

(Time: 3 hours)

Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $t \sqrt{1 + \sin t}$ [5]

(b) Find the constants a, b, c, d, e if [5]

$f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic.

(c) Calculate the Spearman's rank correlation coefficient R [5]

X : 85, 74, 85, 50, 65, 78, 74, 60, 74, 90

Y : 78, 91, 78, 58, 60, 72, 80, 55, 68, 70

(d) Find inverse Laplace transform of $\tan^{-1}\left(\frac{s+a}{b}\right)$. [5]

Q2.

(a) Find the Laplace transform of $e^{-4t} \int_0^t u \sin 3u du$ [6]

(b) find the value of k if the function $f(x) = kx^2(1 - x^3)$, $0 \leq x \leq 1$.

$F(x) = 0$ otherwise

Is a probability density function. find mean and variance. [6]

(c) Obtain the Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ [8]

Q3.

(a) Find the analytic function $f(z) = u + iv$ such that [6]

$$u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y - 2 \cos 2x}}.$$

(b) Using convolution theorem Find inverse Laplace transform of $\frac{s^2}{(s^2+9)(s^2+4)}$. [6]

(c) Fit a second-degree parabolic curve to the following data

Year (x)	: 1974	1975	1976	1977	1978	1979	1980	1981
Production (y)	: 12	14	26	42	40	50	52	53.

[8]

Q4.

(a) Obtain the Fourier series to represent $f(x) = 9 - x^2$ in $(-3, 3)$. [6]

(b) . Find the coefficients of regression and hence obtain the equation of the lines of Regression for the following data

X: 78, 36, 98, 25, 75, 82, 90, 62, 65, 39.

Y: 84, 51, 91, 60, 68, 62, 86, 58, 53, 47. [6]

(c) Prove that $\int_0^\infty e^{-t} \frac{\sin 2t + \sin 3t}{t} dt = \frac{3\pi}{4}$. [8]

Q5.

(a) Find the orthogonal trajectories of the family of curves $3x^2y + 2x^2 - y^3 - 2y^2 = c$. [6]

(b) If X denotes the outcome when a fair die is tossed, find Moment generating function

Of X and hence find the mean and variance of X. [6]

(c) Obtain the half range cosine series of $f(x) = x(\pi - x)$ in $(0, \pi)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \dots$ [8]

Q6.(a) Find inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. [6]

(b) The probability density function of a random variable X is

X	: 0	1	2	3	4	5	6
P (X=x)	: k	3k	5k	7k	9k	11k	13k

Find k, $p(X < 4)$, $P(3 < X \leq 6)$. [6]

(c) Verify Laplace equation for $u = \left(r + \frac{a^2}{r}\right) \cos \theta$. also find v and $f(z)$. [8]
