Lec 4: Gaussian & Friends

$$\left(\frac{\overline{X}_n-\mu}{\sigma/\sqrt{n}}\right)^2 \rightsquigarrow N(0,1) \qquad (\overline{X}_n)^2$$

↳ want $\dfrac{(\overline{X}_n)^2 - 0}{0}$ ← this is what delta func does.

$$\overset{Y_n}{g(\overline{X}_n)} = g(\mu) + g'(\mu)\overset{Y_n-\mu}{(\overline{X}_n-\mu)} + \dots$$

g is differentiable

$$g(\overline{X}_n) - g(\mu) = g'(\mu)(\overline{X}_n-\mu) + \dots$$

$$g(\overline{X}_n) - g(\mu) = g'(\mu)(\overline{X}_n-\mu)$$

$$\frac{g(\overline{X}_n)-g(\mu)}{\sigma/\sqrt{n}} = g'(\mu)\left(\frac{\overline{X}_n-\mu}{\sigma/\sqrt{n}}\right) \rightsquigarrow N(0,\underbrace{(g'(\mu))^2}_{\text{Var.}})$$

★ works if g = differentiable & $g'(\mu) \neq 0$

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if $\sqrt{n}(Y_n-\mu) \rightsquigarrow N(0,\sigma^2)$

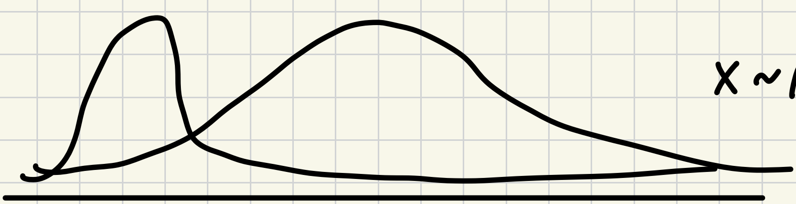
then $\sqrt{n}(g(Y_n)-g(\mu)) \rightsquigarrow N(0,\sigma^2 g(\mu)^2)$

(if g in C' at μ & $g'(\mu) \neq 0$)

C' = differentiable w/ . derivative

Discrete	Continuous	
* Binomial	* Gaussian / Normal	
* Bernoulli	* Exponential	* Gamma
* Poisson	* Uniform	* Beta
	* Log normal	* student-t
* Geometric	* Cauchy	
	* Chi-Squared Dist.	
	* Pareto	

$$\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

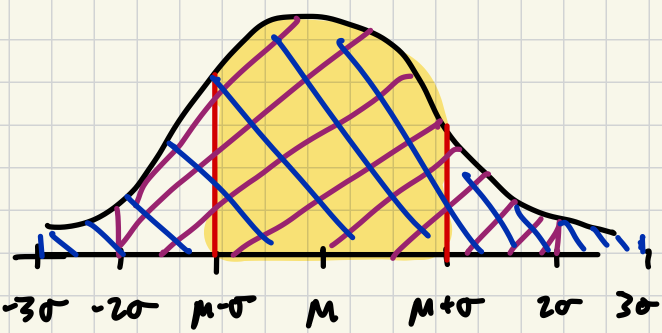


$X \sim N(\mu, \sigma^2)$

$\mathbb{P}(|x-\mu| \leq \sigma) \simeq 0.65$

$\mathbb{P}(|x-\mu| \leq 2\sigma) \simeq \boxed{0.95}$

$\mathbb{P}(|x-\mu| \leq 3\sigma) \simeq 0.997$



$X \sim N(\mu, \sigma^2)$

$aX + b \sim N(a\mu + b,\ a^2\sigma^2)$

$X \sim N(\mu, \sigma^2)$

$X = \mu + \sigma Z$

where $Z \sim N(0,1)$

$\downarrow$

standardization of X

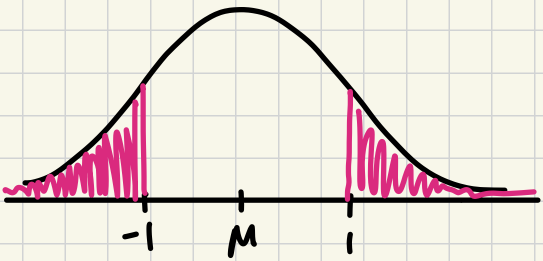
$Z = \dfrac{X-\mu}{\sigma}$

$$\begin{aligned}
&\mathbb{P}(c \le X \le d) \\
&= \mathbb{P}(c-\mu \le X-\mu \le d-\mu) \\
&= \mathbb{P}\left(\frac{c-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{d-\mu}{\sigma}\right) \\
&= \mathbb{P}\left(\frac{c-\mu}{\sigma} \le Z \le \frac{d-\mu}{\sigma}\right) \\
&\ \vdots
\end{aligned}$$

the CDF of $N(0,1) \rightarrow \Phi(x) = \mathbb{P}(Z \le x)$

$$\vdots$$

$$= \Phi\left(\frac{d-\mu}{\sigma}\right) - \Phi\left(\frac{c-\mu}{\sigma}\right)$$



$\Phi(-1) = 1 - \Phi(1)$

$\Phi(-x) = 1 - \Phi(x)$

$$\begin{aligned}
\mathbb{P}(|Z| \ge 1) &= \mathbb{P}(Z \le -1) + \mathbb{P}(Z \ge 1) \\
&= \Phi(-1) + (1 - \Phi(1)) \\
&= 2 - 2\Phi(1)
\end{aligned}$$

	0.00	0.01	...
0.1			
0.2			
0.3			
$\vdots$			
1.1		$0.8665 = \Phi(1.11)$	
1.2			

Think this will be on test to use.

Intro to lec 5

$$v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{pmatrix} \in \mathbb{R}^k = (v_1, \dots, v_k)^T$$

transposed, but we don't really care in this class

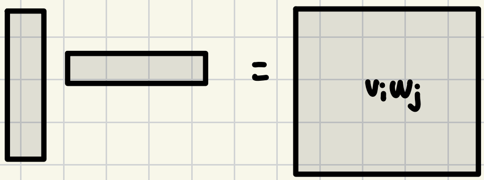
$$A = \begin{pmatrix} a_{11} & \dots & a_{1l} \\ \vdots & & \vdots \\ a_{k1} & \dots & a_{kl} \end{pmatrix} \in \mathbb{R}^{k \times l}$$

vector multiplication

$$v, w \in \mathbb{R}^k, \quad v \times w = v^T w = \langle v, w \rangle = \sum_{i=1}^{k} v_i w_i$$

$v \in \mathbb{R}^k$
$w \in \mathbb{R}^l$

$vw^T =$



why? (SVD thm)

I_k : identity $\in \mathbb{R}^{k \times k}$

Random Vector:

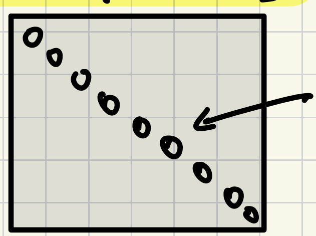
$X = (x_1, \dots, x_k) \in \mathbb{R}^k$ is a random vector.

$$\mathbb{E}[X] = (\mathbb{E}[X_1], \dots, \mathbb{E}[X_k])$$

Covariance Mtx:

$$\Sigma_{ij} = Cov(x_i, x_j)$$
$$= \mathbb{E}[(x_i - \mathbb{E}x_i)(x_j - \mathbb{E}x_j)]$$
$$= \mathbb{E}[x_i x_j] - (\mathbb{E}x_i)(\mathbb{E}x_j)$$

$$Cov(x_i, x_i) = \mathbb{V}[x_i]$$



diag = variance