Spatial-Spectral Unmixing using Fuzzy Local Information

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***Abstract*—Hyperspectral unmixing estimates the proportions of materials represented within a spectral signature. The overwhelming majority of hyperspectral unmixing algorithms are based entirely on the spectral signatures of each individual pixel and do not incorporate the spatial information found in a hyperspectral data cube. In this work, a spectral unmixing algorithm, the Local Information Proportion estimation (LIP) algorithm, is presented. The proposed LIP algorithm incorporates spatial information while determining the proportions of materials found within a spectral signature. Spatial information is incorporated through the addition of a spatial term that regularizes proportion value estimates based on the weighted proportion values of neighboring pixels. Results are shown in the AVIRIS Indian Pines hyperspectral data set.**

I. Introduction

THe convex geometry model (also known as the Linear Mixing Model) states [1, 2],

$$\mathbf{x}_i = \sum_{k=1}^{M} p_{ik}\mathbf{e}_k + \epsilon_i \quad i = 1, \ldots, N \tag{1}$$

where N is the number of pixels in the image, M is the number of endmembers, ϵ_i is an error term, p_{ik} is the proportion (abundance) of endmember k in pixel i, $\mathbf{e}_k$ is the k^{th} endmember, and

$$p_{ik} \geq 0 \quad \forall k = 1, \ldots, M; \quad \sum_{k=1}^{M} p_{ik} = 1. \tag{2}$$

Given this model, spectral unmixing determines the proportions of materials for every data point in the scene. Several endmember detection and spectral unmixing algorithms have been developed in the literature. However, the majority of these methods do not incorporate spatial information while performing spectral unmixing. Recently, studies for developing spatial-spectral unmixing and endmember detection algorithms have been conducted [3–5]. In this work, a method based on the fuzzy location information c-means (FLICM) clustering algorithm [6, 7] is used to estimate proportion values using spatial information and a given set of endmembers.

II. Spatial-Spectral Unmixing Method

In many spectral unmixing algorithms, the objective is to determine the proportion values which minimize the squared-error between the input pixel spectra and their estimates using endmembers and the estimated proportion values.

$$J = \sum_{i=1}^{N} \|\mathbf{x}_i - \mathbf{E}\mathbf{p}_i\|_2^2 \tag{3}$$

where the columns of $\mathbf{E}$ are the endmembers in the image, $\mathbf{x}_i$ is the i^{th} pixel and $\mathbf{p}_i$ is the corresponding proportion vector.

For example, in the Iterated Constrained Endmembers (ICE) and Sparsity Promoting ICE (SPICE) algorithms, the proportions are estimated by minimizing Equation 3 with respect to the constraints in Equation 2 using a quadratic programming step [8] [9]. These approaches, however, do not incorporate spatial information.

The proposed Local Information Proportion estimation algorithm, LIP, adds spatial information during spectral unmixing by incorporating a term into the objective function that promotes setting the proportion value of the current pixel under consideration similar in value to weighted proportion values of neighboring pixels. The proposed algorithm minimizes an objective function that has terms related to the squared error in Equation 3 and spatial smoothing across neighboring proportion values. This objective function is minimized with a solution that trades off between finding proportion values which give the best estimate of a spectrum in terms of the endmembers and finding proportion values for the a spectrum that are similar to the proportion values for neighboring spectra. The proposed spatial smoothing term for spectral unmixing is shown in Equation II.

$$G_{ik} = \sum_{\substack{j \in N_i \\ j \neq i}} \frac{1}{d_{ij}+1}(1-p_{kj})^2 \frac{1}{\|\mathbf{x}_j - \mathbf{E}\mathbf{p}_j\|_2^2 + 1} \tag{4}$$

where N_i is the neighborhood around pixel i, d_{ij} is the distance in pixel space between the spatial location of pixel i and pixel j, p_{kj} is the proportion of the k^{th} endmember in the j^{th} pixel. The size and shape of N_i is a parameter that is set by the user. For example, pixels in a 3x3 neighborhood or a 5x5 neighborhood around a pixel may be considered its neighbors. Furthermore, these neighborhoods can be defined based on additional information such as information from other sensors or *apriori* knowledge about the scene.

The objective function which is minimized during spectral unmixing is then found to be

$$J = \sum_{i=1}^{N} \|\mathbf{x}_i - \mathbf{E}\mathbf{p}_i\|_2^2 + \gamma\mathbf{G}_i^T\mathbf{p}_i. \tag{5}$$

where $\mathbf{G}_i$ is the vector of G values computed centered on pixel i and γ is a parameter used to trade-off between the squared error term and the spatial smoothing term. Larger γ values

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place more emphasis on spatial smoothing of the proportion values whereas smaller γ values place more emphasis on minimizing the residual error between the input data and its estimate using given endmembers and estimated proportion values.

The objective function in Equation 5 is iteratively minimized by alternating between updating proportion values for each data point using fixed $\mathbf{G}_i$ values and, then, updating $\mathbf{G}_i$ values given updated proportion values. Proportion values for each data point are iteratively updated until some stopping criterion in met. In the current implementation of LIP, stopping criteria include a maximum number of iterations and a threshold value on the difference between successive proportion value estimates. The algorithm stops when either criterion is met.

In the current implementation, a quadratic programming step is being used to update each proportion value by minimizing Equation 5 subject to the constraints in Equation 2. The $\mathbf{G}_i$ values are updated by computing the value of Equation for every pixel in the scene. Equation II can be efficiently calculated for each pixel by precomputing neighborhood distance values for a fixed neighborhood size at the start of the algorithm, updating $s_{kj} = (1 - p_{kj})^2 \frac{1}{\|\mathbf{x}_j - \mathbf{E}\mathbf{p}_j\|_2^2 + 1}$ for each pixel j and each endmember k, every iteration, and, then, convolving the image of s_{kj} values with the fixed neighborhood distance matrix.

III. Experimental Results

Results are shown on the AVIRIS Indian Pines data set [10]. Figures 1 and 2 display one band of the data set and the corresponding ground truth, respectively. Results on a subset of the image are shown in Figure 3. Figure 3 shows the proportion estimates without using any spatial information. Figure 4 shows the proportions estimated with a neighborhood window size of [3 3]. Figure 7 shows the proportions estimated with a neighborhood window size of [7 7]. As can be seen, by incorporating the spatial information, the proportion maps become smoother. The proportion maps become more smooth as the window size increases.

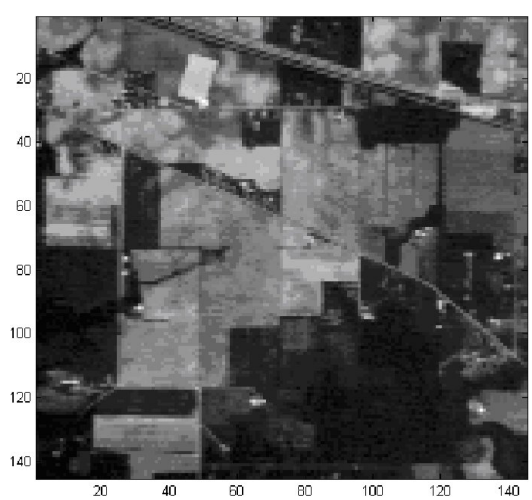

Fig. 1. One band of the AVIRIS Indian Pines data set.

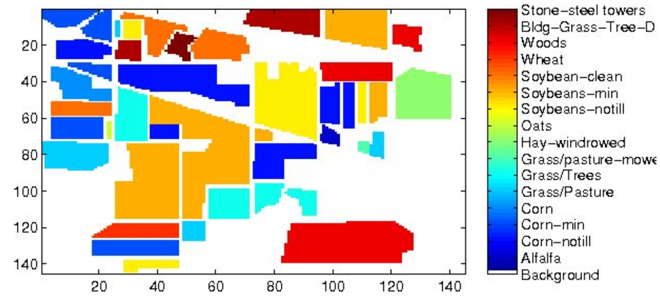

Fig. 2. Groundtruth map of the AVIRIS Indian Pines data set.

Figure 5 and Figure 6 show the results on the entire AVIRIS Indian Pines data set using a neighborhood window size of [5 5] and a γ value of 0.1. Again, incorporating spatial information provides smoother proportion maps. For example, incorporating spatial information isolated the stone steel towers pixels (as shown in the center proportion map in both Figures 5 and 6) and removed much of the noise when compared to proportion maps found without using spatial smoothing.

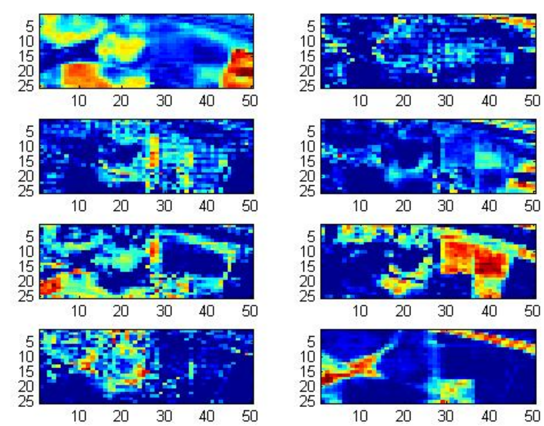

Fig. 3. Results without using spatial smoothing

IV. Conclusions and Future Work

The proposed LIP algorithm provides a spectral unmixing algorithm that incorporates spatial information. Proportion values are encouraged to have similar proportion values as neighboring pixels. Currently, this algorithm is strictly used to estimate proporiton values given a set of endmembers. Extensions to this work can include investigating methods such that a spatial-spectral endmember detection and spectral unmixing algorithm can be developed in which endmember estimates are influenced by spatially-smoothed proportion values. Investigations into incorporating methods that are more effective at preserve long linear structures (such as roads) will be investigated. Also methods of adapting the γ parameter over time or depending on spatial, edge, or texture information will be investigated.

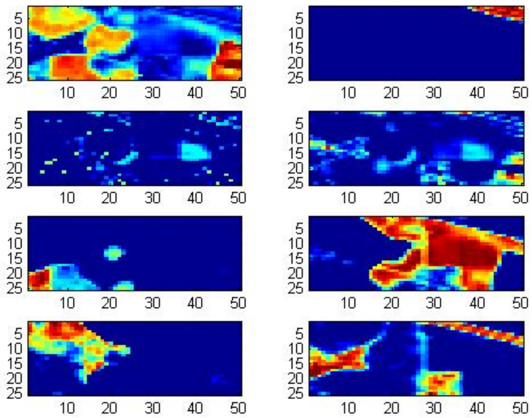

Fig. 4. Results using a window size of [3 3] and γ = 0.1

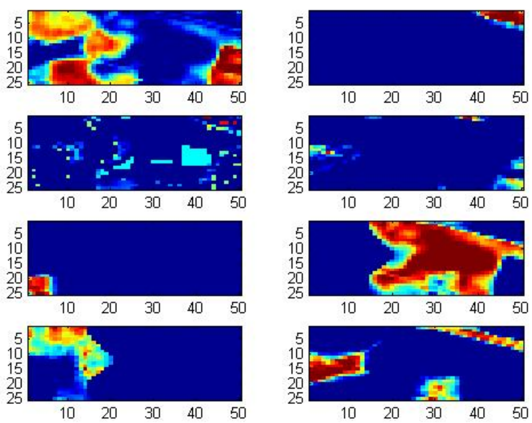

Fig. 5. Results using a window size of [7 7] and γ = 0.1

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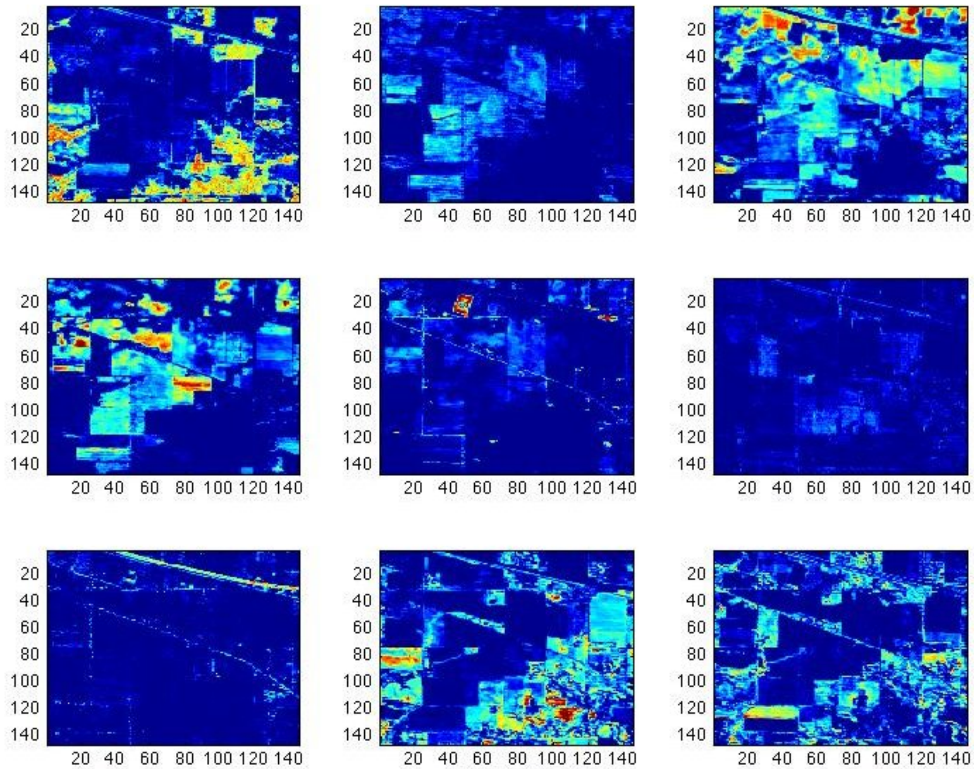

Fig. 6. Results on the full AVIRIS Indian Pines dataset without spatial smoothing.

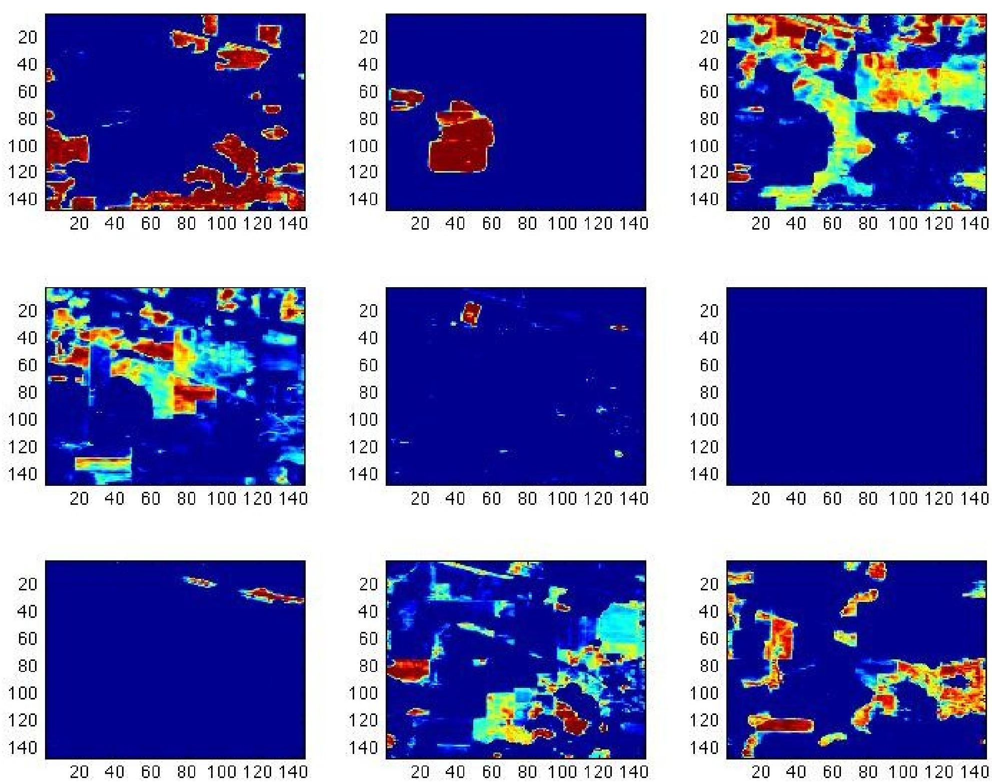

Fig. 7. Results on the full AVIRIS Indian Pines dataset using a window size of [55] and γ= 0.1