Homogeneous Coordinates

The rotation of a point, straight line or an entire image on the screen, about a point other than origin, is achieved by first moving the image until the point of rotation occupies the origin, then performing rotation, then finally moving the image to its original position.

The moving of an image from one place to another in a straight line is called a translation. A translation may be done by adding or subtracting to each point, the amount, by which picture is required to be shifted.

Translation of point by the change of coordinate cannot be combined with other transformation by using simple matrix application. Such a combination is essential if we wish to rotate an image about a point other than origin by translation, rotation again translation.

To combine these three transformations into a single transformation, homogeneous coordinates are used. In homogeneous coordinate system, two-dimensional coordinate positions (x, y) are represented by triple-coordinates.

Homogeneous coordinates are generally used in design and construction applications. Here we perform translations, rotations, scaling to fit the picture into proper position.

Example of representing coordinates into a homogeneous coordinate system: For two-dimensional geometric transformation, we can choose homogeneous parameter h to any non-zero value. For our convenience take it as one. Each two-dimensional position is then represented with homogeneous coordinates (x, y, 1).

Following are matrix for two-dimensional transformation in homogeneous coordinate:

1. Translation $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

2. Scaling $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Rotation (clockwise) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Rotation (anti-clock) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. Reflection against X axis $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. Reflection against Y axis $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. Reflection against origin $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8. Reflection against line Y=X $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9. Reflection against Y= –X $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. Shearing in X direction $\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. Shearing in Y direction $\begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

12. Shearing in both x and y direction $\begin{bmatrix} 1 & Sh_y & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Composite Transformation:

A number of transformations or sequence of transformations can be combined into single one called as composition. The resulting matrix is called as composite matrix. The process of combining is called as concatenation.

Suppose we want to perform rotation about an arbitrary point, then we can perform it by the sequence of three transformations

1. Translation
2. Rotation
3. Reverse Translation

The ordering sequence of these numbers of transformations must not be changed. If a matrix is represented in column form, then the composite transformation is performed by multiplying matrix in order from right to left side. The output obtained from the previous matrix is multiplied with the new coming matrix.

Example showing composite transformations:

The enlargement is with respect to center. For this following sequence of transformations will be performed and all will be combined to a single one

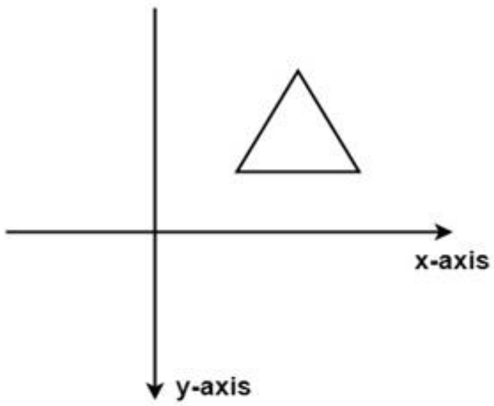
Step1: The object is kept at its position as in fig (a)

Step2: The object is translated so that its center coincides with the origin as in fig (b)

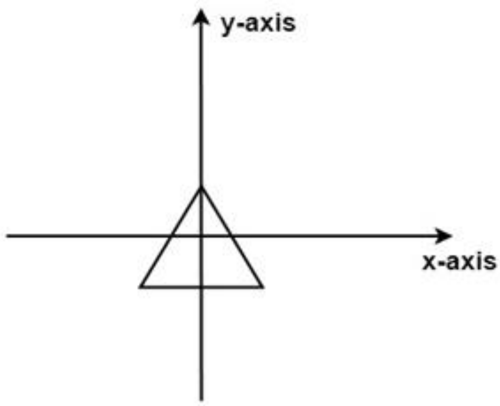
Step3: Scaling of an object by keeping the object at origin is done in fig (c)

Step4: Again translation is done. This second translation is called a reverse translation. It will position the object at the origin location.

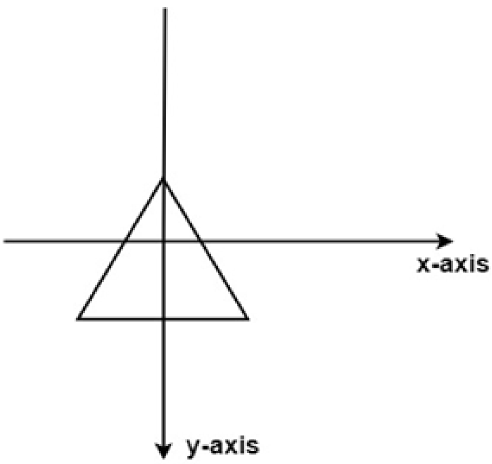
Above transformation can be represented as $T_V.ST_V^{-1}$



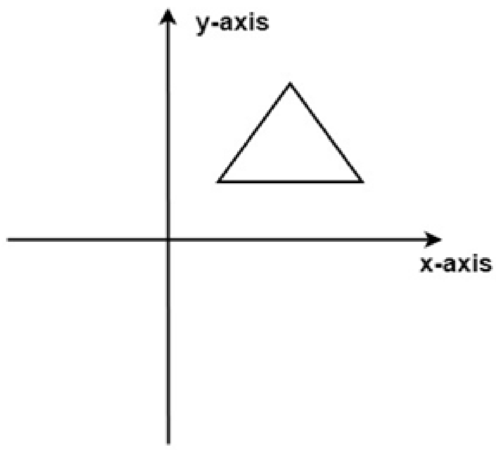
(original position of object)
Fig:(a)



(object is translated to origin)
Fig:(b)



(object is enlarged)
Fig:(c)



(object is retranslated to original position)
Fig:(d)

Advantage of composition or concatenation of matrix:

1. It transformations become compact.
2. The number of operations will be reduced.
3. Rules used for defining transformation in form of equations are complex as compared to matrix.

Composition of two translations:

Let t_1 t_2 t_3 $t_4$are translation vectors. They are two translations P_1 and P_2. The matrix of P_1 and P_2 given below. The P_1 and $P_2$are represented using Homogeneous matrices and P will be the final transformation matrix obtained after multiplication.

$$P_1 = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 1 & 0 & t_3 \\ 0 & 1 & t_4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & t_1 + t_2 \\ 0 & 1 & t_3 + t_4 \\ 0 & 0 & 1 \end{bmatrix}$$

Above resultant matrix show that two successive translations are additive.

Composition of two Scaling: The composition of two scaling is multiplicative. Let S_{11} and $S_{12}$are matrix to be multiplied.

$$S_{11} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{12} = \begin{bmatrix} S_3 & 0 & 0 \\ 0 & S_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = S_{11} * S_{12} = \begin{bmatrix} s_1 * s_3 & 0 & 0 \\ 0 & s_2 * s_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$