Exercises: inference in the linear Gaussian model

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1 Inferring location of a static submarine from its sonar measurements

(a)

The modified code lines appear below and Fig. 1 shows the generated submarine samples, and the mean and 95% confidence ellipse of the samples probability density function.

```
sigma_zx = 1.0
sigma_zy = 2.0
rho_z = 0.7
cov_z_11 = sigma_zx**2
cov_z_12 = rho_z*sigma_zx*sigma_zy
cov_z_21 = rho_z*sigma_zx*sigma_zy
cov_z_22 = sigma_zy**2
```

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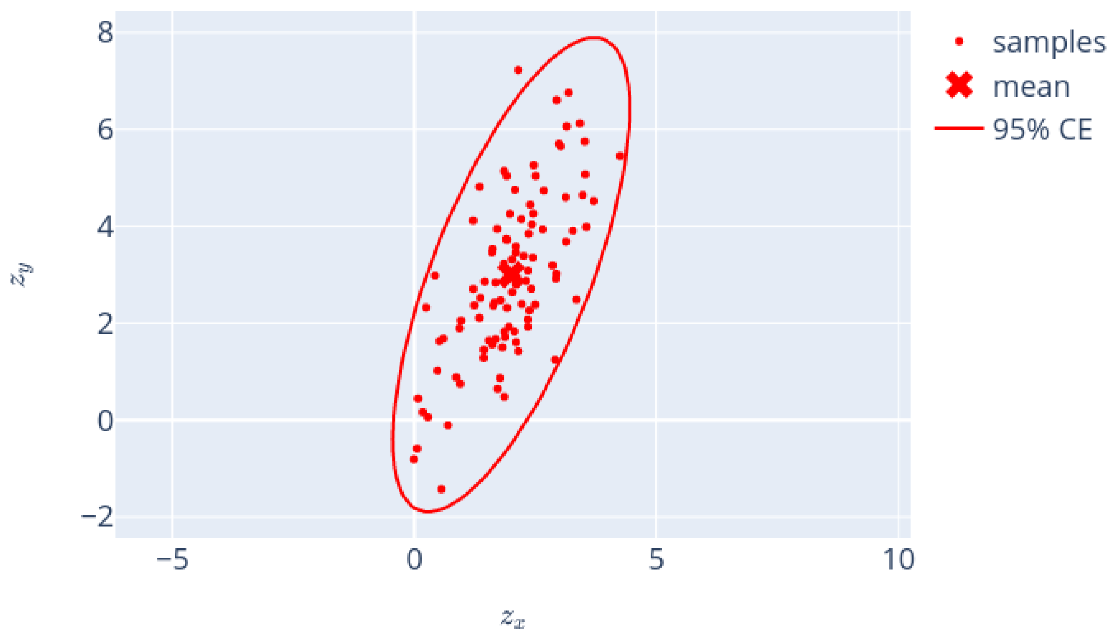

Figure 1: 100 a-priori samples of the submarine location, and the mean and 95% confidence ellipse of the samples probability density function.

(b)

The modified code lines appear below and Fig. 2 shows the generated measurement samples, and the mean and 95% confidence ellipse of the samples probability density function.

```
sigma_y_x = 1.0
sigma_y_y = 1.0
rho_y = 0.0
cov_y_11 = sigma_y_x**2
cov_y_12 = rho_y*sigma_y_x*sigma_y_y
```

```
cov_y_21 = rho_y*sigma_y_x*sigma_y_y
cov_y_22 = sigma_y_y**2
```

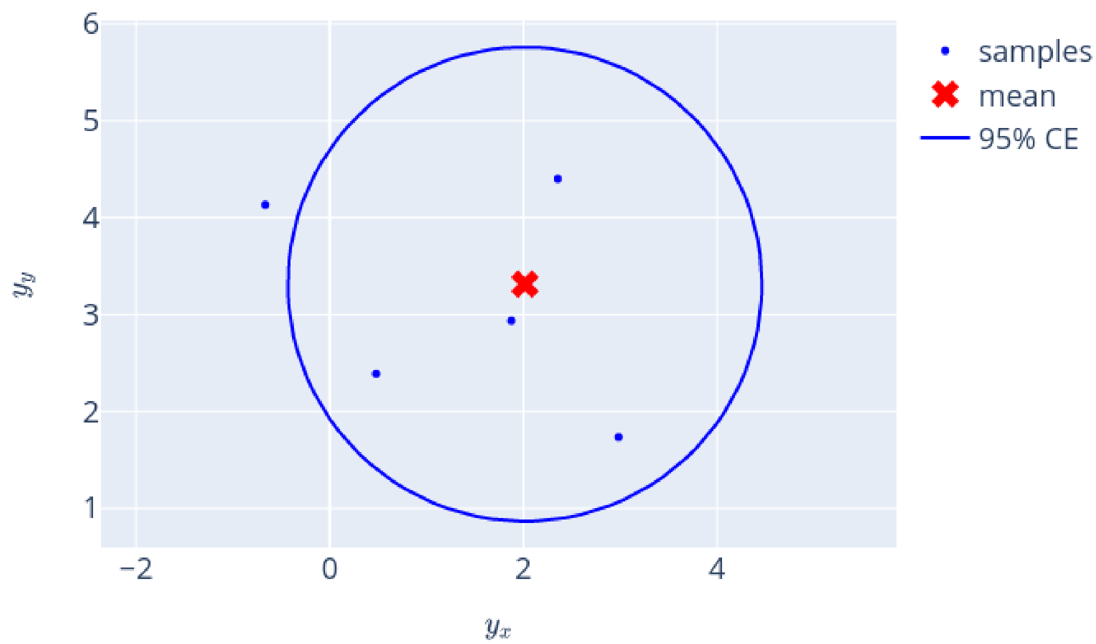

Figure 2: 5 noisy measurements of of the submarine location, and the mean and 95% confidence ellipse of the measurements probability density function.

(c)

$$
\begin{aligned}
p(\mathbf{z}|\mathbf{y}_1,\ldots,\mathbf{y}_N) = \mathcal{N}(\mathbf{z}|\hat{\mu},\hat{\Sigma})& \\
\hat{\mu} &= \left(N\Sigma_y^{-1}+\Sigma_z^{-1}\right)^{-1}\left(N\Sigma_y^{-1}\bar{\mathbf{y}}+\Sigma_z^{-1}\mu_z\right)\\
\hat{\Sigma} &= \frac{1}{N}\left(\Sigma_y^{-1}+\frac{1}{N}\Sigma_z^{-1}\right)^{-1}
\end{aligned}
$$

(d)

The modified code lines appear below and Fig. 3 plots the mean of the measurements, the mean of the posterior and its 95% confidence ellipse.

```
cov_y_inv = np.linalg.inv(cov_y)
cov_z_inv = np.linalg.inv(cov_z)
tmp1 = N * cov_y_inv + cov_z_inv
tmp2 = N * np.matmul(cov_y_inv, sample_mean_y) + \
    np.matmul(cov_z_inv, mean_z)
pos_mean_z = np.linalg.solve(tmp1, tmp2)
pos_cov_z = np.linalg.inv(tmp1)
```

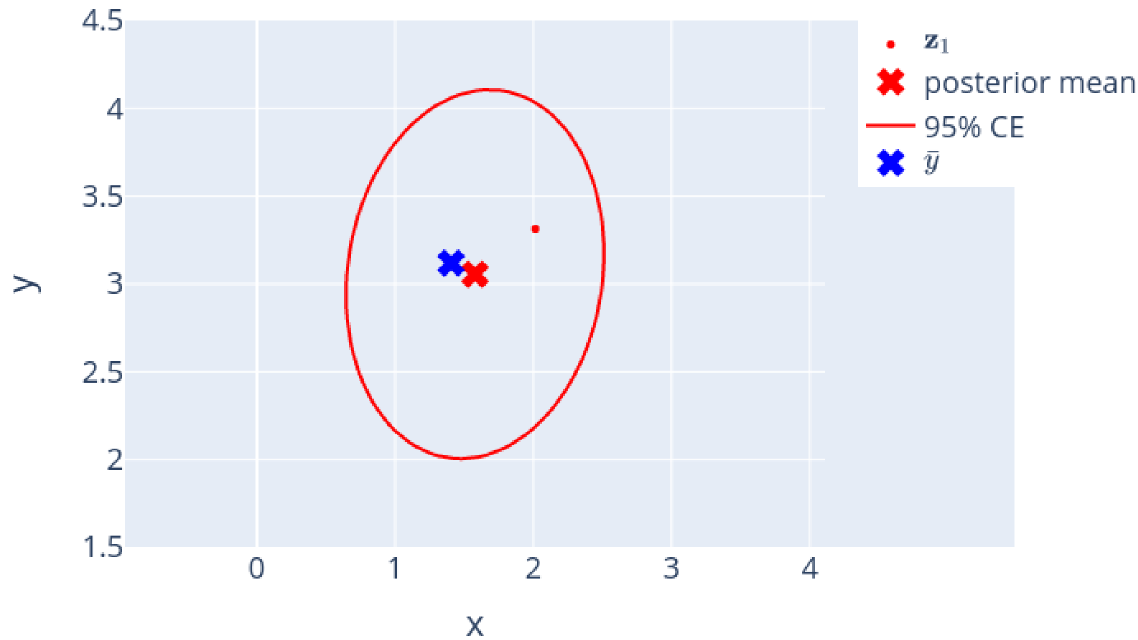

Figure 3: Mean of 5 noisy measurements (blue cross), mean of the posterior distribution and its 95% confidence ellipse.

(e)

Figs. 4-7 plot the posterior estimates computed from an increasing number of measurements.

In these figures we observe that:

1. as the number of measurements increase, the posterior mean approaches the sample mean, and the sample mean approaches the true value, $\mathbf{z}_1$,
2. as the number of measurements increase, the 95% confidence ellipses become smaller,

3. for three measurements (Fig. 4) the posterior 95% confidence ellipse is tilted, as that of the prior (Fig. 1, Σ_z in Eq. 1 of the exercise statement). As the number of measurments increase, the posterior 95% confidence ellipses become more and more spherical, as the 95% confidence ellipse of the measurements likelihood (Fig. 2, Σ_y in Eq. 2 of the exercise statement).

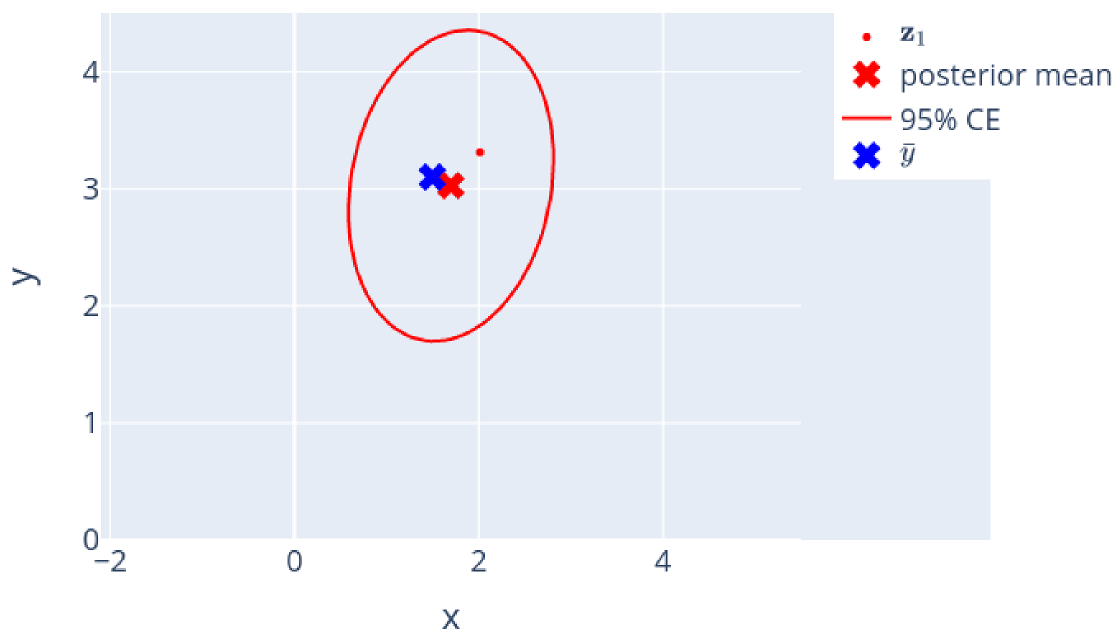

Figure 4: Mean of 3 noisy measurements (blue cross), mean of the posterior distribution and its 95% confidence ellipse.

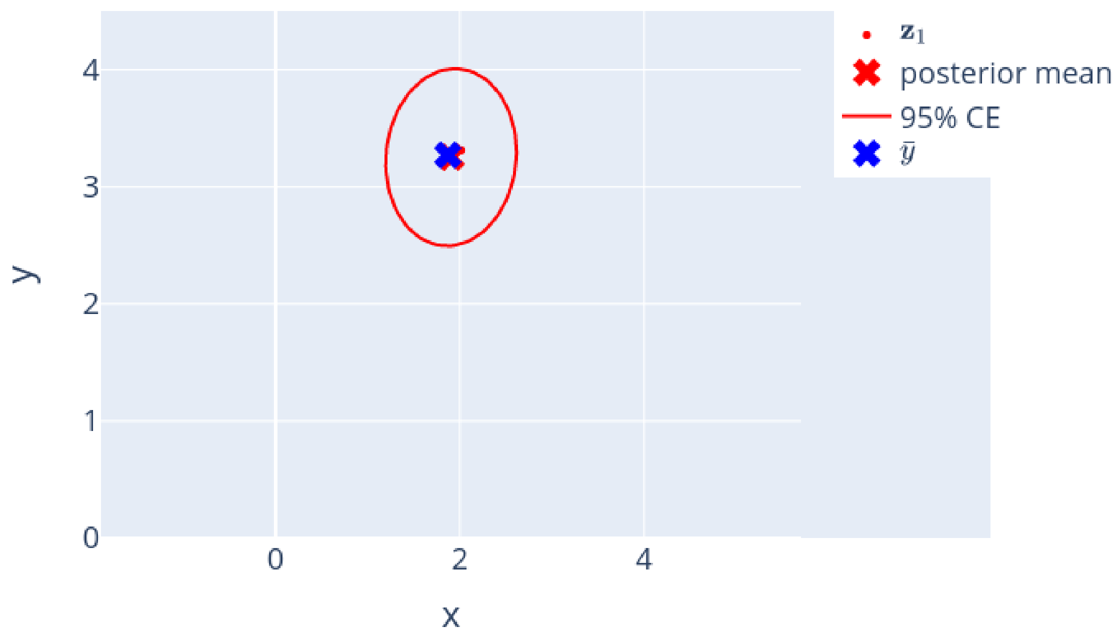

Figure 5: Mean of 10 noisy measurements (blue cross), mean of the posterior distribution and its 95% confidence ellipse.

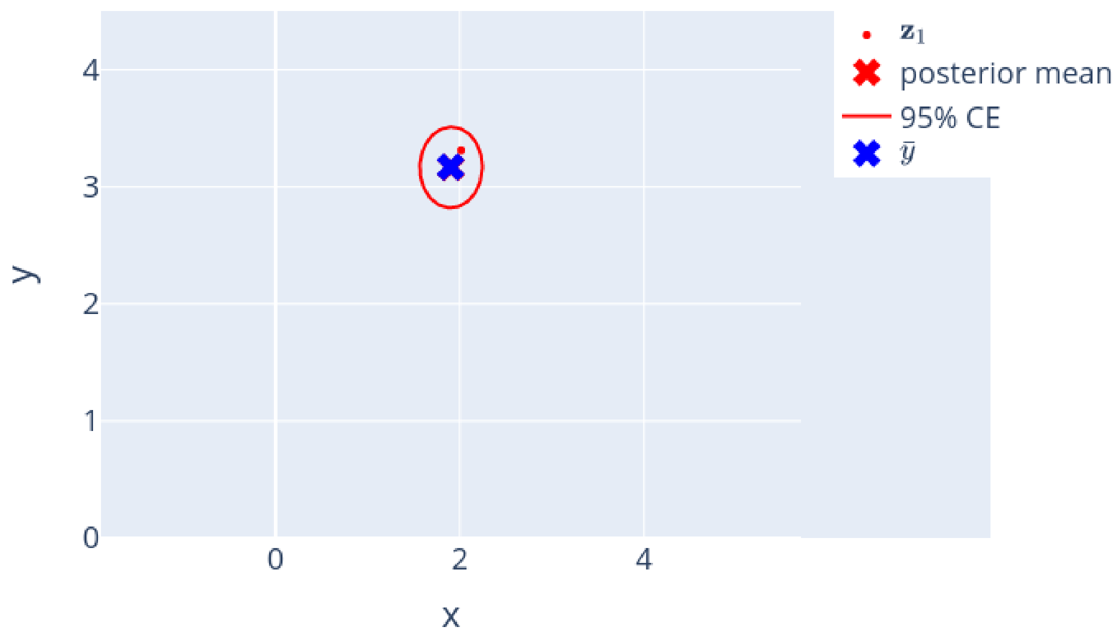

Figure 6: Mean of 50 noisy measurements (blue cross), mean of the posterior distribution and its 95% confidence ellipse.

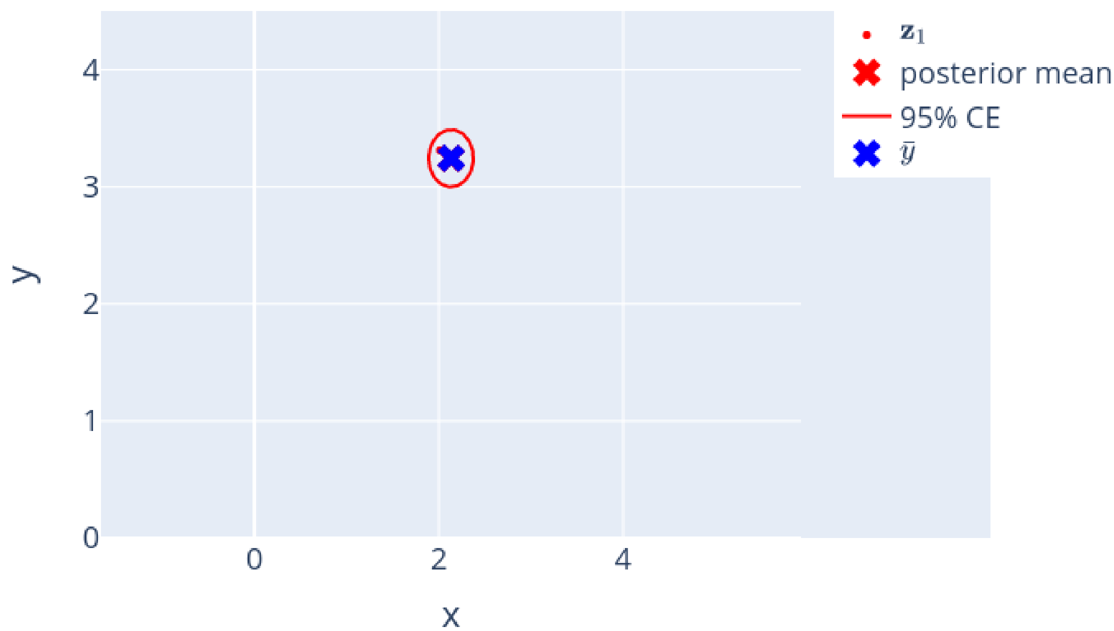

Figure 7: Mean of 100 noisy measurements (blue cross), mean of the posterior distribution and its 95% confidence ellipse.