MULTIPLE MODEL ENDMEMBER DETECTION BASED ON SPECTRAL AND SPATIAL INFORMATION

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ABSTRACT

We introduce a new spectral mixture analysis approach. Unlike most available approaches that only use the spectral information, this approach uses the spectral and spatial information available in the hyperspectral data. Moreover, it does not assume a global convex geometry model that encompasses all the data but rather multiple local convex models. Both the multiple model boundaries and the model's endmembers and abundances are fuzzy. This allows points to belong to multiple groups with different membership degrees. Our approach is based on minimizing a joint objective function to simultaneously learn the underling fuzzy multiple convex geometry models and find a robust estimate of the model's endmembers and abundances.

Index Terms— Hyperspectral imaging, spectral mixture analysis, endmember extraction, fuzzy clustering, convex geometry

1. INTRODUCTION

An hyperspectral image is a three dimensional data cube representing both the spatial and spectral information of a scene. The data cube can be interpreted as a stack of 2D images where each 2D image represents the radiance measure of the hyperspectral scene with respect to one wavelength. More specifically, each pixel in this 2D image represents the radiance measure of the corresponding wavelength at this specific location.

Hyperspectral data is often used to determine what materials are present in a scene. A trivial way to do it is by comparing each pixel of a hyperspectral scene to a material database to determine the type of material constituting the pixel. However, low resolution may cause each pixel to be a mixture of several materials. In fact, each pixel could be a mixed pixel or a pure pixel. While a mixed pixel combines the radiance values of multiple materials, a pure pixel corresponds to radiance values of a single material. These pure pixels are referred to as endmembers.

The process of unmixing mixed pixels into their respective endmembers is called hyperspectral unmixing. A solution to hyperspectral unmixing is to reverse the mixing process by determining the spectra of the materials from the image. This process relies on the definition of the mixing model. The most well known mixing model is the convex geometry model. It assumes that every pixel is a convex combination of endmembers in the scene. It has been found to effectively describe regions where the various pure materials are separated into regions dominated by a single endmember. Generally, mixed pixels in these types of regions are caused by an inadequate spatial resolution of the sensor.

Over the last decade, several algorithms have been developed for automatic extraction of spectral endmembers. However, most available approaches do not take into account the spatial information contained in the image data. Moreover, they assume that a single global convex model holds for all the dataset. However, real hyperspectral data could not be convex globally. Instead, a piece-wise convex model can hold locally.

In this paper, we develop a new approach to incorporate spatial information into the traditional spectral-based endmember search process. As the low resolution of the sensor causes a pure material to be a mixture of its pure material and its neighborhood ones, a local convex geometry model is more appropriate to model the mixing problem then a global one that encompasses all the data. Specifically, the spatial and the spectral information are both used to split the hyperspectral scene into multiple models with fuzzy boundaries in such away that each model shares similar spatial and spectral characteristics . The idea is to fit the hyperspectral scene using a convex geometry model locally. This allows the endmember detection process to be linear locally and non linear globally and benefits from the spatial information.

2. RELATED WORKS

Recently, the automated identification of endmembers has received a considerable interest in the hyperspectral imaging community [3]. Most of these approaches only use the spectral information contained in the image data. However, endmember extraction could greatly benefit from an integrated framework in which both the spectral information and the spatial arrangement of pixel vectors are taken into account. Three representative spatial-spectral techniques for endmember extraction in the have been proposed. These are the automatic

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morphological endmember extraction (AMEE) algorithm [4], the spatial spectral endmember extraction (SSEE) algorithm [7], and the spatial preprocessing (SPP) algorithm [10]. All of these approaches rely on the pixel purity assumption, that is, there exists at least one pixel for each endmember which consists of only that endmember's material. Furthermore, the hyperspectral imaging device must be operating at a spatial resolution that does not combine endmember spectra with the spectra of neighboring materials.

Another assumption made by the majority of published endmember finding algorithms is that the hyperspectral data points lie in a single convex region and can be described by a single set of endmembers which encompass the data set. However, a global convex model could not hold for all the data but it is applicable only to a local subset of it.

The piece-wise convex representation for hyperspectral imagery was first introduced in [5]. This method samples from a Dirichlet process to partition the input hyperspectral data into convex regions. In this approach, convex regions boundaries are crisp. However, this kind of boundaries do not allow to quantitatively distinguish between pixels which are strongly associated with particular convex region from those that have only a marginal association with multiple ones, particularly, along the overlapping boundaries.

The Piece-wise Convex Multiple Model Endmember Detection algorithm P-COMMEND [6] benefits from the fuzzy assignment of pixels. In fact, it represents hyperspectral imagery using several sets of endmembers that allows a robust non-convex hyperspectral image representation. Although this approach presents the advantage of considering piece-wise convex multiple models with fuzzy boundaries, it does not benefit from the spacial information available in the hyperspectral data.

3. PROPOSED APPROACH

Let $\mathbf{x}_j$ denote a $1 \times d$ vector representing the spectral information of the j^{th} pixel of a hyperspectral image, and let $\mathbf{y}_j$ be its spatial coordinate. We propose to simultaneously fit C models to a data and automatically identify the set of endmembers $\left\{\mathbf{e}^i\right\}_{i=1,\dots,C}$ and their corresponding set of abundances $\left\{\mathbf{p}^i\right\}_{i=1,\dots,C}$ by minimizing the following objective function

$$\begin{aligned} J &= \sum_{i=1}^{C}\sum_{j=1}^{N} u_{ij}^{m}\left(\mathbf{x}_j - \mathbf{p}_j^i\mathbf{e}^i\right)\left(\mathbf{x}_j - \mathbf{p}_j^i\mathbf{e}^i\right)^T \\ &+ \rho\sum_{i=1}^{C}\sum_{j=1}^{N} u_{ij}^{m}\left(\mathbf{y}_j - \mathbf{c}_i\right)\left(\mathbf{y}_j - \mathbf{c}_i\right)^T \\ &+ \alpha\sum_{i=1}^{C}\left(M \cdot trace(\mathbf{e}^i\mathbf{e}^{iT}) - 1_{1\times M}\mathbf{e}^i\mathbf{e}^{iT}1_{M\times 1}\right) \end{aligned} \quad (1)$$

subject to

$$\mathbf{p}_{ik} \geq 0 \qquad \forall k = 1,\dots,M, \quad (2)$$

$$\sum_{k=1}^{M}\mathbf{p}_{ik} = 1, \quad (3)$$

and

$$\sum_{j=1}^{N} u_{ij} = 1 \quad (4)$$

In (1), N is the number of pixels, M the number of endmembers per model, $\mathbf{c}_i$ is the spatial center of the points assigned to the i^{th} model, α and ρ are two scaling parameters. The first term of the objective function in (1) corresponds to the model fitting term. Assuming a convex geometry for each model, it simultaneously identifies the set of endmembers $\left\{\mathbf{e}^i\right\}_{i=1,\dots,C}$ and their corresponding set of abundances $\left\{\mathbf{p}^i\right\}_{i=1,\dots,C}$. Each model is wighted by a fuzzy membership that represents the degree of sharing data point j in the i^{th} model. In a sense, the multiple models compete for a given pixel j through a set of weights u_{ij}. The assignment of a point to a model is done according to its spectral information $\mathbf{x}_j$ and its spatial one $\mathbf{y}_j$. Even though the regression residual of a point relative to a convex geometry model framework is not a physical distance, it does represent the goodness of fit of a given model in the neighborhood of a point whenever the model can be described in explicit from.

The third term in (1) constrains the size of the simplex in such a way that the endmembers surround the model tightly. This regularization term was first used in the Iterated Constrained Endmembers approach (ICE) [9].

By using the Lagrange multiplier method and the Karush-Kuhn-Tucker (KKT) Conditions [8], we will alternate the optimization of $\mathbf{e}^i$, $\mathbf{p}^i$, $\mathbf{c}_i$ and u_{ij} and derive an explicit solution for each variable. It can be shown that the corresponding update equations are:

$$\mathbf{e}^i = \left(\sum_{j=1}^{N} u_{ij}^{m}\mathbf{p}_j^{iT}\mathbf{p}_j^i + 2\alpha D\right)^{-1}\left(\sum_{j=1}^{N} u_{ij}^{m}\mathbf{p}_j^{iT}\mathbf{x}_j\right) \quad (5)$$

$$\mathbf{p}_j^{iT} = max\left(\left(\mathbf{e}^i\mathbf{e}^{iT}\right)^{-1}\left(\mathbf{e}^i\mathbf{x}_j^T - \frac{\lambda_i}{2}1_{M\times 1} - \gamma_j\right), 0\right), \quad (6)$$

$$u_{ij} = \frac{\left(\frac{1}{dist_j^i}\right)^{\frac{1}{m-1}}}{\sum_{q=1}^{C}\left(\frac{1}{dist_j^q}\right)^{\frac{1}{m-1}}}, \quad (7)$$

and

$$\mathbf{c}_i = \frac{\sum_{j=1}^{N} u_{ij}^{m}\mathbf{y}_j}{\sum_{j=1}^{N} u_{ij}^{m}}, \quad (8)$$

where

$$D = MI_{M\times M} - 1_{M\times M}, \quad (9)$$

$$\begin{aligned} dist_j^i &= \left(\mathbf{x}_j - \mathbf{p}_j^i\mathbf{e}^i\right)\left(\mathbf{x}_j - \mathbf{p}_j^i\mathbf{e}^i\right)^T \\ &+ \rho\left(\mathbf{y}_j - \mathbf{c}_i\right)\left(\mathbf{y}_j - \mathbf{c}_i\right)^T \end{aligned} \quad (10)$$

and

$$\lambda_i = 2\frac{1_{1\times M}\left(\mathbf{e}^i\mathbf{e}^{iT}\right)^{-1}\mathbf{e}^i\mathbf{x}_j^T - 1}{1_{1\times M}\left(\mathbf{e}^i\mathbf{e}^{iT}\right)^{-1}1_{M\times 1}} \quad (11)$$

It can be seen from equation (10) that we use a more general interpretation of the distance, in particular, the sum of the Euclidean distance of the spatial coordinate of the pixel to the center of the model $(\mathbf{y}_j - \mathbf{c}_i)(\mathbf{y}_j - \mathbf{c}_i)^T$ and the error between the pixel spectra and the pixel estimate found using the endmembers and their proportions $\left(\mathbf{x}_j - \mathbf{p}_j^i \mathbf{e}^i\right)\left(\mathbf{x}_j - \mathbf{p}_j^i \mathbf{e}^i\right)^T$. The proposed Multiple Model endmember finding algorithm is summarized below.

Algorithm 1 The Multiple Model endmember finding algorithm

Fix number of clusters C and m $\in [1\infty)$;
Initialize the fuzzy partition matrix U;
Initialize α and ρ;
Initialize the set of abundance matrices $\left\{\mathbf{p}^i\right\}_{i=1,\ldots,C}$;
REPEAT
  Update the set of endmembers using (5);
  Update the set of abundances using (6) and (11);
  Update the model centers using (8);
  Update the fuzzy membership using (7);
UNTIL convergence

4. EXPERIMENTS

The experiments are conducted on the June 1992 AVIRIS Indian Pines data set [1]. These data were collected over the Indian Pines test site in an agricultural area of northern Indiana.

First, the dimensionality of the data is reduced to 10 using Martinez-Uso et al. band merging algorithm [2]. This algorithm uses information measures and hierarchical clustering. A divergence measure between every pair of bands is computed and used to perform hierarchical clustering of the bands. A band representative is then computed for each cluster. This method presents the advantage of retaining physically meaningful bands that correspond to wavelengths in the original data set.

In order to illustrate the ability of the proposed approach to detect appropriate multiple sets of endmembers and their corresponding abundances values simultaneously, we assume that the ground truth is known, and we compute the normalized distribution of abundance values across the endmembers for each class label with respect to each model.

To assess the performance of the proposed endmember detection approach, we reconstruct the original hyperspectral image, using the set of extracted endmembers and their estimated abundance maps, according to the multiple model spectral mixture model definition and then, we compute the error between the original image and the reconstructed image with respect to each model.

Another way of evaluating the performance of the proposed algorithm is by comparing it to the standard ICE method [9]. For each true label, Shannon's entropy of the normalized distribution of abundance values is computed. A smaller entropy value indicates that the abundances associated with a single class are concentrated onto a smaller number of endmembers. For the case of the multiple model approach the Shannon's entropy is computed with respect to each model and then averaged.

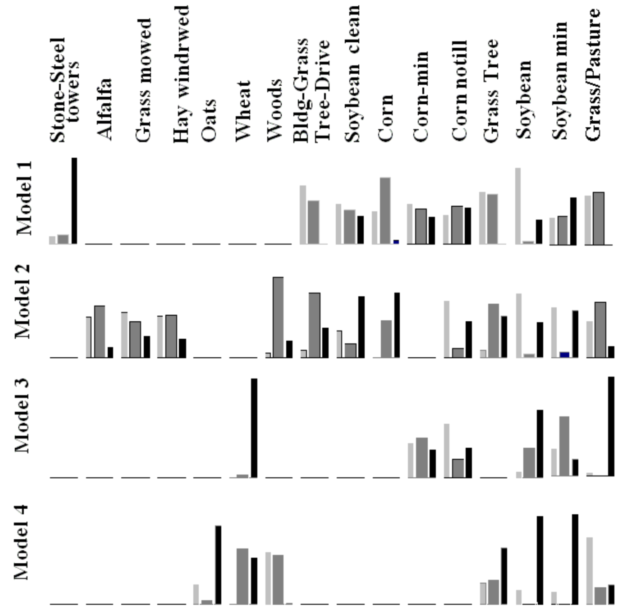

Fig. 1. Normalized distribution of abundance values across the endmembers for each class label with respect to each model.

In the experiments reported in this paper, the number of models C is set to 4, the number of endmembers M is set to 3, and the balancing parameter ρ is set to 0.004.

Figure 1 plots the normalized distribution of abundance values across the endmembers for each class label with respect to each model. Each column shows this distribution with respect to each class label and each row shows it with respect to each model. For instance, the class labels *Stone-Steel-towers*, *Alfalfa*, *Grass-mowed*, *Hay* and *Oats* are fitted by one model. This is explained by the fact that the pixels constituting each one of this material are spatially and spectrally gathered. Thus, a single convex geometric model can fit these materials.

For other classes such as *Soybeans, Corn, Grass* and *Woods*, they are fitted by several models. In fact, as these classes are spread spatially on the dataset, each one of them needs to be fitted locally with a convex model that takes into account only its neighboring pixels. Figure 1 shows that the distribution of each one of these material differ from one model to another. This confirms that these materials are differently affected by their neighbors depending on their location.

Similarly, the *Bldg/Grass/Tree/Drive* and the *Grass/Tree* classes are fitted by several models but for different reason. In fact, as their labels indicate, these classes are a mixture of different materials. Thus, the spectral characteristics of each of them cannot be represented by a single convex geometric model. However a piece-wise convex multiple model can fit these classes.

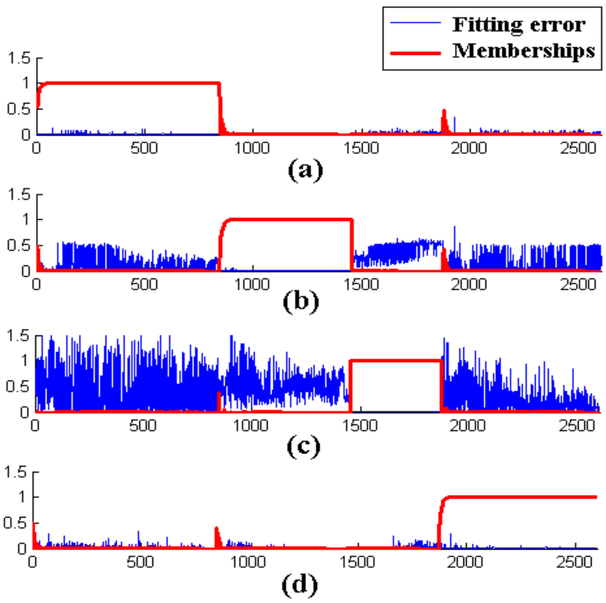

Fig. 2. Models fuzzy memberships and fitting error w.r.t to model 1 (a), model 2 (c), model 3 (e) and model 4 (g).

In figure 2, we illustrate the different models fitting error and their respective fuzzy memberships. We notice that when the fuzzy membership of the pixel is high, its fitting error is null. Therefore, the convex model to which that pixel is assigned provides a good fit to the material constituting it. Figure 2 shows also that some pixels do not have binary memberships. This confirms that the models boundaries are overlapping. Thus, a fuzzy assignment that allows points to belong to different group with different degree of memberships is more appropriate to describe this data.

Figure 3 illustrates the performance comparison between the multiple model proposed approach and the standard ICE approach [9]. The proposed approach results are consistently smaller (except for the *Stone-Steel towers*) indicating that the abundances for each class are concentrated onto a smaller number of endmembers and, therefore, the endmembers provide a better representation of the input data.

5. CONCLUSIONS

We propose an algorithm that provides a robust endmember detection and spectral unmixing method that utilizes multiple convex model representation of hyperspectral imagery. The fuzzy multiple model representation is based on both spatial and spectral information contained in the hyperspectral data. One set of endmembers is determined for each pixel subset sharing similar spectral and spatial characteristics. For each set of endmembers, the convex geometry model is applied and spectral unmixing is performed. As the results indicate, multiple model convex representation of hyperspectral imagery provides a more appropriate representation of the input data sets. Currently, we assume that the number of convex regions and the number of endmembers per cluster are known apriori. We are investigating relaxing this constrain by using an automatic estimation of these two parameters. We also plan to use possibilistic memberships to represent the degree of typicality of each pixel and use it to identify and discard noise points.

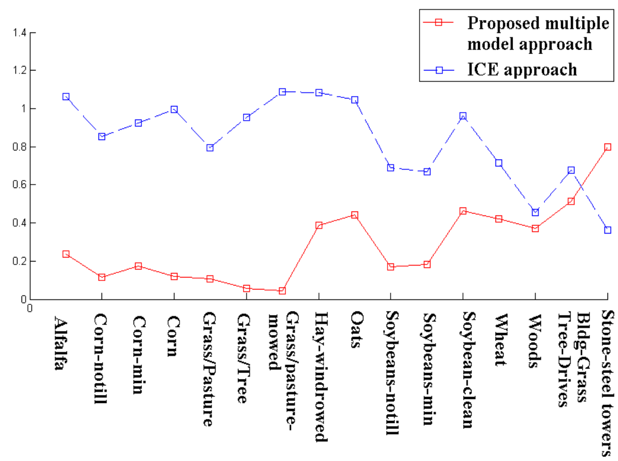

Fig. 3. Shannon's entropy of the abundance distributions.

6. REFERENCES

[1] (2004, Sep) AVIRIS free standard data products. Jet Propulsion Laboratory, California Institute of Technology, Pasedena, CA. URL http://aviris.jpl.nasa.gov/html/aviris.freedata.html.

[2] A. Martinez-Uso, F. Pla, J. M. Sotoca, and P. Garcia-Sevilla,"Clustering-based hyperspectral band selection using information measures", IEEE Transactions on Geoscience and Remote Sensing, 45(12):4158–4171, Dec. 2007.

[3] A. Plaza, P. Martinez, R. Perez, and J. Plaza, "A quantitative and comparative analysis of endmember extraction algorithms from hyperspectral data," IEEE Trans. Geosci. Remote Sens., vol. 42, no. 3, pp. 650–663, 2004.

[4] A. Plaza, P. Martinez, R. Perez, and J. Plazas, "Spatial-spectral endmember extraction by multidimensional morphological operators," IEEE Transactions on Geoscience and Remote Sensing, vol. 40, no. 9, pp. 2025– 2041, Sep. 2002.

[5] A. Zare, "Hyperspectral endmember detection and band selection using bayesian methods," Ph.D. dissertation, University of Florida, 2009.

[6] A. Zare, O. Bchir, H. Frigui, and P. Gader, "Piece-wise convex multiple model endmember detection," IEEE Transactions on Geoscience and Remote Sensing, Submitted.

[7] D. M. Rogge, B. Rivard, J. Zhang, A. Sanchez, J. Harris, and J. Feng, "Integration of spatial–spectral information for the improved extraction of endmembers," Remote Sens. Environ., vol. 110, no. 3, pp. 287–303, 2007.

[8] H. W. Kuhn, A. W. Tucker, "Nonlinear programming", Proceedings of 2nd Berkeley Symposium. Berkeley, pp. 481-492, 1951.

[9] M. Berman, H. Kiiveri, R. Lagerstrom, A. Ernst, R. Donne, and J. F. Huntington, "ICE: A statistical approach to identifying endmembers in hyperspectral images", IEEE Transactions on Geoscience and Remote Sensing, 42:2085–2095, Oct. 2004.

[10] M. Zortea and A. Plaza, "Spatial preprocessing for endmember extraction," IEEE Trans. Geosci. Remote Sens., in press, 2009.