

DEFINICIONES

Matriz triangular superior: $a_{ij} = 0$ para $i > j$

Matriz triangular superior estricta: $a_{ij} = 0$ para $i \geq j$

Matriz triangular inferior: $a_{ij} = 0$ para $i < j$

Matriz triangular inferior estricta: $a_{ij} = 0$ para $i \leq j$

Matriz diagonal: triangular inferior y triangular superior

Matriz identidad: $I_{ij} = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

Matriz simetrica: $A^t = A$

Matriz antisimetrica: $A^t = -A$

Cofactores: $C_{ij} = (-1)^{i+j} |A(i|j)|$

Matriz adjunta: $(adj A)_{ij} = (-1)^{i+j} |A(j|i)| = C^t$

OPERACIONES

$$C = \alpha A \Rightarrow c_{ij} = \alpha a_{ij}$$

$$C = A + B \Rightarrow c_{ij} = a_{ij} + b_{ij}$$

$$C = AB \Rightarrow c_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$

$$(A^t)_{ij} = A_{ji}$$

$$tr A = \sum_{i=1}^n A_{ii}$$

$$dist(A, B) = \sqrt{tr[(B - A)(B - A)^t]}$$

$$|A| = \sum_{\sigma \in S_n} sg(\sigma) \prod_{i=1}^n A_{i\sigma(i)}$$

$$|A| = \sum_{j=1}^n (-1)^{i+j} A_{ij} |A(i|j)|$$

$$A^{-1} = \frac{1}{|A|} adj A$$

PROPIEDADES DE LA SUMA Y EL PRODUCTO

$$0A = 0_{m \times n}$$

$$\alpha 0_{m \times n} = 0_{m \times n}$$

$$A + (B + C) = (A + B) + C$$

$$\alpha(\beta A) = (\alpha\beta)A$$

$$A + 0 = 0 + A = A$$

$$\begin{aligned}
(\alpha + \beta)A &= \alpha A + \beta A \\
\alpha(A + B) &= \alpha A + \alpha B \\
A(BC) &= (AB)C \\
A(B + C) &= AB + AC \\
(A + B)C &= AC + BC \\
IA &= AI = A
\end{aligned}$$

PROPIEDADES DE LAS TRANSPOSICIONES

$$\begin{aligned}
(A^t)^t &= A \\
(\alpha A)^t &= \alpha A^t \\
(A + B)^t &= A^t + B^t \\
\text{Si } A \in \mathbb{F}^{m \times n} \wedge B \in \mathbb{F}^{n \times p} &\Rightarrow (AB)^t = B^t A^t
\end{aligned}$$

PROPIEDADES DE LA TRAZA

$$\begin{aligned}
tr(A + B) &= tr A + tr B \\
tr(\alpha A) &= \alpha tr A \\
tr(AB) &= tr(BA)
\end{aligned}$$

PROPIEDADES DEL DETERMINANTE

$$\begin{aligned}
|I| &= 1 \\
|A| &= |A^t| \\
2 \text{ filas/columnas iguales} &\Rightarrow |A| = 0 \\
|(A_1 \cdots \alpha A_k \cdots A_n)| &= \alpha |(A_1 \cdots A_k \cdots A_n)| \\
|\alpha A| &= \alpha^n |A| \\
|(A_1 \cdots A_{k-1} B_k + C_k A_{k+1} \cdots A_n)| &= |(A_1 \cdots A_{k-1} B_k A_{k+1} \cdots A_n)| + |(A_1 \cdots A_{k-1} C_k A_{k+1} \cdots A_n)| \\
|(A_1 \cdots A_i \cdots A_j \cdots A_n)| &= -|(A_1 \cdots A_j \cdots A_i \cdots A_n)| \\
|(A_1 \cdots A_n)| &= |(A_1 \cdots A_{j-1} [A_j + \alpha A_k] A_{j+1} \cdots A_k \cdots A_n)| \\
|triangular| &= A_{11} A_{22} \cdots A_{nn} \\
|AB| &= |A| |B|
\end{aligned}$$

PROPIEDADES DE LA INVERSA

$$\begin{aligned}
AA^{-1} &= A^{-1}A = I \\
|A^{-1}| &\neq 0 \\
Aadj A &= (adj A)A = |A|I
\end{aligned}$$

OTRAS PROPIEDADES

$$A = A_{sim} + A_{anti}$$