Computer Graphics
Module - 3
Output Primitives
CSC305

By Prof. Sunil Katkar
Department of Computer Engineering, VCET

Module -2 Output Primitives

Objective

To emphasize on implementation aspect of Computer Graphics Algorithms

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Outcome

At the end of the course student will be able to:

apply 2-D geometric transformations on graphical objects and analyze composite transformation.

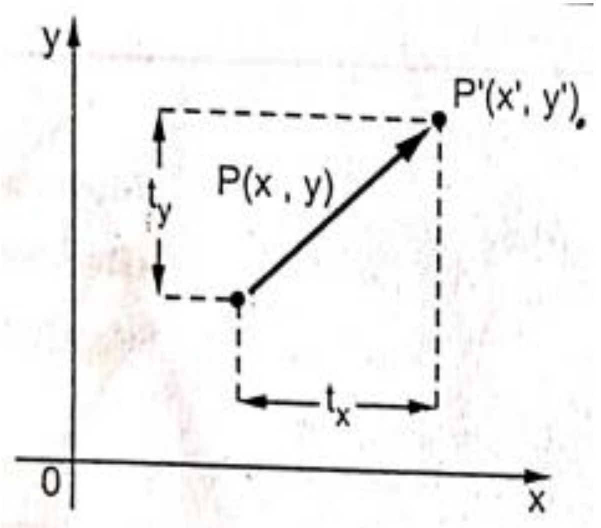
2D Transformations

Translation -

Translate two dimensional point by adding translation distance, tx and ty, to the original coordinate position (x, y) to move to a new point (x’, y’) as

$x' = x + tx$

$y' = y + ty$



$$P = \begin{pmatrix} x \\ y \end{pmatrix} \qquad P' = \begin{pmatrix} x \\ y \end{pmatrix} \qquad T = \begin{pmatrix} x \\ y \end{pmatrix}$$

In matrix form

$P' = P + T$

2D Transformations

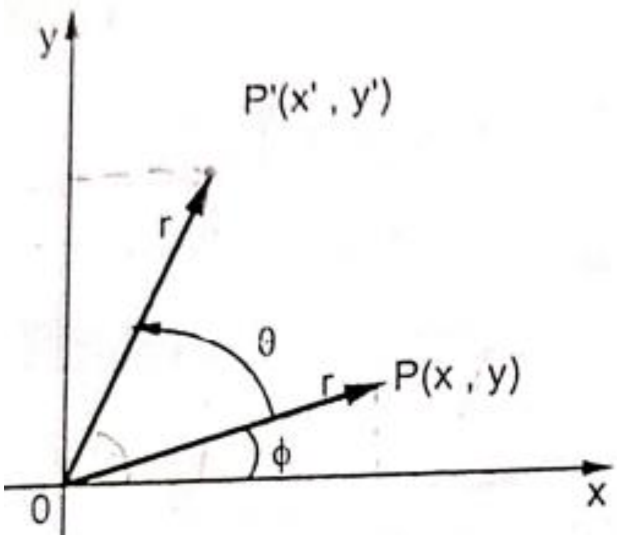
Rotation -

Using standard trigonometric equations, the transformed coordinates in terms of angles θ and ϕ as

$$\left.\begin{aligned} x' &= r\cos(\phi + \theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ y' &= r\sin(\phi + \theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots 1$$

The original coordinates of the point in polar coordinates are given as

$$\left.\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$



From equation-1 using equation-2

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

In matrix form

2D Transformations

Rotation -

$$\begin{pmatrix} x' & y' \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P' = P \;.\; R$$

Where R is rotation matrix and it is given as -

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Note - Positive values for rotation angle define counterclockwise rotations about the rotation point and negative values rotate objects in the clockwise.

$$R' = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

2D Transformations

Scaling -

It changes the size of an object.

Multiply the coordinate values (x, y) of each vertex by scaling factors Sx and Sy to produce the transformed coordinates (x', y').

$$x' = x \cdot Sx$$

$$y' = y \cdot Sy$$

Scaling factor Sx scales an object in the x-direction and scaling factor Sy scales object in the y-direction.

In matrix form

$$\begin{pmatrix} x' & y' \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} Sx & 0 \\ 0 & Sy \end{pmatrix}$$

$$= \begin{pmatrix} x \cdot Sx & y \cdot Sy \end{pmatrix}$$

$$P' = P \cdot S$$

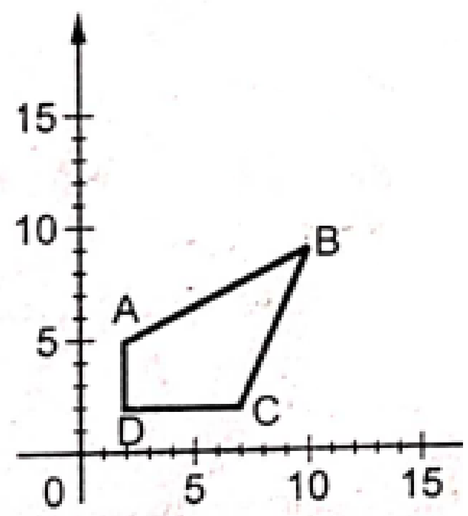
2D Transformations

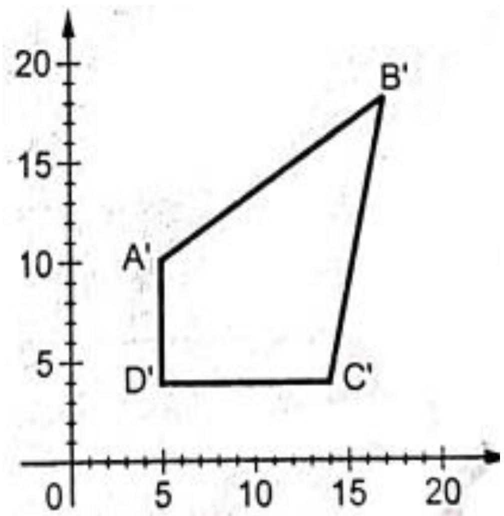
Scaling -

Values of scaling factor Sx and Sy

- ✓ less than 1 reduce the size of an object
- ✓ greater than 1 produce an enlarged object.
- ✓ equal to 1 the size of an object does not change.

To get uniform scaling it is necessary to assign same value for Sx and Sy.





2D Transformations

Ex.1- Translate a polygon with coordinates A(2, 5), B(7, 10) and C(10, 2) by 3 units in x direction and 4 units in y-direction.

We have, P' = P + T

A' = A + T

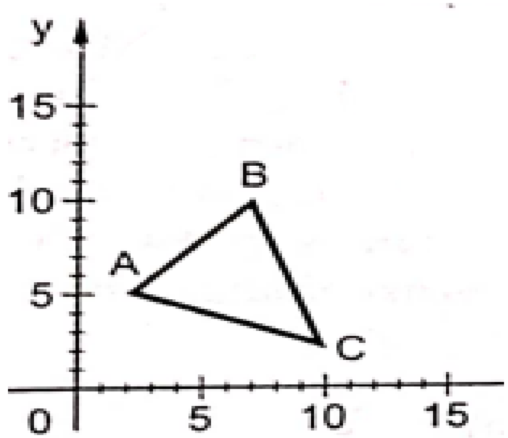
$$A' = \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

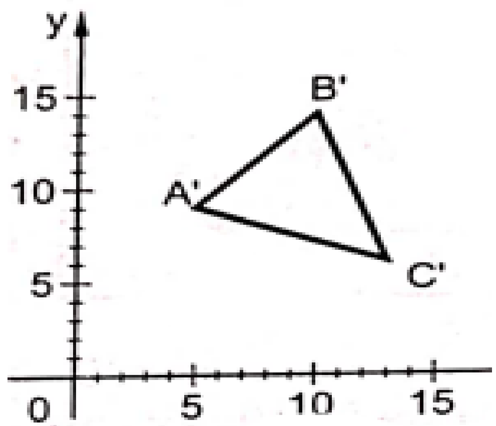
B' = B + T

$$B' = \begin{pmatrix} 7 \\ 10 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

C' = C + T

$$C' = \begin{pmatrix} 10 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 6 \end{pmatrix}$$





2D Transformations

Ex.2- A point (4, 3) is rotated counterclockwise by an angle of 45^0.

Find the rotation matrix and the resultant point.

Given P = [4 3] and $\theta = 45^0$

We have, P' = P . R

Where R is rotation matrix, $R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$P' = \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} \cos 45^0 & \sin 45^0 \\ -\sin 45^0 & \cos 45^0 \end{bmatrix}$$

$$P' = \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$P' = \begin{bmatrix} 4/\sqrt{2} - 3/\sqrt{2} & 4/\sqrt{2} + 3/\sqrt{2} \end{bmatrix}$$

$$P' = \begin{bmatrix} 1/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}$$

Ex.3- Scale the polygon with coordinates A(2, 5), B(7, 10) and C(10, 2)by two units in x-direction and two units in y-direction.

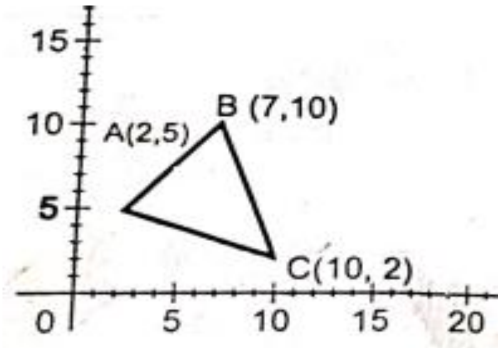
Given Sx = 2 and Sy = 2

We have, P' = P . S

Where S is Scaling matrix, $S = \begin{pmatrix} Sx & 0 \\ 0 & Sy \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

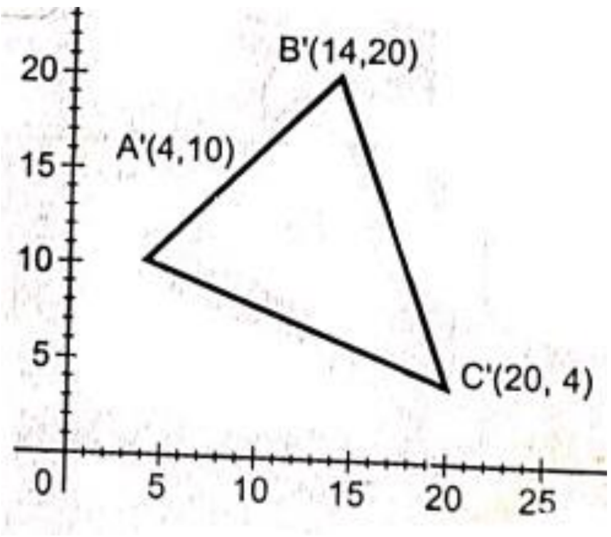
The object matrix is

$$P = \begin{pmatrix} 2 & 5 \\ 7 & 10 \\ 10 & 2 \end{pmatrix}$$



Now,

$$P' = \begin{pmatrix} 2 & 5 \\ 7 & 10 \\ 10 & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 14 & 20 \\ 20 & 4 \end{pmatrix}$$



Homogeneous Coordinates

Translation -

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{pmatrix}$$

$$P' = P.\,T$$

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{pmatrix}$$

$$= \begin{bmatrix} x + tx & y + ty & 1 \end{bmatrix}$$

Homogeneous Coordinates

Rotation -

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P' = P.\,R$$

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta & x\sin\theta + y\cos\theta & 1 \end{bmatrix}$$

Homogeneous Coordinates

Scaling -

$$S = \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P' = P.\ S$$

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} = \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{pmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} x\,Sx & y\,Sy & 1 \end{bmatrix}$$

Ex-1 : Find a transformation of triangle A(1, 0), B(0, 1), C(1, 1) by -

a) rotating 45^0 about the origin and then translating one unit in x-direction and y-direction.

b) translating one unit in x-direction and y-direction and then rotating 45^0 about the origin.

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a) The overall transformation matrix is

$$T_M = R \, . \, T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$P' = P \, . \, T_M$$

$$\begin{pmatrix} A' \\ B' \\ C' \end{pmatrix} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} . \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} A' \\ B' \\ C' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} + 1 & 1/\sqrt{2} + 1 & 1 \\ -1/\sqrt{2} + 1 & 1/\sqrt{2} + 1 & 1 \\ 1 & \sqrt{2} + 1 & 1 \end{pmatrix}$$

b) The overall transformation matrix is

$$T_M = T \cdot R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{pmatrix}$$

$$P' = P \cdot T_M$$

$$\begin{pmatrix} A' \\ B' \\ C' \end{pmatrix} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} \cdot \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{pmatrix}$$

$$\begin{pmatrix} A' \\ B' \\ C' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & \sqrt{2} & 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 3/\sqrt{2} & 1 \\ -1/\sqrt{2} & 3/\sqrt{2} & 1 \\ 0 & 2\sqrt{2} & 1 \end{pmatrix}$$

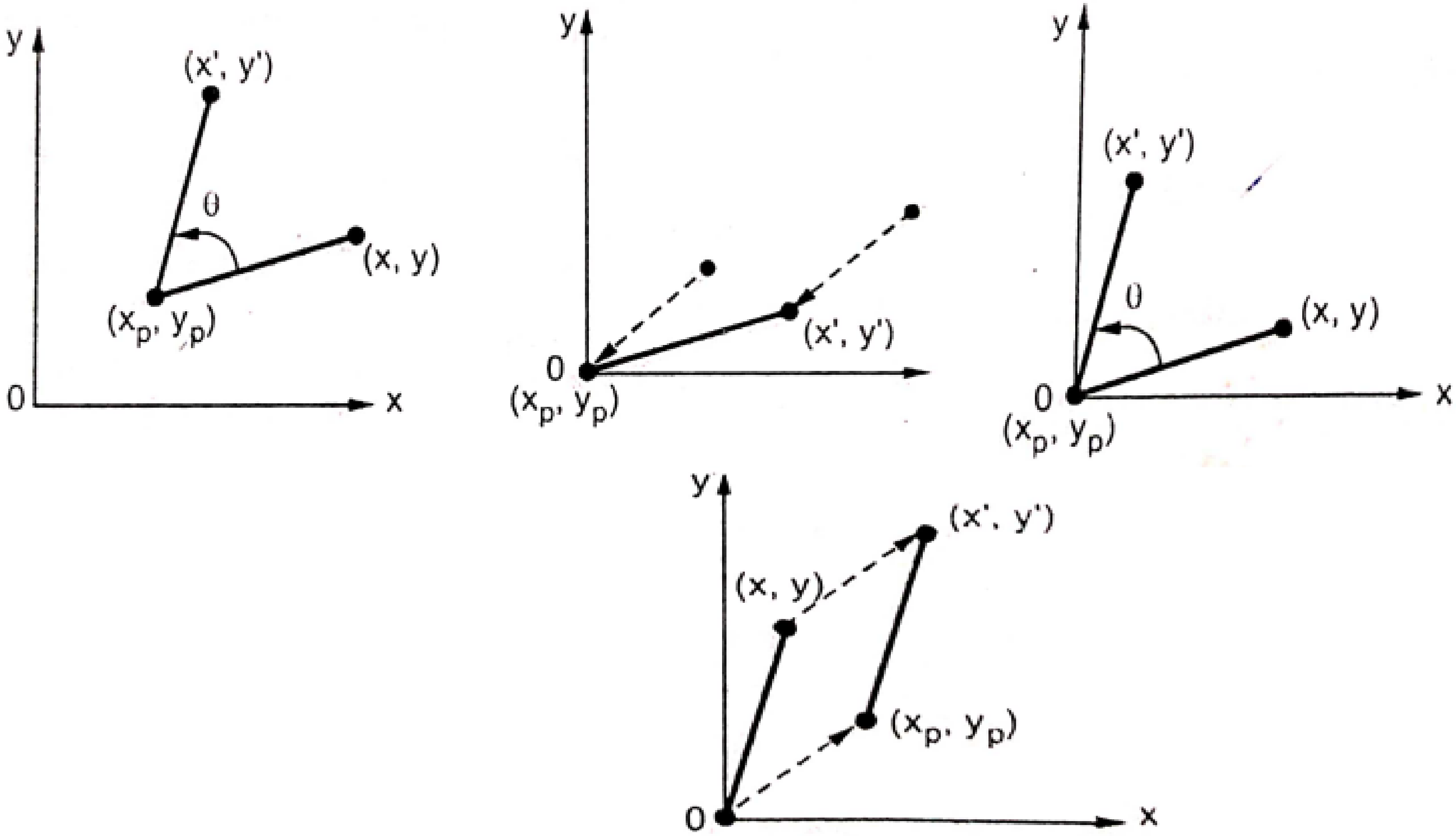
Rotation about an arbitrary point

Steps to rotate an object about an arbitrary point (x_p, y_p)

1. Translate point (x_p, y_p) to the origin.

2. Rotate it about the origin.

3. Finally translate the center of rotation back where it belongs.



Rotation about an arbitrary point

The translation matrix to move point (x_p, y_p) to the origin is given as

$$T1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{pmatrix}$$

The rotation matrix for counter clockwise rotation of point about the origin is given as -

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The translation matrix to move the center point back to its original position is given as -

$$T2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{pmatrix}$$

The overall transformation matrix for a counterclockwise rotation by an angle θ about the point (x_p, y_p) is given as -

$$T_M = T1 \,.\, R \,.\, T2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -y_p & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{pmatrix}$$

$$T_M = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_p\cos\theta + y_p\sin\theta & -x_p\sin\theta - y_p\cos\theta & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{pmatrix}$$

$$T_M = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_p\cos\theta + y_p\sin\theta + x_p & -x_p\sin\theta - y_p\cos\theta + y_p & 1 \end{pmatrix}$$

Ex-2 : Calculate transformation matrix that transforms the given square ABCD to half its size with centre still remaining at the same position. The coordinates of the square are A(1, 1), B(3, 1), C(3, 3), D(1, 3) and centre at (2, 2).

Transformation can be carried out in the following steps

1. Translate the square so that its center coincides with the origin
2. Scale the square with respect to the origin.
3. Translate the square back to the original position.

$$T1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad T2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix}$$

The overall transformation matrix is given as -

$$T_M = T1 \,.\, S \,.\, T2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{pmatrix}$$

$$T_M = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$P' = P \,.\, T_M$$

$$\begin{pmatrix} A' \\ B' \\ C' \\ D' \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 3 & 3 & 1 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1.5 & 1.5 & 1 \\ 2.5 & 1.5 & 1 \\ 2.5 & 2.5 & 1 \\ 1.5 & 2.5 & 1 \end{pmatrix}$$

Ex-3:) A triangle A(2, 2), B91, 1), C(3, 1) is rotated by 90 degree about A. Find new coordinates of a triangle.

Ex-4:) Perform a 45 degree rotation of a triangle A(0, 0), B(1, 1), C(5, 2) about P(-1, -1).

Ex-5:) Find out the final coordinates of a figure bounded by the coordinates (1, 1), (3, 4), (5, 7), (10, 3) when rotated about a point (8, 8) by 30 degree in clockwise direction and scaled by two units in x-direction and three units in y-direction.