Translating PVS to C

SRI International

Gaspard Férey

Ecole Polytechnique

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What is PVS ?

Prototype Verification System

- A specification language
  - Expressive syntax
  - Rich type system
    - uninterpreted types
    - abstract data types
    - predicate subtypes
    - etc
  - Parameterized theories
- A semi automated theorem prover
  - Higher order logic
  - Type system, type-correctness conditions, judgments, ...
  - Theorems, properties, ...
  - SMT solvers and other tools integrated
- A functional programming language (?)

Why translate PVS ?

- To be able to execute PVS
  - Testing
  - Debugging
- To integrate high-insurance PVS code into systems

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Update expression

- In functional programming languages,

$$E := A \ \texttt{WITH [(}x\texttt{) :=}\ y\texttt{]}$$

refers $i \mapsto \begin{cases} A(i) & \text{if } i \neq x \\ y & \text{if } i = x \end{cases}$

- In imperative languages
  - `A[x := y]` a non destructive update using a copy of A.
  - `A[x <- y]` a destructive, in-place update of the aggregate structure representing A.

Two dangers

- Unsafe occurrence

```
LET B = A WITH[(0) := 0] IN B(0) + A(0)
```

The array represented by `A` is used later in the code.

- Trapped reference

```
LET A = B(0) IN f( A WITH [(0) := 0], B(0) )
```

`B` is affected by a destructive update of the array represented by `A`.

Previous analysis

Shankar's analysis relies on sets of *variables*

- *Av* active variables
- *Ov* output variables
- *Fv* free variables
- *Lv* live variables in an *update context*

Cerny and Shankar's analysis adds *flow analysis*.

The intermediate language

Syntax

- Integers, nil pointer
- Variables (`X`, `x`, `y`, ...)
- `newArray(x, y)`
- `X[x := y]`
- `X[x <- y]`
- `X[x]`
- `let x =` a `in` e
- `if(x)` e_1 `else` e_2
- `f(`x_1`, ... ,`x_n`)`

Memory state representation:

- Variables space Var
- Reference space $\mathcal{R}$
- Value space $\mathcal{V} := \mathbb{N} \cup \mathcal{R}$
- Store $R : \mathcal{R} \longrightarrow \mathcal{V}$
- Stack $S : Var \longrightarrow \mathcal{V}$

Evaluation context

- hole $\{\}$
- `let x =` $\{\}$ `in` e
- $E_1\{E_2\}$

Simple reduction rules:

$$
\begin{aligned}
< x|R, S > \quad &\rightarrow \quad < S(x)|R, S > \\
< x[y]|R, S > \quad &\rightarrow \quad < R(S(x))(S(y))|R, S > \\
< \texttt{if(x)}\ a\ \texttt{else}\ b|R, S > \quad &\rightarrow \quad \begin{cases} < b|R, S > & \text{if } S(x) = 0 \\ < a|R, S > & \text{otherwise} \end{cases}
\end{aligned}
$$

Introducing variables

$$
\begin{aligned}
< f(x_1, \dots, x_n)|R, S > \quad &\rightarrow \quad < \texttt{pop}([f])|R, \left(\biguplus_{1 \leq i \leq n} f_i \mapsto S(x_i) \right) :: S > \\
< \texttt{let}\ x\ \texttt{=}\ v\ \texttt{in}\ e|R, S > \quad &\rightarrow \quad < \texttt{pop}(e)|R, (x \mapsto v) :: S > \\
< \texttt{pop}(v)|R, S > \quad &\rightarrow \quad < v|R, pop(S) >
\end{aligned}
$$

Modifying the store

$$
\begin{array}{rcl}
< \texttt{newArray}(x,y)|R,S > & \rightarrow & < r|R \uplus \left(r \mapsto (S(y))_{0\le i<S(x)}\right), S > \\
 & \text{where} & r \text{ is a fresh pointer} \\
< X\,\texttt{[}x\ \ \texttt{:=}\ \ y\texttt{]}|R,S > & \rightarrow & < r|R',S > \\
 & \text{where} & r \text{ fresh pointer} \\
 & \text{and} & R' := R \uplus (r \mapsto A) \\
 & \text{and} & A := R(S(X)) \uplus (S(x) \mapsto S(y)) \\
< X\,\texttt{[}x\ \ \texttt{<-}\ \ y\texttt{]}|R,S > & \rightarrow & < X|R',S > \\
 & \text{where} & R' := R \uplus (S(X) \mapsto A) \\
 & \text{and} & A := R(S(X)) \uplus (s(x) \mapsto S(y))
\end{array}
$$

Reference graph

For a context in the body of a function

```
f(A, B) =
  let C = if(A[0] = 1) then A else B in
  let D =
    let E = C in E[0 := 0] in
  D[0] + A(0)
```

we can define the reference graph $\mathcal{G}(R)(r)$ as

$$\begin{aligned} r &\in \mathcal{G}(r) \\ \mathcal{R} \cap R(\mathcal{G}(r)) &\subset \mathcal{G}(r) \end{aligned}$$

We are interested in

$$\bigcup_R S^{-1}(\ \mathcal{G}(R)(S(x))\ )$$

Reference graph

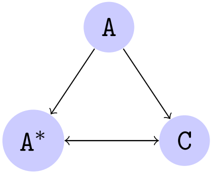
Definition of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0] in
  let E = g(B)        in
  let X = E[0 := 0] in
  let F = if X[0] then D
                  else X in
  let G = F[0 := 0] in
  let H = if B[0] then G
                  else B in
  H[0 := B[0]]
```

Reference graph

```
f(A, B) =
  let C = A[0]        in
  let D = { C }       in
  let E = g(B)        in
  let X = E[0 := 0] in
  let F = if X[0] then D
                  else X in
  let G = F[0 := 0] in
  let H = if B[0] then G
                  else B in
  H[0 := B[0]]
```

Reference graph



Variables lives in the context

$$\{\mathtt{B}, \mathtt{D}, \mathtt{E}, \mathtt{F}, \mathtt{G}, \mathtt{H}, \mathtt{X}\}$$

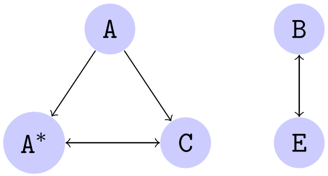
Destructive update would impose too many requirements to f.

Reference graph

Definition of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0] in
  let E = g(B)        in
  let X = { E }       in
  let F = if X[0] then D
                  else X in
  let G = F[0 := 0] in
  let H = if B[0] then G
                  else B in
  H[0 := B[0]]
```

Reference graph



Variables lives in the context

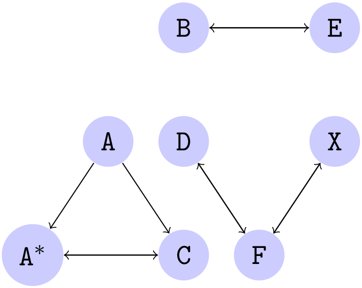
$$\{B, C, D, F, G, H, X\}$$

B is live in the context.

Reference graph

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0]  in
  let E = g(B)        in
  let X = E[0 := 0]  in
  let F = if X[0] then D
                   else X in
  let G = { F }       in
  let H = if B[0] then G
                   else B in
  H[0 := B[0]]
```

Reference graph



Variables lives in the context

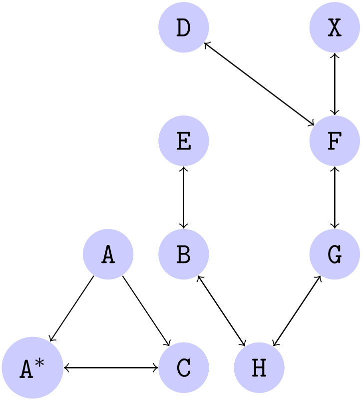
$$\{B, C, G, H\}$$

Destructive update possible.

Reference graph

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0] in
  let E = g(B)        in
  let X = E[0 := 0] in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0] in
  let H = if B[0] then G
                  else B in
  { H }
```

Reference graph



Variables lives in the context

$$\{\}$$

Destructive update possible.

The flags

We implement an approximation of the analysis using three flags

- **mutable**
  - Variable: Every other variable that may point to that variable is not live.
  - Argument: Variables passed as this argument must be flagged **mutable** and **safe** .
  - Function: The result of a call to that function is "fresh".
- **dupl**
  - Argument: Possible active variable in a call to this function.
  - Expression: May be aliased to the return value.
- **safe**
  - Last of occurrence of a variable

The rules

- Two versions for each function
  - destructive
  - non destructive
- **mutable** flag:
  -
- **dupl** flag:
  -

Example

Destructive version of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0] in
  let E = g(B)        in
  let X = E[0 := 0] in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0] in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

Example

C no flags

Destructive version of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0] in
  let E = g(B)        in
  let X = E[0 := 0] in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0] in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

Example

Destructive version of `f`

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0]  in
  let E = g(B)        in
  let X = E[0 := 0]  in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0]  in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

`C` no flags

`D` **mutable**

$\Longleftarrow$ non destructive update

Example

C no flags

D **mutable**

$\Longleftarrow$ non destructive update

E no flags

Destructive version of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0]  in
  let E = g(B)        in
  let X = E[0 := 0]  in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0]  in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

Example

Destructive version of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0] in
  let E = g(B)        in
  let X = E[0 := 0] in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0] in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

C no flags

D **mutable**

$\Longleftarrow$ non destructive update

E no flags

X **mutable**

$\Longleftarrow$ non destructive update

Example

Destructive version of `f`

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0] in
  let E = g(B)        in
  let X = E[0 := 0] in
  let F = if X[0] then D
                   else X in
  let G = F[0 <- 0] in
  let H = if B[0] then G
                   else B in
  H[0 <- 0]
```

C no flags

D **mutable**

$\Longleftarrow$ non destructive update

E no flags

X **mutable**

$\Longleftarrow$ non destructive update

F **mutable**

$\Longleftarrow$ D and X **safe** and **mutable**

Example

Destructive version of `f`

```
f(A, B) =
  let C = A[0]         in
  let D = C[0 := 0] in
  let E = g(B)         in
  let X = E[0 := 0] in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0] in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

`C` no flags

`D` **mutable**

$\Longleftarrow$ non destructive update

`E` no flags

`X` **mutable**

$\Longleftarrow$ non destructive update

`F` **mutable**

$\Longleftarrow$ `D` and `X` **safe** and **mutable**

`G` **mutable**

$\Longleftarrow$ `F` **safe** and **mutable**

Example

Destructive version of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0]  in
  let E = g(B)        in
  let X = E[0 := 0]  in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0]  in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

C no flags

D **mutable**

$\Longleftarrow$ non destructive update

E no flags

X **mutable**

$\Longleftarrow$ non destructive update

F **mutable**

$\Longleftarrow$ D and X **safe** and **mutable**

G **mutable**

$\Longleftarrow$ F **safe** and **mutable**

H **mutable**

$\Longleftarrow$ G and B **safe** and **mutable**

Example

Destructive version of f

```
f(A, B) =
  let C = A[0]        in
  let D = C[0 := 0]   in
  let E = g(B)        in
  let X = E[0 := 0]   in
  let F = if X[0] then D
                  else X in
  let G = F[0 <- 0]   in
  let H = if B[0] then G
                  else B in
  H[0 <- 0]
```

C no flags

D **mutable**

$\Longleftarrow$ non destructive update

E no flags

X **mutable**

$\Longleftarrow$ non destructive update

F **mutable**

$\Longleftarrow$ D and X **safe** and **mutable**

G **mutable**

$\Longleftarrow$ F **safe** and **mutable**

H **mutable**

$\Longleftarrow$ G and B **safe** and **mutable**

$\Longrightarrow$ Arguments B gets **mutable**

Example

Destructive version of f

```
f(A, B) =
  let C = A[0]          in
  let D = C[0 := 0]  in
  let E = g(B)          in
  let X = E[0 := 0]  in
  let F = if X[0] then D
                    else X in
  let G = F[0 <- 0]  in
  let H = if B[0] then G
                    else B in
  H[0 <- 0]
```

C no flags

D **mutable**

$\Longleftarrow$ non destructive update

E no flags

X **mutable**

$\Longleftarrow$ non destructive update

F **mutable**

$\Longleftarrow$ D and X **safe** and **mutable**

G **mutable**

$\Longleftarrow$ F **safe** and **mutable**

H **mutable**

$\Longleftarrow$ G and B **safe** and **mutable**

$\Longrightarrow$ Arguments B gets **mutable**

H **safe**

A reference counting GC

We complete the static analysis with a reference counting garbage collector (GC)

- Easy to implement in C (hashtable of pointers)
- Memory freed soon
- The reference count can allow safe update to be made destructive

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Translation steps

- Typechecking: PVS typechecker
  - TCCs are generated
- Lexical and syntactic analysis: PVS lexer and parser
  - PVS $\longrightarrow$ CLOS representation
- Translation:
  - CLOS representation $\longrightarrow$ intermediate language

Translation steps (2)

- Static analysis:
  - destructive updates added
- Optimizations: Several passes
  - Choosing C types
  - Declaring and freeing variables
- Code generation:
  - intermediate language $\longrightarrow$ compilable C code

Implementation details

- In Common Lisp
- Directly integrated to PVS (soon)
- Require the GMP library to run C code

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Successes

- Working compiler
  - Output C code compilable with gcc, CompCert (?)
  - Use GMP library to represent PVS big integers
- Promising analysis
- Optimization efficient (for simple programs)
  - 100 times faster than Ground Evaluator

Left to be done

- More work on static analysis
  - Reference counting
  - Publication
- Less approximate implementation
- Debugging, optimizing Lisp code
- Wider subset of PVS
  - Closures
  - More types
  - ...

- We compute the array T

$$
\begin{aligned}
T(0) &= 9876 \\
T(i) &= (12345 * T(i)) \mod 59557 \quad \text{for } 0 < i < 10.000
\end{aligned}
$$

- We sort T it using an insertion sort.

```
insert(A, i, v) =
  if    i = 0 or v >= A[i-1]
  then  A[i := v]
  else  insert( A[i := A[i-1]], i-1, v)

sort(A, n) =
  if    n = 100000
  then  A
  else  sort( insert(A, A[n], n), n+1 )

min(A)  = sort(A, 0)[0]
```

- Complexity: time $O(n^2)$ space $O(n)$

Questions ?