# SPM project: Parallel Prefix

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October 31, 2018

### 1 Introduction

In this report we will analyze, theoretically and practically, the resolution of the problem of the (parallel) prefix sum:

Given an input vector  $\langle x_0, x_1, \dots, x_{n-1} \rangle$  and a binary operation  $\oplus$  compute the output vector  $\langle x_0, x_0 \oplus x_1, x_0 \oplus x_1 \oplus x_2, \dots, x_0 \oplus x_1 \oplus \dots \oplus x_{n-1} \rangle$ .

For the analysis of the problem we have to make two assumptions:

- The binary operation  $\oplus$  is associative  $(a \oplus (b \oplus c) = (a \oplus b) \oplus c)$  and commutative  $(a \oplus b = b \oplus a)$ , this is an important assumption as we will see in the next chapters the order of the operations may not be preserved.
- The size of the input vector is a power of 2, not a strong assumptions, as all the algorithms we will present could be easily generalize to all the sizes, but it helps to simplify some operations.

# 2 Sequential implementation

The sequential algorithm compute

The algorithm is optimal in a sequential model as it has a running time of  $\mathcal{O}(n)$  and perform n-1 calls to  $\oplus$  operation.

## 3 Parallel architecture design

- 3.1 Block-based algorithm
- 3.2 Circuit-based algorithm
- 4 Performance modeling
- 5 Implementations structure and details
- 5.1 Sequential algorithm

The sequential implementations

- 5.2 Block-based algorithm
- 5.3 Circuit-based algorithm

## 6 Experimental validation

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#### 6.1 experiments details

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#### 6.2 benchmark results

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### 7 Conclusion

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