19) and (21) which give 11.48 MHz and 58.27° for $\omega_{TR}/2\pi$ and φ_{mR}, respectively, which means errors equal to 4.6% and -6.3%.

The same OTA has been analyzed in noninverting configuration with a 6-dB closed-loop gain ($\beta = 1/2$). This gain, plotted in Fig. 5, shows a 3-dB cutoff frequency of 7.59 MHz and a phase φ_3, of $-69.93°$. From relationships (14) and (15) we get for $\omega_T/2\pi$ and φ_m, 5.72 MHz and 73.74°. The error with respect to the simulated values is of 4.3% and -3.9%, respectively. On the other hand, by using (19) and (21) we get for $\omega_{TR}/2\pi$ and φ_{mR}, 5.5 MHz and 74.31° which lead to errors

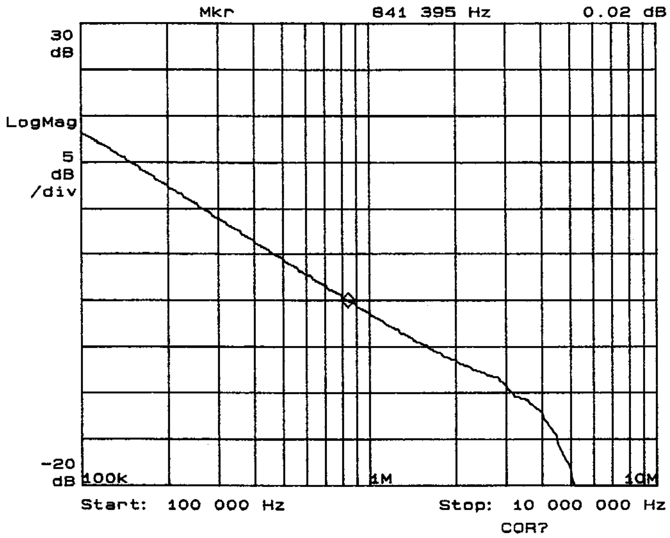

Fig. 6. μA-741 open loop magnitude ($\beta = 1$).

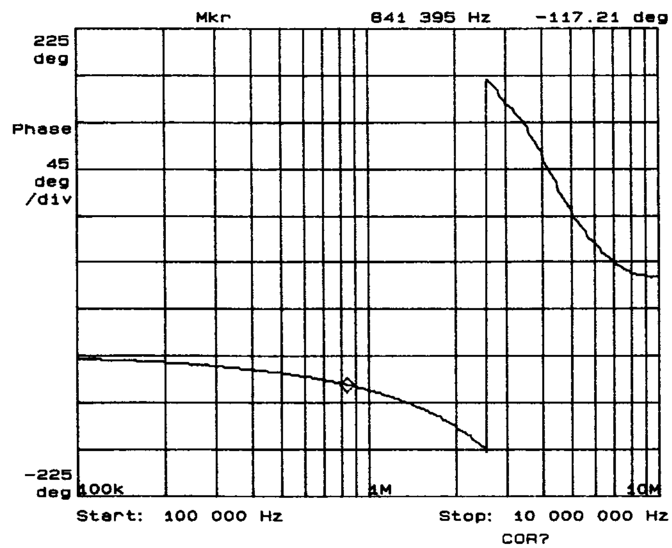

Fig. 7. μA-741 open loop phase ($\beta = 1$).

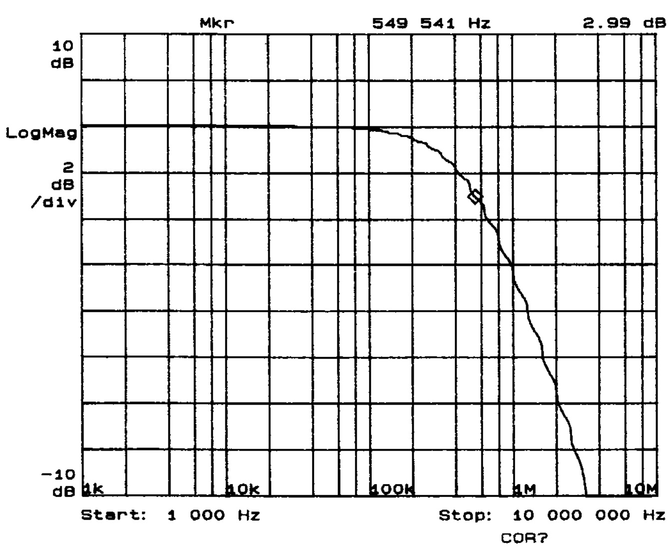

Fig. 8. μA-741 closed loop magnitude ($\beta = 1/2$).

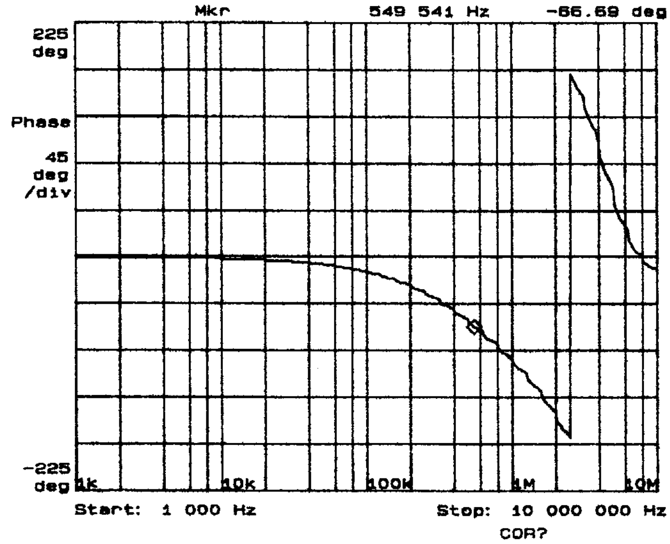

Fig. 9. μA-741 closed loop phase ($\beta = 1/2$).

with respect to the simulated values of 0.2% and -2.4%, respectively. In both cases, all the equations give a highly accurate result since the second pole is less close to the real unity-gain frequency. Moreover, by using (22) the unity-gain frequency, $\omega_{T1}/2\pi$, and the phase margin, φ_{m1}, for the amplifier in unity-gain configuration, result 11.01 MHz and 60.68°, respectively, with an overall error lower than 2.5%.

B. Experimental Results

The approach was also used in a real experimental test. In particular, a μA-741 operational amplifier has been tested. Direct measures of both its transition frequency and phase margin are shown in Figs. 6 and 7, respectively, yielding to a value of 841 kHz and 62.79°.

Indirect measure of these values is performed by characterizing the amplifier in closed loop configuration with a closed loop gain of 6-dB (i.e., $\beta = 1/2$). Specifically, the feedback network was implemented with two equal resistors and the input signal was applied to the noninverting terminal. Figs. 8 and 9 show measured values for $\omega_3/2\pi$ and φ_3 which results to be 549.5 kHz and -66.69°, respectively. By applying relationships (19) and (21) we get for $\omega_{TR}/2\pi$ and φ_{mR}, 410.2 kHz and 75.8°, respectively. Feeding them in (22) we get for the estimated unity-gain frequency and the phase margin the values of 824.2 kHz and of 63.15° with an error of -2% and 0.57%, respectively.

V. Conclusion

A novel method to evaluate the main open-loop parameters of a two-poles feedback amplifier is given. In particular, it is useful to achieve the unity gain frequency and the phase margin of an open-loop amplifier measuring only its closed-loop 3-dB cutoff frequency and the corresponding phase.

The approach was validated using both simulation and experimental results and in any case a high accuracy was demonstrated.

Appendix

In order to analytically evaluate the accuracy of equations (19) and (21) the sensitivity of both the equations with respect to ω_3 and φ_3 is evaluated in the range of interest. In particular for a two-poles feedback amplifier, the phase φ_3 must lie in the range between $-180°$ and $-45°$ (these values represent the case when the closed-loop system realize a double integrator or a single-pole system, respectively). A plot of φ_{mR} versus φ_3, obtained by substituting in (21) the value of K given in

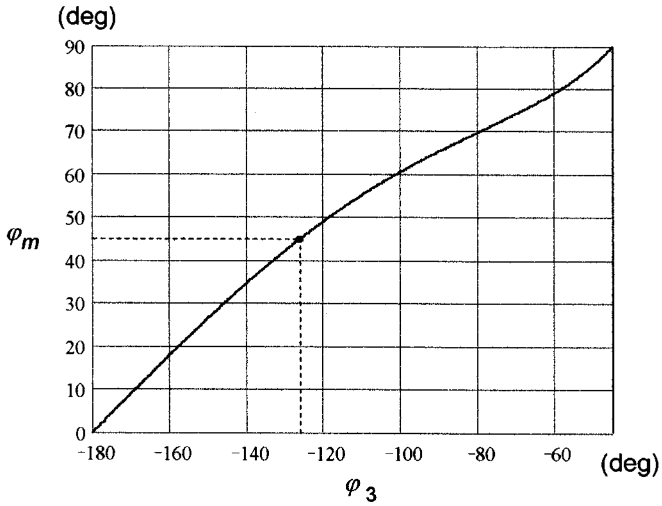

Fig. 10. Phase margin versus cutoff phase φ_3.

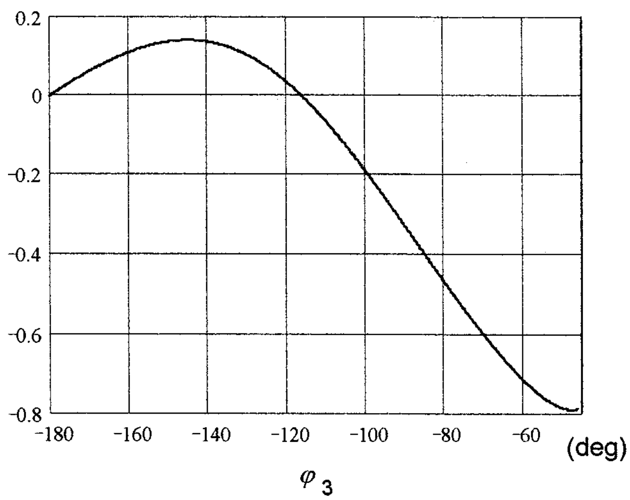

Fig. 11. ω_{TR} sensitivity versus cutoff phase φ_3.

(15), is shown in Fig. 10. It is worth noting that this function grows monotonically and that, in practical cases (that is for well-compensated circuit with a phase margin greater than 45°), the lower bound of φ_3 is reduced to about $-126.3°$.

The percentage variation of ω_{TR} with respect to percentage variations of both ω_3 and φ_3 can be expressed as

$$\frac{\Delta\omega_{TR}}{\omega_{TR}} = S_{\omega_3}^{\omega_{TR}} \cdot \frac{\Delta\omega_3}{\omega_3} + S_{\varphi_3}^{\omega_{TR}} \cdot \frac{\Delta\varphi_3}{\varphi_3}. \tag{A.1}$$

By substituting (14) in (19), it can be simply shown that $S_{\omega_3}^{\omega_{TR}} = 1$. The sensitivity $S_{\varphi_3}^{\omega_{TR}}$ versus φ_3 is depicted in Fig. 11, and it can be shown that $|S_{\varphi_3}^{\omega_{TR}}| \le \pi/4$. As a consequence, errors in the measurement of both ω_3 and φ_3 do not affect the estimation of the transition frequency appreciably.

The phase margin only depends on φ_3 and its percentage error can be expressed as

$$\frac{\Delta\varphi_{mR}}{\varphi_{mR}} = S_{\varphi_3}^{\varphi_{mR}} \cdot \frac{\Delta\varphi_3}{\varphi_3}. \tag{A.2}$$

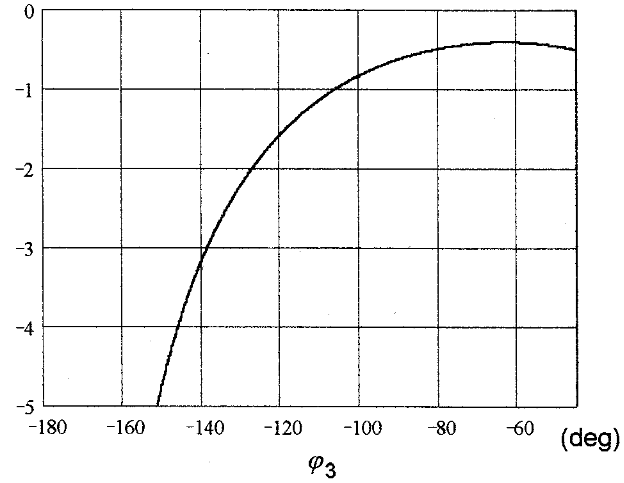

Fig. 12. φ_{mR} sensitivity versus cutoff phase φ_3.

Fig. 12 depicts the sensitivity $S_{\varphi_3}^{\varphi_{mR}}$ versus φ_3. This function decreases drastically when φ_3 approaches $-180°$ thus yielding a high sensitivity to measurement errors. However, in the practical case of a well-compensated circuit, the worst sensitivity is close to -2 thus giving a sufficient accuracy.

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