Modeling Passive Solar House in Massachusetts

By Gati Aher and Zachary Sherman

All measurements are in metric

Design Constraints

- small house

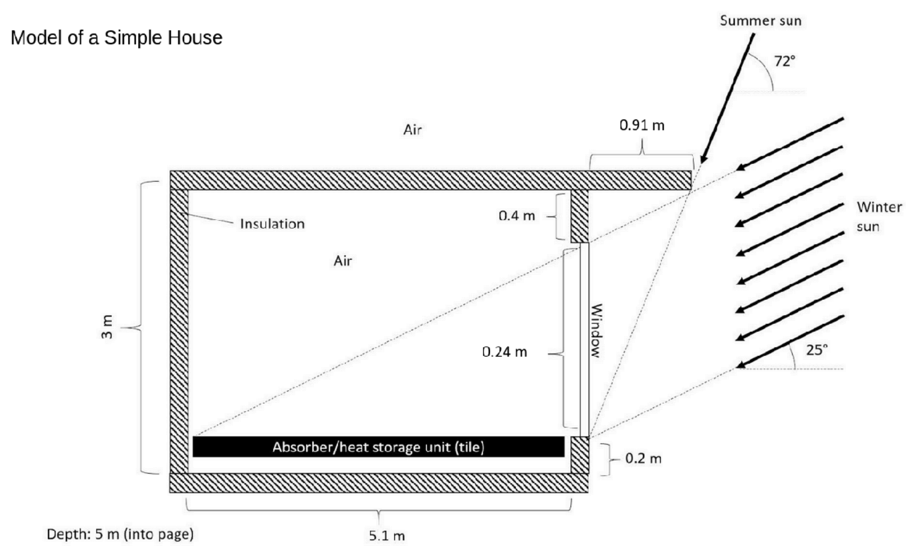
```
min_floor_space = 9.2903; % m^2
max_floor_space = 37.1612; % m^2
```

- reasonably comfortable in winter in Boston climate

```
min_indoor_air_temp = 17; % deg Celsius
max_indoor_air_temp = 25; % deg Celsius
```

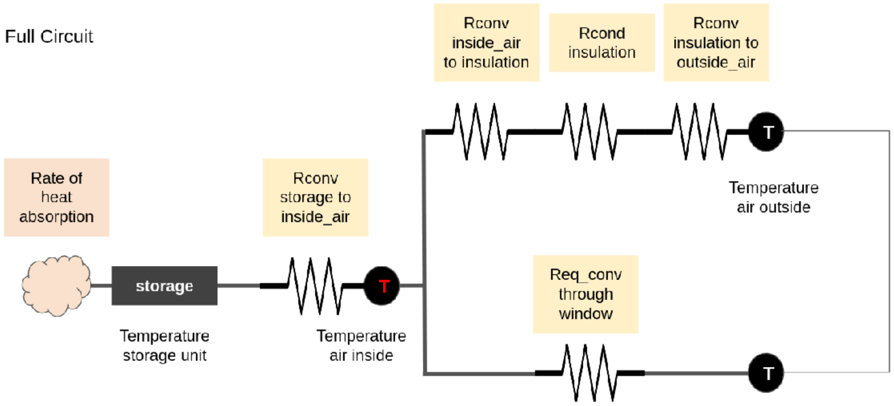
- no direct sunlight through south-facing windows at noon in summer

Minimal Implementation



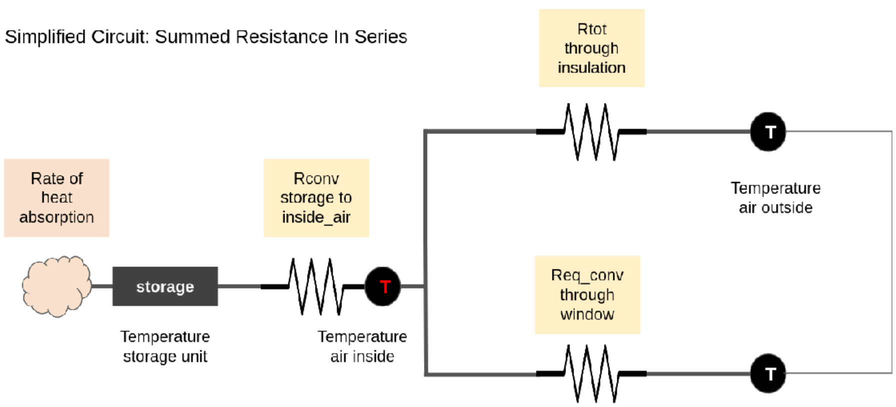
Draw Circuit Diagram to Understand Resistance Network

Full Circuit



Sum resistance in series: $R_{\text{totseries}} = R_1 + R_2 + \ldots + R_n$

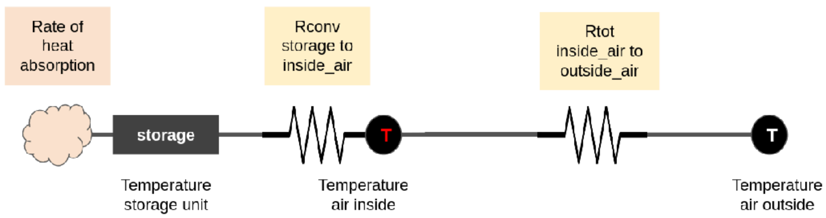
Simplified Circuit: Summed Resistance In Series



Sum the two resistances in parallel

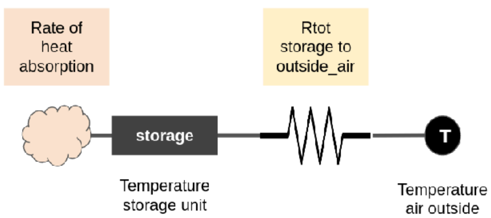
$$R_{\text{totparallel}} = \frac{R_1 R_2}{R_1 + R_2}$$

Simplified Circuit: Summed Resistance In Parallel



Sum in series to get resistance network in most simplified form

Simplified Circuit: Summed Resistance In Series



Find a solution for the house's temperature over the course of serveral days

1. Write an ordinary differential equation to model the heat storage's rate of temperature change over time.

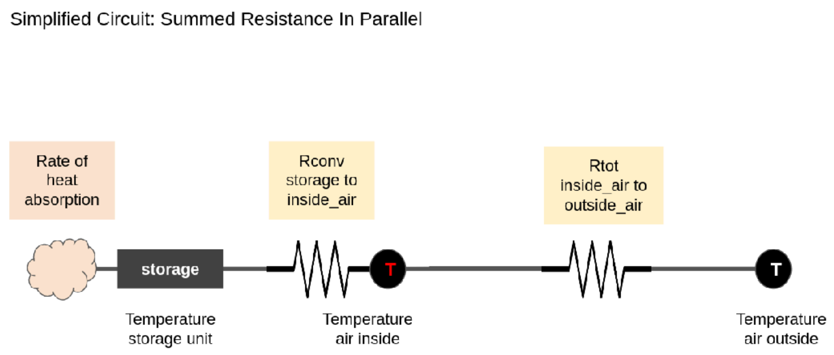
$$C_{\text{storage}} \frac{\mathrm{dT}_{\text{storage}}}{\mathrm{dt}} = \frac{\mathrm{dU}_{\text{storage}}}{\mathrm{dt}} = Q'_{\text{intostorage}} - Q'_{\text{ourofstorage}} = Q'_{\text{in}} - Q'_{\text{out}}$$

$$Q'_{\text{out}} = \frac{T_{\text{storage}} - T_{\text{outsideair}}}{R_{\text{totstoragetooutsideair}}}$$

final ODE for $\frac{\mathrm{dT}_{\text{storage}}}{\mathrm{dt}}$ is y'

$$y' = \frac{\left(Q'_{\text{in}} - \frac{y - T_{\text{outsideair}}}{R_{\text{totstoragetooutsideair}}}\right)}{C_{\text{storage}}}$$

2. Solve for $T_{\text{insideair}}$



Heat flow through a circuit is the same for resistors in series.

$$\frac{T_{\text{storage}} - T_{\text{outsideair}}}{R_{\text{totstoragetooutsideair}}} = \frac{T_{\text{insideair}} - T_{\text{outsideair}}}{R_{\text{totinsideairtooutsideair}}}$$

By rearranging

$$T_{\text{insideair}} = T_{\text{outsideair}} + R_{\text{totinsideairtooutsideair}} \frac{T_{\text{storage}} - T_{\text{outsideair}}}{R_{\text{totstoragetooutsideair}}}$$

Model With Constant vs. Sinusoidal Outside Air Temperature

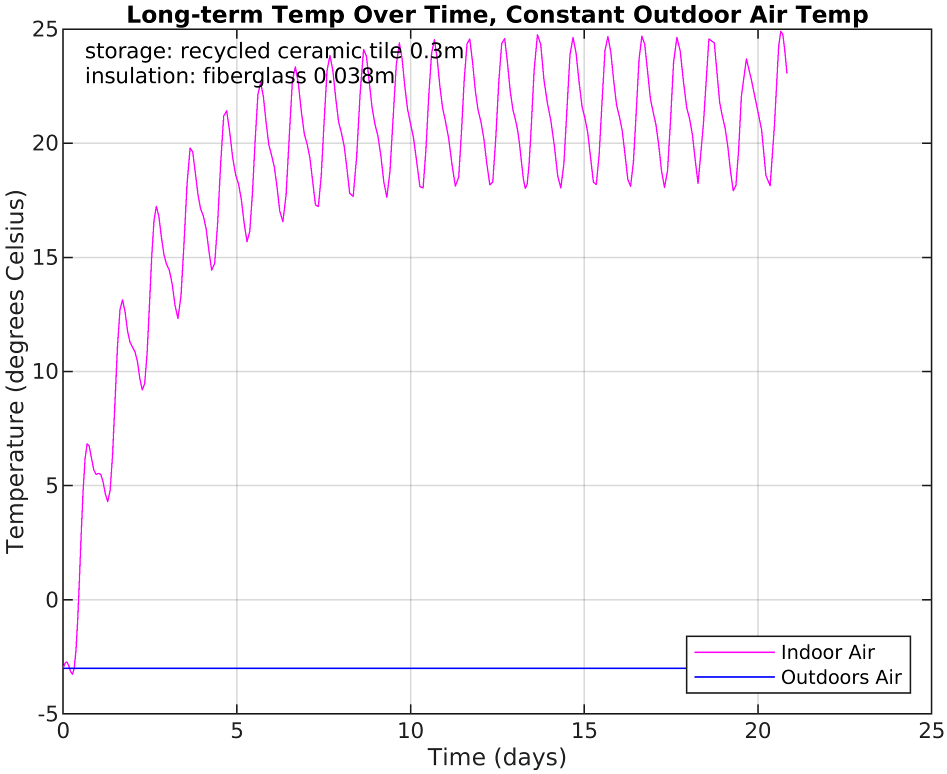
```
tile_storage = PureThermalStorage("recycled ceramic tile", 800, 3000)
```

```
tile_storage = 
  PureThermalStorage with properties:

                LongName: "recycled ceramic tile"
    SpecificHeatCapacity: 800
                 Density: 3000
```

```
fiberglass_insulation = SolidPureThermalResistance("fiberglass", 0.04)
```

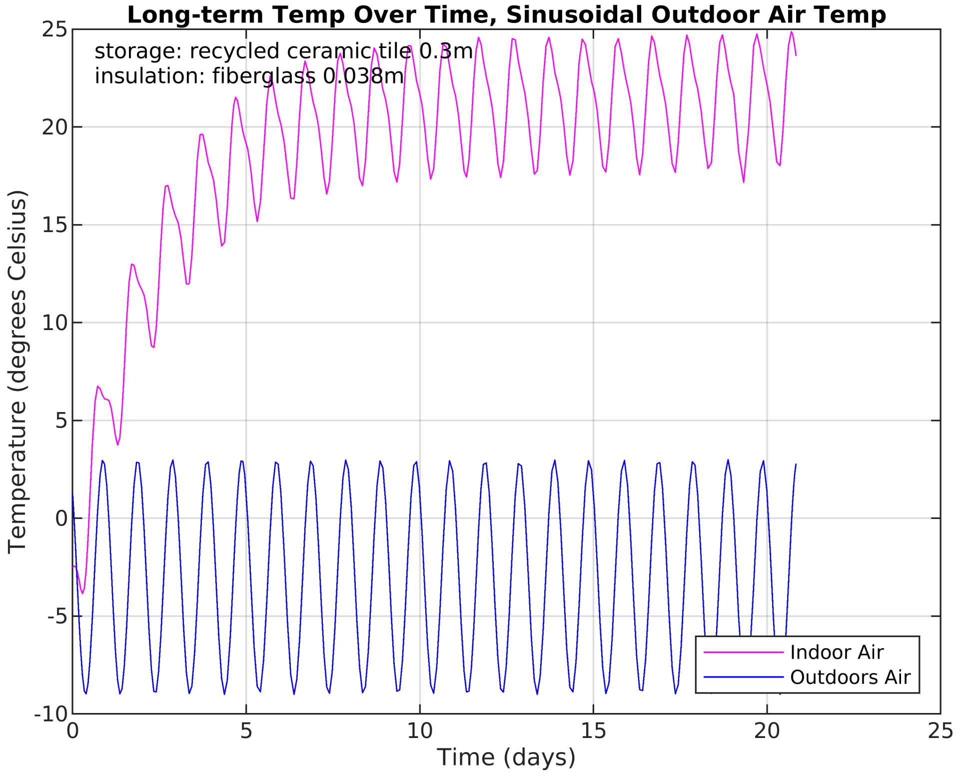
```
fiberglass_insulation = 
  SolidPureThermalResistance with properties:

               LongName: "fiberglass"
    ThermalConductivity: 0.0400
```

```
tile_storage_thickness = 0.30;
fiberglass_insulation_thickness = 0.038;

[t_days_c, T_inside_air_c] = ODEHelper(500, "constant", tile_storage_thickness, tile_st
```

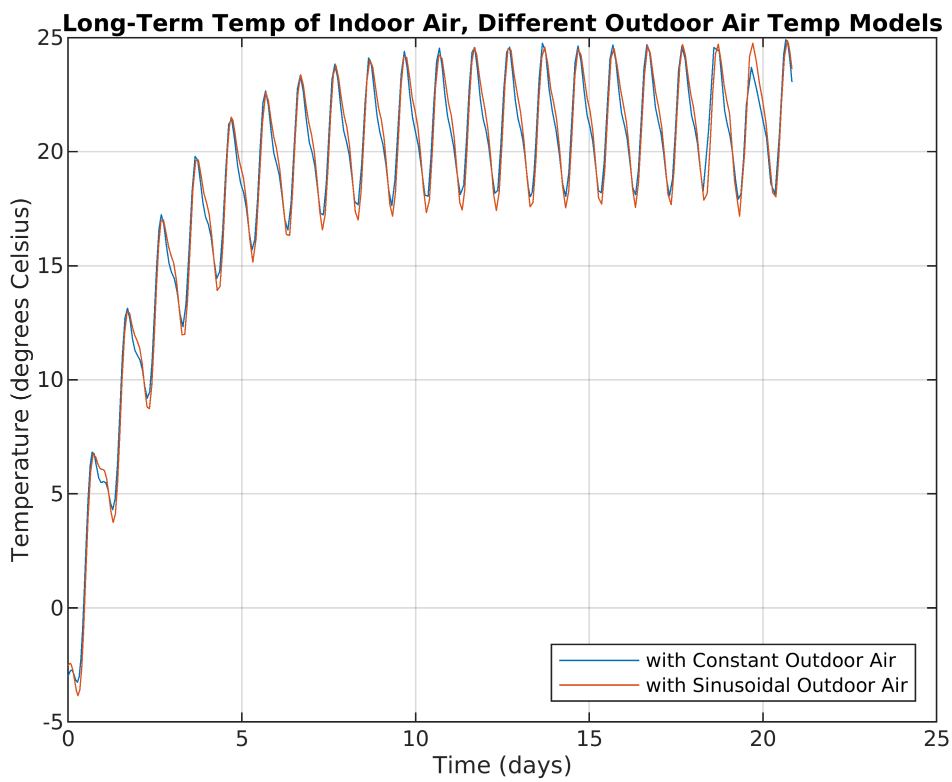


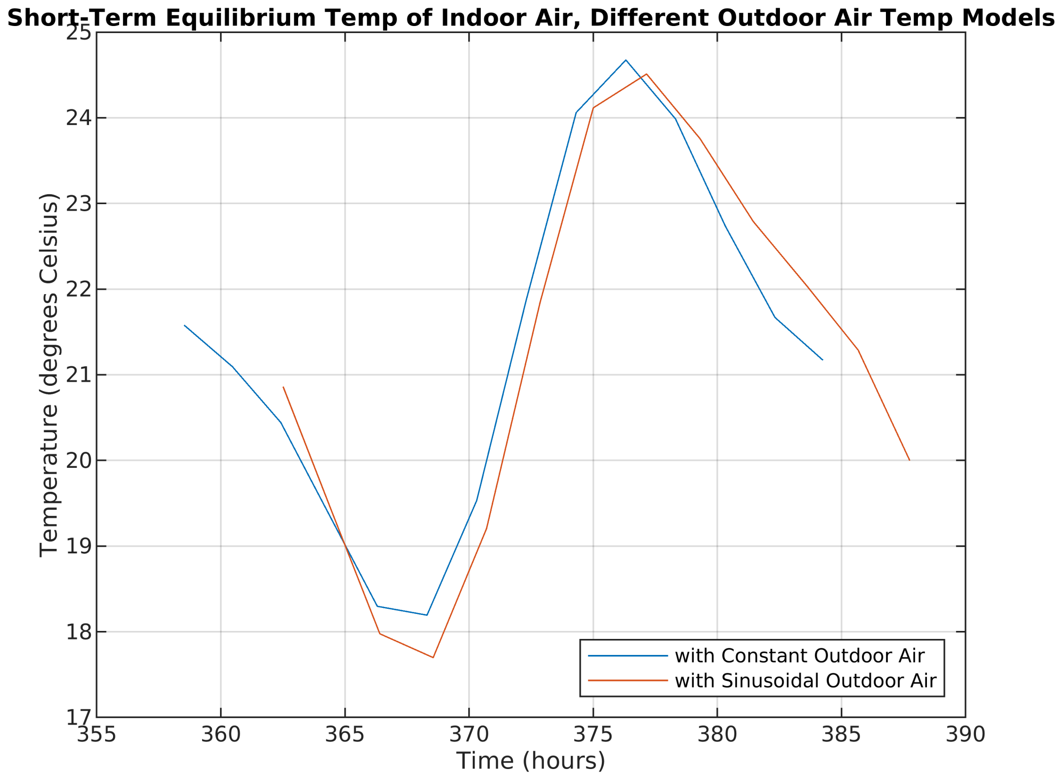
For Model With Sinusoidal Outside Air Temperature

```
[t_days_s, T_inside_air_s] = ODEHelper(500, "sinusoidal", tile_storage_thickness, tile_
```




```
plotComparison(t_days_c, T_inside_air_c, t_days_s, T_inside_air_s);
```





Optimization

Find best thickness for storage material and insulation

Compare different materials

Goal: temperature fluctuation should be between 17-25 degrees Celcius

Fiddle with:

- Number of hours to run simulation
- storage material thickness
- insulation material thickness
- material type

```
masonry_storage = PureThermalStorage("masonry", 1000, 2300)
```

```
masonry_storage = 
  PureThermalStorage with properties:

               LongName: "masonry"
    SpecificHeatCapacity: 1000
                Density: 2300
```

```
water_storage = PureThermalStorage("water", 4200, 1000)
```

```
water_storage = 
  PureThermalStorage with properties:
```

```
              LongName: "water"
  SpecificHeatCapacity: 4200
               Density: 1000
```

```
carpet_insulation = SolidPureThermalResistance("carpet", 0.05)
```

```
carpet_insulation = 
  SolidPureThermalResistance with properties:

               LongName: "carpet"
    ThermalConductivity: 0.0500
```

```
gypsum_plaster_insulation = SolidPureThermalResistance("gypsum plaster", 0.5)
```

```
gypsum_plaster_insulation = 
  SolidPureThermalResistance with properties:

               LongName: "gypsum plaster"
    ThermalConductivity: 0.5000
```

```
aircrete_insulation = SolidPureThermalResistance("aircrete", 0.15)
```

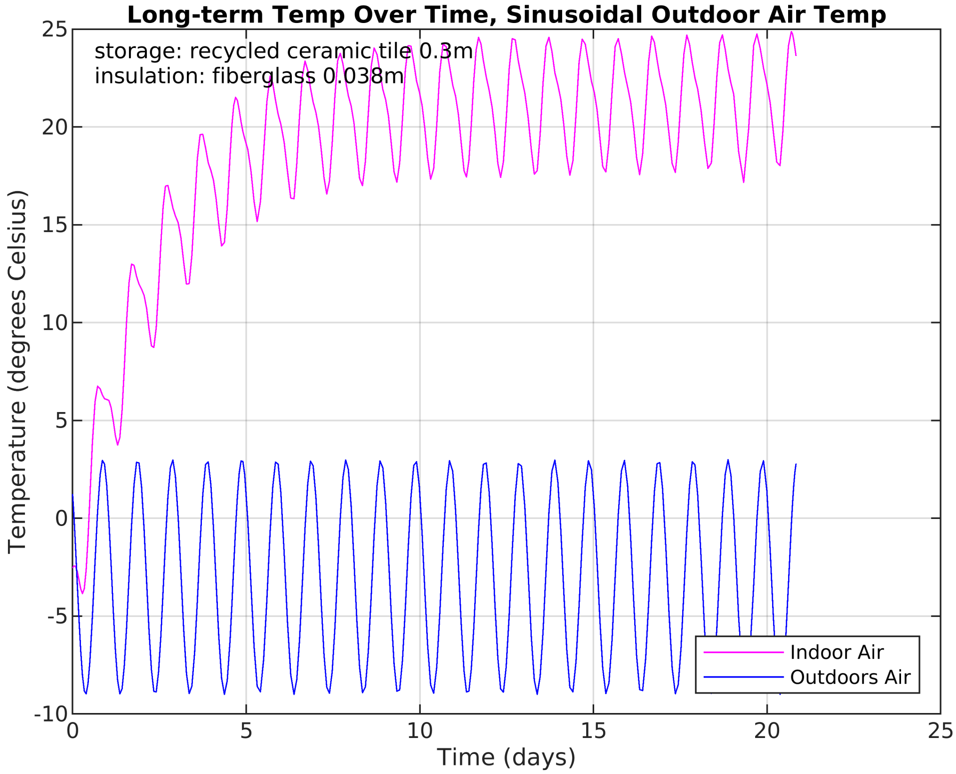
```
aircrete_insulation = 
  SolidPureThermalResistance with properties:

               LongName: "aircrete"
    ThermalConductivity: 0.1500
```

Try out 12 Combinations and compare required thicknesses

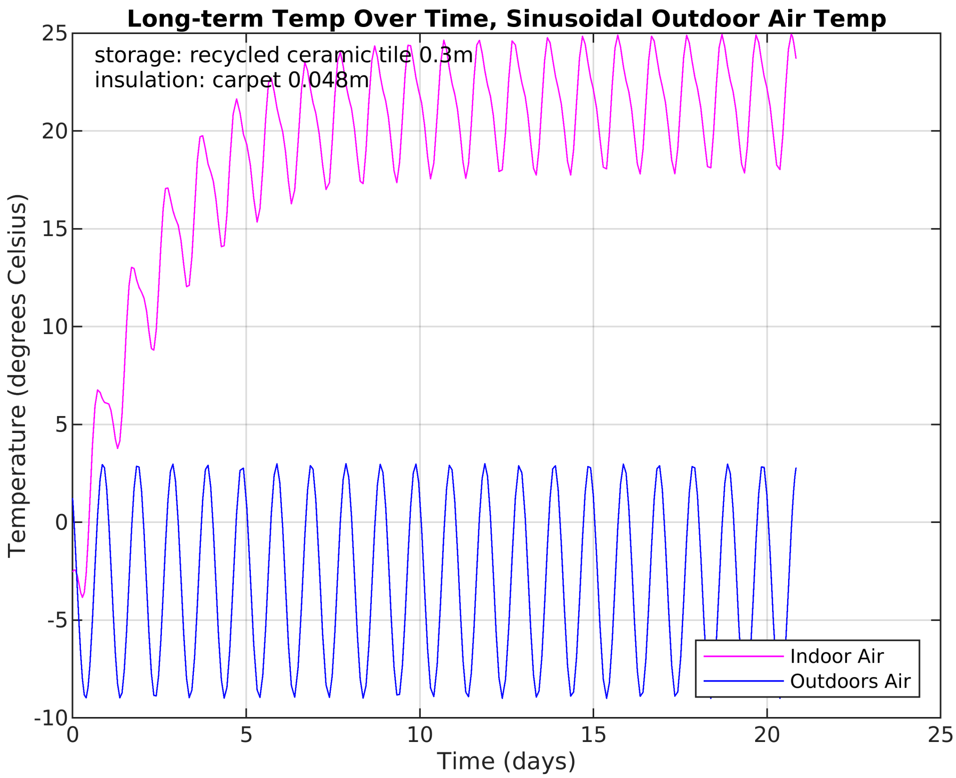
- tile + fiberglass (base)
- tile + carpet
- tile + gypsum plaster
- tile + aircrete
- masonry + fiberglass (base)
- masonry + carpet
- masonry + gypsum plaster
- masonry + aircrete
- water + fiberglass (base)
- water + carpet
- water + gypsum plaster
- water + aircrete

Tests

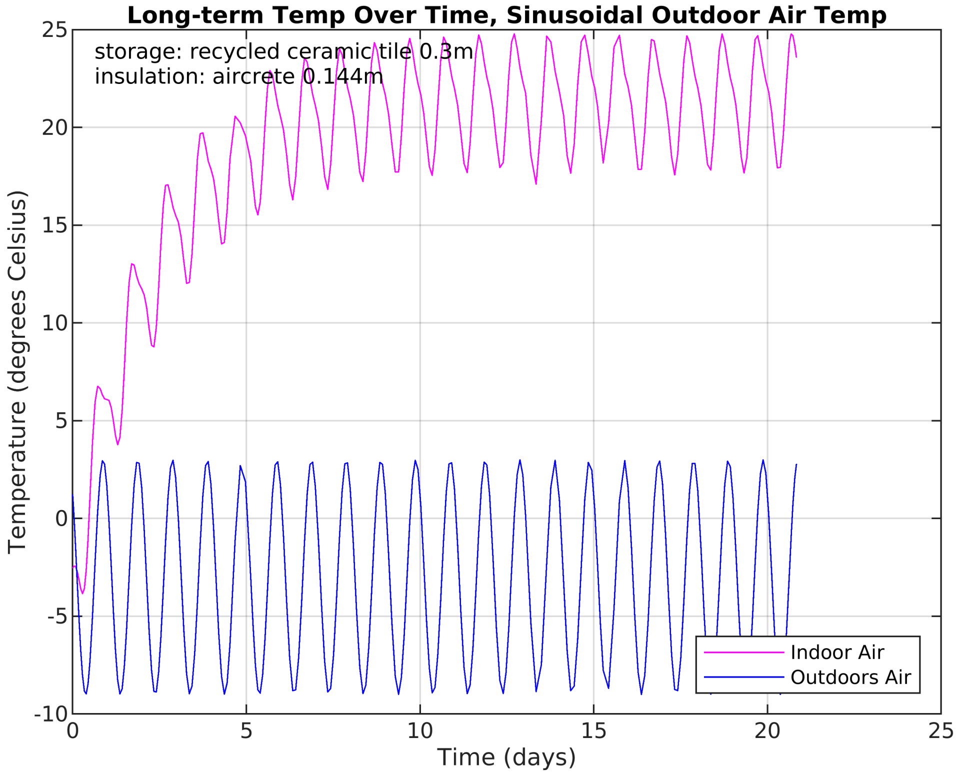
```
masonry_storage_thickness = 0.30;
water_storage_thickness = 0.15;

carpet_insulation_thickness = 0.048;
aircrete_insulation_thickness = 0.144;
gypsum_plaster_insulation_thickness = 0.485;

ODEHelper(500, "sinusoidal", tile_storage_thickness, tile_storage, fiberglass_insulatio
```



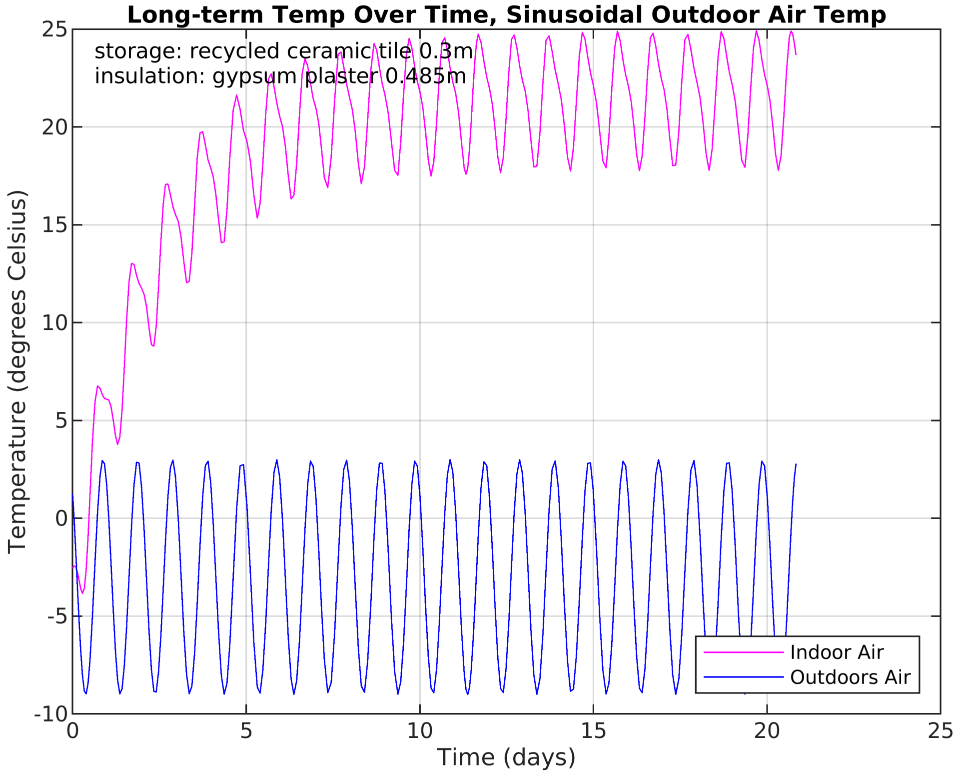

```
ODEHelper(500, "sinusoidal", tile_storage_thickness, tile_storage, carpet_insulation_th
```



```
ODEHelper(500, "sinusoidal", tile_storage_thickness, tile_storage, aircrete_insulation_
```

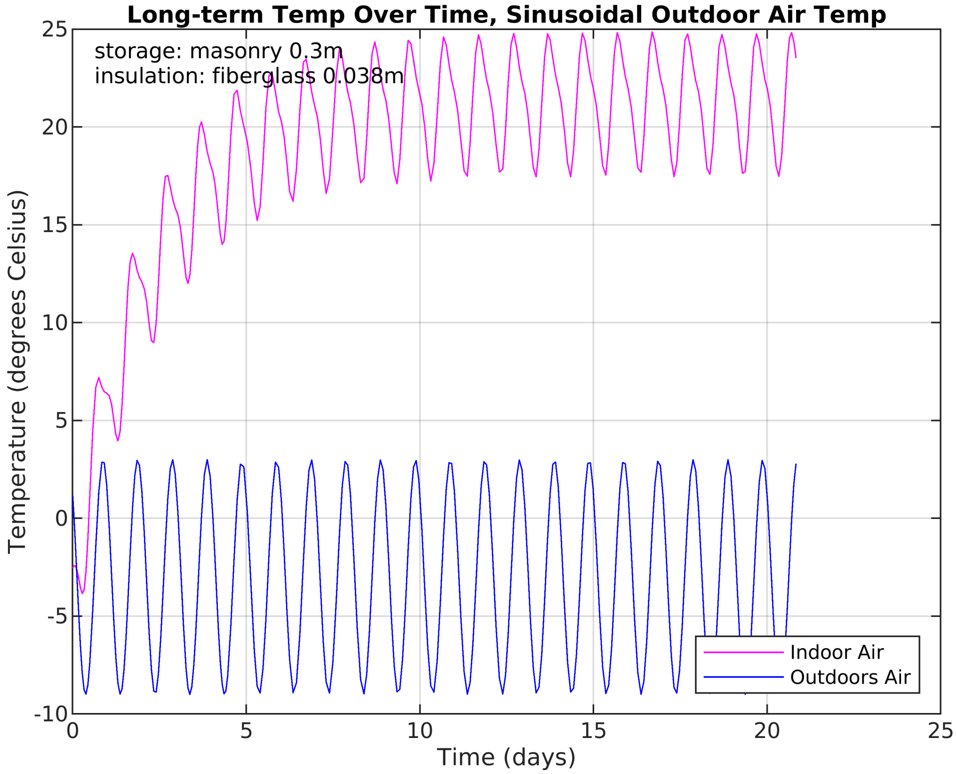



```
ODEHelper(500, "sinusoidal", tile_storage_thickness, tile_storage, gypsum_plaster_insul
```



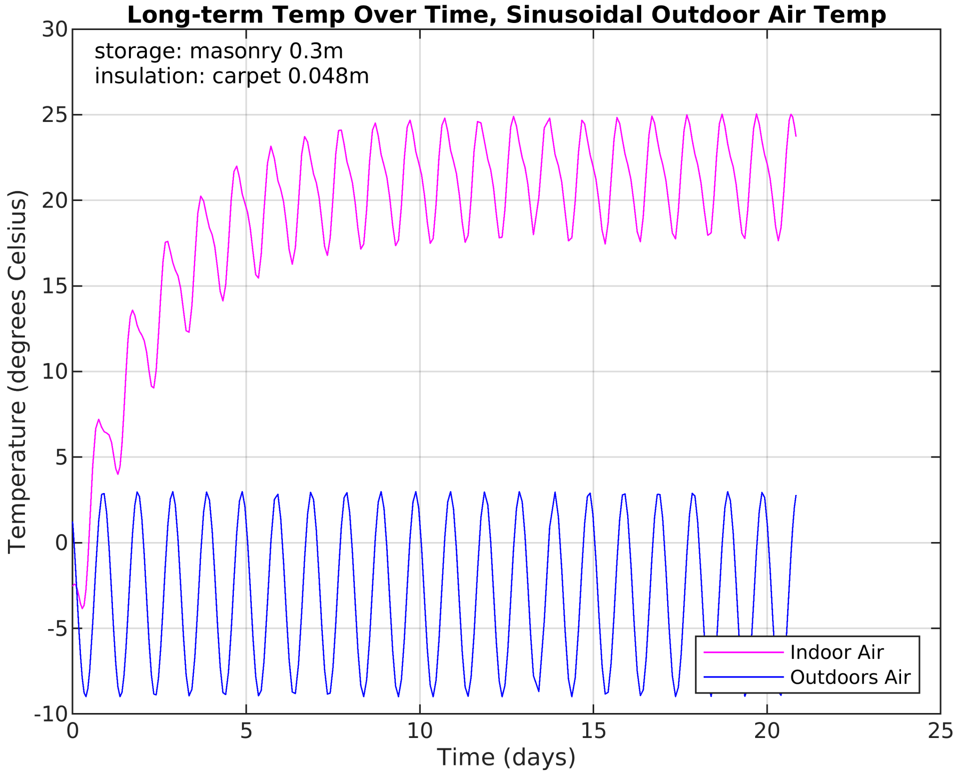
- masonry + fiberglass

```
ODEHelper(500, "sinusoidal", masonry_storage_thickness, masonry_storage, fiberglass_ins
```



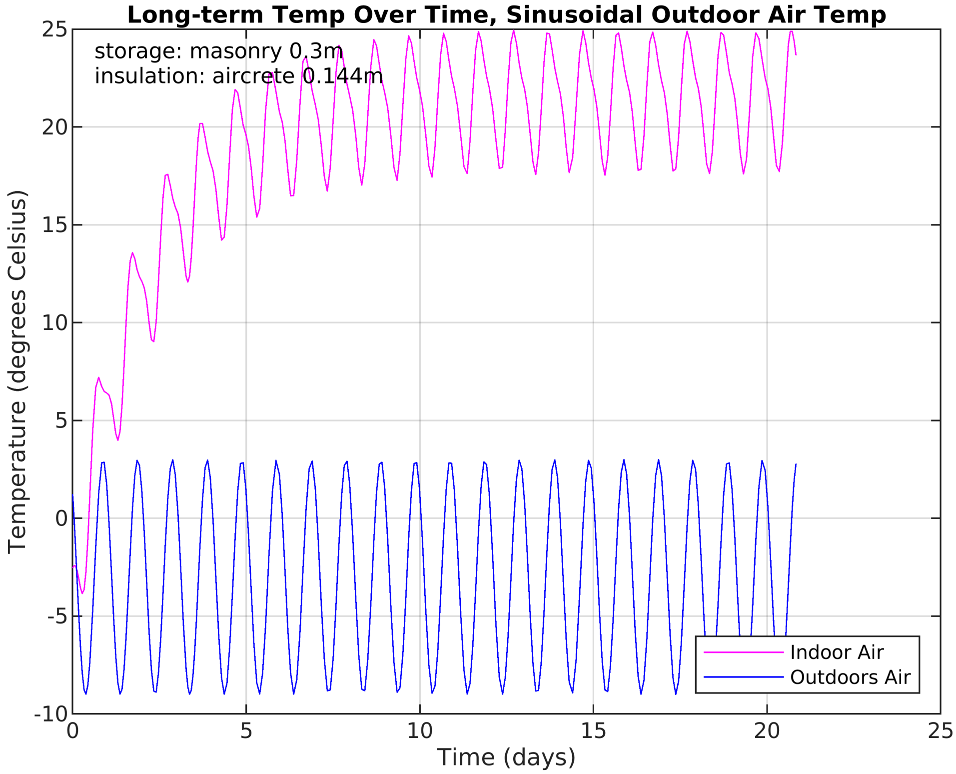
- masonry + carpet

```
ODEHelper(500, "sinusoidal", masonry_storage_thickness, masonry_storage, carpet_insulat
```



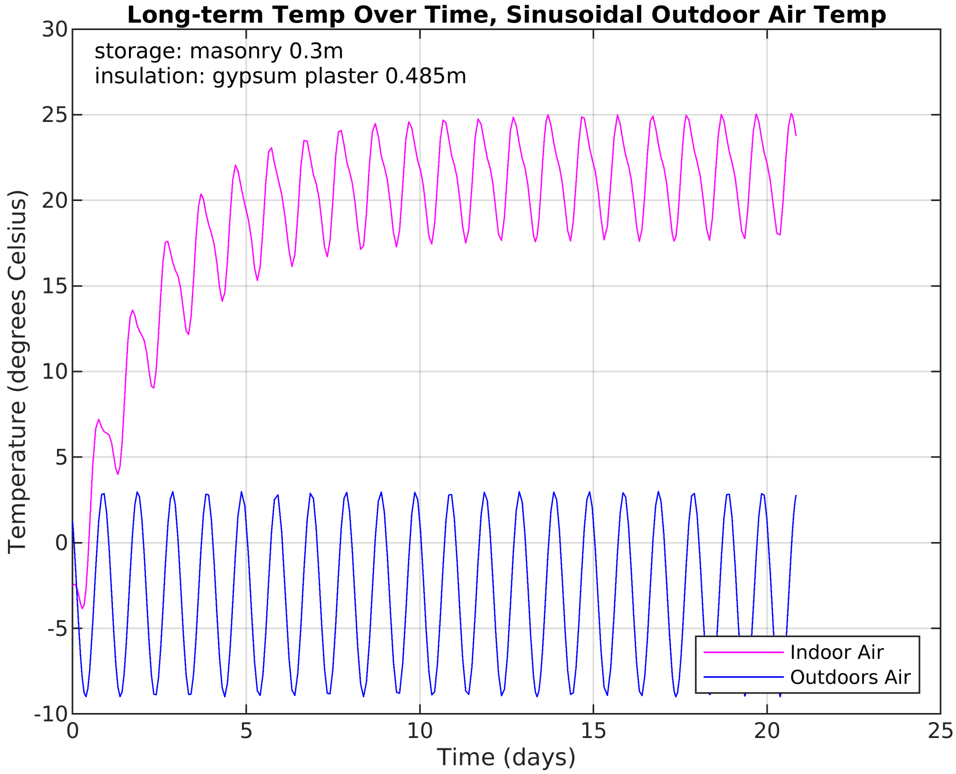
- masonry + aircrete

```
ODEHelper(500, "sinusoidal", masonry_storage_thickness, masonry_storage, aircrete_insul
```



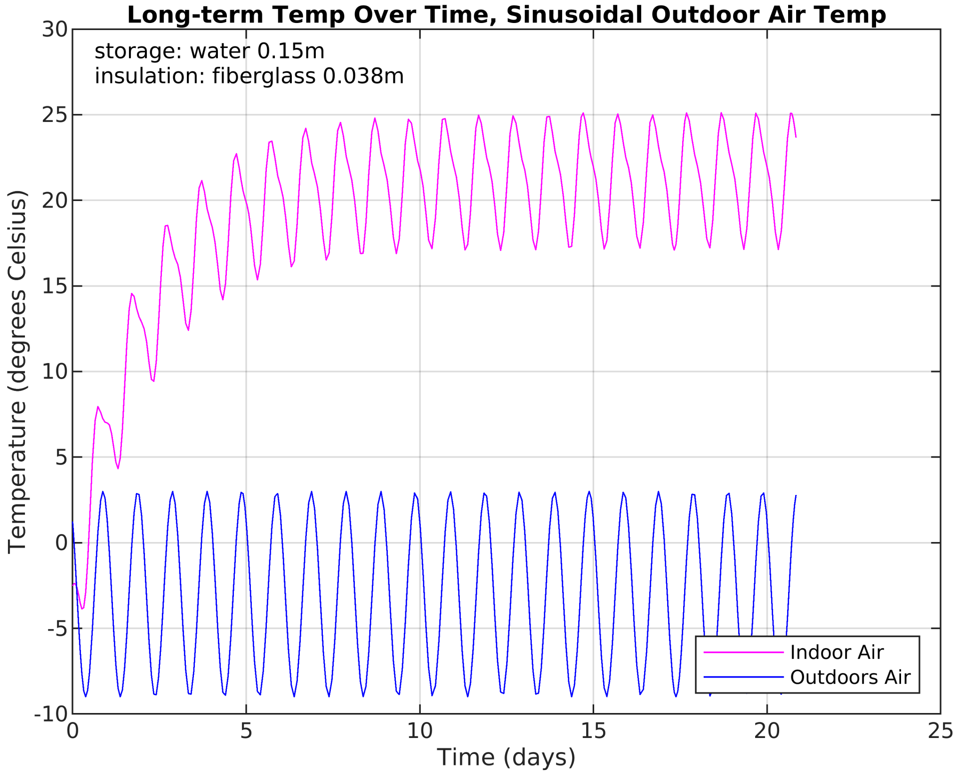
- masonry + gypsum plaster

```
ODEHelper(500, "sinusoidal", masonry_storage_thickness, masonry_storage, gypsum_plaster
```



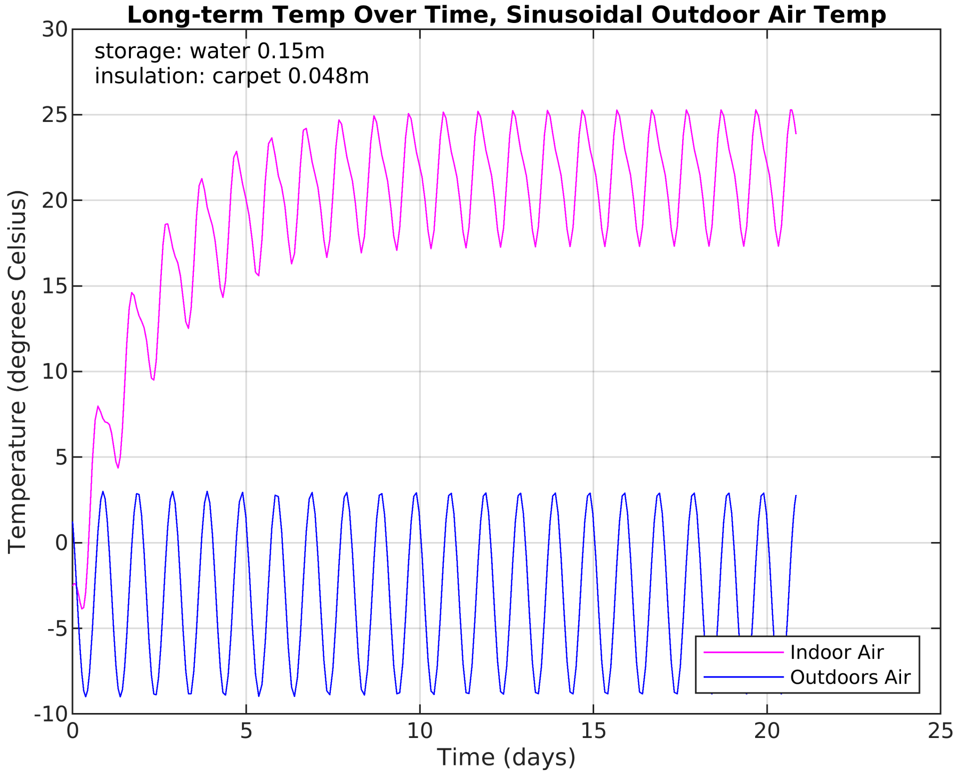
- water + fiberglass

```
ODEHelper(500, "sinusoidal", water_storage_thickness, water_storage, fiberglass_insulat
```



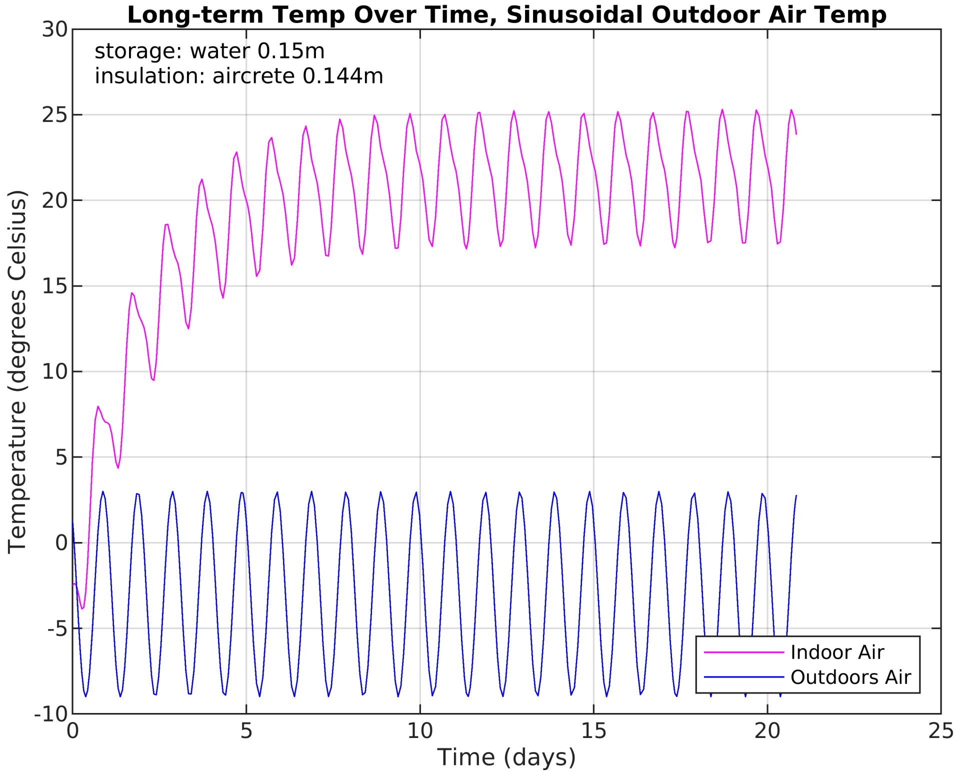
- water + carpet

```
ODEHelper(500, "sinusoidal", water_storage_thickness, water_storage, carpet_insulation_
```



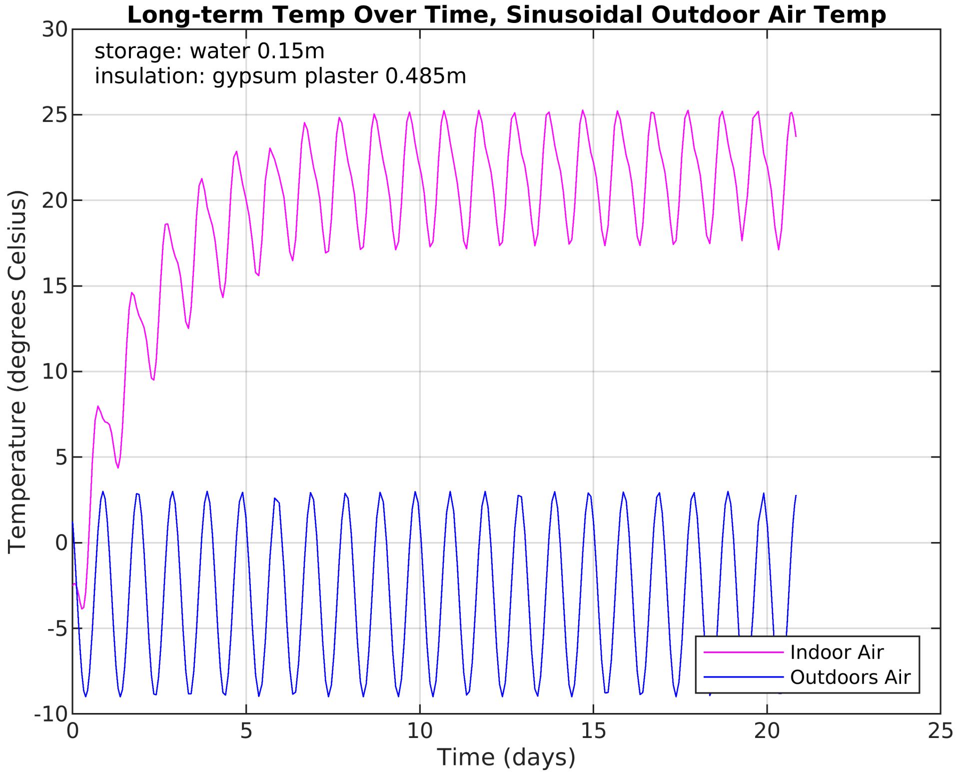
- water + aircrete

```
ODEHelper(500, "sinusoidal", water_storage_thickness, water_storage, 0.144, aircrete_in
```



- water + gypsum plaster

```
ODEHelper(500, "sinusoidal", water_storage_thickness, water_storage, gypsum_plaster_ins
```



Optimization Conclusions

```
% table of ideal thermal storage thicknesses
S.Name = [tile_storage.LongName; masonry_storage.LongName; water_storage.LongName];
S.SpecificHeatCapacity = [tile_storage.SpecificHeatCapacity; masonry_storage.SpecificHe
S.Density = [tile_storage.Density; masonry_storage.Density; water_storage.Density];
S.IdealThickness = [tile_storage_thickness; masonry_storage_thickness; water_storage_th
storage_table = struct2table(S)
```

storage_table = 3×4 table

	Name	SpecificHeatCapacity	Density	IdealThickness
1	"recycled c...	800	3000	0.3000
2	"masonry"	1000	2300	0.3000
3	"water"	4200	1000	0.1500

```
% table of idea thermal insulation thicknesses
I.Name = [fiberglass_insulation.LongName; carpet_insulation.LongName; aircrete_insulati
I.ThermalConductivity = [fiberglass_insulation.ThermalConductivity; carpet_insulation.T
I.IdealThickness = [fiberglass_insulation_thickness; carpet_insulation_thickness; aircr
insulation_table = struct2table(I)
```

insulation_table = 4×3 table

	Name	ThermalConductivity	IdealThickness
1	"fiberglass"	0.0400	0.0380

	Name	ThermalConductivity	IdealThickness
2	"carpet"	0.0500	0.0480
3	"aircrete"	0.1500	0.1440
4	"gypsum pla...	0.5000	0.4850

ODE Solver Helper

```
function [t_days, T_inside_air] = ODEHelper(t_end, air_temp_outside_model_type, storage
    % Solve ODE for the temperature of air inside model house over time
    %Input:
    %   t_end                               (number) number of hours, range of ODE
    %   air_temp_outside_model_type         (string) "sinusoidal" or (default) "constan
    %   storage_thickness:                  (number) thickness of storage material (m)
    %   storage_material:                   (PureThermalStorage)
    %   insulation_thickness:               (number) thickness of insulation material (
    %   insulation_material:                (SolidPureThermalResistance)
    %Output:
    %   t_days                              (column vector) time range measured in days
    %   T_inside_air                        (column vector) temperature of inside air s

    [air_temp_outside_name, air_temp_outside_function] = air_temp_outside_model(air_tem

    [C_storage, Rtot_inside_air_to_outside_air, Rtot_storage_to_outside_air, b_window_a
    yprime = @(t, y) (Qinprime(t, b_window_area) - (y - air_temp_outside_function(t)) /

    % convert T_end from hours to seconds
    t_end_sec = t_end * 60 * 60;

    % ODE constants
    T_storage_0 = -3;
    tspan = [1 t_end_sec];
    % solve ODE for heat storage temp
    [t, y] = ode45(yprime, tspan, T_storage_0);

    % solve for air temp
    T_outside_air = air_temp_outside_function(t);
    T_inside_air = T_outside_air + Rtot_inside_air_to_outside_air * (y - T_outside_air)

    %convert t from seconds to days for readability
    t_days = t / (60 * 60 * 24);

    % make annotation string for readability
    annotation = {"storage: " + storage_material.LongName + " " + storage_thickness + "
        , "insulation: " + insulation_material.LongName + " " + insulation_thickness +

    plotHelper(annotation, "Long-term Temp Over Time, " + air_temp_outside_name, t_days
end
```

Plotting Helpers

```
function [] = plotHelper(annotation, plot_title, t, t_label, ~, T_inside_air, T_outside
```

```
    % Plots Heat Storage, Indoor Air, Outdoors Air over time
    figure
    plot(t, T_inside_air, 'm')
    hold on
    plot(t, T_outside_air, 'b')
    grid on
    xlabel(t_label)
    ylabel('Temperature (degrees Celsius)')
    title(plot_title)
    legend('Indoor Air', 'Outdoors Air', 'Location', 'southeast')

    % for easy identification
    text(0.025,0.95,annotation,'Units','normalized','FontSize',8)

    ax = gca;
    ax.FontSize = 8;
    hold off
end

function [t_hours_section, T_inside_air_section] = getEquilibriumIdx(t_days, T_inside_a
    % Get index of point half-way through time span. Return section of
    % 1-day period starting half-way through time span, measured in hours
    % and degrees Celcius.
    %
    %Input:
    %   t_days                              (column vector) time range measured in days
    %   T_inside_air                        (column vector) corresponding temperature c
    %Output:
    %   t_hours_section                     (column vector) section of time range measu
    %   T_inside_air_section                (column vector) corresponding section of te

    %convert t from days to hours for nice plots
    t_hours = t_days * 24;
    % find indexes of 24 hour range in equilibrium time
    % eyeball check that equilibrium was reached at least 3/4 of the way through the ti
    s = ceil(size(t_hours, 1) * (3/4));
    e = find(t_hours>t_hours(s)+24);
    e = e(1); % end 1-day period in hours
    t_hours_section = t_hours(s:e);
    T_inside_air_section = T_inside_air(s:e);
end

function [] = plotComparison(t_days_c, T_inside_air_c, t_days_s, T_inside_air_s)
    %Plot the solved indor air temperature over time for both
    %   Using a constant outdoor air temperature model
    %   Using a sinusoidal outdoor air temperature model

    %plot long term temp
    figure
    plot(t_days_c, T_inside_air_c)
    hold on
    grid on
    plot(t_days_s, T_inside_air_s)
    xlabel("Time (days)")
```

```
    ylabel("Temperature (degrees Celsius)")
    title("Long-Term Temp of Indoor Air, Different Outdoor Air Temp Models")
    legend("with Constant Outdoor Air", "with Sinusoidal Outdoor Air", "Location", "sou
    ax = gca;
    ax.FontSize = 8;
    hold off

    %plot temperature over course of one day at equilibrium
    [t_hours_section_c, T_inside_air_section_c] = getEquilibriumIdx(t_days_c, T_inside_
    [t_hours_section_s, T_inside_air_section_s] = getEquilibriumIdx(t_days_s, T_inside_
    figure
    plot(t_hours_section_c, T_inside_air_section_c)
    hold on
    grid on
    plot(t_hours_section_s, T_inside_air_section_s)
    xlabel("Time (hours)")
    ylabel("Temperature (degrees Celsius)")
    title("Short-Term Equilibrium Temp of Indoor Air, Different Outdoor Air Temp Models
    legend("with Constant Outdoor Air", "with Sinusoidal Outdoor Air", "Location", "sou
    ax = gca;
    ax.FontSize = 8;
    hold off
end
```