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Satellite Attitude Control Using only Magnetorquers

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Abstract: This study presents a method to control the attitude of spacecraft using only magnetorquers without the gravity gradient boom. The main challenge is that the control torque can only be generated perpendicular to the geomagnetic field. If the dynamic model is expressed in the geomagnetic frame, the system can be divided into two parts: the outer loop and the inner loop. The stability of the outer loop controller is proved by LaSalle's and Floquet's theorems. The inner loop has devastating disturbance torque. Considering the disturbance torque, the sliding mode controller is designed in the inner loop. The results of the simulation of this method are presented herein.

I. INTRODUCTION

Since magnetorquers are relatively reliable, lightweight, and energy efficient, they are found to be attractive for small, inexpensive satellites. Magnetorquers are often used with a gravity gradient boom to control spacecraft's attitude[1][2]. The use of a gravity gradient boom provides a simple way to stabilize a spacecraft in an Earth pointing mode. However, three problems are caused by the gravity gradient boom.

First, a gravity gradient system is bi-stable. The acquisition procedure must be carefully performed to avoid an up-side-down stabilization. The maneuvers for capture of the gravity gradient require extensive ground support.

Second, the boom structure is affected by thermal gradients which result in the boom's bending. The bending alters the attitude's equilibrium point. Consequently, the gravity gradient generates a disturbance torque and reduces pointing precision.

Third, an extendable gravity gradient boom complicates mechanical structure and adds to the weight of the actuator.

It is significant to search for new ways to control a satellite's attitude using only megnetorquers without a gravity gradient boom. However, the attempts at using only magnetorquers in all three-axis stability have been unsuccessful, because the control torque can only be generated perpendicular to the geomagnetic field vector, which results in the system being nonlinear and varying with time.

This study presents a method to control small satellites using only magnetorquers. If the spacecraft is isoinertial, the dynamic equations are divided into two parts in the geomagnetic frame: the outer loop and the inner loop. The outer loop is regarded as a regulation system, which is controlled by the virtual control input. Because the outer loop is a nonlinear and periodical system, its stability is supported by LaSalle's and Floquet's theorems [3][9]. The inner loop controller is designed as a tracking system and produces the signal to track the virtual control. Considering disturbance torque, a sliding mode controller is designed in the inner loop [4][5].

The result of the study shows that three-axis control can be achieved with magnetorquers as the sole actuators. The simulation of a numerical example is presented herein to demonstrate the validity of this method. The control system performs well during both the detumbling phase and the attitude acquisition phase.

II. SPACECRAFT DYNAMICS AND KINEMATICS

1. Definition of Geomagnetic Frame

The coordinate systems used in spacecraft attitude dynamics usually are an Earth center inertial frame (**Fi**), an orbit frame (**Fo**), and a body frame (**Fb**). Their definitions can be found in the reference[6]. Because the control torque generated by magnetorquers depends on the direction of the geomagnetic field, geomagnetic frame (**Fg**) is introduced in this study.

Definition: Suppose the azimuth and elevation of the geomagnetic field (**B**) are α and β respectively in **Fo** (Fig. 1). **Fg** is attained by **Fo**'s principal rotation w.r.t. Zo and Yo axis, about α and β respectively.

$$\mathbf{F}_g = \mathbf{C}_{go}\mathbf{F}_o, \tag{1}$$

where $\mathbf{C}_{go}$ is the **Fg**'s rotation matrix w.r.t. **Fo**

$$\mathbf{C}_{go} = \begin{bmatrix} \cos\beta\cos\alpha & \cos\beta\sin\alpha & -\sin\beta \\ -\sin\alpha & \cos\alpha & 0 \\ \sin\beta\cos\alpha & \sin\beta\sin\alpha & \cos\beta \end{bmatrix}. \tag{2}$$

Obviously, magnetic field is $\mathbf{B} = [b \quad 0 \quad 0]^T$ in **Fg**, and $b = \|\mathbf{B}\|$ is the magnitude of **B**.

Since **B** changes periodically on orbit. α, β, and b are periodical functions. Suppose $\omega_{go} = [a_1 \quad a_2 \quad a_3]^T$ is the **Fg**'s angular velocity w.r.t. **Fo**, and is also a periodical function. For a given orbit, α, β, b and ω_{go} can be calculated on the ground before hand.

2. Rotational Dynamics and Kinematics[6][7]

Provided **Fb** is chosen to be a principal-axis frame, the moment of inertia for the spacecraft is $\mathbf{J} = diag\{I_x \quad I_y \quad I_z\}$. Furthermore, if $I_x = I_y = I_z = \rho$ (the isoinertial case), the Earth pointing satellite's dynamics and kinematics model in **Fb** is

$$\dot{\mathbf{q}} = \frac{1}{2}\mathbf{T}(\mathbf{Q})\omega, \tag{3}$$

$$\dot{q}_4 = -\frac{1}{2}\omega^T\mathbf{q}, \tag{4}$$

$$\dot{\omega} = \omega^{\times}\mathbf{C}_{bo}\omega_{oi} + \frac{1}{\rho}\mathbf{N}_c, \tag{5}$$

where $\mathbf{Q}=[\mathbf{q}^T \quad q_4]^T$ is the quaternions, and $\mathbf{q}=[q_1 \quad q_2 \quad q_3]^T$ is its vector part, which satisfies the restriction $\mathbf{q}^T\mathbf{q}+q_4^2=1$. The matrix $\mathbf{T}$ is defined as $\mathbf{T}(\mathbf{Q})=q_4\mathbf{I}_{3\times3}+\mathbf{q}^\times$. $\omega_{oi}=[0 \quad -\omega_0 \quad 0]^T$ is the orbital velocity, which can be regarded as a constant for a small eccentricity orbit. $\mathbf{N}_c$ is the control torque. $\mathbf{C}_{bo}$ is the **Fb**'s rotation matrix w.r.t. **Fo**;

$$\mathbf{C}_{bo}=\left(q_4^2-\mathbf{q}^T\mathbf{q}\right)\mathbf{I}_{3\times3}+2\mathbf{q}\mathbf{q}^T-2q_4\mathbf{q}^\times . \tag{6}$$

Note: To avoid the singularity in $\mathbf{T}^{-1}$ that occurs at $q_4=0$, the workspace is restricted as follows[6]:

$$\|\mathbf{q}\|\le\gamma<1,\ q_4\ge\sqrt{1-\gamma^2}\ .$$

III. DYNAMICS EQUATION IN GEOMAGNETIC FRAME

The magnetic control torque $\mathbf{N}_c=-\mathbf{B}^\times\mathbf{M}$ is always in the control plane, which is perpendicular to the geomagnetic field vector **B** (Fig. 2). **M** is the magnetic dipole moment generated by magnetorquers. Since only the components of **M,** which are in the control plane, are effective, **M** can be expressed in **Fg** as

$$\mathbf{M}=[0 \quad m_2 \quad m_3]^T . \tag{7}$$

$\mathbf{N}_c$ is described in **Fg** as

$$\mathbf{C}_{go}\mathbf{C}_{bo}^T\mathbf{N}_c=[0 \quad m_3 b \quad -m_2 b]^T . \tag{8}$$

Let $\widetilde{\omega}=\mathbf{C}_{go}\mathbf{C}_{bo}^T\omega$, $\widetilde{\mathbf{q}}=\mathbf{C}_{go}\mathbf{C}_{bo}^T\mathbf{q}$, $u_1=m_3 b/\rho$, and $u_2=-m_2 b/\rho$. Considering that $\dot{\mathbf{C}}_{go}=-\omega_{go}^\times\mathbf{C}_{go}$, $\mathbf{C}_{bo}\mathbf{q}=\mathbf{q}$, and $\mathbf{T}(\mathbf{Q})\mathbf{C}_{bo}=q_4\mathbf{I}_{3\times3}-\mathbf{q}^\times$, Eq. (3) and (5) can be transformed to:

$$\dot{\widetilde{\mathbf{q}}}=-\omega_{go}^\times\widetilde{\mathbf{q}}+\frac{1}{2}\left(q_4\mathbf{I}_{3\times3}-\widetilde{\mathbf{q}}^\times\right)\widetilde{\omega}, \tag{9}$$

$$\dot{\widetilde{\omega}}=-\left(\omega_{go}+\mathbf{C}_{go}\omega_{oi}\right)^\times\widetilde{\omega}+\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}. \tag{10}$$

It is worth to notice that the Eq. (9) and (10) is a cascade structure, which can be rewritten in a scalar notation as:

$$\begin{bmatrix}\dot{\widetilde{q}}_1\\\dot{\widetilde{q}}_2\\\dot{\widetilde{q}}_3\\\dot{\widetilde{\omega}}_1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}0&2a_3&-2a_2&q_4\\-2a_3&0&-2a_1&-\widetilde{q}_3\\2a_2&2a_1&0&\widetilde{q}_2\\0&0&0&0\end{bmatrix}\begin{bmatrix}\widetilde{q}_1\\\widetilde{q}_2\\\widetilde{q}_3\\\widetilde{\omega}_1\end{bmatrix}+\frac{1}{2}\begin{bmatrix}\widetilde{q}_3&-\widetilde{q}_2\\q_4&\widetilde{q}_1\\-\widetilde{q}_1&q_4\\2b_1&2b_2\end{bmatrix}\begin{bmatrix}\widetilde{\omega}_2\\\widetilde{\omega}_3\end{bmatrix}, \tag{11}$$

$$\begin{bmatrix}\dot{\widetilde{\omega}}_2\\\dot{\widetilde{\omega}}_3\end{bmatrix}=\begin{bmatrix}0&0&0&-b_1\\0&0&0&-b_2\end{bmatrix}\begin{bmatrix}\widetilde{q}_1\\\widetilde{q}_2\\\widetilde{q}_3\\\widetilde{\omega}_1\end{bmatrix}+\begin{bmatrix}0&c\\-c&0\end{bmatrix}\begin{bmatrix}\widetilde{\omega}_2\\\widetilde{\omega}_3\end{bmatrix}+\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}, \tag{12}$$

where $b_1=a_3-\omega_0\sin\alpha\sin\beta$, $b_2=-a_2+\omega_0\cos\alpha$, and $c=a_1-\omega_0\cos\beta\sin\alpha$.

IV. PROBLEM FORMULATION

Problem: For the mathematical model (11) and (12), design an control law u_i, i=1,2 such that $\widetilde{q}_i$, $\forall i=\overline{1,3}$ is asymptotically stable:

$$\lim_{t\to\infty}\widetilde{q}_i=0,\ \forall i=\overline{1,3}\ .$$

Remark: An apparent difficulty with the model (11) and (12) is that the state $[\widetilde{q}_1 \quad \widetilde{q}_2 \quad \widetilde{q}_3]^T$ is only indirectly related to the input u_1, u_2, through the state variable $[\widetilde{\omega}_2 \quad \widetilde{\omega}_3]^T$ and the nonlinear state equation (11). Therefore, it is not easy to see how the input u_1, u_2 can be designed to control $[\widetilde{q}_1 \quad \widetilde{q}_2 \quad \widetilde{q}_3]^T$. However, the difficulty of the control design can be reduced if $[\widetilde{\omega}_2 \quad \widetilde{\omega}_3]^T$ is regarded as the virtual control of Eq. (11). The controller's structure is composed of two parts: the outer loop and inner loop, which will be introduced in the order.

V. OUTER LOOP CONTROLLER DESIGN

The goal of the outer loop controller design is to find a control law:

$$\omega_i^*=\widetilde{\omega}_i\left(\widetilde{q}_1 \quad \widetilde{q}_2 \quad \widetilde{q}_3 \quad \widetilde{\omega}_1\right)^T,\ i=2,3, \tag{13}$$

such that $\widetilde{q}_1$, $\widetilde{q}_2$, $\widetilde{q}_3$, $\widetilde{\omega}_1$ converge to zero in terms of Eq. (11).

This task is completed by *detumbling* and *attitude acquisition.* The different control laws on these different time intervals are discussed below.

1. Control law 1 -- Detumbling

After it is released from the launch vehicle, the satellite is tumbling randomly with known bounds on the initial angular rate. The objective of this phase is to reduce angular rate. The angular error **q** will not be considered in this phase. From Eq. (11), the dynamic model of the outer loop in this phase is as follows:

$$\omega_i^*=b_1\omega_2^*+b_2\omega_3^* . \tag{14}$$

The problem of the detumbling is to design the control law ω_i^*, i=2,3, such that $\widetilde{\omega}_1$ converges to zero. The control law is designed as follows:

$$\begin{bmatrix}\omega_2^*\\\omega_3^*\end{bmatrix}=-k^d\begin{bmatrix}b_1\\b_2\end{bmatrix}\widetilde{\omega}_1, \tag{15}$$

where $k^d>0$.

Substituting Eq. (15) into Eq. (14), $\dot{\widetilde{\omega}}_1=-k^d\left(b_1^2+b_2^2\right)\widetilde{\omega}_1$, and $\widetilde{\omega}_1$ is of asymptotic stability.

2. Control law2 -- Attitude Acquisition

After the detumbling phase, $\widetilde{\omega}$ is assumed to be zero. The problem of the attitude acquisition is to design a control law ω_i^*, i=2, 3, such that $\widetilde{q}_1$, $\widetilde{q}_2$, $\widetilde{q}_3$, and $\widetilde{\omega}_1$ are asymptotically stable under the initial condition $\widetilde{\omega}_1(0)=0$.

The dynamic model in this phase is described by Eq.(11). In order to use the Lyapunov technique to design the controller, the following candidate to the Lyapunov function is considered:

$$V=\widetilde{q}_1^2+\widetilde{q}_2^2+\widetilde{q}_3^2+\sigma\widetilde{\omega}_1^2>0, \tag{16}$$

where σ is a positive constant. From Eq.(11), the derivative of V is identified as follows:

$$\dot{V}=q_4(\widetilde{q}_1\widetilde{\omega}_1+\widetilde{q}_2\omega_2^*+\widetilde{q}_3\omega_3^*)+2\sigma\widetilde{\omega}_1\left(b_1\omega_2^*+b_2\omega_3^*\right). \tag{17}$$

If the control law is designed as follows:

$$\begin{aligned}\omega_2^*&=-k_1^a\widetilde{q}_2-b_1k_2^a\left(\widetilde{\omega}_1+k_3^a\widetilde{q}_1\right),\\ \omega_3^*&=-k_1^a\widetilde{q}_3-b_2k_2^a\left(\widetilde{\omega}_1+k_3^a\widetilde{q}_1\right),\end{aligned} \tag{18}$$

where $k_i^a>0$, $i=1,2,3$. Then

$$\dot{V}=-k_i^aq_4\left(\widetilde{q}_2^2+\widetilde{q}_3^2\right)+\delta, \tag{19}$$

where

$$\begin{aligned}\delta&=q_4\widetilde{q}_1\widetilde{\omega}_1-k_2^aq_4\left(\widetilde{\omega}_1+k_3^a\widetilde{q}_1\right)\left(b_1\widetilde{q}_2+b_2\widetilde{q}_3\right)\\ &-2\sigma k_1^a\widetilde{\omega}_1\left(b_1\widetilde{q}_2+b_2\widetilde{q}_3\right)-2\sigma k_2^a\widetilde{\omega}_1\left(b_1^2+b_2^2\right)\left(\widetilde{\omega}_1+k_3^a\widetilde{q}_1\right).\end{aligned} \tag{20}$$

First, when $\delta=0$,

$$\dot{V}=-k_1^aq_4\left(\widetilde{q}_2^2+\widetilde{q}_3^2\right)\le-k_1^a\sqrt{1-\gamma^2}\left(\widetilde{q}_2^2+\widetilde{q}_3^2\right)\le0 \tag{21}$$

is negative semi-definite. According to LaSalle's theorem[3], the solutions of the system satisfy

$$\lim_{t\to\infty}k_1^a\sqrt{1-\gamma^2}\left(\widetilde{q}_2^2+\widetilde{q}_3^2\right)=0. \tag{22}$$

When $\widetilde{q}_2$ and $\widetilde{q}_3=0$, the motion of $\widetilde{q}_1$ and $\widetilde{\omega}_1$ are described by the following equations:

$$\dot{\widetilde{q}}_1=\frac{1}{2}q_4\widetilde{\omega}_1 \tag{23}$$

$$\dot{\widetilde{\omega}}_1=-k_2^a\left(b_1^2+b_2^2\right)\left(\widetilde{\omega}_1+k_3^a\widetilde{q}_1\right). \tag{24}$$

Let $\widetilde{q}_1=\sin\lambda,\left(-\frac{\pi}{2}<\lambda<\frac{\pi}{2}\right)$. Then $q_4=\cos\lambda$, because $q_4=\sqrt{1-\widetilde{q}_1^2}$ when $\widetilde{q}_2$, $\widetilde{q}_3=0$. From Eq.(23) and (24), the following can be derived:

$$\dot{\lambda}=\frac{1}{2}\widetilde{\omega}_1 \tag{25}$$

$$\dot{\widetilde{\omega}}_1=-k_2^a\left(b_1^2+b_2^2\right)\left(\widetilde{\omega}_1+k_3^a\sin\lambda\right). \tag{26}$$

It is easy to select k_2^a and k_3^a such that λ and $\widetilde{\omega}_1$ are asymptotically stable.

Next, assuming $|\delta|\le k_1^a\sqrt{1-\gamma^2}\mu_1$, where μ_1 is a small positive constant, $\widetilde{q}_2$, $\widetilde{q}_3$ will converge to a small bounded domain $\Omega_1:\left\{\left(\widetilde{q}_2^2+\widetilde{q}_3^2\right)\le\mu_1\right\}$[3].

After $\widetilde{q}_2$ and $\widetilde{q}_3$ enter into the domain Ω_1, the motion of $\widetilde{q}_1$ and $\widetilde{\omega}_1$ are described by the following equations:

$$\dot{\widetilde{q}}_1=\frac{1}{2}q_4\widetilde{\omega}_1+\eta_1(\cdot) \tag{27}$$

$$\dot{\widetilde{\omega}}_1=-k_2^a\left(b_1^2+b_2^2\right)\left(\widetilde{\omega}_1+k_3^a\widetilde{q}_1\right)+\eta_2(\cdot), \tag{28}$$

where

$$\eta_1(\cdot)=a_3\widetilde{q}_2-k_2^a\left(b_1\widetilde{q}_3+b_2\widetilde{q}_2\right)\left(\widetilde{\omega}_1+k_3^a\widetilde{q}_1\right), \tag{29}$$

$$\eta_2(\cdot)=-k_2^a\left(b_1\widetilde{q}_2+b_2\widetilde{q}_3\right). \tag{30}$$

If assumed $|\eta_1|\le\mu_2$ and $|\eta_2|\le\mu_3$, where μ_2 and μ_3 are small positive constants, $\widetilde{q}_1$ and $\widetilde{\omega}_1$ will also converge to a small domain Ω_2.

Note: 1) $|b_j|$, j=1, 2 is very small (much less than 1); 2) Since $\widetilde{\omega}_1(0)=0$, and $\widetilde{\omega}_1$ varies very slowly, $|\widetilde{\omega}_1|$ stays very small after detumbling. Therefore, it is reasonable to assume that μ_1 is a very small positive constant. Consequently $\widetilde{q}_1$, $\widetilde{q}_2$, $\widetilde{q}_3$ and $\widetilde{\omega}_1$ will converge to a small domain.

Last, the system's stability in this small domain can be attained by the Floquet theorem[8]. Linearizing the system at the neighborhood of the origin produces the following:

$$\dot{\mathbf{x}}_1(t)=\mathbf{A}_1(t)\mathbf{x}_1(t)+\mathbf{B}_1(t)\mathbf{u}_1(t) \tag{31}$$

$$\mathbf{u}_1(t)=\mathbf{K}_1(t)\mathbf{x}_1(t), \tag{32}$$

where

$$\mathbf{x}_1=\left[\widetilde{q}_1\quad\widetilde{q}_2\quad\widetilde{q}_3\quad\widetilde{\omega}_1\right]^T,\ \mathbf{u}_1(t)=\left[\omega_1^*\quad\omega_2^*\right]^T$$

$$\mathbf{A}_1(t)=\begin{bmatrix}0&a_3&-a_2&0.5\\-a_3&0&-a_1&0\\a_2&a_1&0&0\\0&0&0&0\end{bmatrix},\ \mathbf{B}_1(t)=\begin{bmatrix}0&0\\0.5&0\\0&0.5\\b_1&b_2\end{bmatrix}$$

$$\mathbf{K}_1(t)=\begin{bmatrix}-k_2^ak_3^ab_1&-k_1^a&0&-k_2^ab_1\\-k_2^ak_3^ab_2&0&-k_1^a&-k_2^ab_2\end{bmatrix}.$$

Because

$$\mathbf{A}_1(t+T)=\mathbf{A}_1(t),\ \mathbf{B}_1(t+T)=\mathbf{B}_1(t),\ \mathbf{K}_1(t+T)=\mathbf{K}_1(t),$$

the close loop

$$\dot{\mathbf{x}}_1=\mathbf{A}_c(t)\mathbf{x}_1,\ \mathbf{A}_c(t)=\mathbf{A}_1(t)+\mathbf{B}_1(t)\mathbf{K}_1(t) \tag{33}$$

is a linear periodical system.

According to Floquet's theorem, the transition matrix of the system (33), which is denoted by $\Phi(t_2,t_1)$, has the character

$$\Phi(t+T,t)=\mathbf{C},\ \forall t\in[0,\infty), \tag{34}$$

where $\mathbf{C}$ is a constant matrix named the monodromy matrix of $\mathbf{A}_c(t)$. If all the eigenvalues of $\mathbf{C}$ (called characteristic multipliers of $\mathbf{A}_c(t)$) belong to the open unit circle, the close loop (33) is asymptotically stable.

VI. INNER LOOP CONTROLLER DESIGN

The inner loop is to control $\left[\widetilde{\omega}_2\quad\widetilde{\omega}_3\right]^T$, which tracks the virtual control signal $\left[\omega_2^*\quad\omega_3^*\right]^T$ in terms of Eq. (12). Considering the disturbance torque, a sliding mode controller is used. The dynamics equation of the inner loop is rewritten as follows:

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2(t)\mathbf{x}_2 + \mathbf{f}(\tilde{\omega}_1) + \mathbf{u}_2 + \mathbf{d}\,, \tag{35}$$

where

$\mathbf{x}_2 = [\tilde{\omega}_1 \quad \tilde{\omega}_2]^T$, $\mathbf{f}(\tilde{\omega}_1) = [-b_1\tilde{\omega}_1 \quad -b_2\tilde{\omega}_1]^T$, $\mathbf{u}_2 = [u_1 \quad u_2]^T$, and

$$\mathbf{A}_2(t) = \begin{bmatrix} 0 & c \\ -c & 0 \end{bmatrix},$$

$\mathbf{d} = [d_1 \quad d_2]^T$ is the component of disturbance torque in the control plane.

Let $\mathbf{x}_d = [\omega_1^* \quad \omega_2^*]^T$ is the virtual control profile. The desired sliding surface is one in which

$$\mathbf{S} = \mathbf{x}_d - \mathbf{x}_2 = 0\,. \tag{36}$$

In order to provide asymptotic stability of $\mathbf{S} = 0$, the following candidate Lyapunov function is selected:

$$V = \frac{1}{2}\mathbf{S}^T\mathbf{S}\,. \tag{37}$$

The derivative of V is identified as:

$$\dot{V} = \mathbf{S}^T[\dot{\mathbf{x}}_d - \mathbf{A}_2(t)\mathbf{x}_2 - \mathbf{f} - \mathbf{u}_2 - \mathbf{d}]. \tag{38}$$

The control torque is selected in the following format:

$$\mathbf{u}_2 = \hat{\mathbf{u}}_{\mathbf{eq}} + R\,\mathbf{sign}(\mathbf{S}), \tag{39}$$

where:

$$\hat{\mathbf{u}}_{\mathbf{eq}} = -\dot{\mathbf{x}}_d + \mathbf{A}_2(t)\mathbf{x}_2 + \mathbf{f}\,, \tag{40}$$

is the estimate of the equivalent control, and $R > \|\mathbf{d}\|_\infty$. It is found that

$$\dot{V} = -R\sum_{i=1}^{2}|s_i| - \sum_{i=1}^{2} s_i \cdot d_i < 0\,, \tag{41}$$

which guarantees the state will converge to a sliding surface under the control law (39).

In order to erase chattering, the sign function is replaced with the saturation function[8]. The saturation function is defined as:

$$sat\left(\frac{s_i}{\varepsilon}\right) = \begin{cases} 1 & s_i > \varepsilon \\ \dfrac{s_i}{\varepsilon} & |s_i| \le \varepsilon \\ -1 & s_i < -\varepsilon \end{cases}, \tag{42}$$

where ε is the thickness of the sliding boundary layer.

VII. NUMERICAL EXAMPLES

The validity of this method can be demonstrated by the following example. The moments of inertia about a spacecraft are assumed to be $Ix=Iy=Iz=10$ Kg-m². The spacecraft is assumed to be in circular pole orbit at the altitude of 675 Km, where $\omega_0 = 0.00107$ rad/s. At the pole orbit, the magnetic field in **Fb** can be simply expressed as [7]

$$x_o = -\frac{\mu_m}{r^3}\cos\omega_0(t - t_0)$$

$$y_o = 0$$

$$z_o = -\frac{2\mu_m}{r^3}\sin\omega_0(t - t_0)\,, \tag{43}$$

where r represents distance from the geocenter and $\mu_m = 1\times10^{17}$ Wb-m is Earth's dipole strength. In this case, $a_1(t) = a_3(t) = 0$, $b_1(t) = 0$. $a_2(t)$ and $b_2(t)$ are shown in Fig. 3.

The detumbling controller is designed as follows:

$k^d = 1000.$; $R = 0.01$; $\varepsilon = 0.01$.

Fig.4 shows typical ω trajectories.

On the attitude acquisition phase, the controller is designed as

$k_1^a = 0.002$; $k_2^a = 8$; $k_3^a = 0.4$; $R = 0.01$; $\varepsilon = 0.01$.

If the Eq. (11) and (17) are linearized, the characteristic multipliers of the close loop system (33) are

$0.32 + 0.64i$, $0.32 - 0.64i$, 0.72, and 0.37.

Because all of them are in the open unit circle, the original system is local stable. The simulation of attitude acquisition is shown in fig. 5 and 6.

VIII. CONCLUSION

This study has addressed a practical attitude control method for a small satellite with magnetorquer control only. The controller structure is composed of the outer loop and the inner loop. Different control laws are designed for the phases of detumbling and attitude acquisition respectively. The results of the stability analysis and simulation show that three-axis control can be attained with magnetorquers as the sole actuators in low Earth orbit.

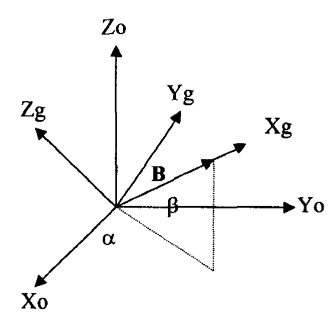


Fig.1 The relation between **Fg** and **Fo**

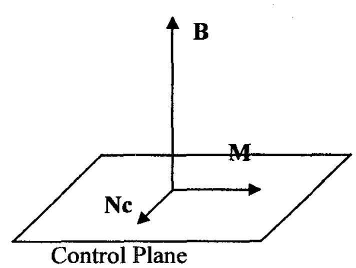


Fig. 2 The control plane

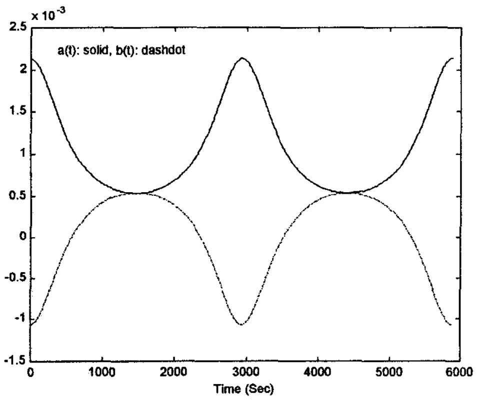
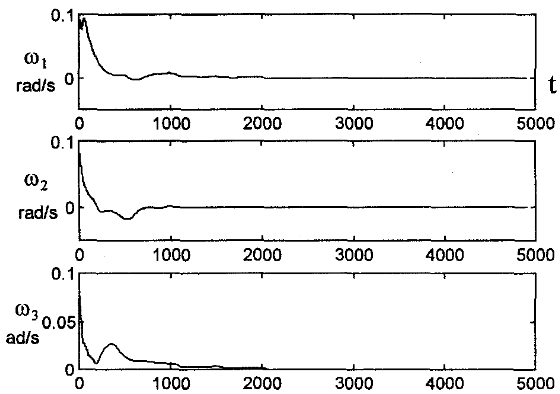


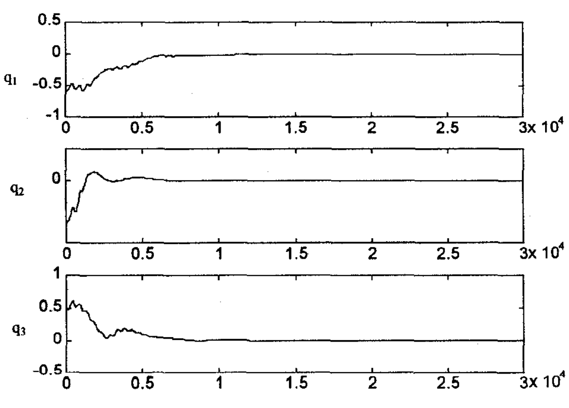
Fig.3 The variety of $a_2(t)$, $b_2(t)$ in the orbit



Time (Sec)

Initial condition: $q = [0.9 \quad 0 \quad 0]^T$, $\omega = [0.09 \quad 0.09 \quad 0.09]^T$.

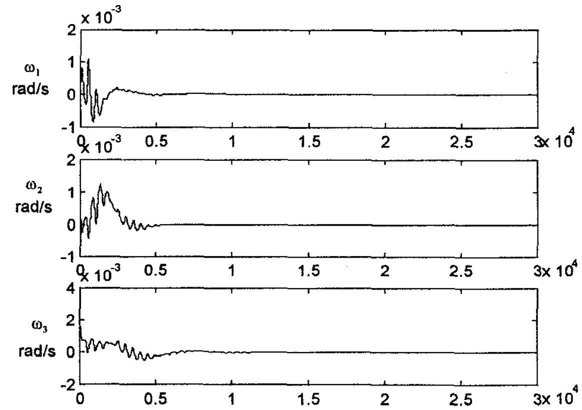
Fig.4 Angular velocity history in detumbling phase.



Time (Sec)

Initial condition: Same to Fig. (6).

Fig. 5. Attitude history in attitude acquisition



Time (Sec)

Initial condition: $q = [-0.5 \quad -0.3 \quad 0.5]^T$,

$\omega = [-0.0001 \quad -0.0002 \quad 0.001]^T$

Fig.6 Angular velocity history in attitude acquisition.

References

[1] Francois Martel, et.al., "Active Magnetic Control System for Gravity Gradient Stabilized Spacecraft," _____.

[2] Wishiewski, R., "Optimal Three-Axis Satellite Attitude Control with Use of Magnetic Torquing," AIAA Guidance, Navigation, and Control Conference, New Orleans, 1997.

[3] Lewis, F. L., Abdallah, C. T., and Dawson, D. M., Control of Robot Manipulators, Macmillan Publishing Company, 1993

[4] Utkin, V. I., Sliding Modes in Control Optimization Springer Verlag, NY, 1992.

[5] Decarlo, R. A., Zak, S.H., and Mathews, G.P., "Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial," Proc. IEEE, vol. 76, no. 3, 1988

[6] Wertz, J. R., Spacecraft Attitude Determination and Control, Reidel, Dordrecht, The Netherlands, 1980.

[7] Hughs, Peter C., Spacecraft Attitude Dynamics, John, Wiley & Sons, Inc. 1986

[8] Slotine, J.J.E. and Li, W., Applied Nonlinear Control, Prentice-Hall, Englewood Cliffs, N.J., 1991

[9] Mohler. R.R. Nonlinear Systems Vol. Dynamics and Control,. Prentice Hall. 1995