SPM project: Parallel Prefix

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1 Introduction

In this report we will analyze, theoretically and practically, the resolution of the problem of the (parallel) prefix sum:

Given a vector $x = \langle x_0, x_1, \ldots, x_{n-1} \rangle$ and a binary operation $\oplus$ compute the vector $y = \langle x_0, x_0 \oplus x_1, x_0 \oplus x_1 \oplus x_2, \ldots, x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1} \rangle$.

In literature this operation is also called (inclusive) scan or partial sum.
For the analysis of the problem we have to make two assumptions:

- The binary operation $\oplus$ is associative ($a \oplus (b \oplus c) = (a \oplus b) \oplus c$) and commutative ($a \oplus b = b \oplus a$), this is an important assumption as we will see in the next chapters the order of the operations may not be preserved.
- The size of the input vector is a power of 2, not a strong assumption, as all the algorithms we will present could be easily generalize to all the sizes, but it only helps to simplify some operations.

2 Sequential algorithm

The sequential algorithm simple compute each element of the vector y as follow:

$$y_i = \begin{cases} x_0 & \text{for } i = 0 \\ x_i \oplus y_{i-1}, & \text{for } 0 < i < n \end{cases}$$

This algorithm is optimal in a sequential model as it has a running time of $\mathcal{O}(n)$, assuming that $\oplus$ is $\mathcal{O}(1)$, and performs $n - 1$ calls to $\oplus$ operation.

3 Parallel architecture design

The problem of computing the prefix sum vector is a classical example of a problem that have an optimal solution in a sequential model but that can be optimized in a parallel model. The optimization is not in terms of total complexity or in the number of ⊕ operations performed (which are already optimals in the sequential algorithm) but in terms of completion time. When the number of available threads is more than one we can trade-off more total work for less completion time.

We will now introduce two different algorithms that solve in an efficient way the prefix sum problem in a parallel model.

3.1 Block-based algorithm

The idea behind the first algorithm is to partitionate the input vector in blocks, compute the prefix vector of each block in parallel .

- Partitionate the input vector in blocks and compute in parallel the prefix vector of each of them.
- second phase
- third phase

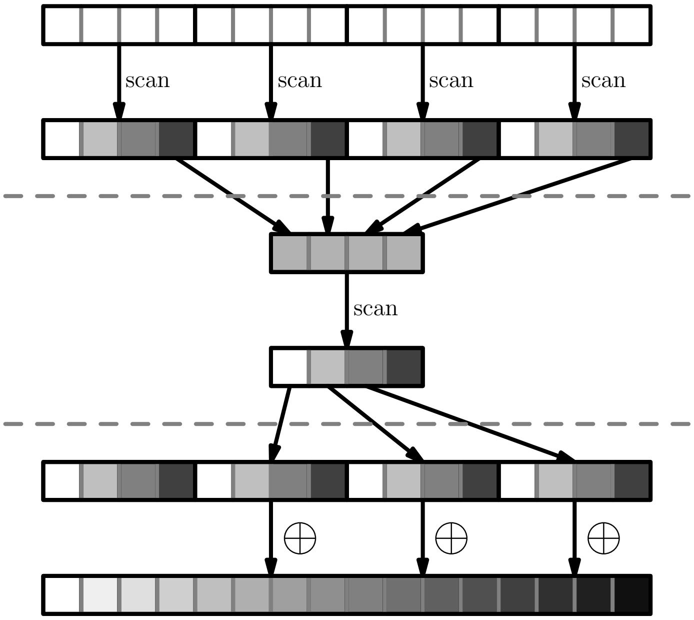

Figure 1: Block-based algorithm graphic representation

3.2 Circuit-based algorithm

Another kind of algorithms are the one based on a circuit-like rappresentation,

4 Performance modeling

4.1 Block-based algorithm

4.2 Circuit-based algorithm

5 Implementations structure and details

All the implementations are written in C++17, the source code is available as attachment with the report or on GitHub (https://github.com/GaspareG/ParallelPrefix).

5.1 Sequential algorithm

The sequential implementations

5.2 Block-based algorithm

TODO

5.3 Circuit-based algorithm

TODO

6 Experimental validation

6.1 experiments details

6.2 benchmark results

7 Conclusion