

# TRAJECTORY OPTIMIZATION FOR MAGNETORQUER-BASED UNDERACTUATED CONTROL OF SMALL SATELLITES

Andrew Gatherer\*, Zac Manchester†

Due to the ever-increasing scope of small satellite missions, there is now significant demand for precise attitude determination and control capabilities onboard CubeSats. Interactions between magnetic torque coils and the Earth's magnetic field have been used for decades onboard satellites to offload momentum from reaction wheels and control-moment gyroscopes. However, magnetorquers are inherently underactuated, and mechanical actuators like reaction wheels are often prohibitively expensive in terms of mass, volume, power, and cost for CubeSat missions. This paper presents a magnetorquer-only attitude control technique that utilizes trajectory optimization to circumvent the under-actuated nature of satellite magnetic field interactions. Given a known orbit and desired attitude state, the method utilizes a simplified dynamics model and a fast constrained trajectory optimization solver based on differential dynamic programming to arrive at a nominal magnetic moment profile that respects the spacecraft's actuator limitations and desired actuation efficiency. This nominal maneuver is then tracked online using a time-varying linear-quadratic regulator (LQR). Using realistic parameters for a 1U CubeSat and 3U CubeSat in low-Earth orbit, this scheme is able to perform an arbitrary 90° reorientation within a few minutes at regardless of satellite location in a 400 km altitude circular orbit. To demonstrate the effectiveness and robustness of the proposed control technique, we present the results of several high-fidelity closed-loop simulations using a simplified IGRF magnetic field model, NRLMSISE-00 atmosphere model, and Earth oblateness effects. The computational complexity of the method and future implementation in a CubeSat flight computer are also discussed.

## INTRODUCTION

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\*Aeronautical and Astronautical Engineering, Stanford University, 496 Lomita Mall, Stanford, CA 94305.

†Aeronautical and Astronautical Engineering, Stanford University, 496 Lomita Mall, Stanford, CA 94305.

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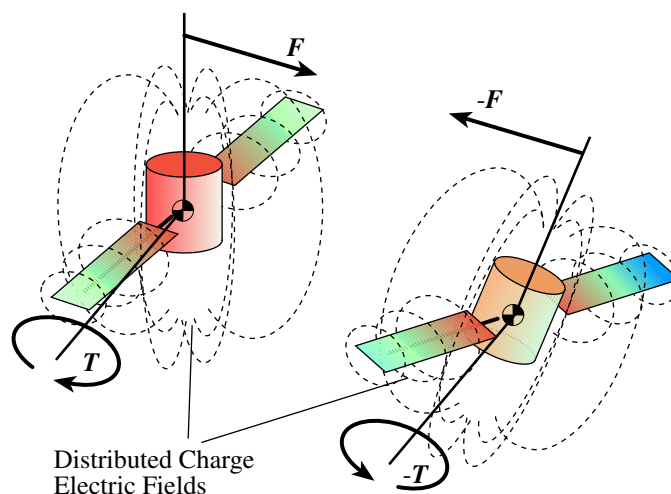
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$$a = b^2 \tag{1}$$

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A section on mathematical notation is provided in the sequel.

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**Table 1. A Caption Goes Here**

Animal	Description	Price (\$)
Gnat	per gram	13.65
	each	0.01
Gnu	stuffed	92.50
Emu	stuffed	33.33
Armadillo	frozen	8.99

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## ACKNOWLEDGMENT

Any acknowledgments by the author may appear here. The acknowledgments section is optional.

## NOTATION

- $a$  a real number
- $b$  the square root of  $a$

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