Delta Hedging with Transaction Costs: Dynamic Multi-Scale Strategy using Neural Nets

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1

Delta Hedging

$$Portfolio \equiv \Pi = C_{(t)} - \Delta_{(t)} S_{(t)}$$

2

with Transaction Costs:

$$\$ \times (1 - \varepsilon)$$

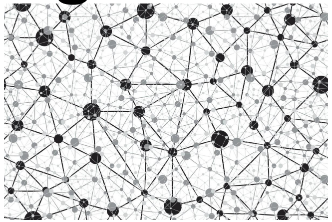
3

Dynamic

$$(time)$$

5

using Neural Nets



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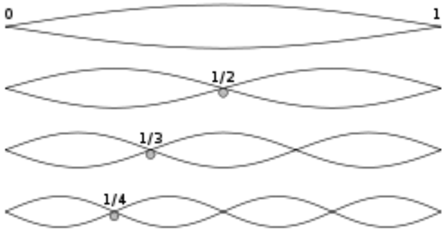
Multi-Scale Strategy



Part 1

1 **Delta Hedging**

$$Portfolio \equiv \Pi = C_{(t)} - \Delta_{(t)} S_{(t)}$$

2 **with Transaction Costs:**

$$\$ \times (1 - \varepsilon)$$

...But first…

Delta Hedging without Transaction Costs

$$Portfolio \equiv \Pi = C_{(t)} - \Delta_{(t)} S_{(t)}$$

Delta Hedging without Transaction Costs

$$Portfolio \equiv \Pi = C_{(t)} - \Delta_{(t)} S_{(t)}$$

S := $$\frac{dS}{S} = \mu dt + \sigma dW$$

C := $$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

Black Scholes, 1973

ASSET DYNAMIC IS MODELLED!

Delta Hedging without Transaction Costs

$$Portfolio \equiv \Pi = C_{(t)} - \Delta_{(t)} S_{(t)}$$

$$\mathbf{S} := \begin{cases} dS_t = \mu S_t \, dt + \sqrt{\nu_t} S_t \, dW_t^S \\ d\nu_t = \kappa(\theta - \nu_t) \, dt + \xi \sqrt{\nu_t} \, dW_t^\nu \end{cases}$$

$$\mathbf{C} := \begin{cases} C = S\Pi_1 - e^{-rt} K \Pi_2 \\ \Pi_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\frac{e^{-iw\ln(K)} \psi_{\ln S_T}(w-i)}{iw\psi_{\ln S_T}(-i)} \right] dw \\ \Pi_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\frac{e^{-iw\ln(K)} \psi_{\ln S_T}(w)}{iw} \right] dw \end{cases}$$

Heston, 1993

ASSET DYNAMIC IS MODELLED!

Delta Hedging with Transaction Costs

$$Portfolio \equiv \Pi = C_{(t)} - \Delta_{(t)} S_{(t)} - \Upsilon_{(t,t-1,t-2,\ldots)}$$

Delta Hedging with Transaction Costs

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Path-dependent term

Delta Hedging with Transaction Costs

S := $$\frac{dS}{S} = \mu dt + \sigma dW$$

C := $$\begin{cases} \dfrac{\partial C}{\partial t} + \dfrac{1}{2}\sigma^2 \dfrac{\partial^2 C}{\partial S^2}\left[1 - \dfrac{c}{\sigma\sqrt{\tau}}\dfrac{2}{\pi} sign(\dfrac{\partial^2 C}{\partial S^2})\right] + \dfrac{\partial C}{\partial S} S - rC = 0 \\ \\ \tilde{\sigma_L}^2 = \sigma^2\left[1 - \dfrac{f}{\sigma\sqrt{\tau}}\sqrt{\dfrac{2}{\pi}}\right] \end{cases}$$

Leland, 1985

ASSET DYNAMIC IS MODELLED!

Part 2

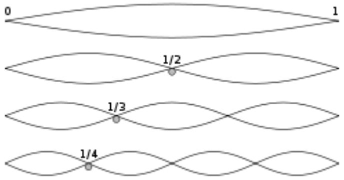
3

Dynamic

$(time)$

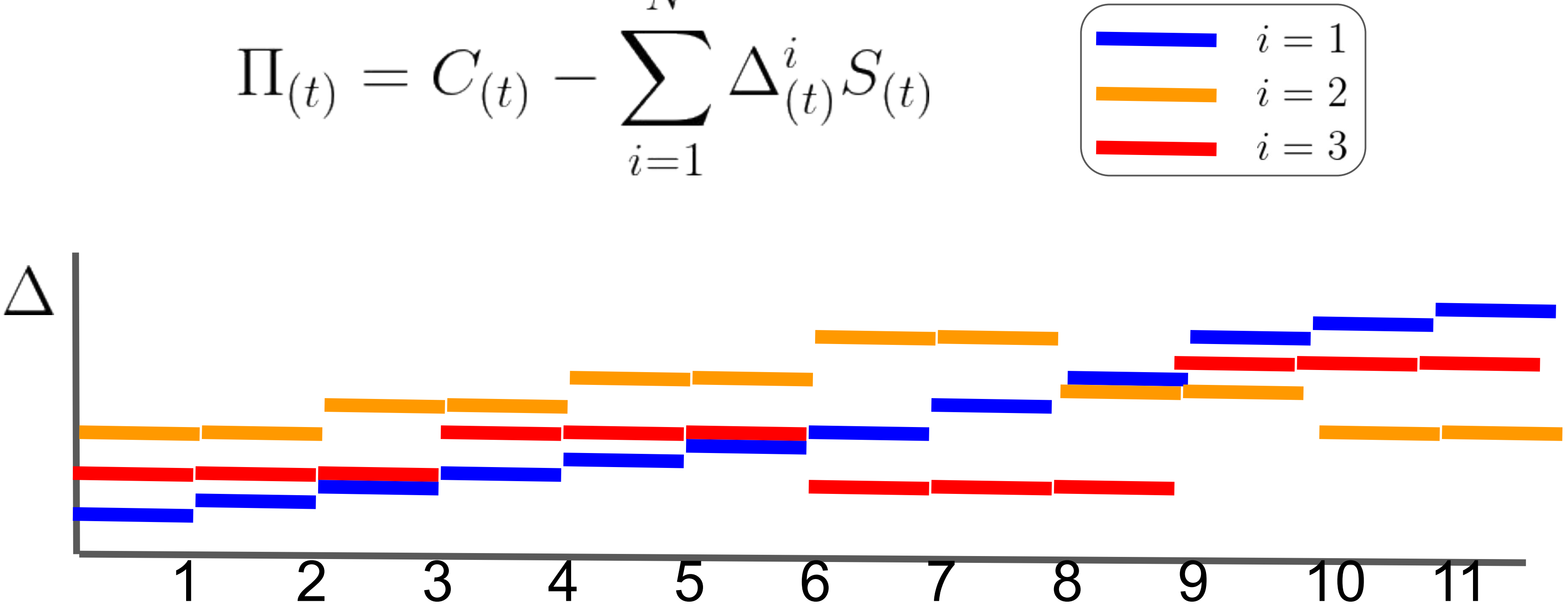
4

Multi-Scale Strategy



Dynamic Multi-Scale Strategy

$$\Pi_{(t)} = C_{(t)} - \sum_{i=1}^{N} \Delta^i_{(t)} S_{(t)}$$



What is the optimal weight-allocation?

The reward function maximizer

$$\left.\begin{array}{l} \dfrac{\Delta \mathrm{Reward}}{\Delta Gain} > 0 \\[2em] \dfrac{\Delta \mathrm{Reward}}{\Delta Risk} < 0 \end{array}\right\} \quad Reward = \frac{Gain}{Risk} \approx \frac{\mathbf{E}(\Pi)}{\mathbf{E}(\sigma)}$$

What is the optimal weight-allocation?

The reward function maximizer

$$\left.\begin{aligned} \frac{\Delta \mathrm{Reward}}{\Delta Gain} > 0 \\ \\ \frac{\Delta \mathrm{Reward}}{\Delta Risk} < 0 \end{aligned}\right\} \quad Reward = \frac{\Pi_{final} - \sum costs}{\gamma + \sqrt{Variance}}$$

What is the optimal weight-allocation?

The reward function maximizer

$$Reward_{(T,f,\gamma)} = \frac{\Pi_0(e^{rTN} - f) - f\sum_{j=1}^{N} S_{(t=jT-T)}|\Delta_{(t=jT)} - \Delta_{(t=jT-T)}|}{\gamma + \sqrt{\frac{\sum_{i=0}^{N}\sum_{j=1}^{T}([C_{(iT)}-\Delta_{(iT)}S_{(iT)}]-[C_{(iT+j)}-\Delta_{(iT+j)}S_{(iT+j)}])^2}{NT}}}$$

$$Reward = \frac{\Pi_{final} - \sum costs}{\gamma + \sqrt{Variance}}$$

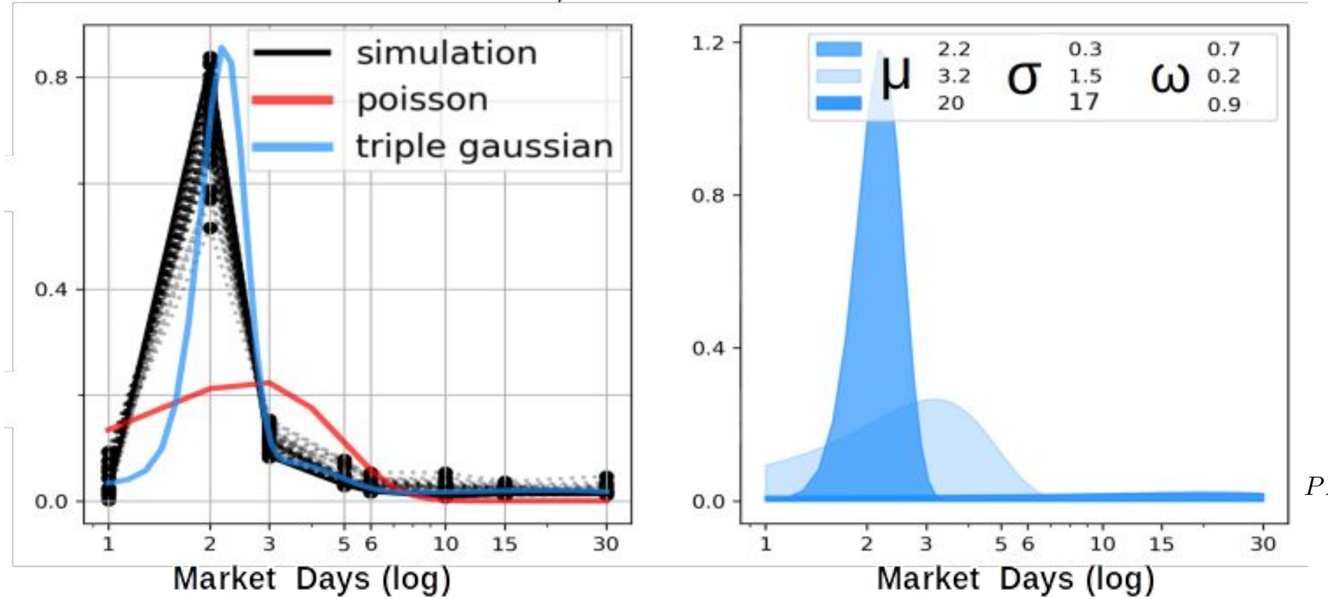
f proportional cost

γ risk aversion

A very risk-averse scenario

Three different models

Normalized line-connected histograms of best rewards for different risk profiles

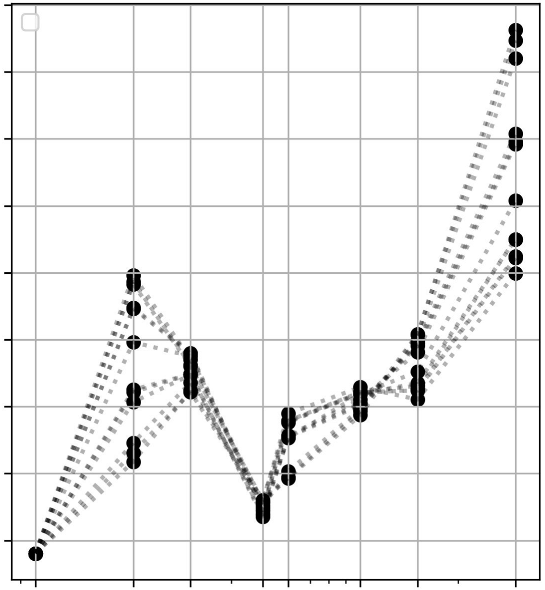


1. 3 gaussians
2. poisson
3. multinomial

$$PMF(x_1 = n_1, ..., x_m = n_m) = \frac{\sum_{i=1}^{m} n_i}{n_1! ... n_m!} p_1^{n_1} ... p_m^{n_m}$$

A risk-tolerant scenario

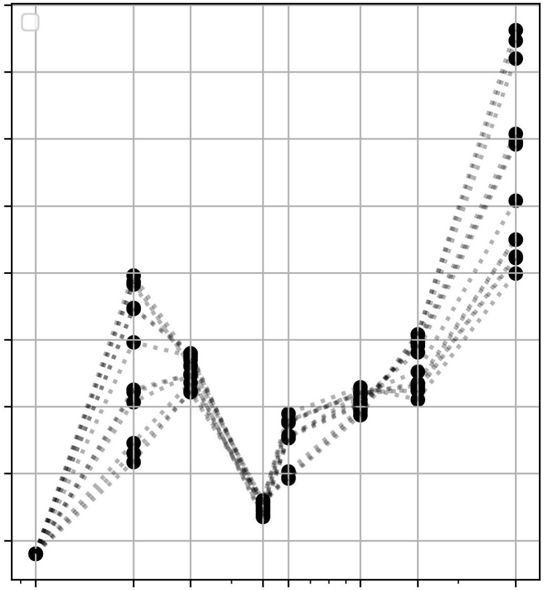
...the previous models should be recomputed



The optimal period is different!!!

A risk-tolerant scenario

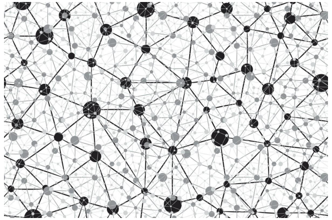
...the previous models should be recomputed



The optimal period is different!!!

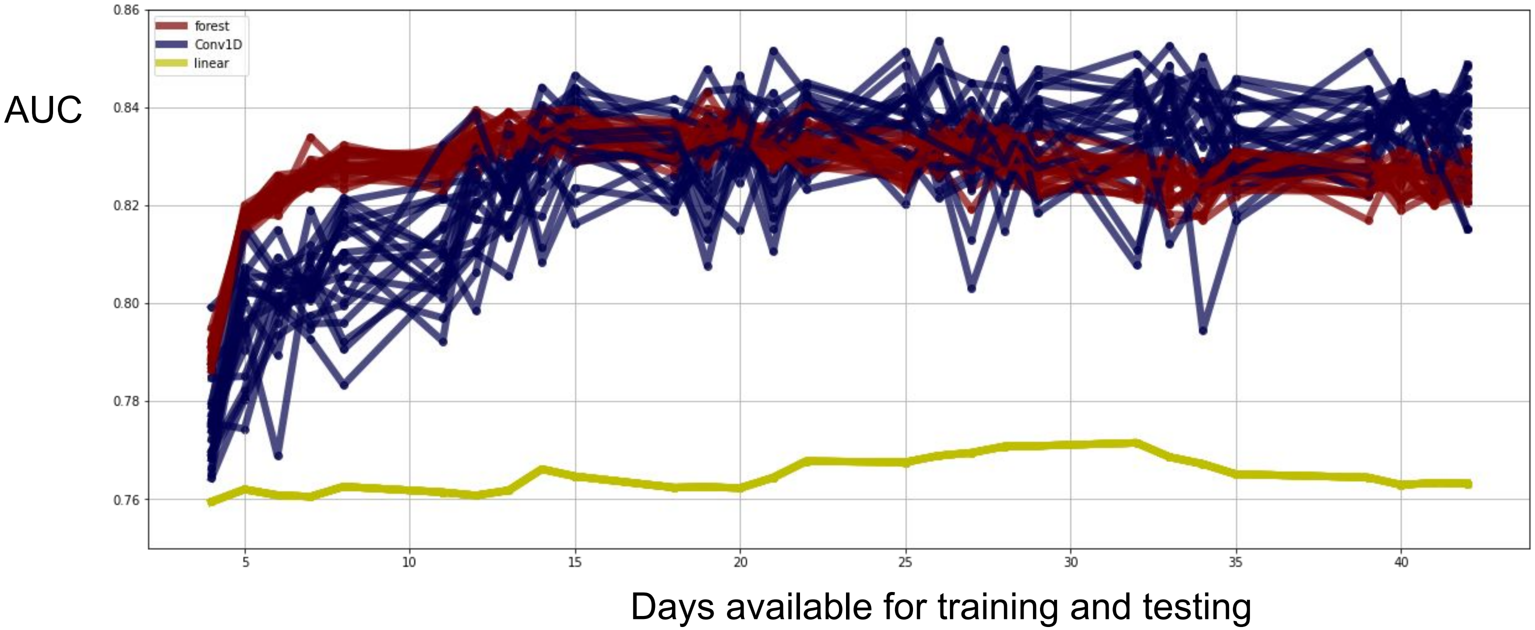
...and it is an average

using Neural Nets

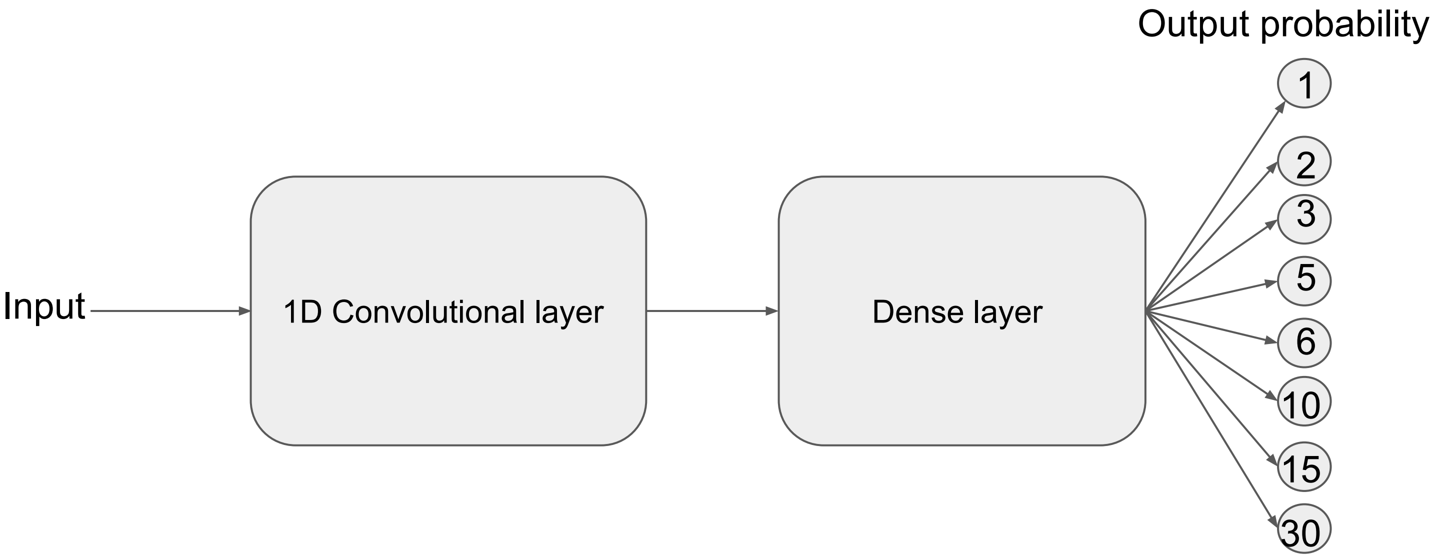


Machine learning models as an alternative

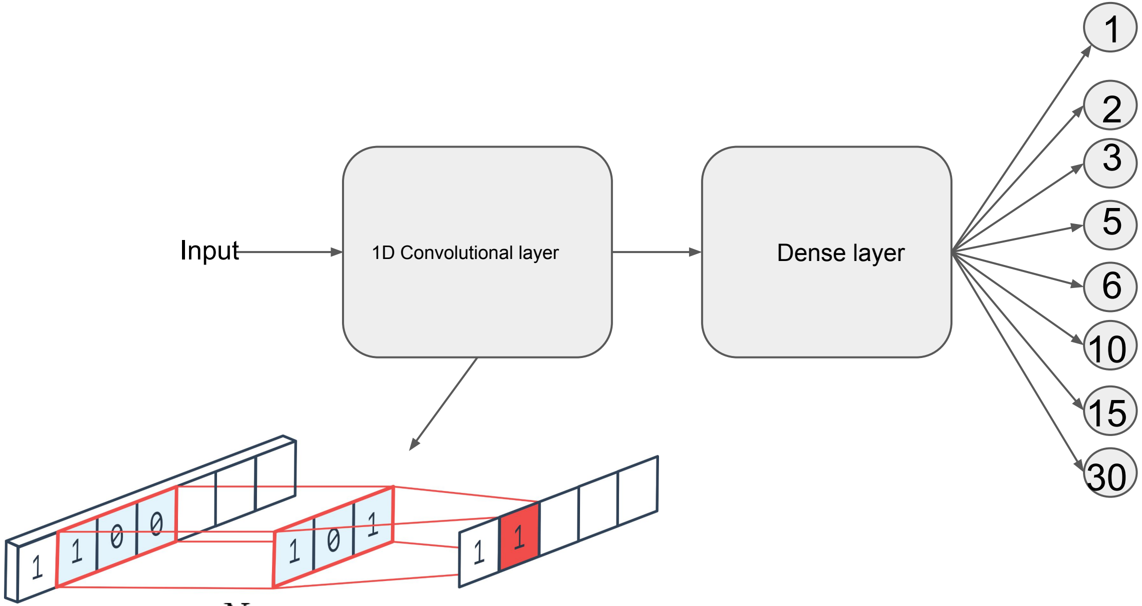
Pattern-seeking over a non-deterministic dataset: the task was to detect the best hedging period



Neural Networks' architecture



Neural Networks’ architecture



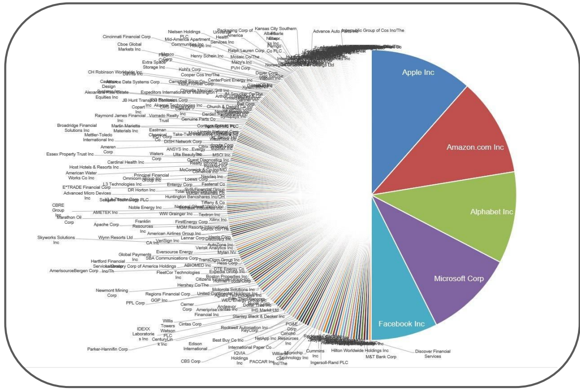
$$\tilde{f}(u) = \sum_{i}^{N} f(u-i)h(i)$$

Output probability, **K** = 8

$$\sigma(\vec{z})_i = \frac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}}$$

Testing the model over real data

Why the S&P 500

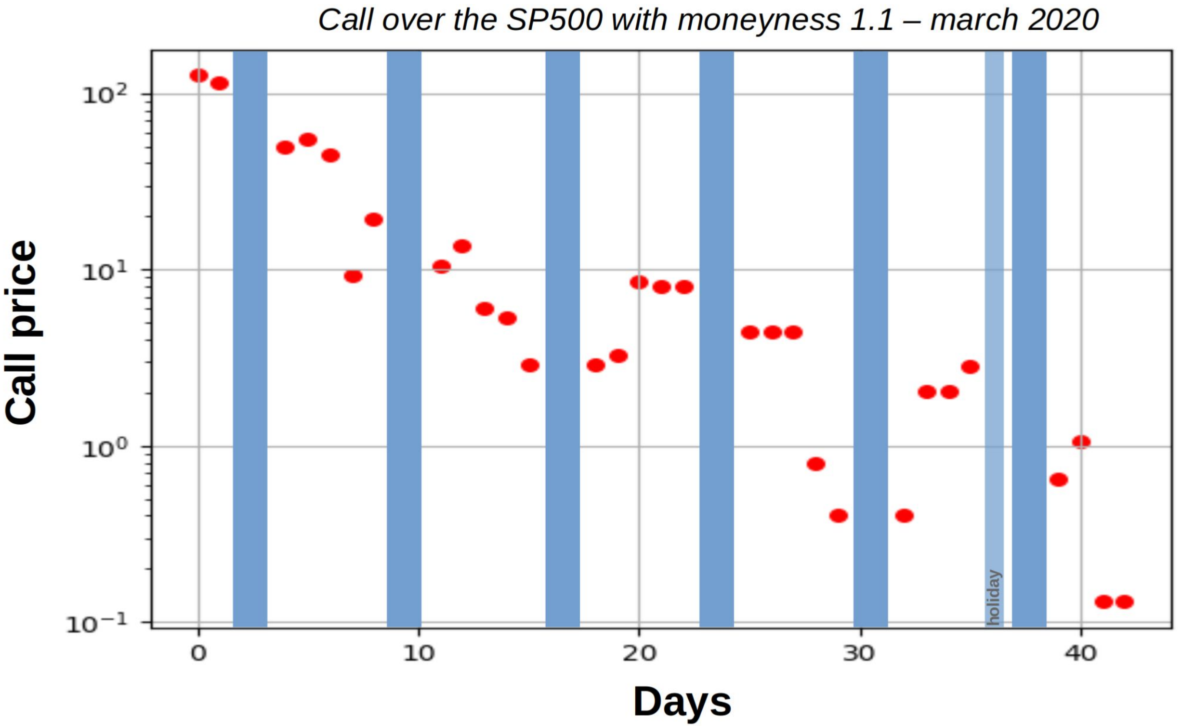


Includes a lot of sectors

Includes a lot of countries

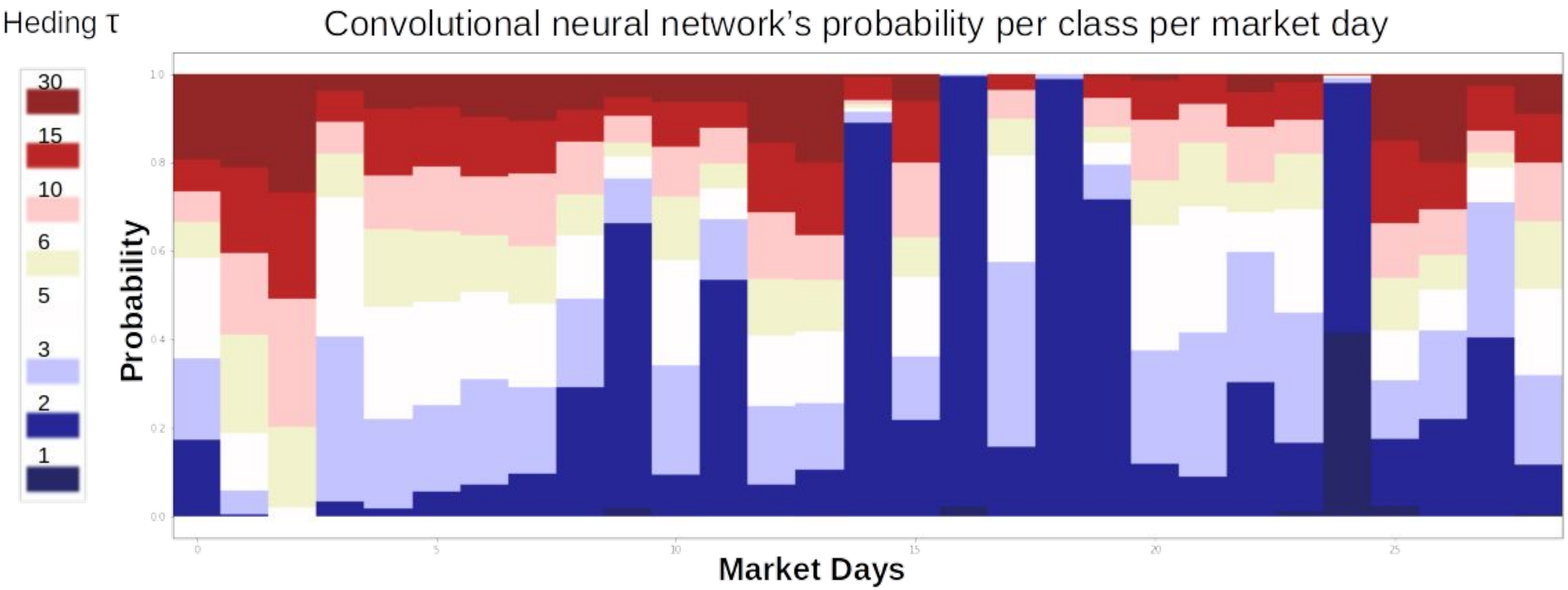
It is a suitable candidate for autoregression

Real 42-day call and possible trading periods



$$\tau \in \{1, 2, 3, 5, 6, 10, 15, 30\}$$

RESULTS (1/3)
Probability estimates are the strategy's weights

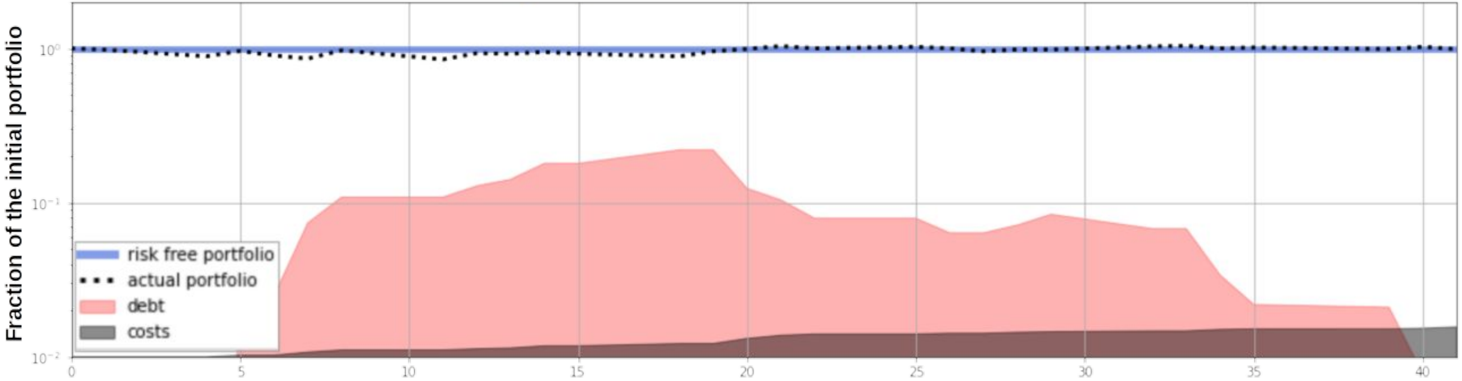


RESULTS (2/3)

Multi-Scale Strategy using the neural net and Comparison between models

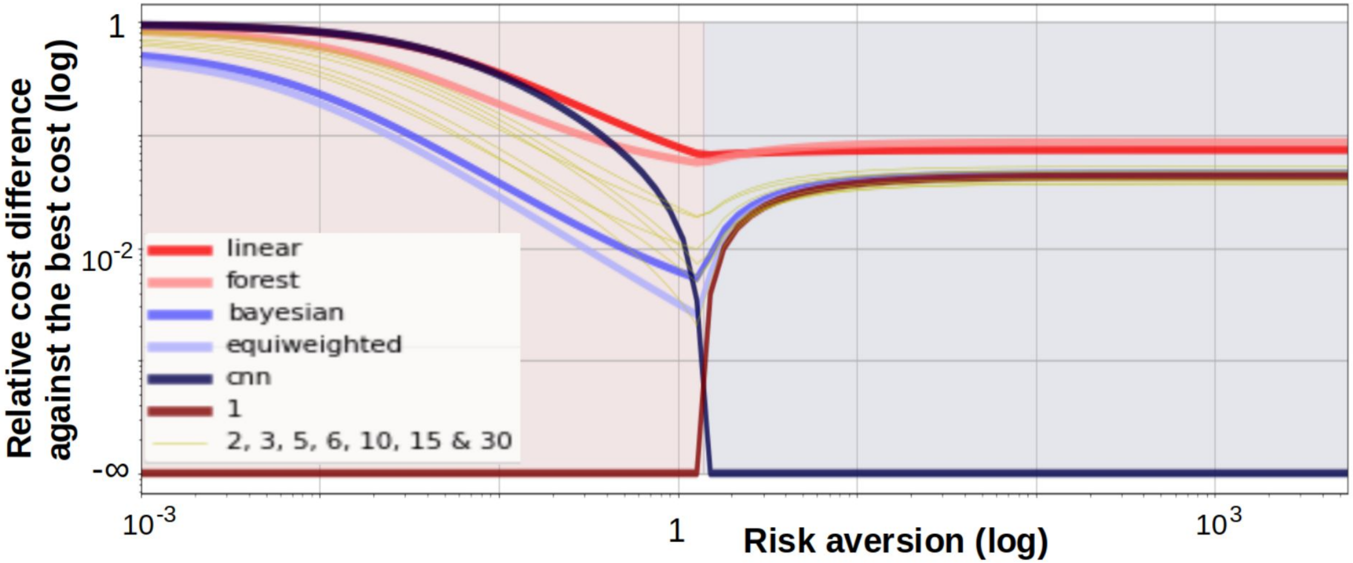
	CNN	forest	linear	bayes	unif	1	2	3	5	6	10	15	30
Final (%)	97.4	82.0	84.0	89.0	89.4	90.1	88.6	89.9	90.0	90.5	90.3	88.3	87.4
Std (%)	12.53	3.76	10.21	1.03	0.92	0.00	1.47	1.97	1.70	3.24	3.67	3.88	2.59
Under (%)	+4.8	-10.6	-8.6	-3.6	-3.2	-2.5	-4.0	-2.7	-2.7	-2.1	-2.3	-4.3	-5.2

Dynamic multi-scale strategy implemented in real time with the Convolutional Network



RESULTS (3/3)
Models' predictions vs risk aversion

Cost difference between all the models and the best one for two different regimes of risk aversion



Thank you for your time!