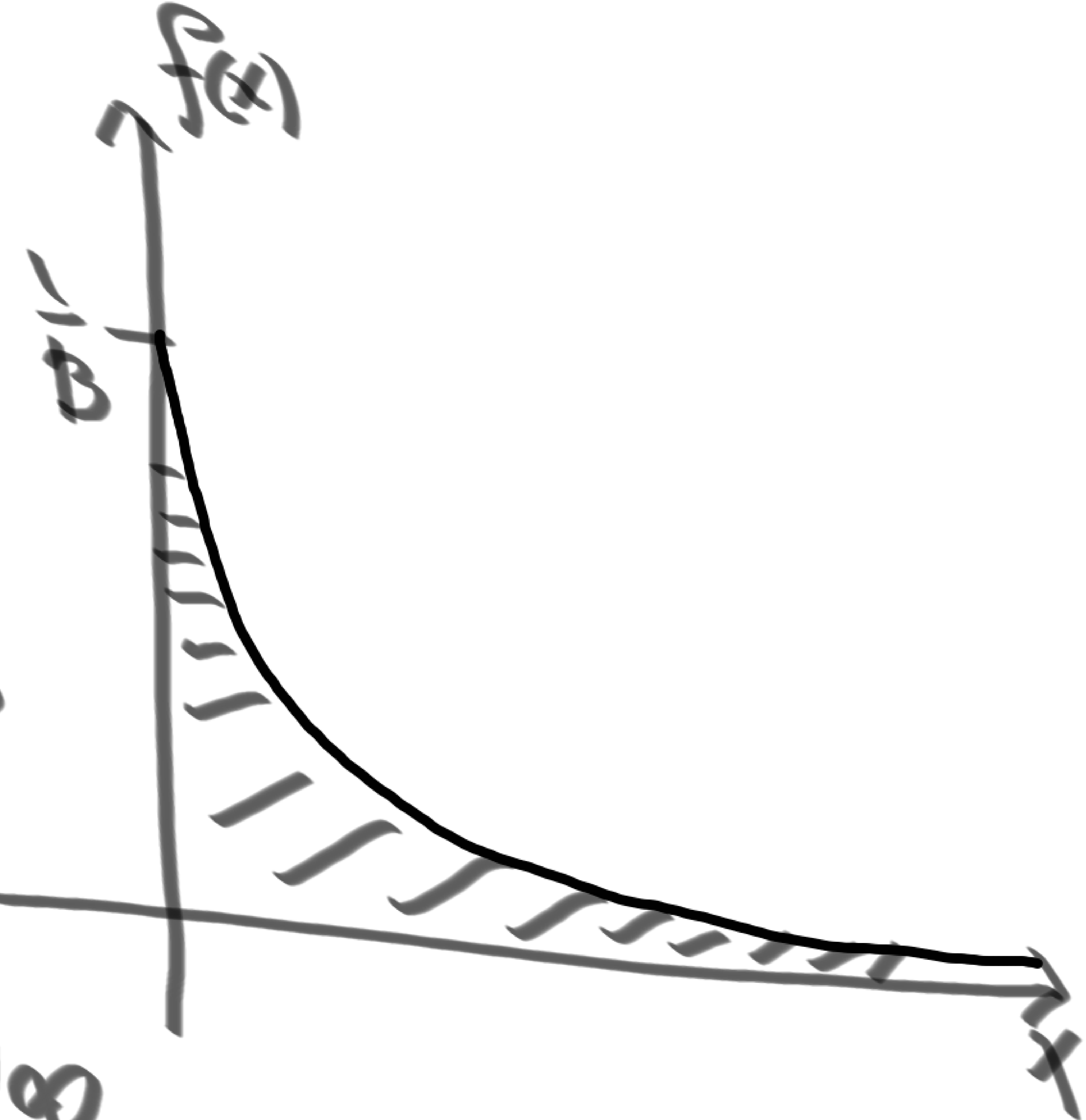
$$X \sim \mathrm{Exp}(\beta)$$

$$\int_0^\infty \underbrace{f(x)}\,dx = 1$$

$$\int_0^\infty \frac{1}{\beta} e^{-x/\beta}\,dx = \left. +e^{-x/\beta} \right]_\infty^0$$

$$= 1 - 0 = 1.$$



$X \sim Exp(\beta)$

① mean of $X \Rightarrow E(X) = \beta$.

$$\Rightarrow E(X) = \int_0^\infty x f(x)\,dx = \int_0^\infty \frac{x}{\beta} e^{-x/\beta}\,dx.$$

$$= \beta \int_0^\infty \frac{x}{\beta} e^{-x/\beta}\, d\frac{x}{\beta}$$

$$= \beta\,\Gamma(2) = \beta$$

Rule

$$\int_0^\infty \square^{?} e^{-\square}\, d\square = \Gamma(?+1)$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma(n) = (n-1)\Gamma(n-1)$$

$$\Gamma(1/2) = \sqrt{\pi}$$

② $Var(X) = \beta^2 \Rightarrow Var(X) = E(X^2) - (E(X))^2$

$$E(X^2) = \int_0^\infty x^2 f(x)\,dx = \beta^2 \int_0^\infty \left(\frac{x}{\beta}\right)^2 e^{-x/\beta} \frac{dx}{\beta}$$

$$= \beta^2 \Gamma(3) = 2\beta^2$$

$2\beta^2$

β^2

$Var(X) = \beta^2$

$$\int_0^\infty \boxed{\text{ص}}^{?} e^{-\boxed{\text{ص}}}\, d\boxed{\text{ص}} = \Gamma(?+1)$$

CDF 의 $F(x)$.

$X \sim Exp(\beta)$

β: mean

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/\beta} & x > 0 \end{cases}$$

$\Rightarrow X \sim Exp(\beta)$

① $E(X) = \beta$

② $Var(X) = \beta^2$

③ $F(x)$

Example

let X be an $\text{Exp}(\overset{\beta}{0.2})$, write The pdf

ⓐ $f(x) = 0.2\, e^{-0.2x}$ $\quad x > 0$

ⓑ $f(x) = \frac{1}{5} e^{-x/5}$ $\quad x > 0$ $\quad \cancel{-\frac{1}{5}x}$

ⓒ $f(x) = 5\,\cancel{x}\left(e^{-5x}\right)$ $\quad x > 0$

ⓓ $f(x) = 5\,e$ $\quad x > 0$

$X \sim \text{Exp}(\beta)$

$$f(x) = \frac{1}{\beta} e^{-x/\beta} \quad x > 0$$

$$= \frac{1}{0.2} e^{-x/0.2} \quad x > 0$$

$$= 5e^{-5x} \quad x > 0$$

find $P(X>1)$ $\quad P(X<2)$

$P(1<X<3)$.

$X \sim Exp(0$

$f(x) = 5e^{-5x}$

$F(x) = 1-e^{-5x}$

$$P(X>1) = 1 - P(X \leq 1)$$

$$= 1 - F(1)$$

$$= 1 - (1 - e^{-5}) = e^{-5}$$

$$P(X<2) = 1 - e^{-5(2)} = 1 - e^{-10}$$

$$P(1<X<3) = F(3) - F(1)$$

$$= \left(1 - e^{-15}\right) - \left(1 - e^{-5}\right)$$

$$= e^{-5} - e^{-15}$$ =).

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & x > 0 \\ 0 & \text{o.w} \end{cases}$$

Construct CDF of X.

$$F(x) = \begin{cases} 0 & X < 0 \\ \int_0^x f(t)\,dt & X \geq 0 \end{cases}$$

$$F(x) = P(X \leq x$$

$$\int_0^x \frac{1}{\beta} e^{-t/\beta}\,dt$$

$$-e^{-t/\beta}\Big]_0^x = e^{-t/\beta}\Big]_x^0$$

$$= 1 - e^{-x/\beta}$$