Network Analysis

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Setting Up the Environment

You will need the following packages installed: brainGraph, igraph, tidyverse, ggplot2, grid, gridExtra, knitr, kableExtra, hrbrthemes

Preparing Data

Load in the required data.

```
# Read in data
load("~/Desktop/School/GatesLab/NetworkAnalysis/FullMatrices.Rdata")

# Preview all data
# head(outReg)
```

Extract Submatrices for each Person

The R data outReg is a list of 40 lists; each list corresponds to an individual and contains a 52 by 52 regression matrix. We want to analyze the first 26 variables for each individual, and we will extract a 26 by 26 regression matrix for each person. The matrices are saved in two different lists. The first list subdf contains the original 26 by 26 regression matrix for each individual person. The second list subdf_pos contains a positive regression matrix, where every negative value is converted into zero.

```
byrow = FALSE,
    dimnames = list(feature_names, feature_names)
  subdf_pos[[x]] = pmax(subdf[[x]], 0)
}
```

Extract Directed Graphs for each Person

Here, we convert both the original and positive matrices for each individual into directed graphs. The original

```
and positive graphs are stored in two separate lists.
dir_graph = list()
dir_graph_pos = list()
for (x in 1:length(outReg)) {
  dir_graph[[x]] = graph.adjacency(subdf[[x]],
                                   mode = "directed",
                                   weighted = TRUE)
  dir_graph_pos[[x]] = graph.adjacency(subdf_pos[[x]],
                                       mode = "directed",
                                       weighted = TRUE)
}
dir_graph[[19]]
## IGRAPH c9afa2b DNW- 26 114 --
## + attr: name (v/c), weight (e/n)
## + edges from c9afa2b (vertex names):
## [1] energetic
                   ->enthusiastic energetic
                                               ->down
## [3] energetic
                    ->fatigue
                                   energetic
                                               ->tension
## [5] energetic
                    ->ruminate
                                   energetic
                                               ->avoid act
## [7] enthusiastic->energetic
                                   enthusiastic->positive
## [9] enthusiastic->tension
                                   enthusiastic->procrast
## [11] content
                    ->irritable
                                   content
                                               ->restless
## [13] content
                    ->worried
                                   content
                                               ->guilty
## [15] content
                    ->angry
                                   content
                                               ->down
## + ... omitted several edges
dir_graph_pos[[19]]
## IGRAPH 042b409 DNW- 26 73 --
## + attr: name (v/c), weight (e/n)
## + edges from 042b409 (vertex names):
## [1] energetic
                    ->enthusiastic enthusiastic->energetic
## [3] enthusiastic->positive
                                   content
                                               ->positive
## [5] irritable ->worried
                                   irritable
                                               ->angry
## [7] irritable
                    ->threatened
                                   irritable
                                               ->ruminate
## [9] restless
                    ->worried
                                   restless
                                               ->angry
## [11] restless
                    ->threatened
                                               ->avoid_people
                                   restless
## [13] worried
                    ->irritable
                                   worried
                                               ->restless
## [15] worried
                    ->guilty
                                   worried
                                               ->afraid
## + ... omitted several edges
```

Metrics for each Person

Now, using the directed graphs, we will calculate the following metrics for each person: Density; Overall Weight; Edge Weight; Global Efficiency; Degree of Each Variable; Strength of Each Variable; Betweenness Centrality; Community Detection.

Note, Global Efficiency, Betweenness Centrality, and Community Detection require the positive directed graphs rather than the original graph.

```
density = list()
overall_weight = list()
edge_weights = list()
global_efficiency = list()
variable_degrees = list()
variable_strengths = list()
betweenness = list()
cluster = list()
for (x in 1:length(outReg)) {
  density[[x]] = edge_density(dir_graph[[x]], loops = FALSE)
  overall_weight[[x]] = sum(strength(dir_graph[[x]]))
  edge_weights[[x]] = edge_attr(dir_graph_pos[[x]], "weight")
  global_efficiency[[x]] = efficiency(dir_graph_pos[[x]],
                                      type = c("global"),
                                      weights = edge_weights[[x]])
  variable_degrees[[x]] = degree(dir_graph[[x]])
  variable_strengths[[x]] = strength(dir_graph[[x]])
  betweenness[[x]] = estimate_betweenness(dir_graph_pos[[x]],
                                          cutoff = -1, weights = edge_weights[[x]])
  cluster[[x]] = cluster_walktrap(
   dir_graph_pos[[x]],
   weights = E(dir_graph_pos[[x]])$edge_weights,
    steps = 4
  )
}
print(paste0("Density of the first person's graph: ", round(density[[1]], 4)))
## [1] "Density of the first person's graph: 0.1077"
print(paste0(
  "Overall Weight of the first person's graph: ",
  round(overall_weight[[1]], 4)
))
## [1] "Overall Weight of the first person's graph: 10.6119"
print(paste0(
  "Global Efficiency of the first person's graph: ",
 round(global efficiency[[1]], 4)
## [1] "Global Efficiency of the first person's graph: 12.5183"
```

```
print("Degree of each variable in the first person's graph: ")
## [1] "Degree of each variable in the first person's graph: "
print(variable_degrees[[1]])
##
      energetic enthusiastic
                                   content
                                              irritable
                                                            restless
                                                                           worried
##
                                        3
##
         guilty
                      afraid
                                anhedonia
                                                  angry
                                                            hopeless
                                                                              down
##
                                        12
                                                                                 8
                                                                   11
##
       positive
                                   tension
                     fatigue
                                            concentrate
                                                            accepted
##
                                        7
##
       ruminate
                   avoid act
                                 reassure
                                               procrast
                                                               hours
                                                                         difficult
##
                                                                                 8
             13
                           3
                                         1
                                                      2
                                                                    2
##
      unsatisfy avoid_people
##
              1
print("Strengths of each variable in the first person's graph: ")
## [1] "Strengths of each variable in the first person's graph: "
print(variable_strengths[[1]])
##
      energetic enthusiastic
                                   content
                                              irritable
                                                            restless
                                                                           worried
     0.76229416
##
                  0.96384765
                               0.47966325
                                             0.83037658
                                                          0.57779129
                                                                        1.20858363
##
                      afraid
                                anhedonia
         guilty
                                                  angry
                                                            hopeless
                                                                              down
     0.02643344 -0.01065439
##
                               0.82426251
                                             0.99845363
                                                          0.89364307
                                                                        0.74160674
##
       positive
                     fatigue
                                  tension concentrate
                                                            accepted
                                                                        threatened
##
     0.95930381
                  0.00000000
                               0.76755289
                                             0.18317240
                                                          0.14779364
                                                                        0.16860533
##
                                                                         difficult
       ruminate
                   avoid_act
                                 reassure
                                               procrast
                                                               hours
##
     0.79906030
                  0.47042696
                                0.04843503 -0.06829517 -1.55552557 -0.56623424
##
      unsatisfy avoid_people
     0.20728066
                  0.75406944
print("Betweenness Centrality of each variable in the first person's graph: ")
## [1] "Betweenness Centrality of each variable in the first person's graph: "
print(betweenness[[1]])
##
      energetic enthusiastic
                                   content
                                              irritable
                                                            restless
                                                                           worried
##
                           5
                                                     40
##
                      afraid
                                                                              down
         guilty
                                anhedonia
                                                            hopeless
                                                  angry
##
                                                                                37
                          10
                                        49
                                                                   85
                                                            accepted
                                                                        threatened
##
       positive
                     fatigue
                                   tension
                                            concentrate
##
             46
                           0
                                         2
                                                      0
                                                                    0
##
       ruminate
                   avoid_act
                                  reassure
                                               procrast
                                                               hours
                                                                         difficult
##
            111
                                         0
                                                      0
                                                                    0
##
      unsatisfy avoid people
print("Clustering Walktrap of each variable in the first person's graph: ")
## [1] "Clustering Walktrap of each variable in the first person's graph: "
print(cluster[[1]])
## IGRAPH clustering walktrap, groups: 11, mod: 0.27
```

+ groups:

```
$11
##
##
     [1] "concentrate" "difficult"
##
     $`2`
##
##
     [1] "angry"
                         "down"
                                         "ruminate"
                                                         "avoid act"
                                                                        "avoid people"
##
##
     $`3`
##
     [1] "content" "positive" "accepted"
##
##
##
     + ... omitted several groups/vertices
```

Computing Metric Summary Statistics

Here, we calculate the mean, standard deviation, and range for Density, Overall Weight, Global Efficiency, Degree of Each Variable, Strength of Each Variable, and Betweenness Centrality. The summary statistics for Density, Overall Weight, and Global Efficiency are stored within a table for accessible viewing.

```
density_mean = mean(unlist(density))
density_sd = sd(unlist(density))
density_range = range(unlist(density))
overall_weight_mean = mean(unlist(overall_weight))
overall weight sd = sd(unlist(overall weight))
overall_weight_range = range(unlist(overall_weight))
global efficiency mean = mean(unlist(global efficiency))
global efficiency sd = sd(unlist(global efficiency))
global_efficiency_range = range(unlist(global_efficiency))
variable_degrees_mat = do.call(rbind, variable_degrees)
variable_degrees_mean = apply(variable_degrees_mat, 2, mean)
variable_degrees_sd = apply(variable_degrees_mat, 2, sd)
variable_degrees_range = apply(variable_degrees_mat, 2, range)
variable_strengths_mat = do.call(rbind, variable_strengths)
variable_strengths_mean = apply(variable_strengths_mat, 2, mean)
variable_strengths_sd = apply(variable_strengths_mat, 2, sd)
variable_strengths_range = apply(variable_strengths_mat, 2, range)
betweenness mat = do.call(rbind, betweenness)
betweenness_mean = apply(betweenness_mat, 2, mean)
betweenness sd = apply(betweenness mat, 2, sd)
betweenness range = apply(betweenness mat, 2, range)
# Store all summary stats into a DataFrame
# Variables as Columns
net_stat = data.frame(
  "Density" = c(density_mean, density_sd, density_range),
  "Overall Weight" = c(overall_weight_mean, overall_weight_sd, overall_weight_range),
  "Global Efficiency" = c(
    global_efficiency_mean,
   global_efficiency_sd,
```

	Density	Overvall Weight	Global Efficiency
Mean	0.1651923	10.90231	24.557177
Standard Deviation	0.0643016	10.72964	33.724137
Minimum	0.0461538	-32.41157	1.450913
Maximum	0.3230769	33.45451	187.580449

	Mean	Standard Deviation	Minimum	Maximum
Density	0.1651923	0.0643016	0.0461538	0.3230769
Overvall Weight	10.9023141	10.7296397	-32.4115689	33.4545141
Global Efficiency	24.5571769	33.7241375	1.4509135	187.5804487

```
global_efficiency_range
  )
)
row.names(net_stat) = c("Mean", "Standard Deviation", "Minimum", "Maximum")
kt = knitr::kable(
 x = net_stat,
 row.names = TRUE,
  col.names = c("Density", "Overvall Weight", "Global Efficiency")
kable_classic(kt, full_width = FALSE, html_font = "Cambria")
# Variables as Rows
net_stat_transposed = data.frame(
  "Mean" = c(density_mean, overall_weight_mean, global_efficiency_mean),
  "Standard Deviation" = c(density_sd, overall_weight_sd, global_efficiency_sd),
  "Minimum" = c(
    density_range[1],
   overall_weight_range[1],
    global_efficiency_range[1]
  ),
  "Maximum" = c(
    density_range[2],
    overall_weight_range[2],
    global_efficiency_range[2]
row.names(net_stat_transposed) = c("Density", "Overvall Weight", "Global Efficiency")
kt_transposed = knitr::kable(
 x = net_stat_transposed,
 row.names = TRUE,
 col.names = c("Mean", "Standard Deviation", "Minimum", "Maximum")
kable_classic(kt_transposed, full_width = FALSE, html_font = "Cambria")
```

Plots and Distribution

Distribution across each network-wide metric using data

```
# Combine network-wide metrics into a dataframe
network_df = data.frame(
  "Density" = unlist(density),
 "Overall Weight" = unlist(overall weight),
 "Global Efficiency" = unlist(global_efficiency)
# Violin Plots
density_plot = ggplot(network_df, aes(y = Density, x = "")) +
  geom_violin() +
  stat_summary(
   fun = mean,
    geom = "point",
   shape = 23,
   size = 2
  ) +
  xlab("Density") +
  ylab("")
efficiency_plot = ggplot(network_df, aes(y = Global.Efficiency, x = "")) +
  geom_violin() +
  stat summary(
   fun = mean,
    geom = "point",
   shape = 23,
   size = 2
  ) +
  xlab("Global Efficiency") +
  ylab("")
overall_weight_plot = ggplot(network_df, aes(y = Overall.Weight, x = "")) +
  geom_violin() +
  stat_summary(
   fun = mean,
    geom = "point",
   shape = 23,
   size = 2
  xlab("Overall Weight") +
  ylab("")
grid.arrange(
  density_plot,
  efficiency_plot,
 overall_weight_plot,
 ncol = 3,
  top = textGrob("Network-Wide Distributions")
)
```

Network-Wide Distributions

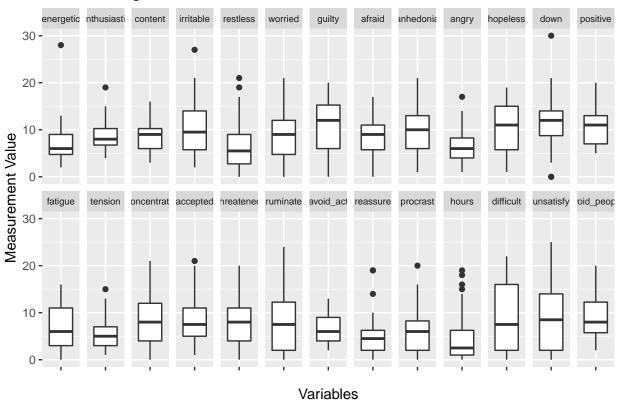


The above plot illustrates that the density, global efficiency, and overall weight measurements are set at different scales and show different distributions. Density shows a relatively symmetrical distribution, whereas Global Efficiency shows a right-skewed distribution and Overall Weight shows a left-skewed distribution.

Plot distributions across each variable-level metric

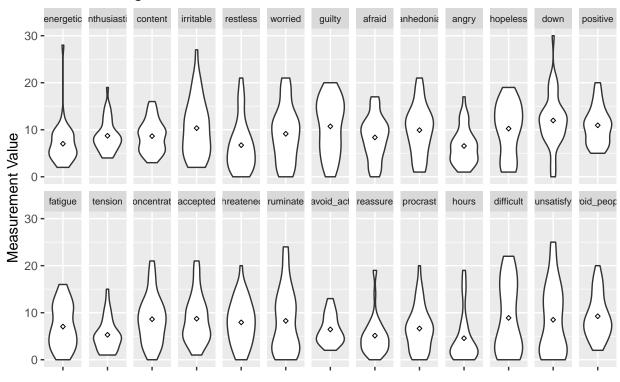
```
# Variable Degrees Boxplot
ggplot(stack(data.frame(variable_degrees_mat)), aes(x = "" , y = values)) +
  geom_boxplot() +
  facet_wrap(~ ind, ncol = 13) +
  ggtitle("Variable Degrees Distributions") +
  xlab("Variables") +
  ylab("Measurement Value") +
  theme(strip.text = element_text(size = 7))
```

Variable Degrees Distributions



```
# Variable Degrees Violin Plot
ggplot(stack(data.frame(variable_degrees_mat)), aes(x = "" , y = values)) +
    geom_violin() +
    facet_wrap(~ ind, ncol = 13) +
    ggtitle("Variable Degrees Distributions") +
    xlab("Variables") +
    ylab("Measurement Value") +
    theme(strip.text = element_text(size = 7)) +
    stat_summary(
    fun = mean,
        geom = "point",
        shape = 23,
        size = 1
    )
```

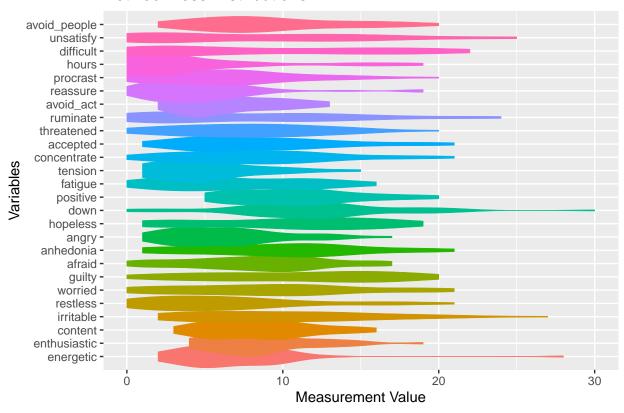
Variable Degrees Distributions



Variables

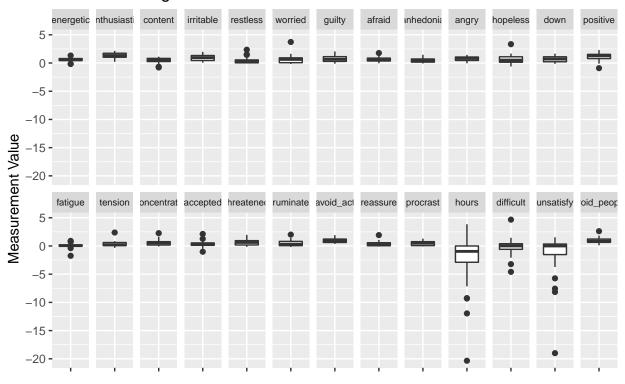
```
# Variable Degrees Combined Violin Plot
ggplot(stack(data.frame(variable_degrees_mat)), aes(
    x = ind ,
    y = values,
    fill = ind,
    color = ind
)) +
    geom_violin(width = 2, size = .1) +
    coord_flip() +
    theme(legend.position = "none") +
    ggtitle("Betweenness Distributions") +
    xlab("Variables") +
    ylab("Measurement Value")
```

Warning: position_dodge requires non-overlapping x intervals



```
# Variable Strengths Boxplot
ggplot(stack(data.frame(variable_strengths_mat)), aes(x = "" , y = values)) +
  geom_boxplot() +
  facet_wrap(~ ind, ncol = 13) +
  ggtitle("Variable Strengths Distributions") +
  xlab("Variables") +
  ylab("Measurement Value") +
  theme(strip.text = element_text(size = 7))
```

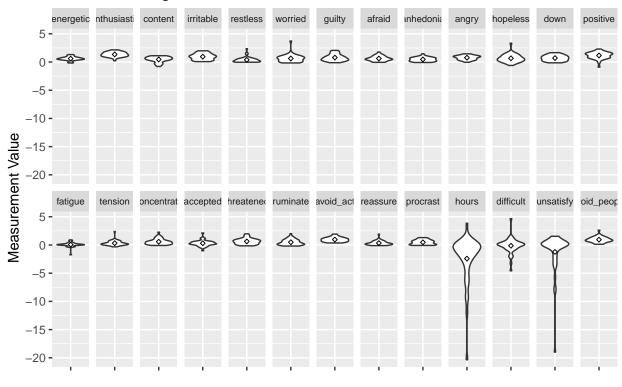
Variable Strengths Distributions



Variables

```
# Variable Strengths Violin Plot
ggplot(stack(data.frame(variable_strengths_mat)), aes(x = "" , y = values)) +
    geom_violin() +
    facet_wrap(~ ind, ncol = 13) +
    ggtitle("Variable Strengths Distributions") +
    xlab("Variables") +
    ylab("Measurement Value") +
    # ylim(-5,5) +
    theme(strip.text = element_text(size = 7)) +
    stat_summary(
    fun = mean,
    geom = "point",
    shape = 23,
    size = 1
)
```

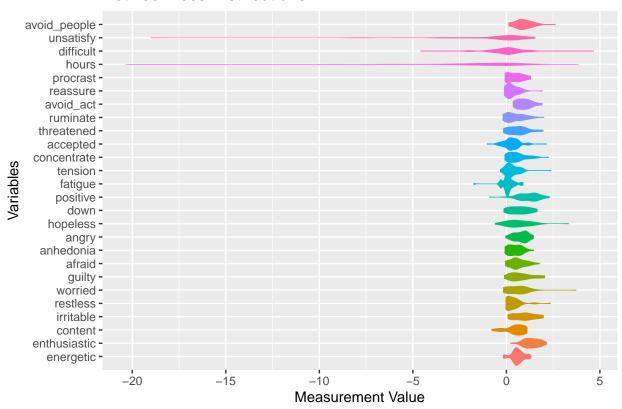
Variable Strengths Distributions



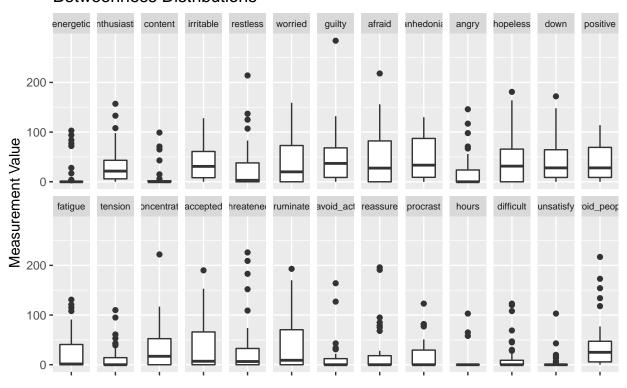
Variables

```
# Variable Strengths Combined Violin Plot
ggplot(stack(data.frame(variable_strengths_mat)), aes(
    x = ind ,
    y = values,
    fill = ind,
    color = ind
)) +
    geom_violin(width = 2, size = .1) +
    coord_flip() +
    theme(legend.position = "none") +
    ggtitle("Betweenness Distributions") +
    xlab("Variables") +
    ylab("Measurement Value")
```

Warning: position_dodge requires non-overlapping x intervals

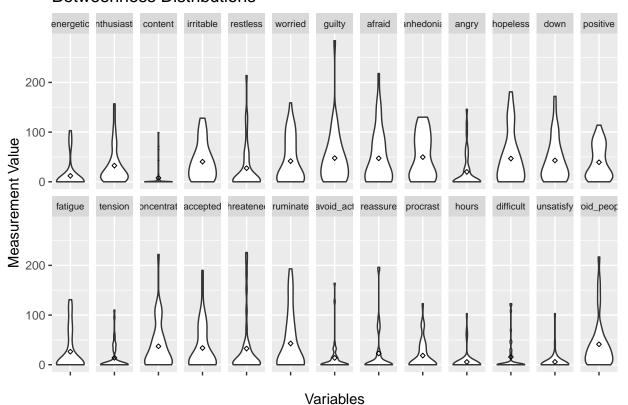


```
# Betweenness Boxplot
ggplot(stack(data.frame(betweenness_mat)), aes(x = "" , y = values)) +
  geom_boxplot() +
  facet_wrap(~ ind, ncol = 13) +
  ggtitle("Betweenness Distributions") +
  xlab("Variables") +
  ylab("Measurement Value") +
  theme(strip.text = element_text(size = 7))
```



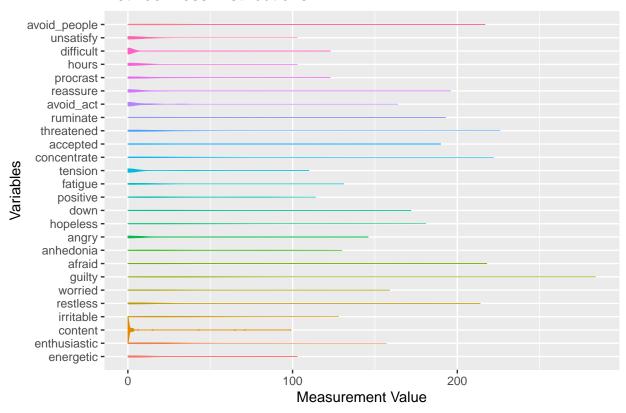
Variables

```
# Betweenness Violin Plot
ggplot(stack(data.frame(betweenness_mat)), aes(x = "" , y = values)) +
geom_violin() +
facet_wrap(~ ind, ncol = 13) +
ggtitle("Betweenness Distributions") +
xlab("Variables") +
ylab("Measurement Value") +
theme(strip.text = element_text(size = 7)) +
stat_summary(
fun = mean,
geom = "point",
shape = 23,
size = 1
)
```



```
# Betweenness Combined Violin Plot
ggplot(stack(data.frame(betweenness_mat)), aes(
    x = ind ,
    y = values,
    fill = ind,
    color = ind
)) +
    geom_violin(width = 2, size = .1) +
    coord_flip() +
    theme(legend.position = "none") +
    ggtitle("Betweenness Distributions") +
    xlab("Variables") +
    ylab("Measurement Value")
```

Warning: position_dodge requires non-overlapping x intervals



The above violin plots help to visualize the skewness in some of the variable distributions. Specifically, many of the variables show a right-skewed distribution for betweenness. However, these distributions are not visibly apparent when we graph only the average measurement value, such as in the comparison plot below.

Plot the average measurement value for variable-level metrics

```
var level df = data.frame(variable = feature names,
                          variable_degrees_mean,
                          variable strengths mean,
                          betweenness_mean)
colnames(var_level_df) =
  c("Variable",
    "Variable Degrees",
    "Variable Strengths",
    "Betweenness")
ggplot(data.frame(
  stack(var_level_df),
  variable = c(feature_names, feature_names, feature_names)
),
aes(x = values, y = variable)) +
  geom_point() +
  geom_line(group = 1, orientation = "y") +
  facet_wrap(~ ind, scales = "free") +
  ggtitle("Average Comparisons on Variable Level Metrics") +
 ylab("Variables") +
  xlab("Average Measurement Value")
```

Warning in stack.data.frame(var_level_df): non-vector columns will be ignored

