Limitations of scaling momentum control strategies to small spacecraft

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Abstract—**As a spacecraft becomes smaller, a number of physical effects scale both favorably and unfavorably for passive stabilization of the craft. Unfortunately, two separate quantities both scale unfavorably for the use of traditional spinning rotor actuators (e.g. reaction wheels, momentum wheels, control moment gyros) for momentum and attitude control. First, the dominant disturbance torques on small spacecraft in low earth orbit, aerodynamic drag and solar radiation pressure, both become relatively larger as spacecraft size decreases. Second, the effectiveness of spinning rotors reduces as the rotor inertia decreases with the square or the wheel radius. These two factors conspire to greatly reduce the effectiveness of rotor-based momentum control systems at small scales. This reduction requires small spacecraft designers to either devote a significantly larger mass fraction to momentum control or adopt alternative momentum control systems. In this study we examine this problem from two viewpoints. First, empirical data is used to find a relationship between spacecraft size and mass fraction devoted to attitude control. While the International Space Station can devote less than 1% of its mass fraction to momentum control effectors, GEO telecom spacecraft tend to need around 1-2% of available mass, and some CubeSats must devote greater than 50% of their mass fraction. Second, we derive an expression for the smallest spacecraft that can use a reaction wheel for effective momentum management. For reasonable assumptions, this lower limit is on the order of 1 cm length scale, which is in good agreement with the empirical trend. Finally, we list some alternative momentum management strategies and discuss how they apply to spacecraft at the smallest size: the centimeter scale ChipSat.**

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1. INTRODUCTION AND MOTIVATION

The number of small spacecraft launched in the past decade has grown significantly and is expected to continue to grow [1]. With the proliferation of the CubeSat standard and CubeSat launch opportunities, the barriers to spaceflight have been greatly reduced. However, inherent challenges to CubeSat production and flight have led to mixed mission success, with approximately 25% of all CubeSat flights failing to achieve all mission objectives. [2]. While low budgets, use of non-space rated parts, and inexperienced teams can all contribute to CubeSat failure, this study examines a more fundamental issue with the scaling of large spacecraft technology down to smaller classes of satellites.

As a spacecraft becomes smaller, a number of physical effects scale both favorably and unfavorably for easy stabilization. Unfortunately, two separate phenomena both scale unfavorably for the use of traditional spinning rotor actuators for momentum and attitude control. First, the dominant disturbance torques for small spacecraft in low earth orbit both become relatively larger as spacecraft size decreases. Second, the effectiveness of spinning rotors is reduced as the rotor inertia decreases with the square or the wheel radius. These two factors conspire to reduce the effectiveness of rotor-based momentum control systems at small scales. This reduction requires small spacecraft designers to either devote a significantly larger mass fraction to momentum control or adopt alternative momentum control system. CubeSats are found to be a particularly challenging spacecraft size, requiring large rotor sizes to overcome environmental effects.

However, at sizes smaller than CubeSats, additional opportunities are found for exploiting these environmental effects rather than overcoming them. In a size range closer to 1 cm, environmental forces and torques become large enough to be used on reasonable timescales for completing a missions. A number of studies of centimeter-scale spacecraft, termed ChipSats, have shown their unique mission utility across a wide range of domains [3–5], and the KickSat mission was built to show their feasibility [6]. While this study shows that rotor-based momentum control will never be useful at the ChipSat scale, it also highlights that the scaling of environmental effects can become beneficial for small satellite attitude control.

This study examines the scaling of rotor-based attitude control systems (ACS) through two methods: first, empirical data is collected across spacecraft scales to show how spacecraft ACS grow relative to overall size as spacecraft get smaller. Second, a minimum size for spacecraft that can hold a useful rotor system is derived and found to be in good agreement with the empirical data. These conclusions are followed by discussion and an overview of some alternative means of attitude control without rotors.

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2. SCALING OF MOMENTUM CONTROL SYSTEMS

A two-pronged approach is taken to study the scaling of momentum control systems for different classes of spacecraft. First, historical vehicles and current commercial offerings are use to create an empirical data set of spacecraft dimensions compared to mass fractions used by their attitude control systems. This data shows a clear negative trend toward increasing mass devoted to ACS as spacecraft size decreases. Second, to back up the empirical data, a derivation is given to calculate the smallest spacecraft size for which, under reasonable assumptions, a reaction wheel is no longer a practical momentum control system. Both methods show excellent agreement in this spacecraft scale below which an alternative momentum control system is required.

Two main factors serve to increase ACS size (inversely with spacecraft length): the scaling of environmental disturbance torques and the quadratic form of the relationship between ACS rotor radius and its momentum storage capacity. Specifically considering the low-Earth orbit environment, the dominant disturbance torques on spacecraft smaller than 1m in size arise from aerodynamic drag and solar radiation pressure. As both of these disturbances are due to pressure effects, the angular acceleration imparted from them on to the spacecraft scales with the spacecraft's area-to-mass ratio. Considering a cubic spacecraft shape, this area-to-mass ratio grows inversely with the spacecraft side length. For an extensive study of the scaling of forces and torques with spacecraft length scale, see Atchison and Peck [7].

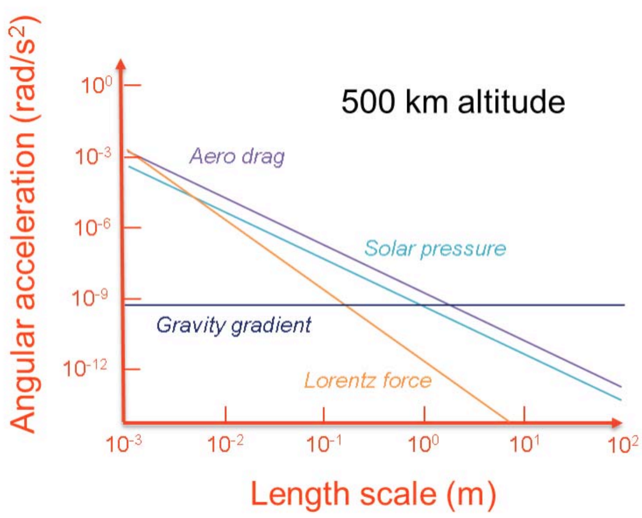

Figure 1. Growth of disturbance torque magnitude as spacecraft size decreases. Recreated from [7].

This scaling of major disturbance angular accelerations is depicted in Fig. 1, which is recreated with data from [7]. This data applies at a 500km altitude for a cubic spacecraft. Drag and solar pressure torques overtake gravity gradient as the major disturbances at sizes below ~1m and continue to grow in size as the spacecraft gets smaller. Electromagnetic effects, such as torque from the Lorentz force, tend to scale as one over the side length squared, allowing them to become the dominant disturbance at even smaller length scales. As is clearly evidenced by Fig. 1, as spacecraft size is reduced down to the CubeSat (10^{-1} m) and ChipSat (10^{-2} m) length scales, any ACS system will have to become more capable in order to overcome the increased disturbance accelerations.

While this study focuses exclusively on angular accelerations and rotor-based ACS, similar scaling occurs in translational accelerations. For detail on the scaling of forces, see Atchison and Peck [7], and for more focus on the orbit evolution of systems across all length scales, see McInnes, et al. [8]

The second driver of ACS size is the declining effectiveness of a rotor to store momentum at smaller length scales. As the moment of inertia of a rotor scales with the square of its radius, a decrease in wheel radius increasingly constrains the capability of an ACS; this adverse relationship continues to grow as spacecraft get smaller. As the rotors become less effective, their size relative to smaller spacecraft must increase.

Empirical Trend of ACS Sizing

To examine how these pressures on momentum-based ACS sizing affect real world spacecraft design, an empirical study is performed using known spacecraft and commercially available ACS and spacecraft buses. Data from the largest scale, the International Space Station, to the smallest scale, 1U CubeSats and below, is gathered to study any trend in size of ACS relative to spacecraft size.

Figure 2 shows the results of the empirical study. A clear trend is seen with ACS mass fraction increasing substantially as the spacecraft becomes smaller. While the ISS uses significantly less than 1% of its mass fraction for ACS and large geosynchronous communications satellites use only 1-2%, some CubeSat designs allocate 50% of their mass fraction for ACS.

The data in Fig. 2 comes from a variety of publicly available sources. The largest spacecraft ever built, the ISS, uses a set of dual-gimbal CMGs to manage momentum growth. While these CMGs are quite massive, they only make up a small fraction of the overall ISS mass. The circles in Fig. 2 represent GEO telecommunication spacecraft. These points do not reference specific spacecraft, but rather combinations of commercially available spacecraft buses and ACS packages from major satellite vendors including Lockheed Martin, Boeing, and Honeywell. Sharing the same mission and having several generations of designs, the GEO spacecraft are both highly optimized and tightly clustered.

The midsize Earth-observation and science spacecraft have a wider array of missions and designs and, thus, a wider spread of ACS mass fractions. The labeled point for the CYGNSS microsatellite represents the spacecraft characteristics of the recently launched CYGNSS mission. The other points are combinations of buses available from Lockheed Martin and Ball paired with reaction wheel systems from Honeywell. The CubeSat data point in Fig. 2 are drawn from the standard definition of 3U and 1U CubeSats paired with Blue Canyon ACS packages.

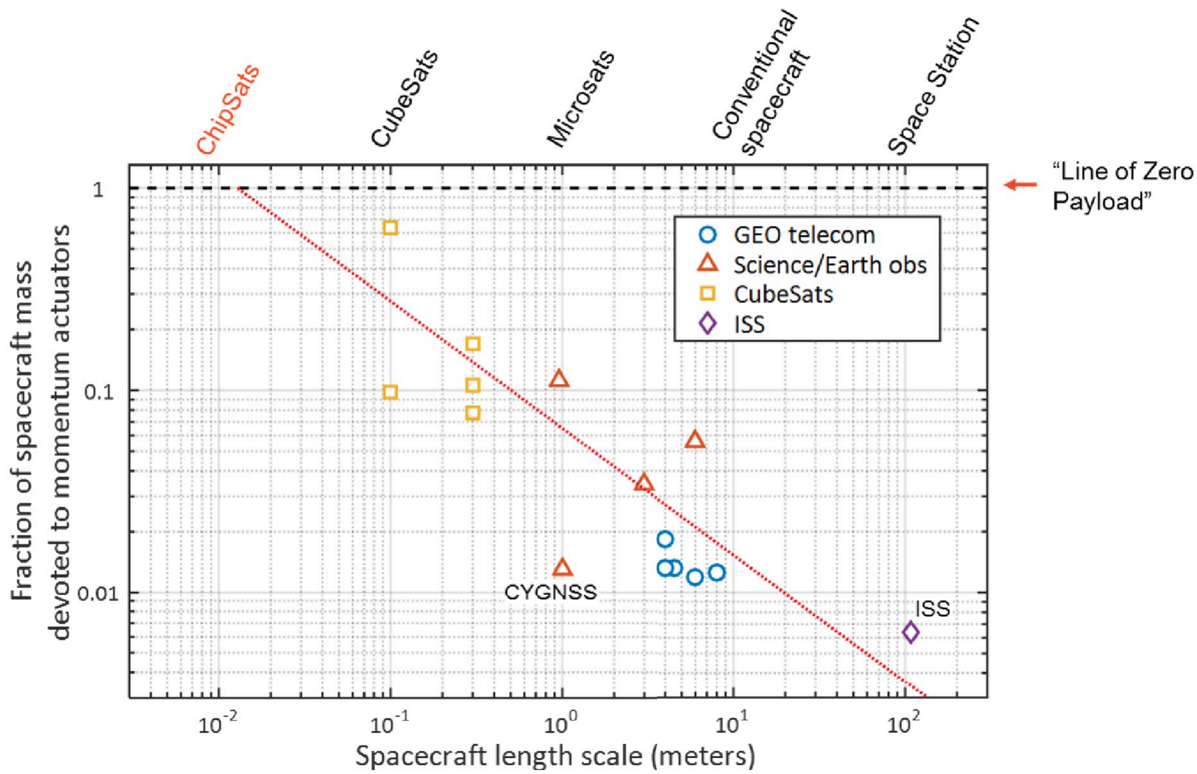

Figure 2. Empirical data showing the growth of spacecraft mass devoted to attitude control system as as overall spacecraft size decreases. A regression line predicts that ACS mass will overwhelm all payload mass for spacecraft smaller than 1.3 cm.

Fitting a regression line to the data in Fig. 2, the best fit for the relationship between spacecraft side length, λ, and mass fraction devoted to ACS, μ_{ACS} , is found to be:

$$\mu_{ACS} = 0.065\lambda^{-0.6277} \tag{1}$$

Equation 1 provides a simple rule of thumb for how much of spacecraft mass allowance must be devoted to a rotor-based ACS as the dimension of the spacecraft change. In the limit where this regression line intersects what is termed the line of zero payload, $\mu_{ACS} = 1$ and the ACS system fills the entire spacecraft. This intersection is effectively a calculation of the lower limit of spacecraft size that can support a traditional ACS. Solving equation 1 for this point, this lower limit is found to be a spacecraft side length of 1.28 cm.

An Analytical Reaction Wheel Limit

To provide further insight into how rotor size needs to scale with spacecraft size, a derivation is given for the necessary reaction wheel sizing to operate under single disturbance torque for a simplified spacecraft model. Given a cubic spacecraft with side length λ that contains a reaction wheel with radius r, what is the size of r relative to λ such the wheel can compensate for the torque induced by aerodynamic drag for some period of time Δt?

The aerodynamic torque on the vehicle is

$$\tau = \frac{1}{2}\rho_a V^2 C_D A(c_g - c_p) \tag{2}$$

where ρ_a is the atmospheric density, V is spacecraft velocity, C_D is drag coefficient, A is drag area projected in the direction of the velocity vector, and $(c_g - c_p)$ is the moment arm computed as the difference between the locations of center of gravity and center of pressure. The $(c_g - c_p)$ difference is assumed to be a fraction of the side length of the spacecraft:

$$c_g - c_p = f_\Delta \lambda \tag{3}$$

With the drag area as λ^2, the aerodynamic torque is

$$\tau = \frac{1}{2}\rho_a V^2 C_D f_\Delta \lambda^3 \tag{4}$$

To tie the spacecraft size to the rotor size, an equivalence is sought between the angular momentum imparted to the spacecraft by drag and the momentum storage capability of the rotor. The scalar angular momentum of the the spacecraft is given by:

$$L_{s/c} = I\omega \tag{5}$$

where ω is the angular rate of the body. Over the period of interest, Δt, the growth in angular momentum of the spacecraft due to the aerodynamic torque is given by

$$\Delta L_{s/c} = I\Delta\omega = \tau\Delta t \tag{6}$$

giving

$$\Delta L_{s/c} = \frac{1}{2}\rho_a V^2 C_D f_\Delta \Delta t \lambda^3 \tag{7}$$

To counteract this growth in momentum the reaction wheel must be able to supply an equal and opposite angular momentum before reaching its upper limit on rotation rate. The momentum stored in the wheel is given by

$$L_w = I_w \omega_w \tag{8}$$

where I_w is the inertia of the rotor and ω_w is its spin rate. The wheel is assumed a solid disk with radius r and thickness h. To further cast the problem in terms of r, the rotor thickness is assumed to be a fraction of its radius, with $h = f_h r$. Assuming a uniform density of the wheel, ρ_w, this gives the moment of inertia of the wheel as:

$$I_w = \frac{\pi}{4}\rho_w f_h r^5 \tag{9}$$

giving

$$L_w = \frac{\pi}{4}\rho_w f_h \omega_w r^5 \tag{10}$$

Given that we want to find how the necessary wheel size scales with the spacecraft size, we search for a parameter k such that

$$r = k\lambda \tag{11}$$

Acceptable attitude stabilization requires that the cumulative angular momentum accrued by the spacecraft, $\Delta L_{s/c}$, is exactly transferred to the reaction wheel so that the spacecraft angular momentum state is regulated to 0. This stable equilibrium requires the identity:

$$\Delta L_{s/c} = -\Delta L_w \tag{12}$$

Combining Eqs. 7, 10, and 11 in accordance with this identity and solving for k gives us an expression for how wheel size must scale to maintain the same performance as spacecraft size changes:

$$k = \left(\frac{\frac{1}{2}\rho_a V^2 C_D f_\Delta \Delta t}{\frac{\pi}{4} f_h \rho_w \omega_w}\right)^{\frac{1}{5}} \lambda^{-\frac{2}{5}} \tag{13}$$

Thus, as λ decreases the $-2/5$ exponent causes k to grow larger, meaning the reaction wheel size as a fraction of side length must continue to increase as the spacecraft gets smaller in order to maintain the same performance.

In fact, this relationship can give us an absolute limit for the smallest spacecraft size that can maintain reasonable performance with a reaction wheel. Based on the definition of k, when λ decreases to the point that k is 0.5, the diameter of the reaction wheel is the same as the side length of the spacecraft. Below this size, deemed λ_c, the spacecraft cannot hold a reaction that gives acceptable performance. Inserting $k = 0.5$ into Eq. 13, λ_c is solved for as

$$\lambda_c = 8\left(\frac{\rho_a V^2 C_D f_\Delta \Delta t}{\pi f_h \rho_w \omega_w}\right)^{\frac{1}{2}} \tag{14}$$

To calculate an example λ_c, some reasonable assumptions are made for a typical low-Earth orbit spacecraft: a 500km altitude circular orbit, a desired continuous operation time of 24 hours, an aluminum rotor with a max speed of 10,000 rpm, a rotor thickness to radius ratio of 10%, and a $c_p - c_g$ difference of 25%. The numerical values associated with these assumptions are presented in Table 1. Given these assumptions, λ_c is 0.99 cm. This critical size which agrees closely with the empirical results seen in Eq. 1, where the spacecraft design does not close at or below 1.3 cm side length.

Table 1. Values used to calculate λ_c

Variable	Value
ρ_a	$4.89e^{-13}$ kg/m^3
V	7613 m/s
C_D	2.2
Δt	86400 s
ρ_w	2700 kg/m^3
ω_w	1047 rad/s
f_Δ	0.25
f_h	0.1

3. Discussion

The growth of disturbance torques as size scales downward changes how ACS must be designed for very small spacecraft. Given that a rotor-based ACS becomes inappropriate below certain spacecraft scales, alternative means of attitude control must be used. Fig. 3 recreates the disturbance magnitudes from Fig. 1 and [7] with new information. Along the right side of the graph, the angular acceleration capability of several known spacecraft (the ISS, Hubble Space Telescope, the CYGNSS microsatellites, and a typical CubeSat) are shown and labeled as the agility of the systems. This agility is the maximum angular acceleration that the rotor-based ACS

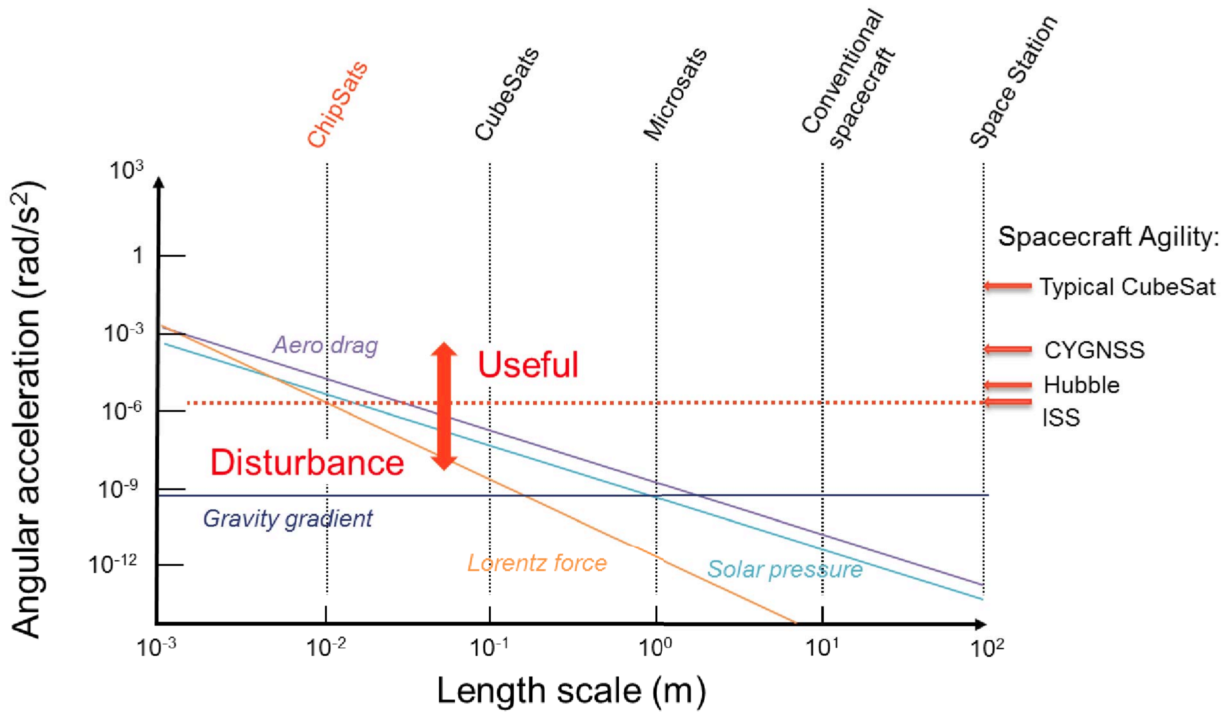

Figure 3. **Growth of disturbance torque magnitude as spacecraft size decreases. Underlying disturbance data drawn from [7]. Angular acceleration capability for several spacecraft shown along the right axis. The plot is divided between useful accelerations (defined as larger than the ISS's capability) and disturbance accelerations. The crossover between useful and disturbance generally occurs between the CubeSat and ChipSat scales.**

on each satellite can produce. A line is also drawn at the agility level of the slowest moving system, the ISS, and any acceleration above this level is deemed useful in spacecraft design. Any acceleration below this is deemed a disturbance, a torque acting on the system at a level too small to produce controlled motion of a spacecraft on a useful timescale. As is seen in the figure, the aerodynamic drag and solar radiation pressure torques both move into the useful region at scales between CubeSats and ChipSats, giving hope that they can be used as the primary attitude actuation system at ChipSat scales.

Figure 3 also illuminates the distinct difficulties of controlling CubeSat attitude with a rotor-based ACS. CubeSats lie at the size where the dominant disturbances are large, but not quite large enough to provide angular accelerations of a useful time scale. Thus, a CubeSat ACS must be powerful enough to both overcome the disturbances and provide ample acceleration capability above that level to perform a mission, leading to designs that have the high agility seen in Fig. 3. Unfortunately, at the CubeSat scale, rotor-based systems have lost much of their capability due to shrinking rotor size, lead to the high CubeSat ACS mass fractions seen earlier in Fig. 2. Overall, the CubeSat size scale is a particularly challenging case for attitude control, where rotors are largely ineffective and disturbances are large but not large enough to be useful.

At the smaller ChipSat scale, while rotor-based systems are not possible, environmental effects are available that are now large enough to provide attitude motion on useful time scales. These useful environmental accelerations come in three main types: magnetic, aerodynamic, and solar. While each disturbance effect may not be able to provide a torque in any arbitrary direction a given instant in time, full controllability may be found over time with an individual effect or most robustly with a combination of the three.

All of three of these effects has been studied for attitude control of small spacecraft. Using the Earth's magnetic field, the simplest and most ubiquitous method of actuation is the magnetic torque coil. Torque coils have been used across the spectrum of spacecraft sizes, from the Hubble to ChipSats. Fortunately, the effectiveness of torque coils scales along with the disturbances to larger values for smaller scales [7].

A second technology for using the magnetic field for attitude control is the electrodynamic tether. As shown in Weis and Peck [9], tether systems also scale favorably for ChipSat-sized systems in both current required and necessary tether length. Tether to provide useful attitude control are shown to be on the order of 10cm long with μA currents, avoiding the traditional problems with macro-scale tethers: many kilometer lengths with high current draw. A micro-tether system has the added advantage of being able to provide a translational acceleration to the spacecraft in addition to angular accelerations.

Methods to create a differential in solar radiation pressure can also be used to provide attitude actuation. Generally, a form of solar sail is employed with either passive or active differential elements. Atchison explored a passively sun-pointing solar sail that provides a restoring torque to maintain a sun-aligned attitude [10]. The IKAROS solar sail demonstration mission flew a system to control the attitude of a solar sail using electrochromic panels [11], although at a larger scale than a ChipSat design. Electrochromic materials are able to control their reflectively in an active manner, allowing for some measure of control over the differential solar torque. An additional benefit of an electrochromic sail is that it can modulate the solar pressure force as well, allowing for active orbital control [12].

The use of aerodynamic drag torque for attitude control is the third major environmental effect control possibility. Studies have been performed to generate passively stable attitudes using aerodynamics by creating a shuttlecock like structure [13]. Fewer studies of active attitude control using drag modulation panels have performed, although there are a small number of examples [14, 15]. The potential utility of drag modulation is much greater at the ChipSat scale and should be studied further. Similar to the magnetic and solar effects, drag modulation can control the spacecraft orbits as well. Due to the parasitic nature of the drag force, there are fewer possibilities for orbit control, but extensive studies have been performed for relative motions such as constellation or formation keeping [16] or rendezvous and docking [17].

4. CONCLUSION

While rotor-based momentum control is the standard for large spacecraft, it does not scale well to small spacecraft. Due to both the physics of rotor-based systems and the relative growth of environmental effects at small scales, reaction wheels and CMGs become much less effective. CubeSats occupy a particular challenging scale where environmental torques are large enough that significant spacecraft mass must be devoted to overcoming them, but are not large enough to be useful on reasonable timescales. Both empirical and analytical data show that spacecraft smaller than 1-2 cm in side length will not be able to effectively use rotors at all and must rely on alternative means of attitude control. However, at this scale environmental effects become large enough to operate at useful timescales, allowing ChipSat spacecraft to take advantage of the environment rather than try to overcome it.

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