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## 1 Template

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 #define forr(i, a, b) for (int i = int(a); i < int(b); i++)
5 #define forn(i, n) forr(i, 0, n)
6 #define dforr(i, a, b) for (int i = int(b)-1; i >= int(a); i--)
7 #define dforn(i, n) dforr(i, 0, n)
8 #define all(v) begin(v), end(v)
9 #define sz(v) (int(size(v)))
10 #define pb push_back
11 #define fst first
12 #define snd second
13 #define mp make_pair
14 #define endl '\n'
15 #define dprint(v) cerr << __LINE__ << ":" "#v" = " << v << endl
16
17 using ll = long long;
18 using pii = pair<int,int>;
19
20 int main() {
21     ios::sync_with_stdio(0); cin.tie(0);
22 }
```

### 1.1 run.sh

```

1 clear
2 make -s $1 &&
3 for CASE in ./cases/$1/*; do
4     ./$1 < $CASE
5 done
```

### 1.2 comp.sh

```

1 clear
2 make -s $1 2>&1 | head -$2
```

## 1.3 Makefile

```

1 CXXFLAGS = -std=gnu++2a -O2 -g -Wall -Wextra -Wshadow
    -Wconversion\
2 -fsanitize=address -fsanitize=undefined
```

## 2 Estructuras de datos

### 2.1 Sparse Table

```

1 #define oper min
2 ELEM st[K][1<<K]; // K tal que (1<<K) > n
3 void st_init(vector<ELEM>& a) {
4     int n = sz(a); // assert(K >= 31-__builtin_clz(2*n));
5     forn(i,n) st[0][i] = a[i];
6     forr(k,1,K) forn(i,n-(1<<k)+1)
7         st[k][i] = oper(st[k-1][i], st[k-1][i+(1<<(k-1))]);
8 }
9 ELEM st_query(int l, int r) { // assert(l<r);
10    int k = 31-__builtin_clz(r-l);
11    return oper(st[k][l], st[k][r-(1<<k)]);
12 }
13 // si la operacion no es idempotente
14 ELEM st_query(int l, int r) {
15     int k = 31-__builtin_clz(r-l);
16     ELEM res = st[k][l];
17     for (l+=(1<<k), k--; l<r; k--) {
18         if (l+(1<<k)<=r) {
19             res = oper(res, st[k][l]);
20             l += (1<<k);
21         }
22     }
23     return res;
24 }
```

### 2.2 Segment Tree

```

1 // Dado un array y una operacion asociativa con neutro, get(i,j)
2 // opera en [i,j]
3 #define oper(x, y) max(x, y)
4 const int neutro=0;
5 struct RMQ{
6     int sz;
7     tipo t[4*MAXN];
8     tipo &operator[](int p){return t[sz+p];}
9     void init(int n){ // O(nlg n)}
```

```

9         sz = 1 << (32-__builtin_clz(n));
10        forn(i, 2*sz) t[i]=neutro;
11    }
12    void updall(){dforn(i, sz) t[i]=oper(t[2*i], t[2*i+1]);} // O(N)
13    tipo get(int i, int j){return get(i,j,1,0,sz);}
14    tipo get(int i, int j, int n, int a, int b){ // O(lgn)
15        if(j<=a || i>=b) return neutro;
16        if(i<=a && b<=j) return t[n];
17        int c=(a+b)/2;
18        return oper(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
19    }
20    void set(int p, tipo val){ // O(lgn)
21        for(p+=sz; p>0 && t[p]!=val;){
22            t[p]=val;
23            p/=2;
24            val=oper(t[p*2], t[p*2+1]);
25        }
26    }
27 } rmq;
28 // Usage:
29 cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();
```

### 2.3 Segment Tree Lazy

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i,
2 // j) opera sobre el rango [i, j].
3 typedef int ELEM;//Elem de los elementos del arreglo
4 typedef int ALT;//Elem de la alteracion
5 #define oper(x,y) x+y
6 #define oper2(k,a,b) k*(b-a)//Aplicar actualizacion sobre [a, b]
7 const ELEM neutro=0; const ALT neutro2=-1;
8 struct RMQ{
9     int sz;
10    ELEM t[4*MAXN];
11    ALT dirty[4*MAXN];//las alteraciones pueden ser distintas a
12    ELEM Ele;
13    ELEM &operator[](int p){return t[sz+p];}
14    void init(int n){//O(nlg n)
15        sz = 1 << (32-__builtin_clz(n));
16        forn(i, 2*sz) t[i]=neutro;
17        forn(i, 2*sz) dirty[i]=neutro2;
18    }
19    void push(int n, int a, int b){//propaga el dirty a sus hijos
20        if(dirty[n]!=neutro2){
21            t[n]+=oper2(dirty[n], a, b);//altera el nodo
22            if(n<sz){//cambiar segun el problema
23                dirty[2*n] = dirty[n];
24                dirty[2*n+1] = dirty[n];
25            }
26            dirty[n]=neutro2;
27        }
28    }
29    ELEM get(int i, int j, int n, int a, int b){//O(lgn)
30        if(j<=a || i>=b) return neutro;
31        push(n, a, b);
32        if(i<=a && b<=j) return t[n];
33        int c=(a+b)/2;
34        return oper(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
35    }
36    ELEM get(int i, int j){return get(i,j,1,0,sz);}
37    //altera los valores en [i, j] con una alteracion de val
38    void alterar(ALT val,int i,int j,int n,int a,int b){//O(lgn)
39        push(n, a, b);
40        if(j<=a || i>=b) return;
41        if(i<=a && b<=j){
42            dirty[n]+=(val);
43            push(n, a, b);
44            return;
45        }
46        int c=(a+b)/2;
47        alterar(val, i, j, 2*n, a, c);
48        alterar(val, i, j, 2*n+1, c, b);
49        t[n]=oper(t[2*n], t[2*n+1]);
50    }
51    void alterar(ALT val, int i, int j){alterar(val,i,j,1,0,sz);}
52 } rmq;
```

### 2.4 Segment Tree 2D

```

1 int n,m;
```

```

2 int a[MAXN][MAXN],st[4*MAXN][4*MAXN];
3 void build(){
4     for(i,0,n)for(j,0,m)st[i+n][j+m]=a[i][j];
5     for(i,0,n)for(int j=m-1;j;--j)
6         st[i+n][j]=op(st[i+n][j<<1],st[i+n][j<<1|1]);
7     for(int i=n-1;i;--i)for(j,0,2*m)
8         st[i][j]=op(st[i<<1][j],st[i<<1|1][j]);
9 }
10 void upd(int x, int y, int v){
11     st[x+n][y+m]=v;
12     for(int j=y+m;j>1;j>>=1)st[x+n][j>>1]=op(st[x+n][j],st[x+n][j^1]);
13     for(int i=x+n;i>1;i>>=1)for(int j=y+m;j,j>>=1)
14         st[i>>1][j]=op(st[i][j],st[i^1][j]);
15 }
16 int query(int x0, int x1, int y0, int y1){
17     int r=NEUT;
18     for(int i0=x0+n,i1=x1+n;i0<i1;i0>>=1,i1>>=1){
19         int t[4],q=0;
20         if(i0&1)t[q++]=i0++;
21         if(i1&1)t[q++]=-i1;
22         forr(k,0,q)for(int j0=y0+m,j1=y1+m;j0<j1;j0>>=1,j1>>=1){
23             if(j0&1)r=op(r,st[t[k]][j0++]);
24             if(j1&1)r=op(r,st[t[k]][-j1]);
25         }
26     }
27     return r;
28 }

```

## 2.5 Segment Tree Persistente

```

1 const int LOG2N = 19; // ceil(log2(MAXN))
2 const int STLEN = 1<<LOG2N;
3
4 struct Mono {
5     // TODO agregar data
6     static Mono zero() { /* TODO */ } // neutro de la suma
7 };
8 Mono operator+ (Mono a, Mono b) { /* TODO */ } // asociativo
9
10 struct N {
11     N(Mono x_, N* l_, N* r_)
12     : x{x_}, l{l_}, r{r_} {}
13     Mono x; N* l; N* r;
14 };
15 N empty_node(Mono::zero(), &empty_node, &empty_node);
16
17 deque<N> st_alloc; // optimizacion: >30% mas rapido que 'new N(x, l, r)'
18 N* make_node(Mono x, N* l, N* r) {
19     st_alloc.emplace_back(x, l, r);
20     return &st_alloc.back();
21 }
22
23 N* u_(N* t, int l, int r, int i, Mono x) {
24     if (i+1 <= l || r <= i) return t;
25     if (r-l == 1) return make_node(x, nullptr, nullptr);
26     int m = (l+r)/2;
27     auto lt = u_(t->l, l, m, i, x);
28     auto rt = u_(t->r, m, r, i, x);
29     return make_node(lt->x + rt->x, lt, rt);
30 }
31
32 int ql, qr;
33 Mono q_(N* t, int l, int r) {
34     if (qr <= l || r <= ql) return Mono::zero();
35     if (ql <= l && r <= qr) return t->x;
36     int m = (l+r)/2;
37     return q_(t->l, l, m) + q_(t->r, m, r);
38 }
39
40 // suma en rango: t[l,r)
41 Mono query(N* t, int l, int r) { ql = l; qr = r; return q_(t, 0, STLEN); }
42
43 // asignacion en punto: t[i]=x
44 N* update(N* t, int i, Mono x) { return u_(t, 0, STLEN, i, x); }
45
46 /* uso:
47 auto t = &empty_node;

```

```

48     t = update(t, 0, Mono{10});
49     t = update(t, 5, Mono{5});
50     auto x = query(t, 0, 5); // devuelve Mono{10}
51     auto y = query(t, 0, 6); // devuelve Mono{10} + Mono{5}
52     auto z = query(t, 1, 6); // devuelve Mono{5}
53 */

```

## 2.6 Fenwick Tree

```

1 struct Fenwick{
2     static const int sz=1<<K;
3     ll t[sz]={};
4     void adjust(int p, ll v){
5         for(int i=p+1;i<sz;i+=(i&-i)) t[i]+=v;
6     }
7     ll sum(int p){ // suma [0,p)
8         ll s = 0;
9         for(int i=p;i;i+=(i&-i)) s+=t[i];
10        return s;
11    }
12    ll sum(int a, int b){return sum(b)-sum(a);} // suma [a,b)
13
14    //funciona solo con valores no negativos en el fenwick
15    //longitud del minimo prefijo t.q. suma <= x
16    //para el maximo v+1 y restar 1 al resultado
17    int pref(ll v){
18        int x = 0;
19        for(int d = 1<<(K-1); d; d>>=1){
20            if( t[x|d] < v ) x |= d, v -= t[x];
21        }
22        return x+1;
23    }
24 };
25
26 struct RangeFT { // 0-indexed, query [0, i), update [l, r)
27     Fenwick rate, err;
28     void adjust(int l, int r, int x) { // range update
29         rate.adjust(l, x); rate.adjust(r, -x);
30         err.adjust(l, -x*l); err.adjust(r, x*r);
31     }
32     ll sum(int i) { return rate.sum(i) * i + err.sum(i); }
33 }; // prefix query
34
35
36 struct Fenwick2D{
37     ll t[N][M]={};
38     void adjust(int p, int q, ll v){
39         for(int i=p+1;i<N;i+=(i&-i))
40             for(int j= q+1; j<M; j+=(j&-j))
41                 t[i][j]+=v;
42     }
43     ll sum(int p,int q){ // suma [0,p)
44         ll s = 0;
45         for(int i=p;i;i+=(i&-i))
46             for(int j=q; j; j-=(j&-j))
47                 s+=t[i][j];
48         return s;
49     }
50     ll sum(int x1, int y1, int x2, int y2){
51         return sum(x2,y2)-sum(x1,y2)-sum(x2,y1)+sum(x1,y1);
52     } // suma [a,b)
53 };

```

## 2.7 Treap

```

1 // representa una lista como arbol con el orden implicito
2 struct node {
3     int val, prio, tam;
4     node *l, *r;
5 };
6 node *make(int val) {
7     return new node { val, rand(), 1, nullptr, nullptr };
8 }
9 int tam(node *n) { return n ? n->tam : 0; }
10 void recalc(node *n) { n->tam = tam(n->l) + 1 + tam(n->r); }
11 node* merge(node* s, node* t) {
12     if (s == nullptr) return t;
13     if (t == nullptr) return s;
14     if (s->prio > t->prio) {
15         s->r = merge(s->r, t);
16         recalc(s);

```

```

17     return s;
18 } else {
19     t->l = merge(s, t->l);
20     recalc(t);
21     return t;
22 }
23 }
24 pair<node*, node*> split(node *s, int k) {
25     if (s == nullptr) return {nullptr, nullptr};
26     if (tam(s->l) < k) {
27         if (s->l == nullptr) return {nullptr, nullptr};
28         auto [l, r] = split(s->r, k-tam(s->l)-1);
29         s->r = l;
30         recalc(s);
31         return {s, r};
32     } else {
33         auto [l, r] = split(s->l, k);
34         s->l = r;
35         recalc(s);
36         return {l, s};
37     }
38 } // usage: node *list = nullptr; list = merge(list, make(5))

```

## 2.8 Union Find

```

1 vector<int> uf(MAXN, -1);
2 int uf_find(int x) { return uf[x]<0 ? x : uf[x] =
3     uf_find(uf[x]); }
4 bool uf_join(int x, int y){ // True si x e y estan en !=
5     componentes
6     x = uf_find(x); y = uf_find(y);
7     if(x == y) return false;
8     if(uf[x] > uf[y]) swap(x, y);
9     uf[x] += uf[y]; uf[y] = x; return true;
8 }

```

## 2.9 Chull Trick

```

1 typedef ll tc;
2 struct Line{tc m,h;};
3 struct CHT { // for minimum (for maximum just change the sign of
4     lines)
5     vector<Line> c;
6     int pos=0;
7     tc in(Line a, Line b){
8         tc x=b.h-a.h,y=a.m-b.m;
9         return x/y+(x%y?((x>0)?(y>0):0); // ==ceil(x/y)
10    }
11    void add(tc m, tc h){ // m's should be non increasing
12        Line l=(Line){m,h};
13        if(sz(c)&&m==c.back().m){
14            l.h=min(h,c.back().h);c.pop_back();if(pos)pos--;
15        }
16        while(sz(c)>1&&in(c.back(),l)<=in(c[sz(c)-2],c.back())){
17            c.pop_back();if(pos)pos--;
18        }
19        c.pb(l);
20    }
21    inline bool fbin(tc x, int m){return in(c[m],c[m+1])>x;}
22    tc eval(tc x){
23        // O(log n) query:
24        int s=0,e=c.size();
25        while(e-s>1){int m=(s+e)/2;
26            if(fbin(x,m-1))e=m;
27            else s=m;
28        }
29        return c[s].m*x+c[s].h;
30        // O(1) query (for ordered x's):
31        while(pos>0&&fbin(x,pos-1))pos--;
32        while(pos<c.size()-1&&!fbin(x,pos))pos++;
33        return c[pos].m*x+c[pos].h;
34    };

```

## 2.10 Chull Trick Dinámico

```

1 struct Entry {
2     using It = set<Entry>::iterator;
3     bool is_query;
4     ll m, b; mutable It it, end;
5     ll x;
6 };

```

```

7     bool operator< (Entry const& a, Entry const& b) {
8         if (!a.is_query) return a.m < b.m;
9         auto ni = next(a.it);
10        if (ni == a.end) return false;
11        auto const& c = *ni;
12        return (c.b-a.b) > b.x * (a.m-c.m);
13    }
14    struct ChullTrick {
15        using It = Entry::It;
16        multiset<Entry> lines;
17        bool covered(It it) {
18            auto begin = lines.begin(), end = lines.end();
19            auto ni = next(it);
20            if (it == begin && ni == end) return false;
21            if (it == begin) return ni->m==it->m && ni->b>=it->b;
22            auto pi = prev(it);
23            if (ni == end) return pi->m==it->m && pi->b>=it->b;
24            return (it->m-pi->m)*(ni->b-pi->b) >=
25                (pi->b-it->b)*(pi->m-ni->m);
26        }
27        bool add(ll m, ll b) {
28            auto it = lines.insert({false, m, b});
29            it->it = it; it->end = lines.end();
30            if (covered(it)) { lines.erase(it); return false; }
31            while (next(it) != lines.end() && covered(next(it)))
32                lines.erase(next(it));
33            while (it != lines.begin() && covered(prev(it)))
34                lines.erase(prev(it));
35            return true;
36        }
37    };
38 };

```

## 3 Matemática

### 3.1 criba Lineal

```

1 const int N = 10'000'000;
2 vector<int> lp(N+1);
3 vector<int> pr;
4 for (int i=2; i <= N; ++i) {
5     if (lp[i] == 0) lp[i] = i, pr.push_back(i);
6     for (int j = 0; i * pr[j] <= N; ++j) {
7         lp[i * pr[j]] = pr[j];
8         if (pr[j] == lp[i]) break;
9     }
10 }

```

### 3.2 Pholland's Rho

```

1 ll mulmod(ll a, ll b, ll m) { return ll(_int128(a) * b % m); }
2
3 ll expmod(ll b, ll e, ll m) { // O(log b)
4     if (!e) return 1;
5     ll q=expmod(b,e/2,m); q=mulmod(q,q,m);
6     return e%2 ? mulmod(b,q,m) : q;
7 }
8
9 bool es_primo_prob(ll n, int a) {
10    if (n == a) return true;
11    ll s = 0, d = n-1;
12    while (d%2 == 0) s++, d/=2;
13    ll x = expmod(a,d,n);
14    if ((x == 1) || (x+1 == n)) return true;
15    forn(i,s-1){
16        x = mulmod(x,x,n);
17        if (x == 1) return false;
18        if (x+1 == n) return true;
19    }
20    return false;
21 }
22
23 bool rabin(ll n) { // devuelve true si n es primo
24     if (n == 1) return false;
25     const int ar[] = {2,3,5,7,11,13,17,19,23};
26     forn(j,9) if (!es_primo_prob(n,ar[j])) return false;
27     return true;
28 }
29

```

```

30 ll rho(ll n) {
31     if ((n & 1) == 0) return 2;
32     ll x = 2, y = 2, d = 1;
33     ll c = rand() % n + 1;
34     while (d == 1) {
35         x = (mulmod(x, x, n)+c) %n;
36         y = (mulmod(y, y, n)+c) %n;
37         y = (mulmod(y, y, n)+c) %n;
38         d=gcd(x-y, n);
39     }
40     return d==n ? rho(n) : d;
41 }
42
43 void factRho(map<ll,ll>&prim, ll n){ //0 (lg n)^3. un solo numero
44     if (n == 1) return;
45     if (rabin(n)) { prim[n]++; return; }
46     ll factor = rho(n);
47     factRho(prim, factor); factRho(prim, n/factor);
48 }
49 auto fact(ll n){
50     map<ll,ll>prim;
51     factRho(prim,n);
52     return prim;
53 }
```

### 3.3 Divisores

```

1 // Usar asi: divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
2 void divisores(const map<ll,ll> &f, vector<ll> &divs, auto it,
3     ll n=1){
4     if (it==f.begin()) divs.clear();
5     if (it==f.end()) { divs.pb(n); return; }
6     ll p=it->fst, k=it->snd; ++it;
7     forn(_, k+1) divisores(f,divs,it,n), n*=p;
8 }
9 ll sumDiv (ll n){ //suma de los divisores de n
10    ll rta = 1;
11    map<ll,ll> f=fact(n);
12    for(auto it = f.begin(); it != f.end(); it++) {
13        ll pot = 1, aux = 0;
14        forn(i, it->snd+1) aux += pot, pot *= it->fst;
15        rta*=aux;
16    }
17    return rta;
18 }
```

### 3.4 Inversos Modulares

```

1 pair<ll,ll> extended_euclid(ll a, ll b) {
2     if (b == 0) return {1, 0};
3     auto [y, x] = extended_euclid(b, a%b);
4     y -= (a/b)*x;
5     if (a*x + b*y < 0) x = -x, y = -y;
6     return {x, y}; // a*x + b*y = gcd(a,b)
7 }
8
9 constexpr ll MOD = 1000000007; // tmb es comun 998'244'353
10 ll invmod[MAXN]; // inversos módulo MOD hasta MAXN
11 void invmods() { // todo entero en [2,MAXN] debe ser coprimo con
12     MOD
13     invmod[1] = 1;
14     forn(i, 2, MAXN) invmod[i] = MOD - MOD/i*invmod[MOD%i] %MOD;
15 }
16
17 // si MAXN es demasiado grande o MOD no es fijo:
18 // versión corta, m debe ser primo. O(log(m))
19 ll invmod(ll a, ll m) { return expmod(a,m-2,m); }
20 // versión larga, a y m deben ser coprimos. O(log(a)), en general
21 // más rápido
22 ll invmod(ll a, ll m) { return (extended_euclid(a,m).fst % m +
23     m) % m; }
```

### 3.5 Catalan

```

1 ll Cat(int n){
2     return ((F[2*n] *FI[n+1]) %M *FI[n]) %M;
3 }
```

### 3.6 Lucas

```

1 const ll MAXP = 3e3+10; //68 MB, con 1e4 int son 380 MB
2 ll C[MAXP][MAXP], P; //inicializar con el primo del input <
3 void llenar_C(){
4     forn(i, MAXP) C[i][0] = 1;
5     forn(i, 1, MAXP) forn(j, 1, i+1)
6         C[i][j]=addmod(C[i-1][j-1],C[i-1][j], P);
7     // Calcula nCk (mod p) con n, k arbitrariamente grandes y p primo
8     // <= 3000
9     ll lucas(ll N, ll K){ // llamar a llenar_C() antes
10        ll ret = 1;
11        while(N>K){
12            ret = ret * C[N%P][K%P] % P;
13            N /= P, K /= P;
14        }
15    }
```

### 3.7 Stirling-Bell

```

1 ll STR[MAXN][MAXN], Bell[MAXN];
2 //STR[n][k] = formas de particionar un conjunto de n elementos en
3 //k conjuntos
4 //Bell[n] = formas de particionar un conjunto de n elementos
5 forn(i, 1, MAXN)STR[i][1] = 1;
6 forn(i, 2, MAXN)STR[1][i] = 0;
7 forn(i, 2, MAXN)forn(j, 2, MAXN){
8     STR[i][j] = (STR[i-1][j-1] + j*STR[i-1][j] %MOD) %MOD;
9 }
10 forn(i, MAXN){
11     Bell[i] = 0;
12     forn(j, MAXN){
13         Bell[i] = (Bell[i] + STR[i][j]) %MOD;
14     }
15 }
```

### 3.8 DP Factoriales

```

1 ll F[MAXN], INV[MAXN], FI[MAXN];
2 // ...
3 F[0] = 1; forn(i, 1, MAXN) F[i] = F[i-1]*i %M;
4 INV[1] = 1; forn(i, 2, MAXN) INV[i] = M - (11)(M/i)*INV[M%i] %M;
5 FI[0] = 1; forn(i, 1, MAXN) FI[i] = FI[i-1]*INV[i] %M;
```

### 3.9 Estructura de Fracción

```

1 tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
2 struct frac{
3     tipo p,q;
4     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
5     void norm(){
6         tipo a = mcd(p,q);
7         if(a) p/=a, q/=a;
8         else q=1;
9         if (q<0) q=-q, p=-p;
10    }
11    frac operator+(const frac& o){
12        tipo a = mcd(q,o.q);
13        return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
14    frac operator-(const frac& o){
15        tipo a = mcd(q,o.q);
16        return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
17    frac operator*(frac o){
18        tipo a = mcd(q,o.p), b = mcd(o.q,p);
19        return frac((p/b)*(o.q/a), (q/a)*(o.q/b));}
20    frac operator/(frac o){
21        tipo a = mcd(q,o.q), b = mcd(o.p,p);
22        return frac((p/b)*(o.q/a), (q/a)*(o.p/b));}
23    bool operator<(const frac &o) const{return p*q < o.p*q;}
24    bool operator==(frac o){return p==o.p&&q==o.q;}
```

### 3.10 Gauss

```

1 double reduce(vector<vector<double>> &a){ //Devuelve determinante
2     if (m == n)
3         int m=sz(a), n=sz(a[0]), i=0, j=0; double r = 1.0;
4         while(i < m and j < n){
5             int h = i;
6             forn(k, i+1, m) if(abs(a[k][j]) > abs(a[h][j])) h = k;
7             if(abs(a[h][j]) < EPS){ j++; r=0.0; continue; }
8             if(h != i){ r = -r; swap(a[i], a[h]); }
```

```

8   r *= a[i][j];
9   dforr(k, j, n) a[i][k] /= a[i][j];
10  forr(k, 0, m) if(k != i)
11    dforr(l_, j, n) a[k][l_] -= a[k][j] * a[i][l_];
12  i++; j++;
13 }
14 return r;
15 }

3.11 FFT

1 // MAXN must be power of 2 !!, MOD-1 needs to be a multiple of
2 // MAXN !!
3 typedef ll tf;
4 typedef vector<tf> poly;
5 const tf MOD = 2305843009255636993, RT = 5;
6 const tf MOD = 998244353, RT = 3;
7 // const tf MOD2 = 897581057, RT2 = 3; // Chinese Remainder
8 // Theorem
9 /* FFT */ struct CD {
10   double r, i;
11   CD(double r_ = 0, double i_ = 0) : r(r_), i(i_) {}
12   void operator/=(const int c) { r/=c, i/=c; }
13 };
14 CD operator*(const CD& a, const CD& b){
15   return CD(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);
16 };
17 CD operator+(const CD& a, const CD& b) { return CD(a.r+b.r,
18   a.i+b.i); }
19 CD operator-(const CD& a, const CD& b) { return CD(a.r-b.r,
20   a.i-b.i); }
21 /* NTT */ struct CD { tf x; CD(tf x_) : x(x_) {} CD() {} };
22 CD operator+(const CD& a, const CD& b) { return CD(addmod(a.x,
23   b.x)); } //ETC
24 vector<tf> rts(MAXN+9,-1);
25 CD root(int n, bool inv){
26   tf r = rts[n]<0 ? rts[n] = expmod(RT,(MOD-1)/n) : rts[n];
27   return CD(inv ? expmod(r, MOD-2) : r);
28 }
29 /* AMBOS */ CD cp1[MAXN+9], cp2[MAXN+9];
30 int R[MAXN+9];
31 void dft(CD* a, int n, bool inv){
32   double pi = acos(-1.0);
33   forn(i, n) if(R[i] < i) swap(a[R[i]], a[i]);
34   for(int m = 2; m <= n; m *= 2){
35     /* FFT */ double z = 2*pi/m * (inv?-1:1);
36     /* FFT */ CD wi = CD(cos(z), sin(z));
37     /* NTT */ CD wi = root(m, inv);
38     for(int j = 0; j < n; j += m){
39       CD w(1);
40       for(int k = j, k2 = j+m/2; k2 < j+m; k++, k2++){
41         CD u = a[k]; CD v = a[k2]*w; a[k] = u+v; a[k2] =
42           u-v; w = w*wi;
43     }
44   }
45   /* FFT */ if(inv) forn(i, n) a[i] /= n;
46   /* NTT */ if(inv){
47     CD z(expmod(n, MOD-2));
48     forn(i, n) a[i] = a[i]*z;
49   }
50   poly multiply(poly& p1, poly& p2){
51     int n = sz(p1)+sz(p2)+1;
52     int m = 1, cnt = 0;
53     while(m <= n) m *= 2, cnt++;
54     forn(i, m) R[i] = 0; forn(j, cnt) R[i] =
55       (R[i]<<1)|(i>>j)&1;
56     forn(i, m) cp1[i] = 0, cp2[i] = 0;
57     forn(i, sz(p1)) cp1[i] = p1[i];
58     forn(i, sz(p2)) cp2[i] = p2[i];
59     dft(cp1, m, false); dft(cp2, m, false);
60     // fast eval: forn(i, sz(p1)) p1(expmod(RT, (MOD-1)/m*i)) ==
61     cp1[i].x;
62     forn(i, m) cp1[i] = cp1[i]*cp2[i];
63     dft(cp1, m, true);
64     poly res;
65     n -= 2;
66     /* FFT */ forn(i, n) res.pb((tf)floor(cp1[i].r+0.5));
67     /* NTT */ forn(i, n) res.pb(cp1[i].x);
68   }
69   return res;
70 }


```

```

62 }

4 Geometria
4.1 Punto

1 using T = double;
2 bool iszero(T u) { return abs(u)<=EPS; }
3 struct Pt {
4   T x, y;
5   T z; // only for 3d
6   Pt() {}
7   Pt(T _x, T _y) : x(_x), y(_y) {}
8   Pt(T _x, T _y, T _z) : x(_x), y(_y), z(_z) {} // for 3d
9   T norm2(){ return *this**this; }
10  T norm(){ return sqrt(norm2()); }
11  Pt operator+(Pt o){ return Pt(x+o.x,y+o.y); }
12  Pt operator-(Pt o){ return Pt(x-o.x,y-o.y); }
13  Pt operator*(T u){ return Pt(x*u,y*u); }
14  Pt operator/(T u) {
15    if (iszero(u)) return Pt(INF,INF);
16    return Pt(x/u,y/u);
17  }
18  T operator*(Pt o){ return x*o.x+y*o.y; }
19  Pt operator^(Pt p){ // only for 3D
20    return Pt(y*p.z-z*p.y, z*p.x-x*p.z, x*p.y-y*p.x); }
21  T operator%(Pt o){ return x*o.y-y*o.x; }
22  T angle(Pt o){ return atan2(*this%o, *this)o); }
23  // T angle(Pt o){ // accurate around 90 degrees
24  //   if (*this%o>0) return acos(*this)o;
25  //   return 2*M_PI-acos(*this)o; }
26  Pt unit(){ return *this/norm(); }
27  bool left(Pt p, Pt q){ // is it to the left of directed line
28    pq?
29    return ((q-p)%(*this-p))>EPS; }
30  bool operator<(Pt p)const{ // for convex hull
31    return x<p.x-EPS||(iszero(x-p.x)&&y<p.y-EPS); }
32  bool collinear(Pt p, Pt q){
33    return iszero((p-*this)%(q-*this)); }
34  bool dir(Pt p, Pt q){ // does it have the same direction of
35    pq?
36    return this->collinear(p, q)&&(q-p)*(*this-p)>EPS; }
37  Pt rot(Pt r){ return Pt(*this)r,*this*r); }
38  Pt rot(T a){ return rot(Pt(sin(a),cos(a))); }
39  Pt ccw90(1,0);
40  Pt cw90(-1,0);

4.2 Linea

1 using T = double;
2 int sgn2(T x){return x<0?-1:1;}
3 struct Ln {
4   Pt p,pq;
5   Ln(Pt p, Pt q):p(p),pq(q-p){}
6   Ln(){}
7   bool has(Pt r){return dist(r)<=EPS; }
8   bool seghas(Pt r){return has(r)&&(r-p)*(r-(p+pq))<=EPS; }
9   // bool operator/((Ln l){return
10    (pq.unit()^l.pq.unit()).norm()<=EPS;} // 3D
11    bool operator/((Ln l){return abs(pq.unit()^l.pq.unit())<=EPS;} // 2D
12    bool operator==(Ln l){return *this/l&&has(l.p);}
13    Pt operator^(Ln l){ // intersection
14      if(*this/l)return Pt(INF,INF);
15      T a=-pq.y, b=pq.x, c=p.x*a+p.y*b;
16      T la=-l.pq.y, lb=l.pq.x, lc=l.p.x*la+l.p.y*lb;
17      T det = a * lb - b * la;
18      Pt r((lb*c-b*lc)/det, (a*lc-c*la)/det);
19      return r;
20      // Pt r=l.p+l.pq*((p-l.p)^pq)/(l.pq^pq));
21      if (!has(r)){return Pt(NAN,NAN,NAN);} // check only for 3D
22    }
23    T angle(Ln l){return pq.angle(l.pq);}
24    int side(Pt r){return has(r)?0:sgn2(pq^(r-p));} // 2D
25    Pt proj(Pt r){return p+pq*((r-p)*pq/pq.norm2());}
26    Pt segclosest(Pt r) {
27      T l2 = pq.norm2();
28      if(l2==0.) return p;
29      T t = ((r-p)*pq)/l2;
30      return p+(pq*min(1,max(0,t)));
31    }
32  }


```

```

31     Pt ref(Pt r){return proj(r)*2-r;}
32     T dist(Pt r){return (r-proj(r)).norm();}
33 //  T dist(Ln l){ // only 3D
34 //    if(*this/l) return dist(l.p);
35 //    return abs((l.p-p)*(pq^l.pq))/(pq^l.pq).norm();
36 //}
37     Ln rot(auto a){return Ln(p,p+pq.rot(a));} // 2D
38 };
39     Ln bisector(Ln l, Ln m){ // angle bisector
40     Pt p=l^m;
41     return Ln(p,p+l.pq.unit()+m.pq.unit());
42 }
43     Ln bisector(Pt p, Pt q){ // segment bisector (2D)
44     return Ln((p+q)*.5,p).rot(ccw90);
45 }



### 4.3 Poligono


1 using T = double;
2 struct Pol {
3     int n;vector<Pt> p;
4     Pol(){}
5     Pol(vector<Pt> _p){p=_p;n=p.size();}
6     T area(){
7         ll a = 0;
8         forr(i, 1, sz(p)-1){
9             a += (p[i]-p[0])^(p[i+1]-p[0]);
10        }
11        return abs(a)/2;
12    }
13    bool has(Pt q){ // O(n), winding number
14        forr(i,0,n)if(Ln(p[i],p[(i+1)%n]).seghas(q))return true;
15        int cnt=0;
16        forr(i,0,n){
17            int j=(i+1)%n;
18            int k=sgn((q-p[j])^(p[i]-p[j]));
19            int u=sgn(p[i].y-q.y),v=sgn(p[j].y-q.y);
20            if(k>0&&u<0&&v>0)cnt++;
21            if(k<0&&v<0&&u>0)cnt--;
22        }
23        return cnt!=0;
24    }
25    void normalize(){ // (call before haslog, remove collinear
26        first)
27        if(n>=3&&p[2].left(p[0],p[1]))reverse(p.begin(),p.end());
28        int pi=min_element(p.begin(),p.end())-p.begin();
29        vector<Pt> s(n);
30        forr(i,0,n)s[i]=p[(pi+i)%n];
31        p.swap(s);
32    }
33    bool haslog(Pt q){ // O(log(n)) only CONVEX. Call normalize
34        first
35        if(q.left(p[0],p[1])||q.left(p.back(),p[0]))return false;
36        int a=1,b=p.size()-1; // returns true if point on boundary
37        while(b>a){ // (change sign of EPS in left
38            int c=(a+b)/2; // to return false in such case)
39            if(!q.left(p[0],p[c]))a=c;
40            else b=c;
41        }
42        return !q.left(p[a],p[a+1]);
43    }
44    bool isconvex(){//O(N), delete collinear points!
45        if(n<3) return false;
46        bool isLeft=p[0].left(p[1], p[2]);
47        forr(i, 1, n)
48            if(p[i].left(p[(i+1)%n], p[(i+2)%n])!=isLeft)
49                return false;
50        return true;
51    }
52    Pt farthest(Pt v){ // O(log(n)) only CONVEX
53        if(n<10){
54            int k=0;
55            forr(i,1,n)if(v*(p[i]-p[k])>EPS)k=i;
56            return p[k];
57        }
58        if(n==sz(p))p.pb(p[0]);
59        Pt a=p[1]-p[0];
60        int s=0,e=n,ua=v*a>EPS;
61        if(!ua&&v*(p[n-1]-p[0])<=EPS)return p[0];
62        while(1){

63            int m=(s+e)/2;Pt c=p[m+1]-p[m];
64            int uc=v*c>EPS;
65            if(!uc&&v*(p[m-1]-p[m])<=EPS)return p[m];
66            if(ua&&(uc||v*(p[s]-p[m])>EPS))e=m;
67            else if(ua||uc||v*(p[s]-p[m])>=-EPS)s=m,a=c,ua=uc;
68            else e=m;
69            assert(e>s+1);
70        }
71    }
72    Pol cut(Ln l){ // cut CONVEX polygon by line l
73        vector<Pt> q; // returns part at left of l.pq
74        forr(i,0,n){
75            int d0=sgn(l.pq^(l.pq^(p[i]-l.p)));
76            int d1=sgn(l.pq^(l.pq^(p[(i+1)%n]-l.p));
77            if(d0>0)q.pb(p[i]);
78            Ln m(p[i],p[(i+1)%n]);
79            if(d0*d1<0&&!(1/m))q.pb(l^m);
80        }
81        return Pol(q);
82    }
83    T intercircle(circle c){ // area of intersection with circle
84        T r=0.;
85        forr(i,0,n){
86            int j=(i+1)%n;T w=c.intertriangle(p[i],p[j]);
87            if((p[j]-c.o)^(p[i]-c.o)>EPS)r+=w;
88            else r-=w;
89        }
90        return abs(r);
91    }
92    T callipers(){ // square distance of most distant points
93        T r=0; // prereq: convex, ccw, NO COLLINEAR POINTS
94        for(int i=0,j=n<2?0:1;i<j;++i){
95            for(;j=(j+1)%n{
96                r=max(r,(p[i]-p[j]).norm2());
97                if(((p[(i+1)%n]-p[i])^(p[(j+1)%n]-p[j]))<=EPS)
98                    break;
99            }
100       }
101    };



### 4.4 Circulo


1 using T = double;
2 struct Circle {
3     Pt o;T r;
4     Circle(Pt o, T r):o(o),r(r){}
5     Circle(Pt x, Pt y, Pt
6             z){o=bisector(x,y)^bisector(x,z);r=(o-x).norm();}
7     bool has(Pt p){return (o-p).norm()<=r+EPS;}
8     vector<Pt> operator^(Circle c){ // ccw
9         vector<Pt> s;
10        T d=(o-c.o).norm();
11        if(d>r+c.r+EPS||d+min(r,c.r)+EPS>max(r,c.r))return s;
12        T x=(d*d-c.r*c.r+r*r)/(2*d);
13        T y=sqrt(r*r-x*x);
14        Pt v=(c.o-o)/d;
15        s.pb(o+v*x-v.rot(ccw90)*y);
16        if(y>EPS)s.pb(o+v*x+v.rot(ccw90)*y);
17        return s;
18    }
19    vector<Pt> operator^(Ln l){
20        vector<Pt> s;
21        Pt p=l.proj(o);
22        T d=(p-o).norm();
23        if(d-EPS>r)return s;
24        if(abs(d-r)<=EPS){s.pb(p);return s;}
25        d=sqrt(r*r-d*d);
26        s.pb(p+l.pq.unit()*d);
27        s.pb(p-l.pq.unit()*d);
28        return s;
29    }
30    vector<Pt> tang(Pt p){
31        T d=sqrt((p-o).norm2()-r*r);
32        return *this^Circle(p,d);
33    }
34    bool in(Circle c){ // non strict
35        T d=(o-c.o).norm();
36        return d+r<=c.r+EPS;
}

```

```

36 }
37 T intertriangle(Pt a, Pt b){ // area of intersection with oab
38     if(abs((o-a)%(o-b))<=EPS) return 0.;
39     vector<Pt> q={a},w=*this^Ln(a,b);
40     if(w.size()==2)for(auto p:w)if((a-p)*(b-p)<-EPS)q.pb(p);
41     q.pb(b);
42     if(q.size()==4&&(q[0]-q[1])*(q[2]-q[1])>EPS)
43         swap(q[1],q[2]);
44     T s=0;
45     fore(i,0,q.size()-1){
46         if(!has(q[i])||!has(q[i+1]))
47             s+=r*r*(q[i]-o).angle(q[i+1]-o)/2;
48         else s+=abs((q[i]-o)%(q[i+1]-o)/2);
49     }
50     return s;
51 }
52 };

```

#### 4.5 Convex Hull

```

1 // CCW order
2 // Includes collinear points (change sign of EPS in left to
   exclude)
3 vector<Pt> chull(vector<Pt> p){
4     if(sz(p)<3) return p;
5     vector<Pt> r;
6     sort(p.begin(),p.end()); // first x, then y
7     forr(i,0,p.size()){ // lower hull
8         while(r.size()>=2&&r.back().left(r[r.size()-2],p[i]))
9             r.pop_back();
10    r.pb(p[i]);
11 }
12 r.pop_back();
13 int k=r.size();
14 for(int i=p.size()-1;i>0;--i){ // upper hull
15     while(r.size()>=k+2&&r.back().left(r[r.size()-2],p[i]))
16         r.pop_back();
17     r.pb(p[i]);
18 }
19 r.pop_back();
20 return r;
21 }

```

#### 4.6 Orden Radial

```

1 struct Radial {
2     Pt o;
3     Radial(Pt _o) : o(_o) {}
4     int cuad(Pt p) {
5         if (p.x>0 && p.y>0) return 1;
6         if (p.x<0 && p.y>0) return 2;
7         if (p.x<0 && p.y<0) return 3;
8         if (p.x>0 && p.y<0) return 4;
9         assert(p.x == 0 && p.y == 0);
10        return 0; // origen < todos
11    }
12    bool comp(Pt p, Pt q) {
13        int c1 = cuad(p), c2 = cuad(q);
14        if (c1 == c2) return p%q>EPS;
15        return c1 < c2;
16    }
17    bool operator()(const Pt &p, const Pt &q) const {
18        return comp(p-o,q-o);
19    }
20 };

```

#### 4.7 Par de puntos más cercano

```

1 #define dist(a, b) ((a-b).norm_sq())
2 bool sortx(pt a, pt b) {
3     return mp(a.x,a.y)<mp(b.x,b.y); }
4 bool sorty(pt a, pt b) {
5     return mp(a.y,a.x)<mp(b.y,b.x); }
6 ll closest(vector<pt> &ps, int l, int r) {
7     if (l == r-1) return INF;
8     if (l == r-2) {
9         if (sorty(ps[l+1], ps[l])) swap(ps[l+1], ps[l]);
10        return dist(ps[l], ps[l+1]);
11    }
12    int m = (l+r)/2; ll xm = ps[m].x;
13    ll min_dist = min(closest(ps, l, m), closest(ps, m, r));
14

```

```

15     vector<pt> left(&ps[l], &ps[m]), right(&ps[m], &ps[r]);
16     merge(all(left), all(right), &ps[l], sorty);
17     ll delta = ll(sqrt(min_dist));
18     vector<pt> strip;
19     forr (i, l, r) if (ps[i].x>=xm-delta&&ps[i].x<=xm+delta)
20         strip.pb(ps[i]);
21     forr (i, sz(strip)) forr (j, i, 8) {
22         if (i+j >= sz(strip)) break;
23         min_dist = min(min_dist, dist(strip[i], strip[i+j]));
24     }
25     return min_dist;
26 }
27 ll closest(vector<pt> &ps) { // devuelve dist^2
28     sort(all(ps), sortx);
29     return closest(ps, 0, sz(ps));
30 }

```

#### 4.8 Arbol KD

```

1 // given a set of points, answer queries of nearest point in
   O(log(n))
2 bool onx(pt a, pt b){return a.x<b.x;}
3 bool ony(pt a, pt b){return a.y<b.y;}
4 struct Node {
5     pt pp;
6     ll x0=INF, x1=-INF, y0=INF, y1=-INF;
7     Node *first=0, *second=0;
8     ll distance(pt p){
9         ll x=min(max(x0,p.x),x1);
10        ll y=min(max(y0,p.y),y1);
11        return (pt(x,y)-p).norm2();
12    }
13    Node(vector<pt>&& vp):pp(vp[0]){
14        for(pt p:vp){
15            x0=min(x0,p.x); x1=max(x1,p.x);
16            y0=min(y0,p.y); y1=max(y1,p.y);
17        }
18        if(sz(vp)>1){
19            sort(all(vp),x1-x0>=y1-y0?onx:ony);
20            int m=sz(vp)/2;
21            first=new Node({vp.begin(),vp.begin()+m});
22            second=new Node({vp.begin()+m, vp.end()});
23        }
24    }
25 };
26 struct KDTree {
27     Node* root;
28     KDTree(const vector<pt>& vp):root(new Node(all(vp))) {}
29     pair<ll,pt> search(pt p, Node *node){
30         if(!node->first){
31             //avoid query point as answer
32             //if(p==node->pp) {INF,pt()};
33             return {(p-node->pp).norm2(),node->pp};
34         }
35         Node *f=node->first, *s=node->second;
36         ll bf=f->distance(p), bs=s->distance(p);
37         if(bf>bs)swap(bf,bs),swap(f,s);
38         auto best=search(p,f);
39         if(bs<best.fst) best=min(best,search(p,s));
40         return best;
41     }
42     pair<ll,pt> nearest(pt p){return search(p,root);}
43 };

```

#### 4.9 Suma de Minkowski

```

1 vector<Pt> minkowski_sum(vector<Pt> &p, vector<Pt> &q){
2     int n=sz(p),m=sz(q),x=0,y=0;
3     forr(i,0,n) if(p[i]<p[x]) x=i;
4     forr(i,0,m) if(q[i]<q[y]) y=i;
5     vector<Pt> ans={p[x]+q[y]};
6     forr(it,1,n+m){
7         Pt a=p[(x+it)%n]+q[y];
8         Pt b=p[x]+q[(y+it)%m];
9         if(b.left(ans.back(),a)) ans.pb(b), y=(y+1)%m;
10        else ans.pb(a), x=(x+1)%n;
11    }
12    return ans;
13 }
14 vector<Pt> do_minkowski(vector<Pt> &p, vector<Pt> &q) {
15     normalize(p); normalize(q);

```

```

16     vector<Pt> sum = minkowski_sum(p, q);
17     return chull(sum); // no normalizado
18 }
19 // escalar poligono
20 vector<Pt> operator*(vector<Pt> &p, td u) {
21     vector<Pt> r; forn(i, sz(p)) r.pb(p[i]*u);
22     return r;
23 }

5 Strings
5.1 Hashing

1 struct StrHash { // Hash polinomial con exponentes decrecientes.
2     static constexpr ll ms[] = {1'000'000'007, 1'000'000'403};
3     static constexpr ll b = 500'000'000;
4     vector<ll> hs[2], bs[2];
5     StrHash(string const& s) {
6         int n = sz(s);
7         forn(k, 2) {
8             hs[k].resize(n+1), bs[k].resize(n+1, 1);
9             forn(i, n) {
10                 hs[k][i+1] = (hs[k][i] * b + s[i]) % ms[k];
11                 bs[k][i+1] = bs[k][i] * b % ms[k];
12             }
13         }
14     }
15     ll get(int idx, int len) const { // Hashes en `s[idx, idx+len)`.
16         ll h[2];
17         forn(k, 2) {
18             h[k] = hs[k][idx+len] - hs[k][idx] * bs[k][len] %
19                     ms[k];
20             if (h[k] < 0) h[k] += ms[k];
21         }
22         return (h[0] << 32) | h[1];
23     }

```

## 5.2 Suffix Array

```

1 #define RB(x) ((x) < n ? r[x] : 0)
2 void csort(vector<int>& sa, vector<int>& r, int k) {
3     int n = sz(sa);
4     vector<int> f(max(255, n)), t(n);
5     forn(i, n) ++f[RB(i+k)];
6     int sum = 0;
7     forn(i, max(255, n)) f[i] = (sum += f[i]) - f[i];
8     forn(i, n) t[f[RB(sa[i]+k)]++] = sa[i];
9     sa = t;
10 }
11 vector<int> compute_sa(string& s){ // O(n*log2(n))
12     int n = sz(s) + 1, rank;
13     vector<int> sa(n), r(n), t(n);
14     iota(all(sa), 0);
15     forn(i, n) r[i] = s[i];
16     for (int k = 1; k < n; k *= 2) {
17         csort(sa, r, k), csort(sa, r, 0);
18         t[sa[0]] = rank = 0;
19         forn(i, 1, n) {
20             if(r[sa[i]] != r[sa[i-1]] || RB(sa[i]+k) !=
21                 RB(sa[i-1]+k)) ++rank;
22             t[sa[i]] = rank;
23         }
24         r = t;
25         if (r[sa[n-1]] == n-1) break;
26     }
27     return sa; // sa[i] = i-th suffix of s in lexicographical order
28 }
29 vector<int> compute_lcp(string& s, vector<int>& sa){
30     int n=sz(s),L=0;
31     vector<int> lcp(n),plcp(n),phi(n);
32     phi[sa[0]]=-1;
33     forn(i,1,n)phi[sa[i]]=sa[i-1];
34     forn(i,n){
35         if(phi[i]<0){plcp[i]=0;continue;}
36         while(s[i+L]==s[phi[i]+L])L++;
37         plcp[i]=L;
38         L=max(L-1,0);
39     }
40     forn(i,n)lcp[i]=plcp[sa[i]];

```

```

40     return lcp; // lcp[i]=longest common prefix between sa[i-1]
41         and sa[i]
41 }

5.3 String Functions

1 template<class Char=char>vector<int>
2     pfun(basic_string<Char>const& w) {
3     int n = sz(w), j = 0; vector<int> pi(n);
4     forn(i, 1, n) {
5         while (j != 0 && w[i] != w[j]) {j = pi[j - 1];}
6         if (w[i] == w[j]) {++j;}
7         pi[i] = j;
8     } // pi[i] = length of longest proper suffix of w[0..i] that is
9         also prefix
9     return pi;
9 }

10 template<class Char=char>vector<int> zfun(const
11     basic_string<Char>& w) {
12     int n = sz(w), l = 0, r = 0; vector<int> z(n);
13     forn(i, 1, n) {
14         if (i <= r) {z[i] = min(r - i + 1, z[i - 1]);}
15         while (i + z[i] < n && w[z[i]] == w[i + z[i]]) {++z[i];}
16         if (i + z[i] - 1 > r) {l = i, r = i + z[i] - 1;}
17     } // z[i] = length of longest prefix of w that also begins at
18         index i
18     return z;
18 }

5.4 Manacher

1 struct Manacher {
2     vector<int> p;
3     Manacher(string const& s) {
4         int n = sz(s), m = 2*n+1, l = -1, r = 1;
5         vector<char> t(m); forn(i, n) t[2*i+1] = s[i];
6         p.resize(m); forn(i, 1, m) {
7             if (i < r) p[i] = min(r-i, p[l+r-i]);
8             while (p[i] <= i && i < m-p[i] && t[i-p[i]] ==
9                     t[i+p[i]]) ++p[i];
10             if (i+p[i] > r) l = i-p[i], r = i+p[i];
11         }
11     } // Retorna palindromos de la forma {comienzo, largo}.
12     pii at(int i) const {int k = p[i]-1; return pair{i/2-k/2, k};}
13     pii odd(int i) const {return at(2*i+1);} // Mayor centrado en
14         s[i].
14     pii even(int i) const {return at(2*i);} // Mayor centrado en
15         s[i-1, i].
15 };

5.5 Mínima Rotación Lexicográfica

1 // única secuencia no-creciente de strings menores a sus
2     rotaciones
2 vector<pii> lyndon(string const& s) {
3     vector<pii> fs;
4     int n = sz(s);
5     for (int i = 0, j, k; i < n;) {
6         for (k = i, j = i+1; j < n && s[k] <= s[j]; ++j)
7             if (s[k] < s[j]) k = i; else ++k;
8             for (int m = j-k; i <= k; i += m) fs.emplace_back(i, m);
9     }
10    return fs; // retorna substrings de la forma {comienzo, largo}
11 }
12
13 // último comienzo de la mínima rotación
14 int minrot(string const& s) {
15     auto fs = lyndon(s+s);
16     int n = sz(s), start = 0;
17     for (auto f : fs) if (f.fst < n) start = f.fst; else break;
18 }
19 }

5.6 Trie

1 // trie genérico. si es muy lento, se puede modificar para que
2     los hijos sean
2 // representados con un array del tamaño del alfabeto
3 template<class Char> struct Trie {
4     struct Node {
5         map<Char, Node*> child;
6         bool term;
6

```

```

7     };
8     Node* root;
9     static inline deque<Node> nodes;
10    static Node* make() {
11        nodes.emplace_back();
12        return &nodes.back();
13    }
14    Trie() : root{make()} {}
15    // retorna el largo del mayor prefijo de s que es prefijo de
16    // algún string
17    // insertado en el trie
18    int find(basic_string<Char> const& s) const {
19        Node* curr = root;
20        for(i,sz(s)) {
21            auto it = curr->child.find(s[i]);
22            if (it == end(curr->child)) return i;
23            curr = it->snd;
24        }
25        return sz(s);
26    }
27    // inserta s en el trie
28    void insert(basic_string<Char> const& s) {
29        Node* curr = root;
30        for(i,sz(s)) {
31            auto it = curr->child.find(s[i]);
32            if (it == end(curr->child)) curr = curr->child[s[i]] =
33                make();
34            else curr = it->snd;
35        }
36        curr->term = true;
37    }
38    // elimina s del trie
39    void erase(basic_string<Char> const& s) {
40        auto erase = [&](auto&& me, Node* curr, int i) -> bool {
41            if (i == sz(s)) {
42                curr->term = false;
43                return sz(curr->child) == 0;
44            }
45            auto it = curr->child.find(s[i]);
46            if (it == end(curr->child)) return false;
47            if (!me(me,it->snd,i+1)) return false;
48            curr->child.erase(it);
49            return sz(curr->child) == 0;
50        };
51        erase(erase,root,0);
52    }

```

### 5.7 Aho-Corasick

```

1 struct vertex {
2     map<char,int> next,go;
3     int p,link;
4     char pch;
5     vector<int> leaf;
6     vertex(int p=-1, char pch=-1):p(p),pch(pch),link(-1){}
7 };
8 vector<vertex> t;
9 void aho_init(){ //do not forget!!
10     t.clear();t.pb(vertex());
11 }
12 void add_string(string s, int id){
13     int v=0;
14     for(char c:s){
15         if(t[v].next.count(c)){
16             t[v].next[c]=t.size();
17             t.pb(vertex(v,c));
18         }
19         v=t[v].next[c];
20     }
21     t[v].leaf.pb(id);
22 }
23 int go(int v, char c);
24 int get_link(int v){
25     if(t[v].link<0)
26         if(!v||!t[v].p)t[v].link=0;
27         else t[v].link=go(get_link(t[v].p),t[v].pch);
28     return t[v].link;
29 }
30 int go(int v, char c){

```

```

31     if(!t[v].go.count(c))
32         if(t[v].next.count(c))t[v].go[c]=t[v].next[c];
33         else t[v].go[c]=v==0?0:go(get_link(v),c);
34     return t[v].go[c];
35 }

```

### 5.8 Suffix Automaton

```

1 // Minimal DFA that accepts all suffixes of a string.
2 /// - Any path starting at `0` forms a substring.
3 /// - Every substring corresponds to a path starting at `0`.
4 /// - Each state corresponds to the set of all substrings that
5     // have the same
6     // ending positions in the string, that is, each state `u`
7     // represents an
8     // equivalence class according to their ending positions
9     // `endpos(u)`.

10 // Given a state `u`, we can define the following concepts:
11 // - `longest(u)`: longest substring corresponding to `u`.
12 // - `len(u)`: length of `longest(u)`.

13 // Any state `u` corresponds to all suffixes of `longest(u)` no
14 // shorter
15 // than `minlen(u)`.

16 // For state `u`, `link(u)` points to the state `v` such that
17 // `longest(v)`
18 // is a suffix of `longest(u)` with `len(v) == minlen(u) - 1`.

19 // These links
20 // form a tree with the root in `0` and an inclusion
21 // relationship between
22 // all `endpos`.

23 template<class Char=char>class SuffixAutomaton {
24     using str = basic_string<Char>;
25     void extend(Char c, int& last) {
26         txt.pb(c); int p = last; last = new_state();
27         len[last] = len[p] + 1, firstpos[last] = len[p];
28         do {next[p][c] = last, p = link[p];} while (p >= 0 &&
29             !next[p].count(c));
30         if (p == -1) {link[last] = 0;} else {
31             int q = next[p][c];
32             if (len[q] == len[p] + 1) {link[last] = q;} else {
33                 int cl = copy_state(q);
34                 len[cl] = len[p] + 1; link[last] = link[q] = cl;
35                 do {next[p][c] = cl, p = link[p];} while (p >= 0 &&
36                     next[p].at(c) == q);
37             }
38         }
39     }
40     int new_state() {
41         next.pb({}), link.pb(-1), len.pb(0), firstpos.pb(-1);
42         return size++;
43     }
44     int copy_state(int state) {
45         next.pb(next[state]), link.pb(link[state]);
46         len.pb(len[state]), firstpos.pb(firstpos[state]);
47         return size++;
48     }
49     void dfs(int curr=0) {
50         terminal_paths_from[curr] = term[curr];
51         paths_from[curr] = 1;
52         fore(edge, next[curr]) {
53             int other = edge.snd;
54             if (!paths_from[other]) {dfs(other);}
55             terminal_paths_from[curr] +=
56                 terminal_paths_from[other];
57             paths_from[curr] += paths_from[other];
58             substrings_from[curr] += substrings_from[other];
59         }
60         substrings_from[curr] += terminal_paths_from[curr];
61     }
62     void compute(int last) {
63         term.resize(size);
64         for (int curr = last; curr != -1; curr = link[curr])
65             {term[curr] = true;}
66         inv_link.resize(size);
67         forr(curr, 1, size) {inv_link[link[curr]].pb(curr);}
68     }
69 public:
70     vector<bool> term; // Terminal statuses.

```

```

52     vector<vector<int>> inv_link; // Inverse suffix links.
53     vector<map<Char, int>> next{{}}; // Automaton transitions.
54     vector<int> len{0}; // len[u] = length of longest(u)
55     vector<int> link{-1}; // Suffix links.
56     vector<int> firstpos{-1}; // First endpos element of each
      state.
57     // Number of paths starting at each state and ending in a
      terminal state.
58     // For `0`, this is the number of suffixes (including the
      empty suffix).
59     vector<int> terminal_paths_from;
60     // Number of paths starting at each state. For `0`, this is
      the number of
      // distinct substrings (including the empty substring).
61     vector<ll> paths_from;
62     // Number of substrings starting at each state. For `0`, this
      is the number
      // of substrings counting repetitions (including the empty
      substring
63     // repeated `n+1` times, where `n` is the length of the
      original string).
64     vector<ll> substrings_from;
65     int size = 1; // Number of states.
66     str txt; // Original string.
67     SuffixAutomaton(str const& _txt) {
68         int last = 0;
69         fore(c, _txt) {extend(c, last);}
70         compute(last); terminal_paths_from.resize(size);
71         paths_from.resize(size); substrings_from.resize(size);
72         dfs();
73     }
74     pair<int, int> run(str const& pat) const {
75         int curr = 0, read = 0; // curr = last visited state
76         for (
77             auto it = pat.begin();
78             it != pat.end() && next[curr].count(*it);
79             curr = next[curr].at(*it++))
80         ) {++read;} // read = number of traversed transitions
81         return {curr, read};
82     }
83     bool is_suff(str const& pat) const
84     {auto [state, read] = run(pat); return term[state] && read
85      == sz(pat);}
86     bool is_substr(str const& pat) const {return run(pat).snd ==
87      sz(pat);}
88     int num_occs(str const& pat) const {
89         auto [state, read] = run(pat);
90         return read == sz(pat) ? terminal_paths_from[state] : 0;
91     }
92     int fst_occ(str const& pat) const {
93         int m = sz(pat); auto [state, read] = run(pat);
94         return read == m ? firstpos[state] + 1 - m : -1;
95     }
96     vector<int> all_occs(str const& pat) const {
97         vector<int> occs; int m = sz(pat); auto [node, read] =
98             run(pat);
99         if (read == m) {
100             stack<int> st{{node}};
101             while (!st.empty()) {
102                 int curr = st.top(); st.pop();
103                 occs.pb(firstpos[curr] + 1 - m);
104                 fore(child, inv_link[curr]) {st.push(child);}
105             }
106         }
107         // sort(all(occs)); occs.erase(unique(all(occs)),
108         // occs.end());
109         return occs; // unsorted and nonunique by default
110     }
111 };
112 
```

## 6 Grafos

### 6.1 Dikjstra

```

1     vector<pair<int,int>> g[MAXN]; // u->[(v,cost)]
2     ll dist[MAXN];
3     // complejidad O((E+V)*log(V))
4     void dijkstra(int x){
5         memset(dist,-1,sizeof(dist));
6         priority_queue<pair<ll,int> > q;
7         dist[x]=0;q.push({0,x}); 
```

```

8         while(!q.empty()){
9             x=q.top().snd;ll c=-q.top().fst;q.pop();
10            if(dist[x]!=c)continue;
11            forn(i,g[x].size()){
12                int y=g[x][i].fst; ll c=g[x][i].snd;
13                if(dist[y]<0||dist[x]+c<dist[y])
14                    dist[y]=dist[x]+c,q.push({-dist[y],y});
15            }
16        }
17    } 
```

### 6.2 LCA

```

1     int n;
2     vector<int> g[MAXN];
3
4     vector<int> depth, etour, vtime;
5
6     // operación de la sparse table, escribir `#define oper lca_oper`
7     int lca_oper(int u, int v) { return depth[u]<depth[v] ? u : v; };
8
9     void lca_dfs(int u) {
10        vtime[u] = sz(etour), etour.push_back(u);
11        for (auto v : g[u]) {
12            if (vtime[v] >= 0) continue;
13            depth[v] = depth[u]+1; lca_dfs(v); etour.push_back(u);
14        }
15    }
16    auto lca_init(int root) {
17        depth.assign(n,0), etour.clear(), vtime.assign(n,-1);
18        lca_dfs(root); st_init(etour);
19    }
20
21    auto lca(int u, int v) {
22        int l = min(vtime[u],vtime[v]);
23        int r = max(vtime[u],vtime[v])+1;
24        return st_query(l,r);
25    }
26    int dist(int u, int v) { return
27        depth[u]+depth[v]-2*depth[lca(u,v)]; } 
```

### 6.3 Binary Lifting

```

1     vector<int> g[1<<K]; int n; // K such that 2^K>=n
2     int F[K][1<<K], D[1<<K];
3     void lca_dfs(int x){
4         forn(i, sz(g[x])){
5             int y = g[x][i]; if(y==F[0][x]) continue;
6             F[0][y]=x; D[y]=D[x]+1;lca_dfs(y);
7         }
8     }
9     void lca_init(){
10        D[0]=0;F[0][0]=-1;
11        lca_dfs(0);
12        forn(k,1,K)forn(x,n)
13            if(F[k-1][x]<0)F[k][x]=-1;
14            else F[k][x]=F[k-1][F[k-1][x]];
15    }
16
17    int lca(int x, int y){
18        if(D[x]<D[y])swap(x,y);
19        for(int k = K-1;k>=0;--k) if(D[x]-(1<<k) >=D[y])x=F[k][x];
20        if(x==y) return x;
21        for(int
22            k=K-1;k>=0;--k)if(F[k][x] != F[k][y])x=F[k][x],y=F[k][y];
23    }
24
25    int dist(int x, int y){
26        return D[x] + D[y] - 2*D[lca(x,y)];
27    } 
```

### 6.4 Toposort

```

1     vector<int> g[MAXN];int n;
2     vector<int> tsrt(){ // lexicographically smallest topological
      sort
3     vector<int> r;priority_queue<int> q;
4     vector<int> d(2*n,0);
5     forn(i,n)forn(j,g[i].size())d[g[i][j]]++;
6     forn(i,n)if(d[i])q.push(-i);
7     while(!q.empty()){ 
```

```

8     int x=-q.top();q.pop();r.pb(x);
9     forn(i,sz(g[x])) {
10        d[g[x][i]]--;
11        if(!d[g[x][i]])q.push(-g[x][i]);
12    }
13 }
14 return r; // if not DAG it will have less than n elements
15 }
```

### 6.5 Detección ciclos negativos

```

1 // g[i][j]: weight of edge (i, j) or INF if there's no edge
2 // g[i][i]=0
3 ll g[MAXN][MAXN],int n;
4 void floyd(){ // O(n^3) . Replaces g with min distances
5     forn(k,n)forn(i,n)if(g[i][k]<INF)forn(j,n)if(g[k][j]<INF)
6         g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
7 }
8 bool inNegCycle(int v){return g[v][v]<0;}
9 bool hasNegCycle(int a, int b){ // true iff there's neg cycle in
   between
10    forn(i,n)if(g[a][i]<INF&&g[i][b]<INF&&g[i][i]<0)return true;
11    return false;
12 }
```

### 6.6 Camino Euleriano

```

1 // Directed version (uncomment commented code for undirected)
2 struct edge {
3     int y;
4     list<edge>::iterator rev;
5     edge(int y):y(y){}
6 };
7 list<edge> g[MAXN];
8 void add_edge(int a, int b){
9     g[a].push_front(edge(b)); //auto ia=g[a].begin();
10    // g[b].push_front(edge(a));auto ib=g[b].begin();
11    // ia->rev=ib;ib->rev=ia;
12 }
13 vector<int> p;
14 void go(int x){
15     while(g[x].size()){
16         int y=g[x].front().y;
17         //g[y].erase(g[x].front().rev);
18         g[x].pop_front();
19         go(y);
20     }
21     p.push_back(x);
22 }
23 vector<int> get_path(int x){ // get a path that begins in x
24 // check that a path exists from x before calling to get_path!
25     p.clear();go(x);reverse(p.begin(),p.end());
26     return p;
27 }
```

### 6.7 Camino Hamiltoniano

```

1 constexpr int MAXN = 20;
2 int n;
3 bool adj[MAXN][MAXN];
4
5 bool seen[1<<MAXN][MAXN];
6 bool memo[1<<MAXN][MAXN];
7 // true si existe camino simple en el conjunto s que empieza en u
8 bool hamilton(int s, int u) {
9     bool& ans = memo[s][u];
10    if (seen[s][u]) return ans;
11    seen[s][u] = true, s ^= (1<<u);
12    if (s == 0) return ans = true;
13    forn(v,n) if (adj[u][v] && (s&(1<<v)) && hamilton(s,v))
14        return ans = true;
15    return ans = false;
16 // true si existe camino hamiltoniano. complejidad O((1<<n)*n*n)
17 bool hamilton() {
18     forn(s,1<<n) forn(u,n) seen[s][u] = false;
19     forn(u,n) if (hamilton((1<<n)-1,u)) return true;
20     return false;
21 }
```

### 6.8 Tarjan SCC

```

1 vector<int> g[MAXN], ss;
2 int n, num, order[MAXN], lnk[MAXN], nsc, cmp[MAXN];
3 void scc(int u) {
4     order[u] = lnk[u] = ++num;
5     ss.pb(u); cmp[u] = -2;
6     for (auto v : g[u]) {
7         if (order[v] == 0) {
8             scc(v);
9             lnk[u] = min(lnk[u], lnk[v]);
10        }
11        else if (cmp[v] == -2) {
12            lnk[u] = min(lnk[u], lnk[v]);
13        }
14    }
15    if (lnk[u] == order[u]) {
16        int v;
17        do { v = ss.back(); cmp[v] = nsc; ss.pop_back(); }
18        while (v != u);
19        nsc++;
20    }
21 }
22 void tarjan() {
23     memset(order, 0, sizeof(order)); num = 0;
24     memset(cmp, -1, sizeof(cmp)); nsc = 0;
25     forn (i, n) if (order[i] == 0) scc(i);
26 }
```

### 6.9 Bellman-Ford

```

1 const int INF=2e9; int n;
2 vector<pair<int,int> g[MAXN]; // u->[(v,cost)]
3 ll dist[MAXN];
4 void bford(int src){ // O(nm)
5     fill(dist,dist+n,INF);dist[src]=0;
6     forn(_,_0,n)forn(x,0,n)if(dist[x]!=INF)for(auto t:g[x]){
7         dist[t.fst]=min(dist[t.fst],dist[x]+t.snd);
8     }
9     forn(x,0,n)if(dist[x]!=INF)for(auto t:g[x]){
10        if(dist[t.fst]>dist[x]+t.snd){
11            // neg cycle: all nodes reachable from t.fst have
12            // -INF distance
13            // to reconstruct neg cycle: save "prev" of each
14            // node, go up from t.fst until repeating a node.
15            // this node and all nodes between the two
16            // occurrences form a neg cycle
17        }
18    }
19 }
```

### 6.10 Puentes y Articulación

```

1 // solo para grafos no dirigidos
2 vector<int> g[MAXN];
3 int n, num, root, rootChildren;
4 int order[MAXN], lnk[MAXN], art[MAXN];
5 void bridge_art(int u, int p) {
6     order[u] = lnk[u] = ++num;
7     for (auto v : g[u]) if (v != p) {
8         if (u == root) rootChildren++;
9         if (order[v] == 0) {
10             bridge_art(v, u);
11             if (lnk[v] >= order[u]) // para puntos de
12                 art[u] = 1; // articulacion.
13             if (lnk[v] > order[u]) // para puentes.
14                 handle_bridge(u, v);
15         }
16         lnk[u] = min(lnk[u], lnk[v]);
17     }
18 }
19 void run() {
20     memset(order, 0, sizeof(order));
21     memset(art, 0, sizeof(art)); num = 0;
22     forn (i, n) {
23         if (order[i] == 0) {
24             root = i; bridge_art(i, -1);
25             art[i] = (rootChildren > 1);
26         }
27     }
28 }
```

### 6.11 Kruskal

```

1 int uf[MAXN];
2 void uf_init(){memset(uf,-1,sizeof(uf));}
3 int uf_find(int x){return uf[x]<0?x:uf[x]=uf_find(uf[x]);}
4 bool uf_join(int x, int y){
5     x=uf_find(x);y=uf_find(y);
6     if(x==y) return false;
7     if(uf[x]>uf[y]) swap(x,y);
8     uf[x]+=uf[y];uf[y]=x;
9     return true;
10 }
11 vector<pair<ll,pair<int,int>> > es; // edges (cost, (u,v))
12 ll kruskal(){ // assumes graph is connected
13     sort(es.begin(),es.end());uf_init();
14     ll r=0;
15     forr(i,0,es.size()){
16         int x=es[i].snd.fst,y=es[i].snd.snd;
17         if(uf_join(x,y))r+=es[i].fst; // (x,y,c) belongs to mst
18     }
19     return r; // total cost
20 }

```

### 6.12 Chequeo Bipartito

```

1 int n;
2 vector<int> g[MAXN];
3
4 bool color[MAXN];
5 bool bicolor(){
6     vector<bool> seen(n);
7     auto dfs = [&](auto&& me, int u, bool c) -> bool {
8         color[u] = c, seen[u] = true;
9         for (int v : g[u]) {
10             if (seen[v] && color[v] == color[u]) return false;
11             if (!seen[v] && !me(me,v,!c)) return false;
12         }
13         return true;
14     };
15     forn(u,n) if (!seen[u] && !dfs(dfs,u,0)) return false;
16     return true;
17 }

```

### 6.13 Centroid Decomposition

```

1 int sz[MAXN], ft[MAXN], tk[MAXN];
2 void calcsz(int u, int p) {
3     sz[u] = 1;
4     for (auto v : g[u]) if (v!=p && !tk[v]) {
5         calcsz(v, u);
6         sz[u]+=sz[v];
7     }
8 }
9 int dfs(int u, int p) {
10    int pesado = -1;
11    for (auto v : g[u]) if (v!=p && !tk[v]) {
12        if (pesado==-1 || sz[pesado]<sz[v]) pesado = v;
13    }
14    if (pesado==-1) return u;
15    if (sz[pesado]<=sz[u]/2) {
16        tk[u] = true;
17        for (auto v : g[u]) if (!tk[v]) {
18            int c=dfs(v, u);
19            ft[c]=u;
20        }
21        return u;
22    } else {
23        int sz_pesado=sz[pesado];
24        sz[pesado]=sz[u];
25        sz[u]-=sz_pesado;
26        return dfs(pesado, u);
27    }
28 }

```

### 6.14 HLD

```

1 vector<int> g[MAXN];
2 int wg[MAXN],dad[MAXN],dep[MAXN]; // weight,father,depth
3 void dfs1(int x){
4     wg[x]=1;
5     for(int y:g[x])if(y!=dad[x]){
6         dad[y]=x;dep[y]=dep[x]+1;dfs1(y);
7         wg[x]+wg[y];
8     }

```

```

9 }
10 int curpos,pos[MAXN],head[MAXN];
11 void hld(int x, int c){
12     if(c<0)c=x;
13     pos[x]=curpos++;head[x]=c;
14     int mx=-1;
15     for(int y:g[x])if(y!=dad[x]&&(mx<0||wg[mx]<wg[y]))mx=y;
16     if(mx>=0)hld(mx,c);
17     for(int y:g[x])if(y!=mx&&y!=dad[x])hld(y,-1);
18 }
19 void hld_init(){dad[0]=-1;dep[0]=0;dfs1(0);curpos=0;hld(0,-1);}
20 int query(int x, int y, RMQ& rmq){
21     int r=neutro; //neutro del rmq
22     while(head[x]!=head[y]){
23         if(dep[head[x]]>dep[head[y]])swap(x,y);
24         r=oper(r,rmq.get(pos[head[y]],pos[y]+1));
25         y=dad[head[y]];
26     }
27     if(dep[x]>dep[y])swap(x,y); // now x is lca
28     r=oper(r,rmq.get(pos[x],pos[y]+1));
29     return r;
30 }
31 // hacer una vez al principio hld_init() después de armar el
32 // grafo en g
33 // para querys pasar los dos nodos del camino y un stree que
34 // tiene en pos[x] el valor del nodo x
35 // for updating: rmq.set(pos[x],u);
36 // queries on edges: - assign values of edges to "child" node ()
37 // *** - change pos[x] to pos[x]+1 in query (line 28)
38 // *** if(dep[u] > dep[v]) rmq.upd(pos[u], w) para cada arista
39 // (u,v)

```

### 6.15 Max Tree Matching

```

1 int n, r, p[MAXN]; // número de nodos, raíz, y lista de padres
2 vector<int> g[MAXN]; // lista de adyancencia
3
4 int match[MAXN];
5 // encuentra el max matching del árbol. complejidad O(n)
6 int maxmatch(){
7     fill(match,match+n,-1);
8     int size = 0;
9     auto dfs = [&](auto&& me, int u) -> int {
10        for (auto v : g[u]) if (v != p[u])
11            if (match[u] == me(me,v)) match[u] = v, match[v] = u;
12        size += match[u] >= 0;
13        return match[u];
14    };
15    dfs(dfs,r);
16    return size;
17 }

```

### 6.16 Min Tree Vertex Cover

```

1 int n, r, p[MAXN]; // número de nodos, raíz, y lista de padres
2 vector<int> g[MAXN]; // lista de adyancencia
3
4 bool cover[MAXN];
5 // encuentra el min vertex cover del árbol. complejidad O(n)
6 int mincover(){
7     fill(cover,cover+n,false);
8     int size = 0;
9     auto dfs = [&](auto&& me, int u) -> bool {
10        for (auto v : g[u]) if (v != p[u] && !me(me,v)) cover[u] = true;
11        size += cover[u];
12        return cover[u];
13    };
14    dfs(dfs,r);
15    return size;
16 }

```

### 6.17 2-SAT

```

1 struct TwoSatSolver{
2     int n_vars;
3     int n_vertices;
4     vector<vector<int>> adj, adj_t;
5     vector<bool> used;
6     vector<int> order,comp;
7     vector<bool> assignment;

```

```

8  TwoSatSolver(int _n_vars) : n_vars(_n_vars),
9    n_vertices(2*_n_vars), adj(n_vertices),
10   adj_t(n_vertices), used(n_vertices),
11   order(), comp(n_vertices, -1), assignment(n_vars){
12     order.reserve(n_vertices);
13   }
14   void dfs1(int v){
15     used[v] = true;
16     for(int u : adj[v]){
17       if(!used[u]) dfs1(u);
18     }
19     order.pb(v);
20   }
21   void dfs2(int v, int c1){
22     comp[v] = c1;
23     for(int u : adj_t[v]){
24       if(comp[u] == -1) dfs2(u, c1);
25     }
26   }
27   bool solve_2SAT(){
28     order.clear();
29     used.assign(n_vertices, false);
30     forn(i, n_vertices){
31       if(!used[i]) dfs1(i);
32     }
33     comp.assign(n_vertices, -1);
34     for(int i = 0, j = 0; i < n_vertices; ++i){
35       int v = order[n_vertices - i - 1];
36       if(comp[v] == -1) dfs2(v, j++);
37     }
38     assignment.assign(n_vars, false);
39     for(int i = 0; i < n_vertices; i+=2){
40       if(comp[i] == comp[i+1]) return false;
41       assignment[i/2] = comp[i] > comp[i+1];
42     }
43     return true;
44   }
45   void add_disjunction(int a, bool na, int b, bool nb){
46     a = 2 * a ^ na;
47     b = 2 * b ^ nb;
48     int neg_a = a ^ 1;
49     int neg_b = b ^ 1;
50     adj[neg_a].pb(b);
51     adj[neg_b].pb(a);
52     adj_t[b].pb(neg_a);
53     adj_t[a].pb(neg_b);
54   }
55 };

```

### 6.18 K Colas

```

1 const int K=9999; // en general, K = MAX_DIST+1
2 vector<Datos> colas[K];
3 int cola_actual = 0, ult_col = -1;
4 // push toma la dist actual y la siguiente
5 #define push(d,nd,args...)
6   colas[(cola_actual+nd-d)%K].emplace_back(nd, args)
7 #define pop colas[cola_actual].pop_back
8 #define top colas[cola_actual].back
9 // PUSHEAR POSICION INICIAL
10 for (; ; cola_actual = (cola_actual+1)%K) {
11   if (ult_col == cola) break; // dimos la vuelta
12   if (colas[cola_actual].size()) ult_col = cola;
13   while (colas[cola_actual].size()) {
14   }
15 }

```

### 6.19 Arborescence

```

1 struct RollbackUF{
2   vi e;
3   vector<pii> st;
4   RollbackUF(int n) : e(n, -1) {}
5   int size(int x){return -e[find(x)];}
6   int find(int x) {return e[x] < 0 ? x : find(e[x]);}
7   int time() {return sz(st);}
8   void rollback(int t){
9     for(int i = time(); i-- > t;) e[st[i].fst] = st[i].snd;
10    st.resize(t);
11  }

```

```

12   bool join(int a, int b){
13     a = find(a), b = find(b);
14     if(a==b) return false;
15     if(e[a] > e[b]) swap(a, b);
16     st.emplace_back(a, e[a]), st.emplace_back(b, e[b]);
17     e[a]+=e[b], e[b] = a;
18     return true;
19   }
20 }
21
22 struct Edge{
23   int a, b; ll w;
24 };
25
26 struct Node{
27   Edge key;
28   Node *l, *r;
29   ll delta;
30   void prop(){
31     key.w += delta;
32     if(l) l->delta += delta;
33     if(r) r->delta += delta;
34     delta = 0;
35   }
36   Edge top(){
37     prop();
38     return key;
39   }
40 };
41
42 Node* merge(Node* a, Node*b){
43   if(!a || !b) return a?:b;
44   a->prop(), b->prop();
45   if(a->key.w > b->key.w) swap(a, b);
46   swap(a->l, (a->r = merge(b, a->r)));
47   return a;
48 }
49
50 void pop(Node*& a){
51   a->prop();
52   a = merge(a->l, a->r);
53 }
54
55 pair<ll, vi> dmst(int n, int r, vector<Edge>& g){
56   RollbackUF uf(n);
57   vector<Node*> heap(n);
58   for(Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
59   ll res = 0;
60   vi seen(n, -1), path(n), par(n);
61   seen[r] = r;
62   vector<Edge> Q(n), in(n, {-1,-1}), comp;
63   deque<tuple<int, int, vector<Edge>>> cycs;
64   forn(s, n){
65     int u = s, qi = 0, w;
66     while(seen[u] < 0){
67       if(!heap[u]) return {-1, {}};
68       Edge e = heap[u]->top();
69       heap[u]->delta -= e.w, pop(heap[u]);
70       Q[qi] = e, path[qi+1] = u, seen[u] = s;
71       res+=e.w, u = uf.find(e.a);
72       if(seen[u] == s){
73         Node* cyc = 0;
74         int end = qi, time = uf.time();
75         do cyc = merge(cyc, heap[w=path[--qi]]);
76         while(uf.join(u, w));
77         u = uf.find(u), heap[u] = cyc, seen[u] = -1;
78         cycs.push_front({u, time, {&Q[qi], &Q[end]}});
79       }
80     }
81     forn(i, qi) in[uf.find(Q[i].b)] = Q[i];
82   }
83   for(auto& [u, t, cmp] : cycs){
84     uf.rollback(t);
85     Edge inEdge = in[u];
86     for(auto &e : cmp) in[uf.find(e.b)] = e;
87     in[uf.find(inEdge.b)] = inEdge;
88   }
89   forn(i,n) par[i] = in[i].a;
90   return {res, par};

```

91 }

**7 Flujo****7.1 Dinic**

```

1 // complejidad O(V^2*E)
2 struct Dinic{
3     int nodes,src,dst;
4     vector<int> dist,q,work;
5     struct edge {int to,rev,ll f,cap;};
6     vector<vector<edge>> g;
7     Dinic(int x):nodes(x),g(x),dist(x),q(x),work(x){}
8     void add_edge(int s, int t, ll cap){
9         g[s].pb((edge){t,sz(g[t]),0,cap});
10        g[t].pb((edge){s,sz(g[s])-1,0,0});
11    }
12    bool dinic_bfs(){
13        fill(all(dist),-1);dist[src]=0;
14        int qt=0;q[qt++]=src;
15        for(int qh=0;qh<qt;qh++){
16            int u=q[qh];
17            forn(i,sz(g[u])) {
18                edge &e=g[u][i];int v=g[u][i].to;
19                if(dist[v]<0&&e.f<e.cap)dist[v]=dist[u]+1,q[qt++]=v;
20            }
21        }
22        return dist[dst]>=0;
23    }
24    ll dinic_dfs(int u, ll f){
25        if(u==dst) return f;
26        for(int &i=work[u];i<sz(g[u]);i++) {
27            edge &e=g[u][i];
28            if(e.cap<=e.f) continue;
29            int v=e.to;
30            if(dist[v]==dist[u]+1){
31                ll df=dinic_dfs(v,min(f,e.cap-e.f));
32                if(df>0){e.f+=df;g[v][e.rev].f-=df;return df;}
33            }
34        }
35        return 0;
36    }
37    ll max_flow(int _src, int _dst){
38        src=_src;dst=_dst;
39        ll result=0;
40        while(dinic_bfs()){
41            fill(all(work),0);
42            while(ll delta=dinic_dfs(src,INF))result+=delta;
43        }
44        return result;
45    }
46 };

```

**7.2 Min Cost Max Flow**

```

1 typedef ll tf;
2 typedef ll tc;
3 const tf INFFLOW=1e9;
4 const tc INFCOST=1e9;
5 // complejidad O(V^2*E*log(V))
6 struct MCF{
7     int n;
8     vector<tc> prio, pot; vector<tf> curflow; vector<int>
9         prevedge,prevnode;
10    priority_queue<pair<tc, int>, vector<pair<tc, int>>, greater<pair<tc, int>>> q;
11    struct edge{int to, rev; tf f, cap; tc cost;};
12    vector<vector<edge>> g;
13    MCF(int n):n(n),prio(n),curflow(n),
14        prevedge(n),prevnode(n),pot(n),g(n){}
15    void add_edge(int s, int t, tf cap, tc cost) {
16        g[s].pb((edge){t,sz(g[t]),0,cap,cost});
17        g[t].pb((edge){s,sz(g[s])-1,0,0,-cost});
18    }
19    pair<tf,tc> get_flow(int s, int t) {
20        tf flow=0; tc flowcost=0;
21        while(1){
22            q.push({0, s});
23            fill(all(prio),INFCOST);
24            prio[s]=0; curflow[s]=INFFLOW;
25            while(!q.empty()) {
26                auto cur=q.top();

```

```

26                    tc d=cur.fst;
27                    int u=cur.snd;
28                    q.pop();
29                    if(d!=prio[u]) continue;
30                    for(int i=0; i<sz(g[u]); ++i) {
31                        edge &e=g[u][i];
32                        int v=e.to;
33                        if(e.cap<=e.f) continue;
34                        tc nprio=prio[u]+e.cost+pot[u]-pot[v];
35                        if(prio[v]>nprio) {
36                            prio[v]=nprio;
37                            q.push({nprio, v});
38                            prevnode[v]=u; prevedge[v]=i;
39                            curflow[v]=min(curflow[u], e.cap-e.f);
40                        }
41                    }
42                    if(prio[t]==INFCOST) break;
43                    forr(i,0,n) pot[i]+=prio[i];
44                    tf df=min(curflow[t], INFFLOW-flow);
45                    flow+=df;
46                    for(int v=t; v!=s; v=prevnode[v]) {
47                        edge &e=g[prevnode[v]][prevedge[v]];
48                        e.f+=df; g[v][e.rev].f-=df;
49                        flowcost+=df*e.cost;
50                    }
51                }
52            }
53            return {flow,flowcost};
54        }
55    };

```

**7.3 Hopcroft Karp**

```

1 int n, m;           // número de nodos en ambas partes
2 vector<int> g[MAXN]; // lista de adyacencia [0,n) -> [0,m)
3
4 int mat[MAXN]; // matching [0,n) -> [0,m)
5 int inv[MAXM]; // matching [0,m) -> [0,n)
6 // encuentra el max matching del grafo bipartito
7 // complejidad O(sqrt(n+m)*e), donde e es el número de aristas
8 int hopkarp() {
9     fill(mat,mat+n,-1);
10    fill(inv,inv+m,-1);
11    int size = 0;
12    vector<int> d(n);
13    auto bfs = [&] {
14        bool aug = false;
15        queue<int> q;
16        forn(u,n) if (mat[u] < 0) q.push(u); else d[u] = -1;
17        while (!q.empty()) {
18            int u = q.front();
19            q.pop();
20            for (auto v : g[u]) {
21                if (inv[v] < 0) aug = true;
22                else if (d[inv[v]] < 0) d[inv[v]] = d[u] + 1,
23                                q.push(inv[v]);
24            }
25        }
26        return aug;
27    };
28    auto dfs = [&](auto& me, int u) -> bool {
29        for (auto v : g[u]) if (inv[v] < 0) {
30            mat[u] = v, inv[v] = u;
31            return true;
32        }
33        for (auto v : g[u]) if (d[inv[v]] > d[u] && me(me,inv[v]))
34            {
35                mat[u] = v, inv[v] = u;
36                return true;
37            }
38        d[u] = 0;
39        return false;
40    };
41    while (bfs()) forn(u,n) if (mat[u] < 0) size += dfs(dfs,u);
42    return size;
43}

```

**7.4 Kuhn**

```

1 int n, m;           // número de nodos en ambas partes
2 vector<int> g[MAXN]; // lista de adyacencia [0,n) -> [0,m)

```

```

3
4 int mat[MAXN]; // matching [0,n) -> [0,m)
5 int inv[MAXM]; // matching [0,m) -> [0,n)
6 // encuentra el max matching del grafo bipartito
7 // complejidad O(n*e), donde e es el número de aristas
8 int kuhn() {
9     fill(mat,mat+n,-1);
10    fill(inv,inv+m,-1);
11    int root, size = 0;
12    vector<int> seen(n,-1);
13    auto dfs = [&](auto&& me, int u) -> bool {
14        seen[u] = root;
15        for (auto v : g[u]) if (inv[v] < 0) {
16            mat[u] = v, inv[v] = u;
17            return true;
18        }
19        for (auto v : g[u]) if (seen[inv[v]] < root &&
20            me(me,inv[v])) {
21            mat[u] = v, inv[v] = u;
22            return true;
23        }
24    }
25    forn(u,n) size += dfs(dfs,root=u);
26    return size;
27 }

```

## 7.5 Min Vertex Cover Bipartito

```

1 // requisito: max matching bipartito, por defecto Hopcroft-Karp
2
3 vector<bool> cover[2]; // nodos cubiertos en ambas partes
4 // encuentra el min vertex cover del grafo bipartito
5 // misma complejidad que el algoritmo de max matching bipartito
6 // elegido
6 int konig() {
7     cover[0].assign(n,true);
8     cover[1].assign(m,false);
9     int size = hopkarp(); // alternativamente, también funciona
10    con Kuhn
11    auto dfs = [&](auto&& me, int u) -> void {
12        cover[0][u] = false;
13        for (auto v : g[u]) if (!cover[1][v]) {
14            cover[1][v] = true;
15            me(me,inv[v]);
16        }
17    };
18    forn(u,n) if (mat[u] < 0) dfs(dfs,u);
19    return size;
19 }

```

## 7.6 Hungarian

```

1 typedef long double td; typedef vector<int> vi; typedef
2     vector<td> vd;
2 const td INF=1e100;//for maximum set INF to 0, and negate costs
3 bool zero(td x){return fabs(x)<1e-9;}//change to x==0, for ints/ll
4 struct Hungarian{
5     int n; vector<vd> cs; vi L, R;
6     Hungarian(int N, int M):n(max(N,M)),cs(n,vd(n)),L(n),R(n){
7         forn(x,0,N)forn(y,0,M)cs[x][y]=INF;
8     }
9     void set(int x,int y,td c){cs[x][y]=c;}
10    td assign(){
11        int mat = 0; vd ds(n), u(n), v(n); vi dad(n), sn(n);
12        forn(i,0,n)u[i]==min_element(all(cs[i]));
13        forn(j,0,n){
14            v[j]=cs[0][j]-u[0];
15            forn(i,1,n)v[j]=min(v[j],cs[i][j]-u[i]);
16        }
17        L=R=vi(n, -1);
18        forn(i,0,n)forn(j,0,n) {
19            if(R[j]==-1&&zero(cs[i][j]-u[i]-v[j])){
20                L[i]=j;R[j]=i;mat++;break;
21            }
22        }
23        for(;mat<n;mat++){
24            int s=0, j=0, i;
25            while(L[s] != -1)s++;
26            fill(all(dad),-1);fill(all(sn),0);
27            forn(k,0,n)ds[k]=cs[s][k]-u[s]-v[k];
28            forn(;){

```

```

29                j = -1;
30                for(k,0,n)if(!sn[k]&&(j===-1||ds[k]<ds[j]))j=k;
31                sn[j] = 1; i = R[j];
32                if(i == -1) break;
33                auto new_ds=ds[j]+cs[i][k]-u[i]-v[k];
34                if(ds[k] > new_ds){ds[k]=new_ds;dad[k]=j;}
35            }
36            for(k,0,n)if(k!=j&&sn[k]){
37                auto w=ds[k]-ds[j];v[k]+=w,u[R[k]]-=w;
38                u[s] += ds[j];
39                while(dad[j]>=0){int d =
40                    dad[j];R[j]=R[d];L[R[j]]=j;j=d;
41                    R[j]=s;L[s]=j;
42                }
43                td value=0;forn(i,0,n)value+=cs[i][L[i]];
44                return value;
45            };
45 }

```

## 8 Optimización

### 8.1 Ternary Search

```

1 // mínimo entero de f en (l,r)
2 ll ternary(auto f, ll l, ll r) {
3     for (ll d = r-l; d > 2; d = r-l) {
4         ll a = l+d/3, b = r-d/3;
5         if (f(a) > f(b)) l = a; else r = b;
6     }
7     return l+1; // retorna un punto, no un resultado de evaluar f
8 }
9
10 // mínimo real de f en (l,r)
11 // para error < EPS, usar iters = log((r-l)/EPS)/log(1.618)
12 double golden(auto f, double l, double r, int iters) {
13     constexpr double ratio = (3-sqrt(5))/2;
14     double x1 = l+(r-l)*ratio, f1 = f(x1);
15     double x2 = r-(r-l)*ratio, f2 = f(x2);
16     while (iters--) {
17         if (f1 > f2) l=x1, x1=x2, f1=f2, x2=r-(r-l)*ratio,
18             f2=f(x2);
19     else r=x2, x2=x1, f2=f1, x1=l+(r-l)*ratio, f1=f(x1);
20 }
21 return (l+r)/2; // retorna un punto, no un resultado de evaluar f
21 }

```

### 8.2 Longest Increasing Subsequence

```

1 // subsecuencia creciente más larga
2 // para no decreciente, borrar la línea 9 con el continue
3 template<class Type> vector<int> lis(vector<Type>& a) {
4     int n = sz(a);
5     vector<int> seq, prev(n,-1), idx(n+1,-1);
6     vector<Type> dp(n+1,INF); dp[0] = -INF;
7     forn(i,n) {
8         int l = int(upper_bound(all(dp),a[i])-begin(dp));
9         if (dp[l-1] == a[i]) continue;
10        prev[i] = idx[l-1], idx[l] = i, dp[l] = a[i];
11    }
12    dfor(i,n+1) {
13        if (dp[i] < INF) {
14            for (int k = idx[i]; k >= 0; k = prev[k]) seq.pb(k);
15            reverse(all(seq));
16            break;
17        }
18    }
19    return seq;
20 }

```

## 9 Otros

### 9.1 Mo

```

1 int n,sq,nq; // array size, sqrt(array size), #queries
2 struct qu{int l,r,id;};
3 qu qs[MAXN];
4 ll ans[MAXN]; // ans[i] = answer to ith query
5 ll qcomp(const qu &a, const qu &b){
6     if(a.l/sq==b.l/sq) return a.l<b.l;
7     return (a.l/sq)&1?a.r<b.r:a.r>b.r;
8 }

```

```

9 void mos(){
10    forn(i,nq)qs[i].id=i;
11    sq=sqrt(n)+.5;
12    sort(qs,qs+nq,qcomp);
13    int l=0,r=0;
14    init();
15    forn(i,nq){
16        qu q=qs[i];
17        while(l>q.l)add(--l);
18        while(r<q.r)add(r++);
19        while(l<q.l)remove(l++);
20        while(r>q.r)remove(--r);
21        ans[q.id]=get_ans();
22    }
23 }

9.2 Divide and Conquer Optimization

1 vector<ll> dp_ant, dp_curr;

3 void compute(int l, int r, int optl, int optr){
4    if(l == r) return;
5    int m = (l+r)/2;
6    ll dpm = 1e17;
7    int optm = -1;
8    forr(i, max(m+1, optl), optr+1){
9        ll cost = C(m, i) + (i == n ? 0 : dp_ant[i]);
10       if(cost < dpm) dpm = cost, optm = i;
11    }
12    dp_curr[m] = dpm;
13    compute(l, m, optl, optm);
14    compute(m+1, r, optm, optr);
15 }
16
17 forn(i, k){
18    compute(0, n, 0, n);
19    dp_ant = dp_curr;
20 }
22 cout << dp_curr[0] << endl;

9.3 Fijar el numero de decimales

1 // antes de imprimir decimales, con una sola vez basta
2 cout << fixed << setprecision(DECIMAL_DIG);

9.4 Hash Table (Unordered Map/ Unordered Set)

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 template<class Key, class Val=null_type>using
4     hashtable=gp_hash_table<Key,Val>;
4 // como unordered_map (o unordered_set si Val es vacio), pero sin
5 // metodo count

9.5 Indexed Set

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 template<class Key, class Val=null_type>
4 using indexed_set = tree<Key, Val, less<Key>, rb_tree_tag,
5     tree_order_statistics_node_update>;
6 // indexed_set<char> s;
7 // char val = *s.find_by_order(0); // acceso por indice
8 // int idx = s.order_of_key('a'); // busca indice del valor

9.6 Subconjuntos

1 // iterar por mascaraas 0(2^n)
2 for(int bm=0; bm<(1<<n); bm++)
3 // subconjuntos de una mascara 0(2^n)
4 for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
5 // iterar por submascaras 0(3^n)
6 for(int bm=0; bm<(1<<n); bm++)
7     for(int sbm=bm; sbm; sbm=(sbm-1)&(bm))
8 // para superconjuntos (que contienen a bm),
9 // negar la mascara: bm=~bm

9.7 Simpson

1 // integra f en [a,b] llamandola 2*n veces
2 double simpson(double f, double a, double b, int n=1e4) {
3     double h = (b-a)/2/n, s = f(a);
4     forn(i,1,2*n) s += f(a+i*h) * ((i%2)?4:2);

```

```

5     return (s+f(b))*h/3;
6 }


```

### 9.8 Pragmas

```

1 #pragma GCC target("avx2")
2 #pragma GCC optimize("O3")
3 #pragma GCC optimize("unroll-loops")


```

### 9.9 Random

```

1 unsigned seed =
2     std::chrono::steady_clock::now().time_since_epoch().count();
3 mt19937 generator(seed);
4 generator(); // generar un número aleatorio entre 0 y 4294967295
4 // existe mt19937_64 para la versión de 64 bits, que
5 // probablemente sea más rápido
5
6 /*
7 // también se puede hacer lo siguiente para una versión hasta 3x
8 // más rápida:
9 #include <ext/random>
10 using namespace __gnu_cxx;
11 unsigned seed =
12     std::chrono::steady_clock::now().time_since_epoch().count();
13 sfmt19937 generator(seed); // existe también sfmt19937_64
14 */
15 uniform_int_distribution<ll> dist_int(L, R);
16 dist_int(generator); // generar un entero en [L, R]
17 // cerrado-cerrado) con prob uniforme
18 uniform_real_distribution<double> dist_real(0.0, 1.0);
19 dist_real(generator); // generar un real en [0, 1)
20 // cerrado-abierto) con prob uniforme


```

### 9.10 Utilidades de strings

```

1 getline(cin, linea); // tomar toda la linea
2 stringstream ss(linea); // tratar una linea como stream
3 ss >> s; ss << s; // leer solo hasta un espacio, escribir a ss
4 tipo n; ss >> n; // leer de un stringstream (float, int, etc.)
5 int pos = s.find_first_of("aeiou"); // devuelve -1 si no encuentra
6 int next = s.find_first_of("aeiou", pos);
7 // s.find_first_not_of("aeiou"); s.find_last_of();
8 s.substr(pos, next-pos); // substr(pos, len)
9 s.c_str(); // devuelve un puntero de C
10 ss.str(); // devuelve el string en ss
11 // isspace(); islower(); isupper(); isdigit(); isalpha();
12 // tolower(); toupper();


```

## Apéndice

Para el regional elegimos nombre

18 de febrero de 2026

Dinitz en una red unitaria:  $O(\sqrt{V} \cdot E)$

Lista de números con mayor cantidad de divisores hasta  $10^n$ :

(1, 6, 4) (2, 60, 12) (3, 840, 32) (4, 7560, 64) (5, 83160, 128)  
(6, 720720, 240) (7, 8648640, 448) (8, 73513440, 768) (9, 735134400, 1344)  
(10, 6983776800, 2304) (11, 97772875200, 4032) (12, 963761198400, 6720)  
(13, 9316358251200, 10752) (14, 97821761637600, 17280)  
(15, 866421317361600, 26880) (16, 8086598962041600, 41472)  
(17, 74801040398884800, 64512) (18, 897612484786617600, 103680)

Teorema de Hall: En un grafo bipartito existe un matching perfecto si para cualquier subconjunto de vértices W, la vecindad de W es mayor o igual que W.

$$|W| \leq |N_G(W)|$$

Teorema de Konig: El numero de aristas en un matching máximo es igual al número de vértices en un cubrimiento por vertices mínimo.

Teorema de Dilworth: En todo poset finito, el maximo numero de elementos en una anticadena es igual al tamaño de la minima particion en cadenas del conjunto.

Ley de cosenos: Dados dos lados de un triangulo  $a, b$  y el ángulo entre ellos  $\alpha$ , la longitud del otro lado  $c$  es:

$$c^2 = a^2 + b^2 - 2ab \cos(\alpha)$$

Ley de senos: En un triángulo la razón, entre cada lado y el seno de su ángulo opuesto, es constante e igual al diámetro de la circunferencia circunscrita.

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)} = 2R$$

Valor de  $\pi$ :

$$\pi = \arccos(-1,0) \quad \text{o} \quad \pi = 4 \cdot \arctan(1,0)$$

Longitud de una cuerda: Sea  $\alpha$  el ángulo descripto por una cuerda de longitud  $l$  en un círculo de radio  $r$ .

$$l = \sqrt{2r^2(1 - \cos(\alpha))}$$

Fórmula de Herón: Sea un triángulo con lados  $a, b, c$  y semiperímetro  $s = \frac{a+b+c}{2}$ . El área del triángulo es

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Teorema de Pick: Sean  $A$  el área de un polígono,  $I$  la cantidad de puntos de coordenadas enteras en su interior, y  $B$  la cantidad de puntos de coordenadas enteras en el borde.

$$A = I + \frac{B}{2} - 1$$

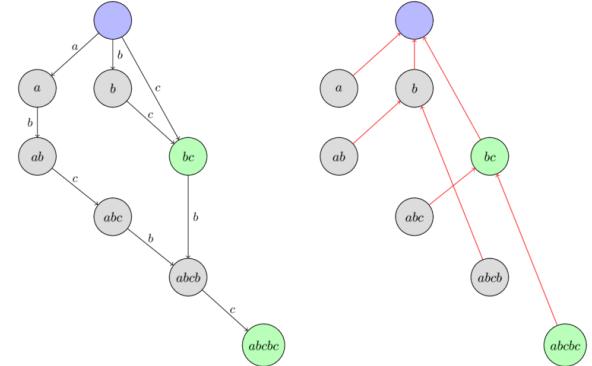


Figura 1: Suffix automaton de  $abcbc$ .

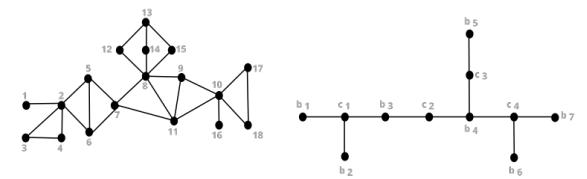


Figura 2: Ejemplo de block-cut tree