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Global Magnetic Attitude Control of Inertially Pointing Spacecraft

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I. Introduction

ATTITUDE control for rigid spacecraft has been extensively studied, and a wide body of results is available in the literature for both the regulation and the tracking problem.[1–3]

The problem of spacecraft equipped with magnetic actuators, however, cannot be faced directly with the preceding results, but raises specific issues.

The use of magnetic coils for control purposes has been the subject of extensive study because the early years of satellite missions, both for attitude control and for momentum management on spacecraft controlled with reaction wheels. Until recent years, however, only approximate solutions to the problem of dealing with such time-varying actuators were available.[4,5] Concerning the problems of analysis and design of magnetic control laws in the linear case, that is, control laws for nominal operation of a satellite near its equilibrium attitude, nominal and robust stability and performance have been studied, using either tools from periodic control theory exploiting the (quasi) periodic behavior of the system near an equilibrium.[6–9]

On the other hand, limited attention has been dedicated to global formulations of the magnetic attitude control problem. The attitude regulation problem for Earth-pointing spacecraft has been addressed exploiting periodicity assumptions on the system,[5,10,11] hence resorting to standard passivity arguments to prove local asymptotic stabilisability of open-loop stable equilibria. Similar arguments have been used to analyze a state feedback control law for the particular case of an inertially spherical spacecraft.[12]

The aim of this Note is to provide an almost global stabilization result for the case of full state feedback (see also Ref. 13 for the case of attitude feedback only). [Given a system $\dot{x} = f(x)$, we say that an equilibrium x_0 is almost globally asymptotically stable if it is locally asymptotically stable, all of the trajectories of the system are bounded, and the set of initial conditions giving rise to trajectories that do not converge to x_0 has zero Lebesgue measure.] In particular, with respect to previous work,[14–16] the results presented in this Note can deal with a generic magnetically actuated satellite, do not rely on restrictive assumptions on the controller parameters, and guarantee that there are no trajectories of the closed-loop system along which average controllability can be lost. Note that the results presented herein do not rely on the (frequently adopted) periodicity assumption for the geomagnetic field along the considered orbit.

In addition, the proposed framework for closed-loop stability analysis of magnetically controlled spacecraft can be also exploited to predict the effect of actuator faults on the behavior of the controlled satellite.

II. Spacecraft Model

The attitude dynamics can be expressed by the well-known Euler's equations[17]

$$J_0\dot{\omega}_b = S(\omega_b)J_0\omega_b + T_{\text{coils}} + T_{\text{dist}} \tag{1}$$

where $\omega_b = [\omega_{bx} \;\; \omega_{by} \;\; \omega_{bz}]^T \in \mathbb{R}^3$ is the vector of spacecraft angular rates, expressed in body frame, $J_0 \in \mathbb{R}^{3\times 3}$ is the inertia matrix, $S(\omega_b)$ is given by

$$S(\omega_b) = \begin{bmatrix} 0 & \omega_{bz} & -\omega_{by} \\ -\omega_{bz} & 0 & \omega_{bx} \\ \omega_{by} & -\omega_{bx} & 0 \end{bmatrix} \tag{2}$$

$T_{\text{coils}} \in \mathbb{R}^3$ is the vector of external torques induced by the magnetic coils, and $T_{\text{dist}} \in \mathbb{R}^3$ is the vector of external disturbance torques.

In turn, the attitude kinematics can be described by means of the four Euler parameters (or quaternions), which lead to the following representation:

$$\dot{\boldsymbol{q}} = W(\boldsymbol{q})\omega_b \tag{3}$$

where

$$\boldsymbol{q} = [q_1 \;\; q_2 \;\; q_3 \;\; q_4]^T = [\boldsymbol{q}^T \;\; q_4]^T$$

$$W(\boldsymbol{q}) = \frac{1}{2}\begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{4}$$

Note that the attitude of inertially pointing spacecraft is usually referred to the Earth centered inertial reference frame.

The magnetic attitude control torques are generated by a set of three magnetic coils, aligned with the spacecraft principal inertia axes, which generate torques according to the law

$$T_{\text{coils}} = m_{\text{coils}} \times \tilde{b}(t) = S[\tilde{b}(t)]m_{\text{coils}} \tag{5}$$

where $\times$ denotes the vector cross product, $m_{\text{coils}} \in \mathbb{R}^3$ is the vector of magnetic dipoles for the three coils, and $\tilde{b}(t) \in \mathbb{R}^3$ is the vector formed with the components of the Earth's magnetic field in the body frame of reference. Note that the vector $\tilde{b}(t)$ can be expressed

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in terms of the attitude matrix $A(\boldsymbol{q})$ (Ref. 17) and of the magnetic field vector expressed in the inertial coordinates, namely, $\tilde{b}_0(t)$, as

$$\tilde{b}(t) = A(\boldsymbol{q})\tilde{b}_0(t) \tag{6}$$

and that the orthogonality of $A(\boldsymbol{q})$ implies $\|\tilde{b}(t)\| = \|\tilde{b}_0(t)\|$. Because $S[\tilde{b}(t)]$ is structurally singular, as mentioned in the Introduction, magnetic actuators do not provide full controllability of the system at each time instant. In particular, it is easy to see that rank $\{S[\tilde{b}(t)]\} = 2$ [since $\|\tilde{b}_0(t)\| \neq 0$ along all orbits of practical interest for magnetic control] and that the kernel of $S[\tilde{b}(t)]$ is given by the vector $\tilde{b}(t)$ itself, that is, at each time instant it is not possible to apply a control torque along the direction of $\tilde{b}(t)$.

If a preliminary feedback of the form

$$m_{\text{coils}} = \left[1\big/\|\tilde{b}_0(t)\|^2\right] S^T[\tilde{b}(t)]v \tag{7}$$

is applied to the system, where $u \in \mathbb{R}^3$ is a new control vector, the overall dynamics can be written as

$$\dot{\boldsymbol{q}} = W(\boldsymbol{q})\omega_b, \qquad J_0\dot{\omega}_b = S(\omega)J_0\omega + \Gamma(t)v \tag{8}$$

where $\Gamma(t) = S[b(t)]S^T[b(t)] \geq 0$ and $b(t) = [1/\|\tilde{b}_0(t)\|]\tilde{b}(t) = [1/\|\tilde{b}(t)\|]\tilde{b}(t)$. Similarly, let $\Gamma_0(t) = S[b_0(t)]S^T[b_0(t)] \geq 0$ and $b_0(t) = [1/\|\tilde{b}_0(t)\|]\tilde{b}_0(t)$. Note, also, that $\Gamma(t)$ can be written as $\Gamma(t) = \mathcal{I}_3 - b(t)b(t)^T$, where $\mathcal{I}_3$ is the 3×3 identity matrix.

Interestingly enough, the formulation of the magnetic attitude stabilization problem turns out to be remarkably simple if the attitude dynamics is represented with respect to the inertial frame rather than with respect to the body frame. It is easy to verify that with this choice the dynamics of the magnetically controlled spacecraft can be written as

$$\dot{\boldsymbol{q}} = \tilde{W}(\boldsymbol{q})\omega, \qquad \dot{\widehat{J\omega}} = A(\boldsymbol{q})^T\Gamma(t)v \tag{9}$$

where $\omega \in \mathbb{R}^3$ is the vector of spacecraft angular rates, expressed in the inertial frame, $J = A(\boldsymbol{q})^T J_0 A(\boldsymbol{q})$ and $\tilde{W}(\boldsymbol{q})$ is given by[18]

$$\widetilde{W}(\boldsymbol{q}) = \frac{1}{2}\begin{bmatrix} q_4 & q_3 & -q_2 \\ -q_3 & q_4 & q_1 \\ q_2 & -q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{10}$$

Let now $v = A(\boldsymbol{q})u$ and note that $\Gamma(t) = A(\boldsymbol{q})\Gamma_0(t)A(\boldsymbol{q})^T$. Therefore, Eq. (9) can be written as

$$\dot{\boldsymbol{q}} = \tilde{W}(\boldsymbol{q})\omega, \qquad \dot{\widehat{J\omega}} = \Gamma_0(t)u \tag{11}$$

Assumption 1: The considered orbit for the spacecraft satisfies the condition

$$\bar{\Gamma}_0 = \lim_{T\to\infty}\frac{1}{T}\int_0^T \Gamma_0(t)\,\mathrm{d}t = \lim_{T\to\infty}\frac{1}{T}\int_0^T S[b_0(t)]S^T[b_0(t)]\,\mathrm{d}t > 0 \tag{12}$$

Assumption 1 is satisfied for most orbits of practical interest for low-Earth-orbit spacecraft.

III. State Feedback Stabilization

In this section a general stabilization result for a spacecraft with magnetic actuators is given in the case of full state feedback (attitude and rate). Without loss of generality in the following, we assume that the equilibrium to be stabilized is given by $(\bar{\boldsymbol{q}}, 0)$, where $\bar{\boldsymbol{q}} = [0\ \ 0\ \ 0\ \ 1]^T$.

Proposition 1: Consider the magnetically actuated spacecraft described by Eq. (11) and the control law

$$u = -J^{-1}\left(\varepsilon^2 k_p \boldsymbol{q} + \varepsilon k_v \omega\right) \tag{13}$$

with $k_p > 0$ and $k_v > 0$. Then, under Assumption 1, there exists $\varepsilon^\star > 0$ such that for any $0 < \varepsilon < \varepsilon^*$ the control law (13) ensures that $(\bar{\boldsymbol{q}}, 0)$ is a locally exponentially stable equilibrium for the closed-loop system (11–13). Moreover, all trajectories of Eqs. (11–13) are such that $\boldsymbol{q} \to 0$ and $\omega \to 0$.

Proof: Introduce the coordinates transformation

$$\boldsymbol{z}_1 = \boldsymbol{q}, \qquad z_2 = \omega/\varepsilon \tag{14}$$

(so that $\boldsymbol{z}_1 = \boldsymbol{q}$ and $z_{14} = q_4$) in which the system (11) is described by the equations

$$\dot{\boldsymbol{z}}_1 = \varepsilon\tilde{W}(\boldsymbol{z}_1)z_2, \qquad \dot{\widehat{Jz_2}} = \varepsilon\Gamma_0(t)J^{-1}(-k_p\boldsymbol{z}_1 - k_v z_2) \tag{15}$$

System (15) satisfies all of the hypotheses for the applicability of the generalized averaging theory,[19] which yields the averaged system

$$\dot{\bar{z}}_1 = \varepsilon\tilde{W}(z_1)z_2, \qquad \dot{\widehat{Jz_2}} = \varepsilon\bar{\Gamma}_0 J^{-1}(-k_p z_1 - k_v z_2) \tag{16}$$

[In particular, it is easy to verify that the Jacobian of the difference between the right-hand sides of Eqs. (15) and (16) has zero average.] As a result, there exists $\varepsilon^\star > 0$ such that for any $0 < \varepsilon < \varepsilon^\star$ the trajectories of system (16) are close to the trajectories of system (15).

Consider now the Lyapunov function[1,2]

$$V_1(z_1, z_2) = k_p\left[z_1^T z_1 + (z_{14} - 1)^2\right] + \tfrac{1}{2}(Jz_2)^T\bar{\Gamma}_0^{-1}(Jz_2) \tag{17}$$

Its time derivative

$$\dot{V}_1 = -\varepsilon k_v z_2^T z_2 \tag{18}$$

is negative semidefinite; therefore, $z_2 \to 0$ and, applying La Salle invariance principle, $z_1 \to 0$.

Finally, consider the linear approximation of system (16) around the equilibrium $(\bar{\boldsymbol{q}}, 0)$, which is given by

$$\dot{\boldsymbol{z}}_1 = \tfrac{1}{2}\varepsilon z_2, \qquad \dot{\widehat{Jz_2}} = -\varepsilon\bar{\Gamma}_0 J^{-1}(k_p\boldsymbol{z}_1 + k_v z_2) \tag{19}$$

It is easy to verify that

$$V_L(z_1, z_2) = k_p\boldsymbol{z}_1^T\boldsymbol{z}_1 + \tfrac{1}{2}(Jz_2)^T\bar{\Gamma}_0^{-1}(Jz_2) \tag{20}$$

is a Lyapunov function for the linear system (19), and so the convergence of the trajectories of the closed loop system is locally exponential. □

Finally, note that the actual control variable in body frame is given by $v = A(\boldsymbol{q})u$ and because $J = A(\boldsymbol{q})^T J_0 A(\boldsymbol{q})$, $A(q)\boldsymbol{q} = \boldsymbol{q}$, and $\omega_b = A(\boldsymbol{q})\omega$, one has that the control law (13) can be easily implemented as

$$v = -J_0^{-1}\left(\varepsilon^2 k_p\boldsymbol{q} + \varepsilon k_v\omega_b\right) \tag{21}$$

Remark 1: It is well known[2] that whenever three independent torques are available the state feedback problem can be solved via a proportional-derivative control law and that almost global stability of the closed-loop system can be guaranteed for any choice of $k_p > 0$ and $k_v > 0$. This is not the case for magnetic attitude control, as the proportional and derivative actions must meet the scaling condition defined by ε in order to guarantee closed-loop stability. In this respect, Proposition 1 provides a very useful guideline for the design of magnetic controllers in practical cases, as it combines the simplicity of a state feedback control law[2] with an explicit stability condition. In particular, the scaling factor ε, which is introduced in the control law, implies that the control action must be sufficiently slow with respect to the natural variability of the geomagnetic field along the spacecraft orbit in order to be able to prove closed-loop stability.

IV. Actuator Saturation

The state feedback magnetic control law proposed in Sec. III can be readily modified to deal with saturation of the magnetic coils, as expressed in the following statement.

Corollary 1: Consider the system (11) and the state feedback control law

$$u = -J^{-1}\left\{\varepsilon^2 k_p \boldsymbol{q} + \varepsilon\beta \operatorname{sat}[k_v(\omega/\beta)]\right\} \tag{22}$$

with $\beta > 0$, $k_p > 0$, and $k_v > 0$. [By sat $(\cdot)$ we indicate a continuous saturating function limited between -1 and 1.] Then for any $\rho > 0$ there exist $\varepsilon^\star > 0$ such that for any $0 < \varepsilon < \varepsilon^\star$ the control law (22) ensures that $(\bar{\boldsymbol{q}}, 0)$ is a locally exponentially stable equilibrium for the closed-loop system (11–22), all trajectories of (11–22) are such that $\boldsymbol{q} \to 0$, $\omega \to 0$, and

$$|u_i| \leq \rho \tag{23}$$

Proof: The proof of the first two statements is similar to the proof of Proposition 1. To prove the bound (23), note that

$$|u_i| \leq [1/\sigma_{\min}(J_0)]\left(\varepsilon^2 k_p + \varepsilon\beta\right) \tag{24}$$

and this can be made arbitrarily small by a proper selection of the design parameters. □

Remark 2: The bound (23) on the signals u implies the bound

$$|m_{\text{coils}_i}| \leq \|b_0\|\, \rho \tag{25}$$

on the actual control inputs m_{coils}.

V. Actuator Faults

The proposed approach to the stability analysis of magnetically actuated spacecraft lends itself also to the treatment of the effect of actuator faults on the closed-loop system dynamics, corresponding to situations in which one or more of the magnetic torquers becomes unavailable. For, consider matrix $S[\tilde{b}(t)]$ in Eq. (5), and let

$$S(\tilde{b}(t)) = [S_1 \;\; S_2 \;\; S_3] \tag{26}$$

where S_i, $i = 1, \ldots, 3$ are the columns of $S[\tilde{b}(t)]$. Then, assuming that only two of the three coils are actually available as actuators, we can rewrite the overall dynamics of the magnetically controlled satellite as

$$\dot{\boldsymbol{q}} = \tilde{W}(\boldsymbol{q})\omega, \qquad \hat{J}\dot{\omega} = \Gamma_{0ij}(t)u \tag{27}$$

where

$$\Gamma_{0ij}(t) = S_{ij}[\tilde{b}_0(t)]S_{ij}[\tilde{b}_0(t)]^T \tag{28}$$

$$S_{ij}[\tilde{b}(t)] = [S_i \;\; S_j] \tag{29}$$

for $i, j = 1, \ldots, 3$; $i \neq j$. The approach to closed-loop stability analysis developed in the preceding sections for the cases of full state feedback and output feedback remains applicable provided that also in the case of one faulty actuator one has that

$$\begin{aligned}\bar{\Gamma}_{0ij} &= \lim_{T\to\infty} \frac{1}{T}\int_0^T \Gamma_{0ij}(t)\,\mathrm{d}t \\ &= \lim_{T\to\infty} \frac{1}{T}\int_0^T S_{ij}[b_0(t)]S_{ij}^T[b_0(t)]\,\mathrm{d}t > 0\end{aligned} \tag{30}$$

that is, if the averaged system associated with Eq. (27) is completely controllable. Clearly, condition (30) might or might not hold depending on the considered combination between orbit characteristics, desired nominal equilibrium attitude, and considered fault. What matters is that the effect of actuator faults can be predicted a priori by checking the condition number of Eq. (30) and possibly by developing suitable tunings for the state and output feedback control laws in order to cope in a more efficient way with the reduced control authority.

Remark 3: It is easy to verify that the case of a spacecraft equipped with only one magnetic coil {i.e., the case of $\Gamma_{0ii}(t) = S_i[\tilde{b}_0(t)]S_i[\tilde{b}_0(t)]^T$, $i = 1, \ldots, 3$} corresponds to a structurally singular average gain and is therefore equivalent, in the average sense, to the case of an underactuated spacecraft.

VI. Conclusions

The problem of inertial attitude regulation for a small spacecraft by using only magnetic coils as actuators has been analyzed, and a global solution has been proposed to the problems of static attitude and rate feedback. The proposed approach can be also used to deal with Earth-pointing magnetic attitude control problems.

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