

Exercises: inference in the linear Gaussian model

Joaquín Rapela*

October 26, 2022

1 Inferring location of a static submarine from its sonar measurements

(a)

The modified code lines appear below and Fig. 1 shows the generated submarine samples, and the mean and 95% confidence ellipse of the samples probability density function.

```
sigma_zx = 1.0
sigma_zy = 2.0
rho_z = 0.7
cov_z_11 = sigma_zx**2
cov_z_12 = rho_z*sigma_zx*sigma_zy
cov_z_21 = rho_z*sigma_zx*sigma_zy
cov_z_22 = sigma_zy**2
```

*j.rapela@ucl.ac.uk

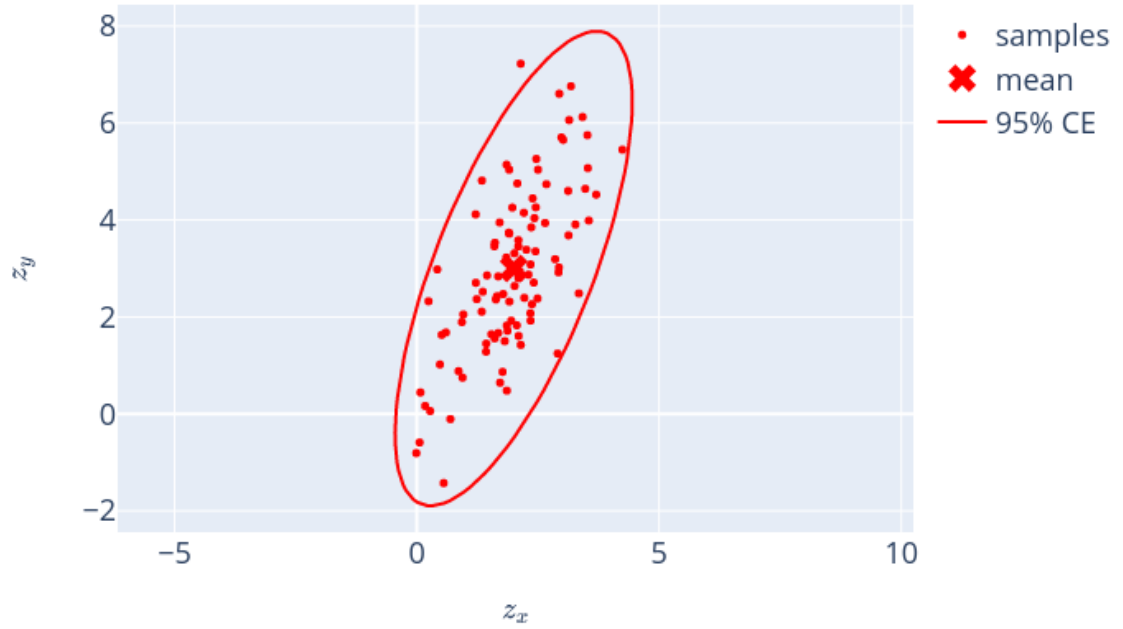


Figure 1: 100 a-priori samples of the submarine location (dots), and the mean (cross) and 95% confidence ellipse (line) of the samples probability density function.

(b)

The modified code lines appear below and Fig. 2 shows the generated measurement samples, and the mean and 95% confidence ellipse of the samples probability density function.

```
sigma_y_x = 1.0
sigma_y_y = 1.0
rho_y = 0.0
cov_y_11 = sigma_y_x**2
```

```

cov_y_12 = rho_y*sigma_y_x*sigma_y_y
cov_y_21 = rho_y*sigma_y_x*sigma_y_y
cov_y_22 = sigma_y_y**2

```

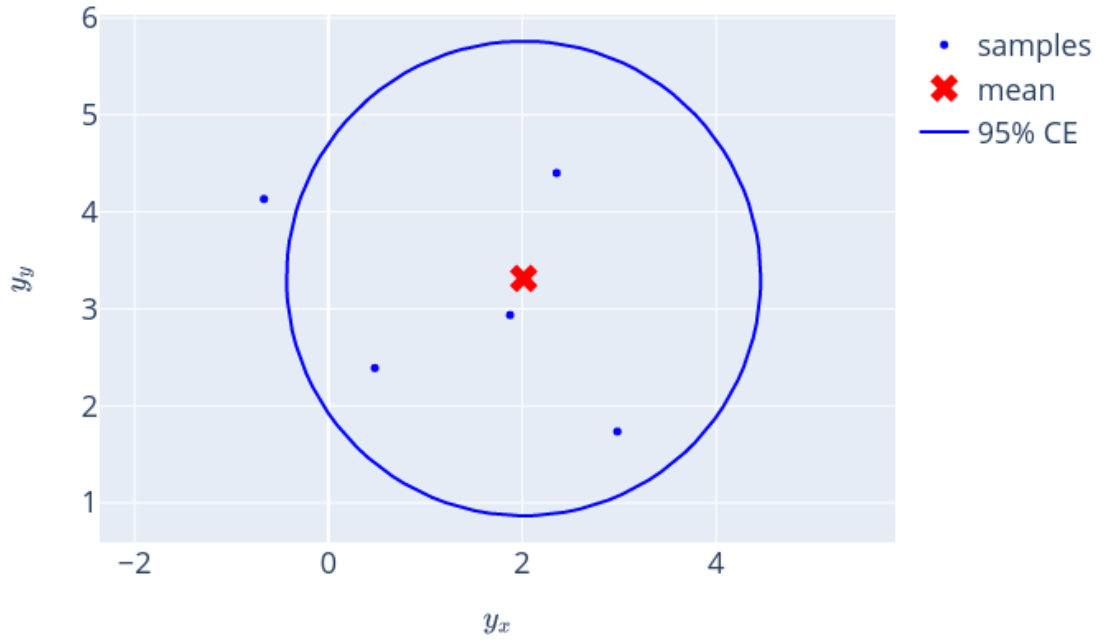


Figure 2: 5 noisy measurements of of the submarine location (dots), and the mean (\mathbf{z}_1 , cross) and 95% confidence ellipse (line) of the measurements probability density function.

(c)

$$\begin{aligned} p(\mathbf{z}|\mathbf{y}_1, \dots, \mathbf{y}_N) &= \mathcal{N}(\mathbf{z}|\hat{\mu}, \hat{\Sigma}) \\ \hat{\mu} &= (N\Sigma_y^{-1} + \Sigma_z^{-1})^{-1} (N\Sigma_y^{-1}\bar{\mathbf{y}} + \Sigma_z^{-1}\mu_z) \\ \hat{\Sigma} &= \frac{1}{N} \left(\Sigma_y^{-1} + \frac{1}{N}\Sigma_z^{-1} \right)^{-1} \end{aligned}$$

(d)

The modified code lines appear below and Fig. 3 plots the mean of the measurements, the mean of the posterior and its 95% confidence ellipse.

```
cov_y_inv = np.linalg.inv(cov_y)
cov_z_inv = np.linalg.inv(cov_z)
tmp1 = N * cov_y_inv + cov_z_inv
tmp2 = N * np.matmul(cov_y_inv, sample_mean_y) + \
      np.matmul(cov_z_inv, mean_z)
post_mean_z = np.linalg.solve(tmp1, tmp2)
post_cov_z = np.linalg.inv(tmp1)
yBar_mean = z
yBar_cov = 1.0/N*cov_y
```

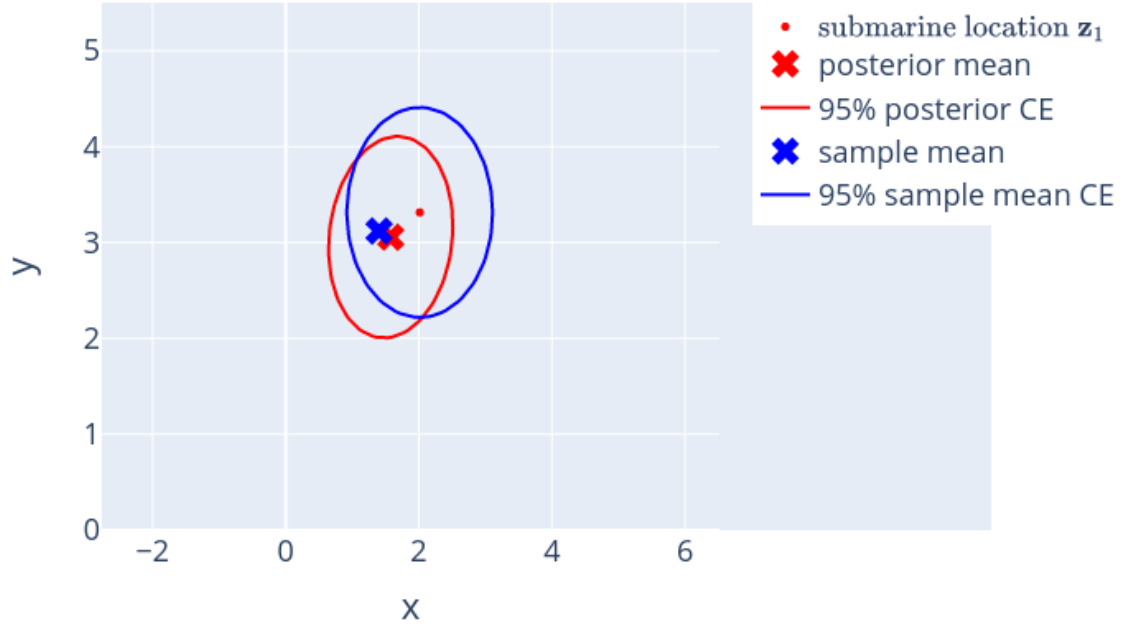


Figure 3: Sample average of 5 noisy measurements (blue cross), 95% confidence ellipse of this average (blue line), mean of the posterior distribution (red cross), its 95% confidence ellipse (red line), and submarine location (\mathbf{z}_1 , red dot)

(e)

Figs. 4-7 plot the posterior estimates computed from an increasing number of measurements.

In these figures we observe that:

1. as the number of measurements increase, the posterior mean approaches the sample mean, and the sample mean approaches the true value, \mathbf{z}_1 ,

2. as the number of measurements increase, the 95% confidence ellipses become smaller,
3. for three measurements (Fig. 4) the posterior 95% confidence ellipse is tilted, as that of the prior (Fig. 1, Σ_z in Eq. 1 of the exercise statement). As the number of measurements increase, the posterior 95% confidence ellipses become more and more spherical, as the 95% confidence ellipse of the measurements likelihood (Fig. 2, Σ_y in Eq. 2 of the exercise statement).

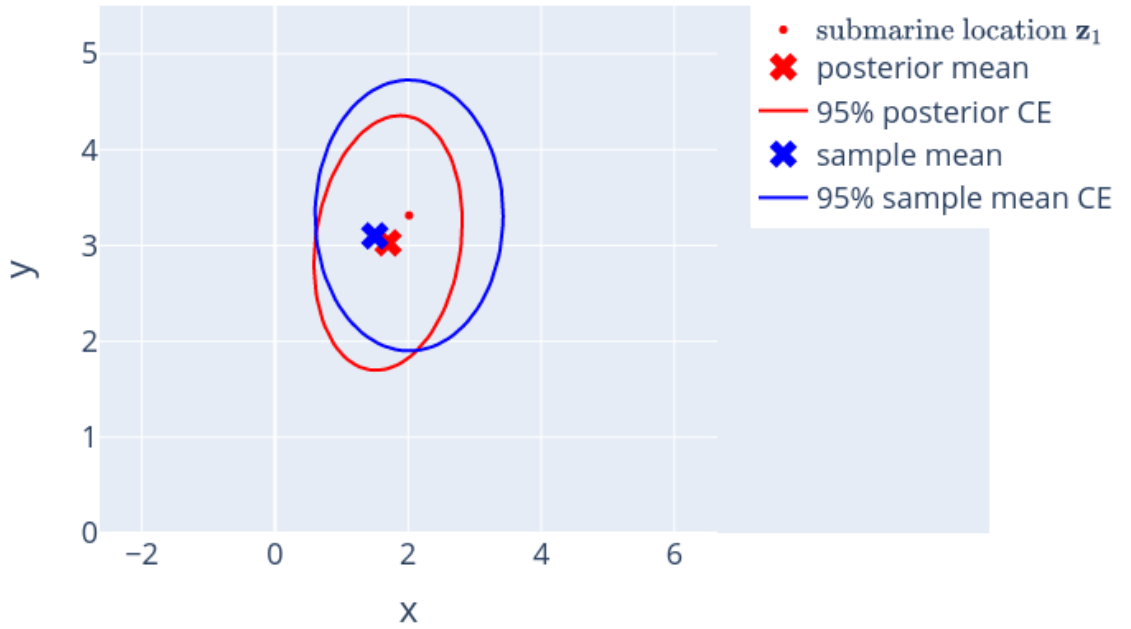


Figure 4: Sample average of 3 noisy measurements (blue cross), 95% confidence ellipse of this average (blue line), mean of the posterior distribution (red cross), its 95% confidence ellipse (red line), and submarine location (\mathbf{z}_1 , red dot)

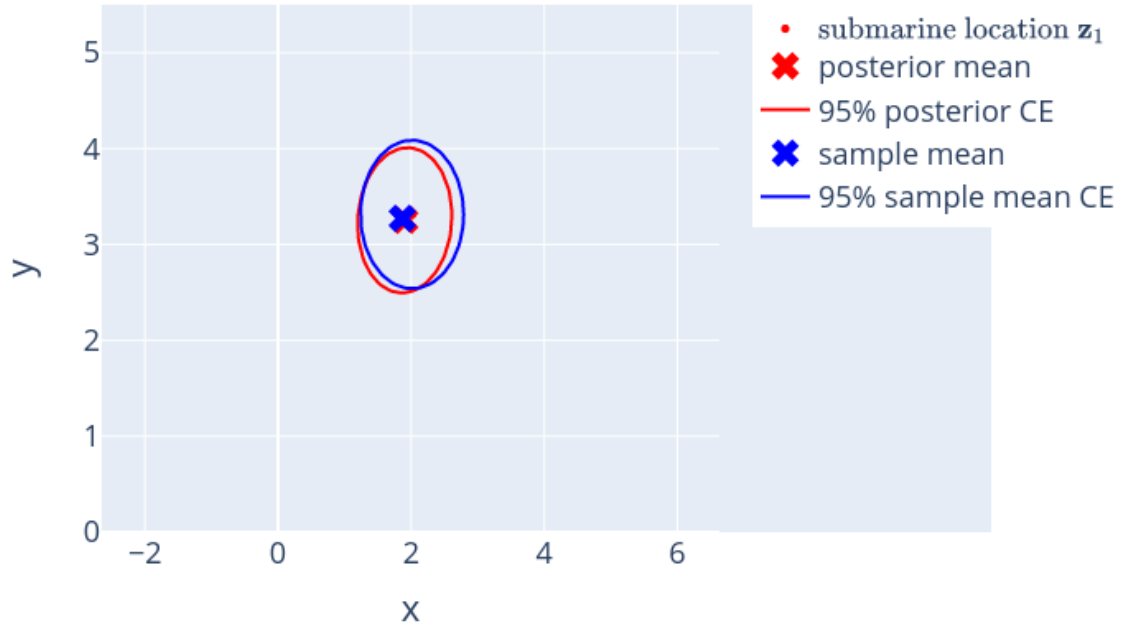


Figure 5: Sample average of 10 noisy measurements (blue cross), 95% confidence ellipse of this average (blue line), mean of the posterior distribution (red cross), its 95% confidence ellipse (red line), and submarine location (\mathbf{z}_1 , red dot)

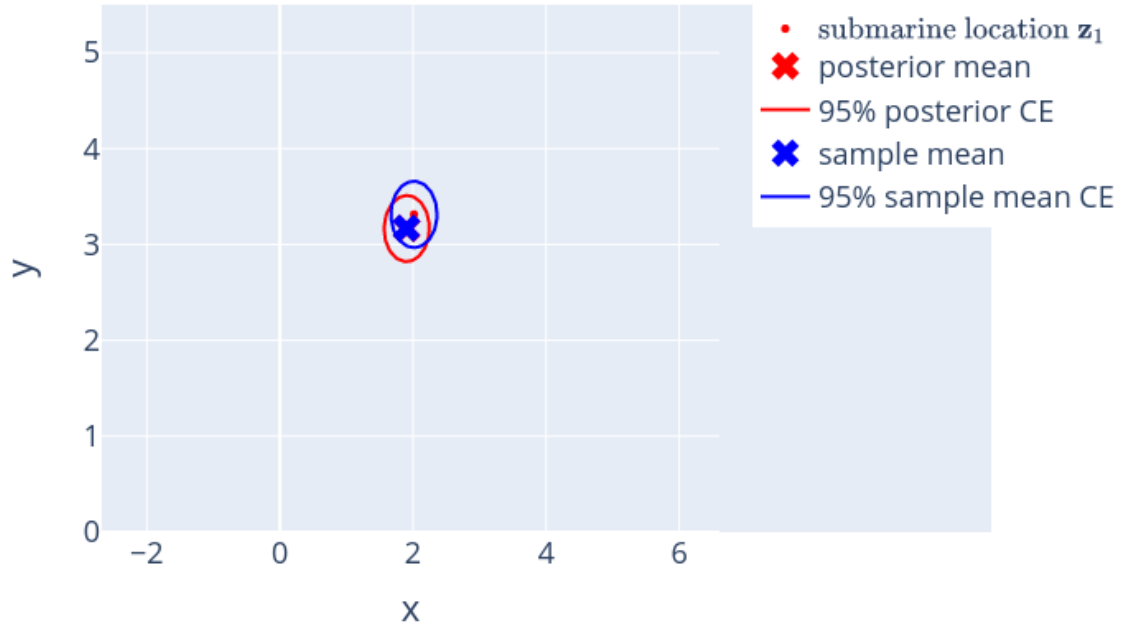


Figure 6: Sample average of 50 noisy measurements (blue cross), 95% confidence ellipse of this average (blue line), mean of the posterior distribution (red cross), its 95% confidence ellipse (red line), and submarine location (\mathbf{z}_1 , red dot)

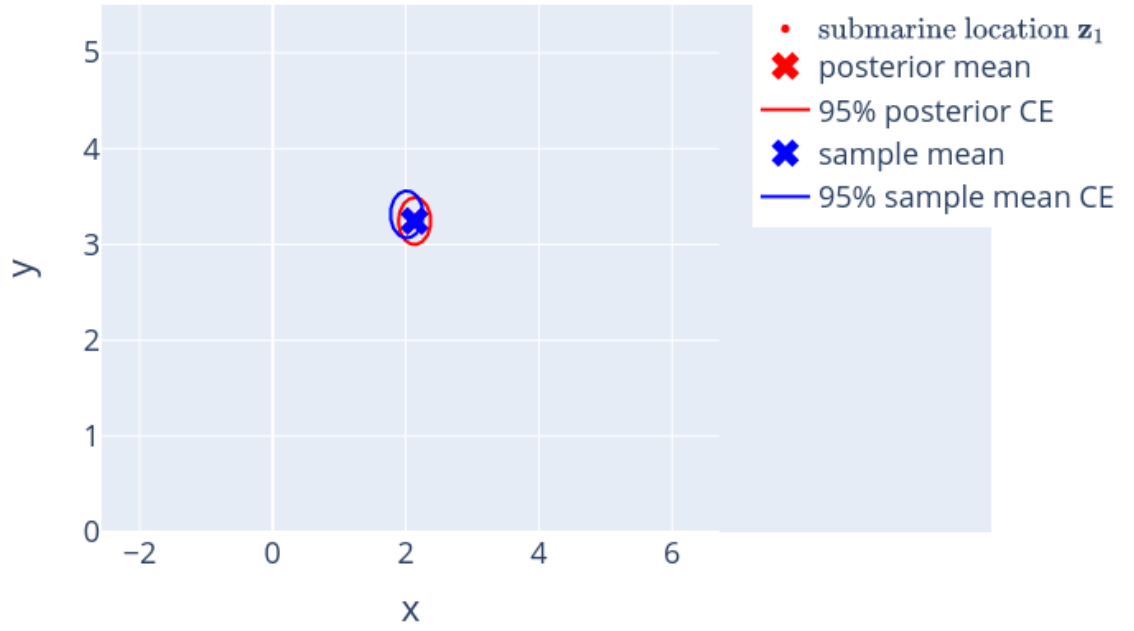


Figure 7: Sample average of 100 noisy measurements (blue cross), 95% confidence ellipse of this average (blue line), mean of the posterior distribution (red cross), its 95% confidence ellipse (red line), and submarine location (\mathbf{z}_1 , red dot)

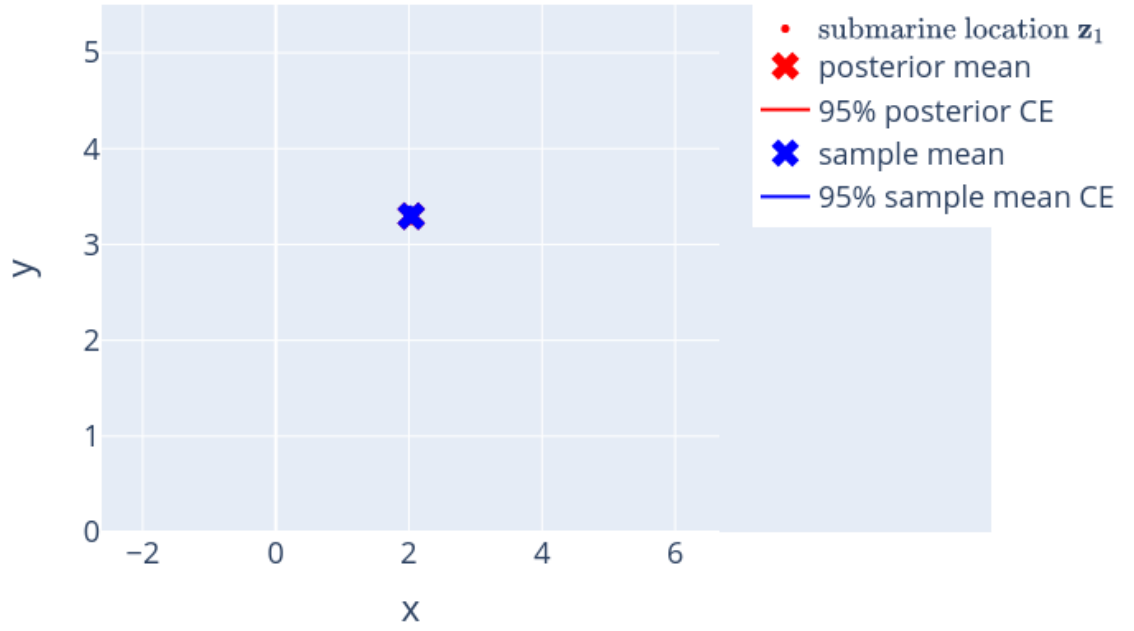


Figure 8: Sample average of 1000 noisy measurements (blue cross), 95% confidence ellipse of this average (blue line), mean of the posterior distribution (red cross), its 95% confidence ellipse (red line), and submarine location (\mathbf{z}_1 , red dot)