A Framework for Computing Crowd Emotions Using Agent Based Modeling

Mihail Popescu, James M. Keller, Alina Zare
Electrical and Computer Engineering Dept.
University of Missouri
Columbia, MO, USA
{popescum, kellerj, zarea}@missouri.edu

***Abstract*— We present a computational framework for modeling geospatial dynamics of crowd emotions as part of anticipatory analysis. The framework is based on agent based modeling (ABM) and evaluation-activation space for emotion representation. ABM is a simulation technique that uses the actions and interactions of individual agents to represent the behavior of a given population or system. The agent based models used in this paper employ geospatial data such as elevation, population density and locations of interest to define the context of agent decisions. We apply our framework to model the emotional dynamic of a population during disaster evacuation. In our examples we use a simple evacuation scenario: the agents learn of an incoming hurricane and evacuate for safety to the nearest shelter. At each time step, an agent chooses the direction of motion using a bounded rationality model that accounts for his or her emotional state (an evaluation-activation tuple) and geospatial context. After t time steps we use fuzzy set methods to analyze the crowd emotions in each of the N_S shelters.**
Keywords—crowd emotion; agent based modeling; bounded rationality; linguistic summarization

I. INTRODUCTION

Human Geography is concerned with how human-related factors, e.g. cultural, economic, religious, and political, influence the behavior of individuals and groups of people. The study of Human Geography is important in a number of applications including, for example, preparing for disaster response and relief. In order to study the influence of these human-related factors, mathematical models along with meaningful visualization need to be developed. Agent based approaches show promise for both modeling and visualization of geospatial events for such anticipatory reasoning.

Various agent based models for crowd disaster behavior in closed spaces have been described in the literature [1-6]. However, more realistic models need to account for factors related to agent personality and geospatial context. For example, agents in a human geographic model will not all have the same goal in mind, like exiting a burning room or crossing a narrow bridge. Rather, their goal will depend on factors like their psychosocial personality (e.g. age, health, income) and geographic location (e.g. mountain, plain, urban, rural). Moreover, population sentiment during evacuation is not pure irrational panic [2] but a more complex mixture of rational sentiments [7]. Furthermore, since disaster scenarios unfold over longer time frame than emergency exits, needs such as food and supplies must be also considered.

In previous [8-10] papers we proposed extensions to the closed space models [1-6] to make them usable for human geography investigation. In [8] we introduced a large space ABM where agents employ a factorial decision model that can account for personal or geographic context. In [9] we proposed a modified agent decision model based on bounded rationality. In [10] we introduced several agent communication models for ABM for simulating rumor spreading during disaster evacuation.

In this paper we extend the model proposed in [9] by introducing agent sentiments. The sentiments are modeled using the evaluation-activation framework proposed by Whissell [11]. In fact, similar models are wide spread in Psychology literature. For example, Russell [12] introduces a "circumplex" model in which 8 emotions are arranged on a circle in the arousal (activation) - pleasure (evaluation) plane. Other models that use two orthogonal dimensions, such as pleasantness and intensity, are mentioned in [13]. In this paper, agent affective state is coupled with the evacuation model and aggregated sentiment states are computed at evacuation centers with the goal of predicting potential conflicts.

The rest of our paper is structured as follows: in section II we describe the ABM framework used in this paper; in section III we give a short overview of the Whissell model; in section IV we describe our experiments; in section V we show experimental results; and in section VI we provide conclusions and future work.

II. CONTEXTUAL BOUNDED RATIONALITY (COBRA) MODELS

In [9] we introduced a contextual bounded rationality (CoBRa) ABM framework. Here we use the COBRA framework to model the dynamics of affect in a population during a disaster evacuation event. The diagram of the CoBRa model is shown in figure 1.

CoBRa models are contextual because the agents react to temporal and spatial changes in the environment. Environmental context is captured by a set of geospatial layers, C_i, i=1,M. Geospatial layers in raster format represent variables

978-1-4673-6010-4/13/$31.00

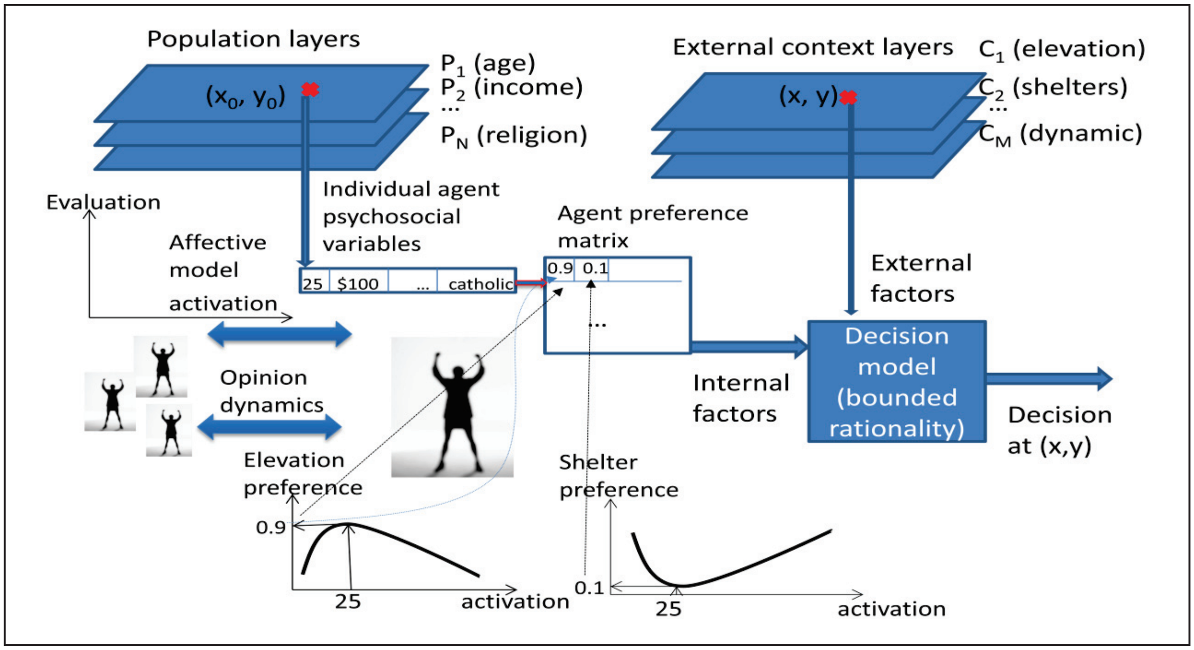

Figure 1. Contextual bounded rationality (CoBRa) model diagram

such as elevation, population density and distance to shelter. At the same time, agent reaction is mediated by its rationality and a set of individual psychosocial variables P_i, i=1,N, such as age, religion, income, etc. Each agent i moves to the adjacent grid cell j by assessing a set of probabilities $\{p_{ij}\}_{j=1,4}$. The combined influence of the agent variables (P_i) and context (C_i) on the motion probabilities p_{ij} can be described as [8]:

$$p_{ij}(C_1,...,C_M,P_1,...,P_N,t)=\left(\prod_{m=1}^{M}\frac{1}{1+K_{C_m}(P_1,...,P_N,t)C_{m,ij}}\right)n_{ij}\xi_{ij} \tag{1}$$

where n_{ij} is an occupancy factor (equal to 0 if the cell is occupied by another agent, and 1 else) and ξ_{ij} is a forbidden cell factor (equal to 0 if no agent is allowed to move there, and 1 else). Forbidden cells may be due to impassable areas (e.g. extremely rocky terrain, buildings, etc.). Equation (1) can be also written as [9]:

$$p_{ij}=\left(\prod_{m=1}^{M}A_m\right)n_{ij}\xi_{ij} \tag{2}$$

where A_m are the decision factors related to motion probabilities p_{ij}. The bounded rationality decision model introduced in [9] modifies (2) by employing an ordered geometric average operator (OGA) [14] as:

$$p_{ij}=\left(\prod_{m=1}^{M}A_{(m)}^{w_m}\right)n_{ij}\xi_{ij} \tag{3}$$

where $A_{(1)}>A_{(2)}>\ldots>A_{(M)}$ and $w_1+\ldots+w_M=1$.

Each individual variable P_i influences agent's propensity of reacting to a change in the external context variable C_m through a model term K_{C_m}. Here, we make the simplifying assumption that the individual personal factors P_i are independent, i.e.

$$K_{C_m}=\prod_{i=1}^{N}K_{C_m}(P_i,t) \tag{4}$$

III. WHISSELL'S ACTIVATION-EVALUATION MODEL

Agent emotions are modeled using Whissell's activation-evaluation model [11]. Each emotion is mapped in a 2D space with axes activation and evaluation, respectively. Activation represents the amount of "energy" of the emotional state, while evaluation carries a value and a sign (positive for emotions such as joy or negative for those like despair). Table I shows activation-evaluation values for 8 emotions commonly found during disaster evacuation events. We note that while in the original model [11] the values of the two dimensions (variables) varies between [-3, 3], here for convenience we scale both axes to [-1, 1].

TABLE I. EIGHT AFFECTIVE STATES AND THEIR ACTIVATION-EVALUATION VALUES

Affective state	Activation (passive/ active)	Evaluation (negative/ positive)
Calm	-0.7	0.7
Sad	-0.4	-0.3
Fearful	0.2	-0.2
Desperate	-0.4	-0.7
Furious	0.7	-0.4
Depressed	-0.7	-0.3
Joy	0.4	0.7
Neutral	0	0

The affective state of an agent influences his evacuation behavior in a complex fashion. We employ a simplified model in which we assume that agent's activation state influences his ability to walk uphill (represented by the elevation layer) and his tendency (or lack thereof) to follow others (dynamic layer).

Examples of activation influence on two context variables, C_1=propensity to climb and C_3=following behavior (dynamic), are shown in figure 2. The influence can also be expressed as rules such as "If the agent is passive, he doesn't like to climb" and "If the agent is passive, he is more likely to follow others".

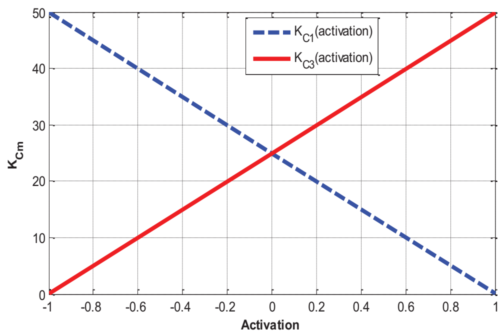

Figure 2. Functions used in our experiments to model the influence of activation on agent decisions. High values of K_{C_m} make the variable C_m less relevant in the decision process.

IV. METHODS

A. Evacuation Model Description

There are several ABM software packages available for rapid model building such as Repast (http://repast.sourceforge.net), Anylogic (http://www.anylogic.com) and Netlogo (http://ccl.northwestern.edu/netlogo/). Their advantage is that they allow fast model development with minimal programming. However, these packages are somewhat inflexible and do not easily allow the manipulation of geospatial data. In [10] we used Repast to model rumor spreading in case of disaster evacuation using a custom software toolbox. However, in this paper we used MATLAB to implement the affect dynamics models because of its flexibility. Once the models are developed and validated, we plan to deploy them in Repast.

The experiments use the map of a hypothetical island (shown in figure 3). We employed an island example to avoid dealing with border assumption and to speed up the computations. The island picture has 1219 × 1985 pixels with the land surface being about 320,000 pixels. We chose the location of a number of evacuation shelters (N_S=8) by clustering the island land pixels using a hard C-means procedure in which the feature vectors used during clustering were spatial location of the island land pixels. The evacuation shelters were placed in the resulting cluster centers.

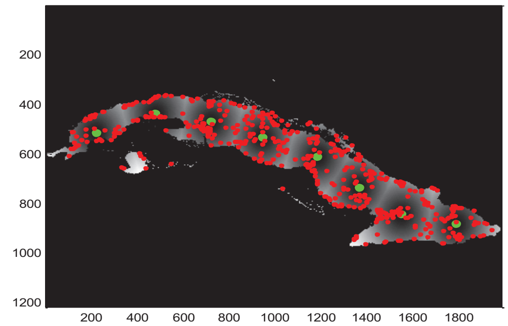

Figure 3. Hypothetical island map used in this paper (axes represent pixel dimensions of the matlab map)

CoBRa models are stochastic in nature. The stochastic behavior is obtained by considering the uncertainty in geospatial distribution of both psychosocial $\rho(P_i)$ and contextual $\rho(C_i)$ variables. For example, in figure 5 we show an example of initial agent distribution. In figure 5, the agent distribution for of the activation variable, ρ(Activation), is a Gaussian with mean=0.5 and standard deviation=0.5 while ρ(Evaluation) is a Gaussian with mean = -0.5 and standard deviation = 0.5. At each CoBRa run, we sample the activation of each of the N_P agents from ρ(Activation). Similarly, we independently sample the evaluation of each agent from a Gaussian distribution ρ(Evaluation).

B. Agent Affective State Representation

We use Whissell's model to represent the affective state (denoted henceforth as "Whissell affective state") of an agent, g_i, that is $g_i = (a_i, e_i)$ where a_i is its activation value and e_i is its evaluation. As mentioned earlier, we consider that each agent has a complex affective state where many emotions are present. To model this reality we propose to represent the affective state (denoted henceforth as possibilistic affective state) as $g'_i = (\mu_{i1},\ldots, \mu_{ij,\ldots,}\mu_{i,Ne})$ where μ_{i1} is the (possibilistic) membership of g_i in emotion E_j. Emotions E_j are emotions possible in the event considered, which for our example (disaster evacuation) are the ones listed in Table I (hence N_e=8). The memberships μ_{ij} are computed as:

$$\mu_{ij} = \frac{1}{1 + d(g_i, E_j)^2} \quad , \qquad (5)$$

where $d(g_i, E_j)$ is the Euclidean distance between Whissell affective state g_i and emotion E_j. While here we chose the Euclidean distance for convenience, we note that further research is needed to determine the best distance measure for comparing affective states.

C. Linguistic Summarization of Crowd Emotions

A linguistic summarization procedure similar to the one presented in [15] is then used to describe the emotional state of agents in a shelter. The summarization is based on templates (protoforms) of the form, "Q y's are S", where Q is a quantifier, S is a summarizer and y's represent the objects that are summarized. In our case the objects are agents. Examples of quantifiers include *most*, *many*, *almost all*, etc. Summarizers are terms like *calm*, *sad*, etc. For example, in "*many* agents are *furious*", "*many*" is the quantifier and "*furious*" is the summarizer. For every summary, we calculate the truth value which is the basic criterion for evaluating the quality of the linguistic summaries defined by:

$$T(Q\ y\text{'s are } S) = \mu_Q\left(\frac{1}{n}\sum_{i=1}^{n}\mu_S(y_i)\delta(y_i > T_M)\right) \qquad (6)$$

where n is the number of objects y_i that are summarized, $\delta(.)$ is the Kronecker symbol (i.e. $\delta(x) = 1$ if x = True and $\delta(x) = 0$ otherwise), T_M is a membership threshold and μ_S and μ_Q are the membership functions of the summarizer and quantifier, respectively. To represent a fuzzy set with a trapezoidal membership function (Trap) we need four numbers, a, b, c and d. We use 5 linguistic quantifiers: *almost all* (Trap[0.8, 1, 1, 1]), *many* (Trap[0.6, 0.75, 0.75, 9]), *about a half* (Trap[0.3, 0.5, 0.5, 0.7]), *some* (Trap[0.1, 0.25, 0.25, 0.4]), and *a few* (Trap[0.0 0.0 0.2 0.2]). We chose the membership threshold T_M=0.5 to denote the presence of a sentiment, that is if $\mu_S(y_i)>T_M$ then agent y_i has the sentiment S.

V. Experiments

A. Model validation: simple panic behavior model

Validation of ABM geospatial crowd models is challenging [5] due to the difficulty in capturing large scale population behavior data for groundtruth. Among few methods found in the literature is the capacity of the model to simulate known crowd behaviors such as "faster is slower", which means that the more the agents rush, the slower they move (for example, entering a store at Black Friday opening).

To demonstrate this, we used equation (1) and two contextual fields: C_2- the distance to shelter and C_3- following behavior (the dynamic field). Since we wanted to keep the model as simple as possible, no emotional effects and elevation influence were considered during the actual evacuation for this experiment. We show the results for constants about midrange, that is K_{C2} =10 and K_{C3}=10, but similar results were obtained for most value combinations. We used an initial agent population of N_P=1000.

Figure 4 shows the variation of the percent of population evacuated, across all 8 shelters after t=100 time steps, with agent speed (all agents have the same speed).

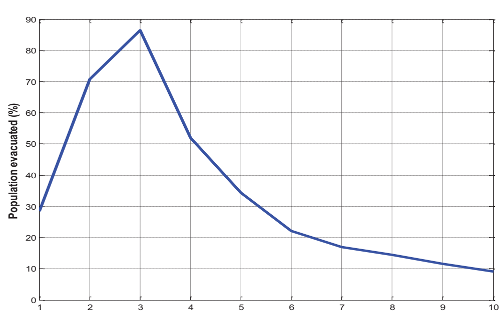

Figure 4. "Faster is slower" effect during disaster evacuation: when the agent speed increases over 3cells/step, the total population evacuated decreases.

In figure 4 we see a clear example of "faster is slower": when the agent speed increases above 3 cells/step, the total population evacuated after t=100 time steps decreases rapidly. We conclude from this experiment that our model has the possibility to capture meaningful crowd behavior.

B. Linguistic Summarization of Crowd Emotions for a Homogeneous Population

Just knowing the number of people in a shelter is not enough for the emergency response teams to take adequate measures. For example, in [8] we showed how the need for food can be estimated from model runs. However, an equally important variable in emergency response is crowd emotions. Shelters where people are furious or desperate might require attention sooner than places where they are neutral or calm.

In this section we used three context fields, C_1=elevation, C_2=distance to shelter and C_3=following behavior in the evacuation model. The values of the C_1 and C_3 constants were computed individually for each agent (as explained in Section III) based on his activation value, a_i. The value of C_2 was set to 1 for all agents. We used a population N_P=10,000 agents and t=200 time steps. As described in section IV.A, we initialize the activation of the population randomly using a Gaussian ρ(Activation) with mean 0.5 and standard deviation 0.5. We chose a mean activation of 0.5 here since in an evacuation, we conjecture that more people are active in searching for a shelter. Similarly, we initialize the evaluation of each agent with ρ(Evaluation) with mean -0.5 and standard deviation 0.5. The agent initialization follows the intuition that the crowd during a disaster has somewhat negative sentiments (fear, fury, despair) and a large range of energy levels (from passive to active). The activation-evaluation values for the agents used in this experiment are shown in figure 5.

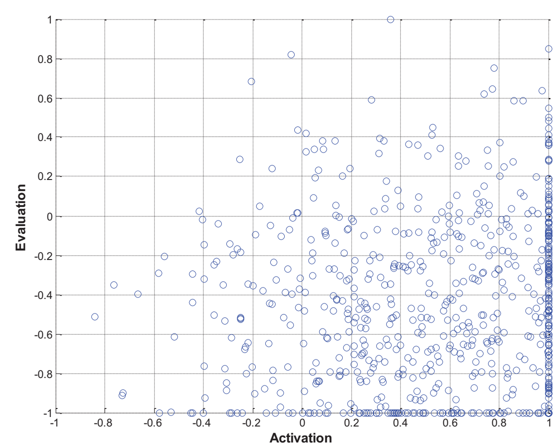

Figure 5. EA values for the population used in Homogeneous Experiment

Once the simulation was terminated at t=200, we considered the total population and those at each of the evacuation shelter. This would represent a snapshot in time during the evacuation, or simply that the crisis passed and there was no more movement. The affective summary for the entire evacuated population (637 agents) is shown in Table II. Since linguistic protoform summaries are calculated for all combinations of quantifiers and summarizers, we record only those whose truth values exceed a threshold (0.5 here).

TABLE II. LINGUISTIC SUMMARIZATION OF THE EMOTION OF THE ENTIRE EVACUATED POPULATION

Summary statement	Truth, T
a few people are *calm*	1
some people are *sad*	0.74
some people are *desperate*	0.69
many people are *furious*	0.84
a few people are *depressed*	1
a few people are *joyful*	1

As expected, many people are fearful and furious but a few are joyful and a few are calm. The overall sentiment is clearly negative. The influence of the initial activation on the evacuation behavior can be observed by comparing the crowd sentiments from shelter 3 (located in a plains region, 52 agents) shown in Table III to the ones from shelter 8 (located in a mountain region, hence more "activation" is needed to reach the shelter, 67 agents) shown in Table IV. We only chose two shelters to show the linguistic summaries in the interest of space. In addition, the fact that one shelter is situated in the mountains and another in the plains should make a difference in the type of agents that can reach them.

TABLE III. LINGUISTIC SUMMARIZATION OF THE EMOTIONS OF AGENTS FROM SHELTER 3

Summary statement	Truth, T
a few people are *calm*	1
some people are *sad*	0.84
some people are *desperate*	0.69
many people are *furious*	0.67
a few people are *depressed*	1
a few people are *joyful*	1

TABLE IV. LINGUISTIC SUMMARIZATION OF THE EMOTIONS OF AGENTS FROM SHELTER 8

Summary statement	Truth, T
a few people are *calm*	1
some people are *sad*	0.81
some people are *desperate*	0.79
many people are *furious*	0.75
a few people are *depressed*	1
some people are *joyful*	0.75

As we see from Table III and IV the summaries are almost identical, as expected in a homogeneous population. However, the amount of joyful, desperate and furious people is seen to be more numerous in shelter 8 than in shelter 3 (higher truth values). The explanation might be that shelter 8 was reached by agents with higher activation levels, hence extreme sentiments, are favored (recall from Table I that fury has activation value of 0.7, the highest of all sentiments considered).

C. *Linguistic Summarization of Sentiments for a Heterogeneous Population*

In the previous experiment we assumed that the crowd contained a population described by a single activation-evaluation profile, centered around (0.5, -0.5) in the EA space. Now, we assume that the initial agent population consists of two subpopulations (see figure 6): one urban, centered around (-0.5, 0.5) and another one rural, centered around (0.5, -0.5). The assumption here is that while the "urban" population has more positive feelings than the "rural" one, it is less energetic. All standard deviations were picked to be 0.2.

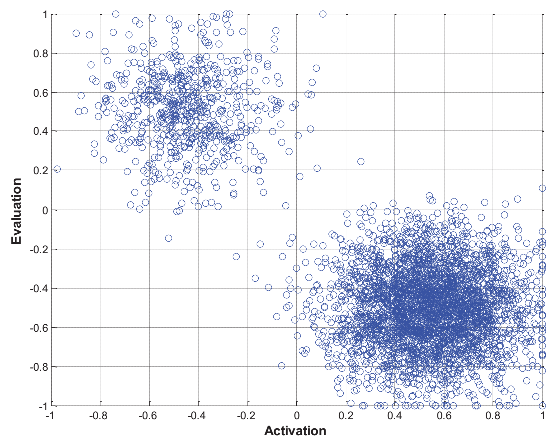

Figure 6. EA values for agents in Heterogeneous Experiment

The experiment was similar to the one in the previous section (N_P = 10,000, same context layers), except agent initialization and number of time steps t = 1000 (more time steps allowed more agents to reach the shelters). All agents who came from a 2 mile area of a city (using the city location layer from our dataset) were initialized with the urban trait (about a third). All other agents were initialized using the rural profile. The emotion summaries in this case are shown in Table V for the global shelter population (3775 agents), in table VI for shelter 3 (plain area, close to a big city, 277 agents) and in Table VII for shelter 8 (mountain area, mostly rural, 287 agents).

TABLE V. LINGUISTIC SUMMARIZATION OF THE EMOTIONS OF AGENTS OF TWO SUBPOPULATIONS

Summary statement	Truth, T
a few people are *calm*	1
many people are *fearful*	0.75
some people are *desperate*	0.85
many people are *furious*	0.97
a few people are *depressed*	1
a few people are *joyful*	1
about half of the people are *neutral*	0.53

From Table V we see that the summaries are somewhat similar to the one from the homogeneous experiment (Table II), except the increase in fearful people caused by the fact that mostly the rural agents made it to the shelter (due to their much higher activation). The similarity of the two summaries is also a proof that the summarization procedure obeys Plutchik's rule (see next section).

TABLE VI. LINGUISTIC SUMMARIES FOR SHELTER 3 AND TWO SUBPOPULATIONS

Summary statement	Truth, T
a few people are *calm*	1
some people are *sad*	0.64
some people are *desperate*	0.83
many people are *furious*	0.7
a few people are *depressed*	1
a few people are *joyful*	1
about half of the people are *neutral*	0.69

TABLE VII. LINGUISTIC SUMMARIES FOR SHELTER 8 AND TWO SUBPOPULATIONS

Summary statement	Truth, T
some people are *calm*	0.71
about half of the people are *sad*	0.51
some people are *desperate*	0.99
about half of the people are *furious*	0.54
some people are *depressed*	0.75
a few people are *joyful*	1

By comparing Table VI and VII we note that the summaries of the two shelters look very different and are somewhat difficult to compare and interpret. Both shelters seem to be dominated by unpleasant sentiment such as fury and fear which might be surprising at the first glance. This is due to the fact that city agents (with more positive feeling but negative activation) were not able to reach shelter 3 in great numbers. There is however, an increase in the number of neural emotional people in shelter 3. So while intuition might suggest that the emotional makeup of the city shelter should be calmer than the other, the simulation points to the fact that there may be issues that need to be addressed there also.

D. Alternative crowd summarization procedure

The previous linguistic summarization procedure might be too verbose if the number of emotions considered is large. An alternative procedure that can produce a summary like "the dominant sentiment is fear" was suggested in [16]. First, the aggregate crowd Whissell affective state is computed. To aggregate activation and evaluation values, [16] employs a heuristic approach based on Plutchik's rule [17] that forbids averaging "emotional incompatible" states. If not, by averaging furious (0.7, -0.4) (see Table I) and calm (-0.7, 0.7), we end up with (0, 0.3) which is close to a neutral state (0,0).

Our heuristic aggregation procedure inspired from [16] consists in finding the dominant activation (passive or active) and evaluation (positive or negative) and then averaging only the agents that have the dominant values. Then, we compute the memberships of the aggregated state in the sentiments considered (Table I) using formula 5. The sentiment with the highest membership is declared dominant for the entire crowd.

Employing this procedure we found a Whissell state value of (0.69 -0.5) for shelter 3 in the homogeneous population experiment (Table III). The related memberships in the states from Table I are: (0.23 0.44 0.75 0.44 0.98 0.33 0.39 0.57), and hence the dominant sentiment is *fury*. This can somewhat observed from the membership histogram for shelter 3 shown in figure 7.

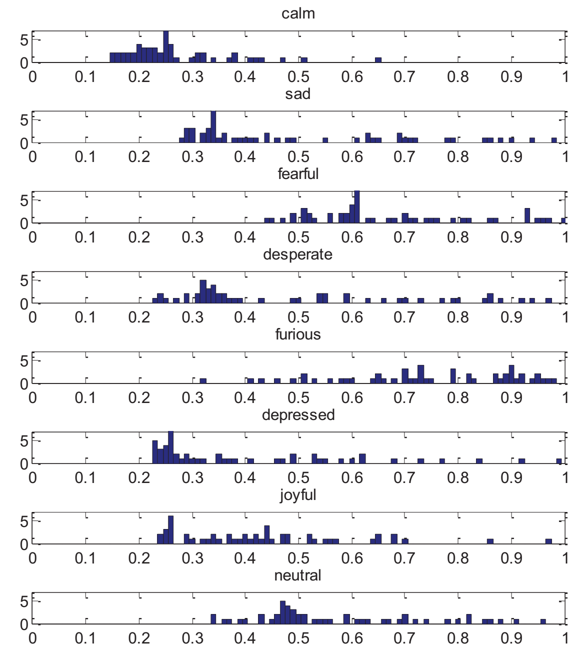

Figure 7. Sentiment membership histograms for shelter 3 in experiment 1.

In figure 7 we see that f*ury* seem to have most high memberships (above our T=0.5 threshold), though membership in *desperate* is also strong.

While succinct, this procedure is less sensitive than the linguistic summaries presented in the previous section as the same answer (*fury*) is obtained for shelter 8 and the entire population.

VI. CONCLUSIONS

In this paper, models to perform anticipatory analysis of crowd behavior in an evacuation scenario were presented. The models utilized multiple agents who moved through a geographic environment in a stochastic simulation, motivated by cultural and emotional factors. In such complex situations, fuzzy set theory provides a natural mechanism to summarize and explain the results of the models. Here, we proposed linguistic summaries and an affect aggregation to display emotional states of people who arrived at shelters during a simulated hurricane evacuation. While the results are encouraging, more work is needed to refine and extend the models, to study the form and parameters of the linguistic summaries, to investigate alternate sentiment comparison and aggregation methods, and to study the effect of the dynamics of sentiment change when agents aggregate at shelters or even when they meet and influence each other on the way.

References

[1] A. Kirchner, A. Schadschneider, “Simulation of evacuation processes using a bionics-inspired cellular automaton model for pedestrian dynamics”, Physica A 312, 2002, pp. 260 – 276.

[2] D. Helbing, I. Farkas, T. Vicsek, “Simulating dynamical features of escape panic”, Nature 407, 2000, pp. 487–490.

[3] C.M. Henein, T. White, “Macroscopic effects of microscopic forces between agents in crowd models”, Physica A 373, 2007, pp. 694-712.

[4] Z. Xiaoping, L. Wei, G. Chao, “Simulation of evacuation processes in a square with a partition wall using a cellular automaton model for pedestrian dynamics”, Physica A 389, 2010, pp. 2177-2188.

[5] D. Helbing, A. Johansson, “Pedestrian, Crowd and Evacuation Dynamics, Encyclopedia of Complexity and Systems Science, 16, 6476-6495, 2010.

[6] M. Moussaïd, D. Helbing, and G. Theraulaz, “How simple rules determine pedestrian behavior and crowd disasters. PNAS 108 (17), 2011, pp. 6884-6888.

[7] E.L. Quarantelli, “The sociology of panic”, Disaster Research Center, University of Delaware, 1999, 24 pages.

[8] J. Keller, M. Popescu, D. Gibeson, “An extension of a confined space evacuation model to human geography”, Proceedings, IEEE Geoscience and Remote Sensing Symposium, IGARSS 2012, Munich, Germany, July, 2012, pp. 531-534.

[9] M. Popescu, M., J. Keller, “Implementing bounded rationality in disaster agent behavior using OGA operators”, *Proceedings, IEEE Geoscience and Remote Sensing Symposium, IGARSS 2012, Munich*, Germany, July, 2012, pp.5379-5381.

[10] A. Zare, Z. Fields, J. Keller, and J. Horton, “Agent-based rumor spreading models for human geography”, Proceedings, IEEE Geoscience and Remote Sensing Symposium, IGARSS 2012, Munich, Germany, July, 2012, pp. 5394-5397.

[11] C. Whissell, “Using the revised dictionary of affect in language to quantify the emotional undertones of samples of natural language”, Psychological Reports, 2009, 105, pp. 509-521.

[12] J.A. Russell, “A circumplex model of affect”, Journal of Personality and Social psichology, 1980, vol 39, no. 6, 1161-1178.

[13] L.E. Bourne, B.R. Ekstrand, W.L.S. Dunn, “Psychology. A concise introduction”, 1988, Holt, Rinehart and Winston Inc., New York.

[14] F. Herrera, E. Herrera-Viedma, F. Chiclana, “A Study of the Origin and Uses of the Ordered Weighted Geometric Operator in Multicriteria Decision Making”, Int. J. of Intelligent Systems, vol. 18, 2003, pp. 689–707.

[15] A. Wilbik, J.M. Keller, “A fuzzy measure similarity between sets of linguistic summaries”, to appear IEEE TFS, 2013.

[16] I. Hupont, E. Cerezo, S. Baldassarri, “Sensing Facial Emotions in a Continuous 2D Affective Space”, Proc. of SMC, pp 2045-2051, Oct. 2010.

[17] R. Plutchik, "Emotion: a psychoevolutionary synthesis, " New York: Harper & Row, 1980.